

What have we learned
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Spatial models
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Gaussian Random Field models
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The SPDE approach
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Fitting spatial
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Spatial modeling with INLA and `inlabru`

University of Zurich, March, 2022

Instructor: Sara Martino

Department of Mathematical Science (NTNU)



Norwegian University of
Science and Technology

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Space-time Modeling

Prediction in space

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What have we learned

INLA in a nutshell

- many data sets these days are complex, resulting in complex models, e.g. with complex dependence structures (spatial, temporal, etc..)
 - usually Markov chain Monte Carlo (MCMC) methods have been used to fit these models
 - (realistically) complex models result in very long running times
 - often impossible (or unrealistic) to fit
 - INLA (Integrated nested Laplace approximation) is an alternative to MCMC
 - much, much **faster**
 - suitable for a specific (but very large!) class of models

INLA in a nutshell

Three main ingredients in INLA

- Gaussian Markov random fields
- Latent Gaussian models
- Laplace approximations

which together (with a few other things) give a very nice tool for Bayesian inference

- quick
- accurate

Models amenable to INLA: Latent Gaussian Models

$$\mathbf{y}|\mathbf{x}, \theta \sim \prod_i \pi(y_i|x_i, \theta) \quad \text{Likelihood}$$

$$\mathbf{x}|\theta \sim \exp\left(-\frac{1}{2}\mathbf{x}^T \mathbf{Q}(\theta) \mathbf{x}^T\right) \quad \text{Latent field (GMRF)}$$

$$\theta \sim \pi(\theta) \quad \text{Hyperparameter}$$

- Each data point depends on only one of the elements in the latent Gaussian field \mathbf{x} .

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- Each data point depends on only one of the elements in the latent Gaussian field \mathbf{x} .
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 - The latent field \mathbf{x} , can be large but it is endowed with some conditional independence (Markov) properties so that the precision matrix $Q(\theta)$ is sparse.

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- The linear predictor depends **linearly** on the unknown smooth function of covariates.

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 - The latent field \mathbf{x} , can be large but it is endowed with some conditional independence (Markov) properties so that the precision matrix $Q(\theta)$ is sparse.
 - The linear predictor depends **linearly** on the unknown smooth function of covariates.
 - The inferential interest lies in the univariate posterior marginals $\pi(x_i|\mathbf{y})$

INLA allows us to compute in a **fast** and **efficient** way an **accurate** approximation to the posterior marginals for the hyperparameters

$$\tilde{\pi}(\theta_j | \mathbf{y})$$

and for the latent field

$$\tilde{\pi}(x_i | \mathbf{y})$$

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and for the latent field

$$\tilde{\pi}(x_i | \mathbf{y})$$

All of this is implemented in the **INLA** library in R.

Yesterday we saw how to implement simple models with **INLA**

Today we are going to look at how to implement continuous spatial models.

Such model are **a lot** easier to implement in **inlabru!!!!**

inlabru in a nutshell

`inlabru` is a friendlier version of R-INLA

- it makes INLA more accessible to the user
- makes complex features and predictions (especially for spatial data) a lot easier
- is a softer-wrapper around INLA
- in some cases it allows to release some of the constraints of R-INLA (the linearity of the predictor)

inlabru

- Installation:

- There is a CRAN version

```
install.packages("inlabru")
```

- You can also install the development version of inlabru from GitHub (recommended)

```
# install.packages("remotes")
remotes::install_github("inlabru-org/inlabru",
                      ref="devel")
```

- Documentation

- Web site: <https://sites.google.com/inlabru.org/inlabru>
 - Github: <https://github.com/inlabru-org/inlabru>

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Spatial models

Types of Spatial Data

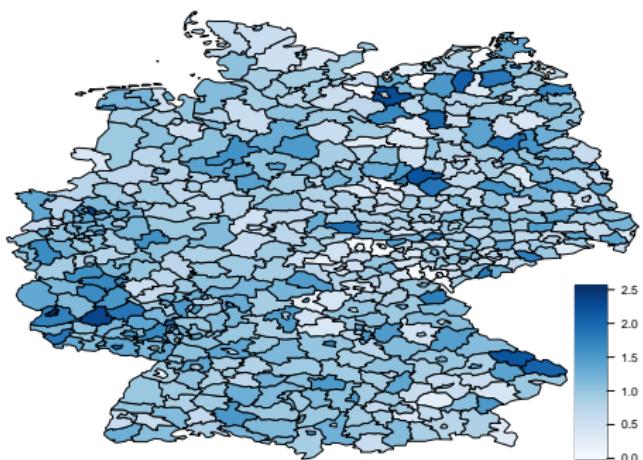
We can distinguish three types of spatial data

- Discrete space
 - data on a spatial grid
- Continuous space:
 - geostatistical data
 - spatial point data

Discrete Counts:

Data on a spatial grid

- examples: number of individuals in a region, average rainfall in a province
- (originally geostatistical or point data; gridded for practical reasons)

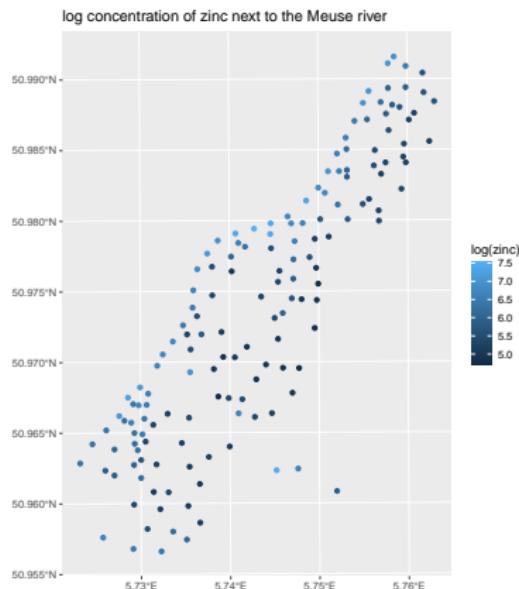


Observed response(s):

- Measurement over each grid cell (e.g. number of individuals in cell; rainfall in province)

Continuous Space: Geostatistics

- phenomenon that is continuous in space
- examples: nutrient levels in soil, salinity in the sea
- measurements at a given set of locations that are determined by surveyor



Observed response(s):

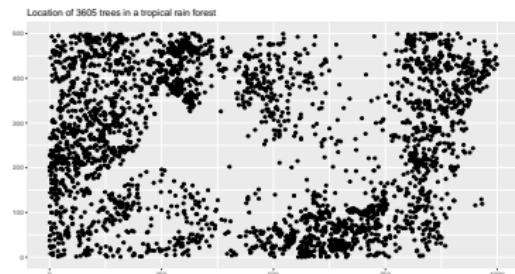
- measurement(s) taken at given locations

Continuous Space: Point Process

- locations of objects (individuals) in space (typically 2D)
- examples: locations of trees in a forest, groups of animals

Observed response(s):

- x, y coordinates of points (individuals/groups)
- maybe also properties of individuals/groups (“marks”)



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Gaussian Random Field models

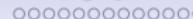
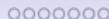
Gaussian Random fields

Definition: A random function $u(x) : R^d \rightarrow R$ is a Gaussian random field if for any finite collection of locations, (x_1, \dots, x_n) , $x_i \in R^d$, the joint distribution of $\mathbf{u} = (u(x_1), \dots, u(x_n))$ is $\mathbf{u} \sim N(0, \Sigma)$, and

$$E(u(x)) = 0,$$

$$Cov(u(x), u(x')) = R(x, x'), \quad \Sigma_{ij} = R(x_i, x_j)$$

for some expectation function $\mu(\cdot)$ and positive definite covariance function $R(\cdot, \cdot)$. Σ is the covariance matrix for the specific location collection.



Matern covariance function

The Matern covariance function is a popular model in spatial problems:

$$c_\nu(d; \sigma, \rho) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{8\nu} \frac{d}{\rho} \right)^\nu K_\nu \left(\sqrt{8\nu} \frac{d}{\rho} \right)$$

* ρ and σ are the range and the marginal standard deviation

- K_ν modified Bessel function of the second kind, order ν .

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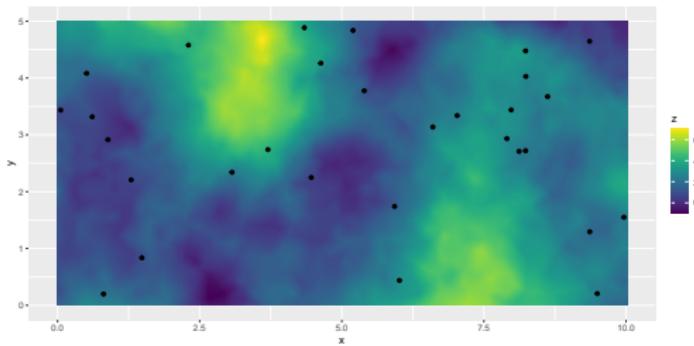
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Example



Gaussian Random field

GRF are a very popular model

- Flexible and easy to use
- Can be part of the latent Gaussian field in a LGM

However:

- **computationally inefficient** (the precision matrix is dense)
- not flexible enough (complicated boundary, barrier, . . .)

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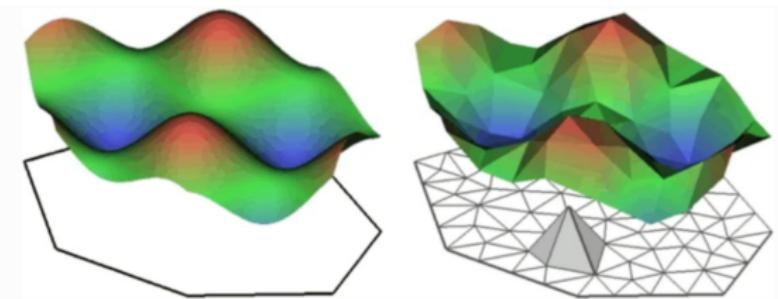
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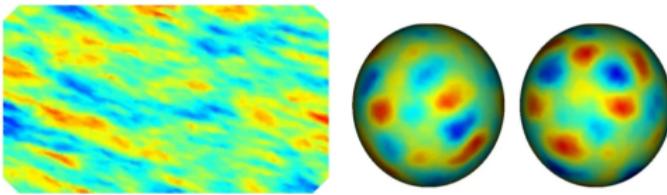
The SPDE approach

- Matern fields can be seen as solution to a PDE
- Using finite element methods such solution can be represented using a GRMF



Advantages of the SPDE approach

- Computationally fast
- Allows for flexible modeling
 - non-stationary models (anisotropy)
 - models on a sphere
 - non separable models



All these models (and many more) can be fitted with **R-INLA** and **inlabru**

Learning about the SPDE approach

- F. Lindgren, H. Rue, and J. Lindström. *An explicit link between Gaussian fields and Gaussian Markov random fields: The SPDE approach (with discussion)*. In: Journal of the Royal Statistical Society, Series B 73.4 (2011), pp. 423–498.
- H. Bakka, H. Rue, G. A. Fuglstad, A. Riebler, D. Bolin, J. Illian, E. Krainski, D. Simpson, and F. Lindgren. *Spatial modelling with R-INLA: A review*. In: WIREs Computational Statistics 10:e1443.6 (2018). (Invited extended review). DOI: 10.1002/wics.1443.
- E. T. Krainski, V. Gómez-Rubio, H. Bakka, A. Lenzi, D. Castro-Camilo, D. Simpson, F. Lindgren, and H. Rue. *Advanced Spatial Modeling with Stochastic Partial Differential Equations using R and INLA*. Github version www.r-inla.org/spde-book. CRC press, Dec. 20

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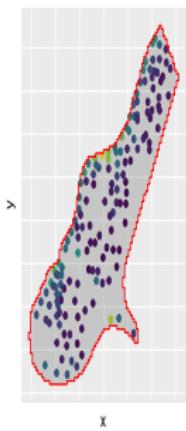
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Fitting spatial models

Example: Meuse Data

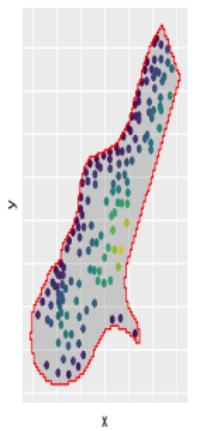
Measures of zinc concentration.

Zinc concentration

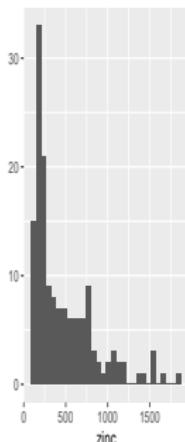


zinc
1500
1000
500
0

Dist from river



dist
0.8
0.6
0.4
0.2
0.0



0 500 1000 1500
zinc

The model

$$\log(Y(s)) \sim \mathcal{N}(\eta(s), \sigma_y^2)$$

$$\eta(s) = \alpha + \beta x(s) + u(s)$$

where

- $Y(s)$ is the measure of zinc in location s
- α a common intercept
- β a model parameter
- $x(s)$ distance from the river at location s
- $u(s)$ the Matern Gaussian field

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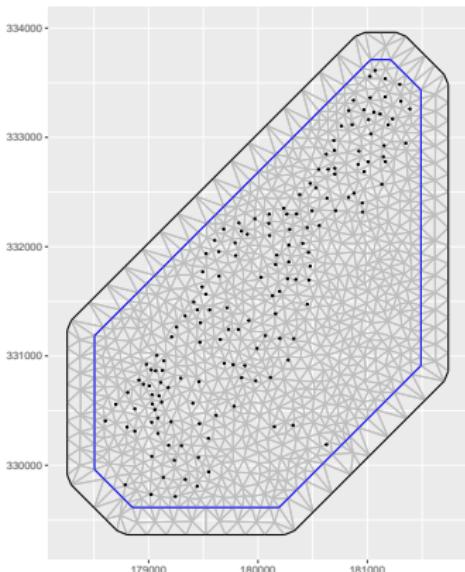
Step 1: Define the SPDE representation

- The mesh
- The SPDE model

Define the SPDE representation: The mesh

1. Define the mesh

```
mesh <- inla.mesh.2d(loc.domain = cbind(meuse$x, meuse$y),  
                      max.edge = c(150, 500),  
                      offset = c(100, 250) )
```



Define the SPDE representation: The mesh

- All random field models need to be discretised for practical calculations.
- The SPDE models were developed to provide a consistent model definition across a range of discretisations.
- We use finite element methods with local, piecewise linear basis functions defined on a triangulation of a region of space containing the domain of interest.
- Deviation from stationarity is generated near the boundary of the region.
- The choice of region and choice of triangulation affects the numerical accuracy.

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Two separate issues:

- Continuous space with bounded domain: Boundary effect
- Discretised model: Numerical accuracy

Sometimes the boundary effect may be desireable.

Define the SPDE representation: The mesh

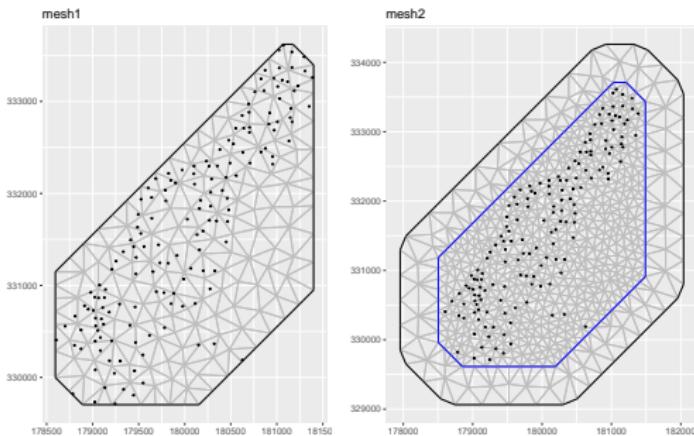
- Too fine meshes → heavy computation
- Too coarse mesh → not accurate enough

Some guidelines

- Create triangulation meshes with `inla.mesh.2d()`
- Move undesired boundary effects away from the domain of interest by extending to a smooth external boundary (`inla.nonconvex.hull(loc, convex)`, `convex ≥ correlation range`)
- Use a coarser resolution in the extension to reduce computational cost (`max.edge=c(inner, outer)`)
- Use a fine resolution (subject to available computational resources) for the domain of interest (inner correlation range) and filter out small input point clusters (`0 < cutoff < inner`)
- Coastlines and similar can be added to the domain specification in `inla.mesh.2d()`

Define the SPDE representation: The mesh

```
mesh1 = inla.mesh.2d(loc.domain = cbind(meuse$x, meuse$y),  
                      max.edge = 350,  
                      offset = 10)  
  
mesh2 = inla.mesh.2d(loc.domain = cbind(meuse$x, meuse$y),  
                      max.edge = c(150, 500),  
                      cutoff = 100,  
                      offset = c(100, 550) )
```



Define the SPDE representation: The SPDE model

```
meuse.spde <- inla.spde2.pcmatern(mesh = mesh,  
                                     prior.sigma = c(1, 0.1),  
                                     prior.range = c(1000, 0.5))
```

PC-priors for the range ρ and the standard deviarion σ

- Define the prior for the range `prior.range` = `(range0,Prange)` $\text{Prob}(\rho < \rho_0) = p_\rho$
- Define the prior for the range `prior.sigma` = `(sigma0,Psigma)` $\text{Prob}(\sigma > \sigma_0) = p_\sigma$

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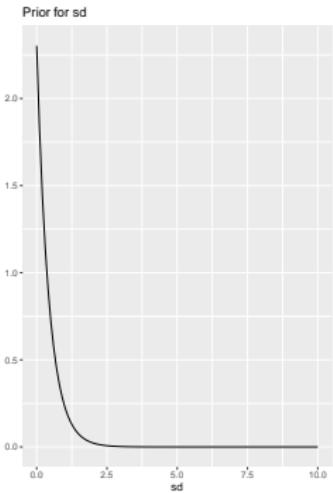
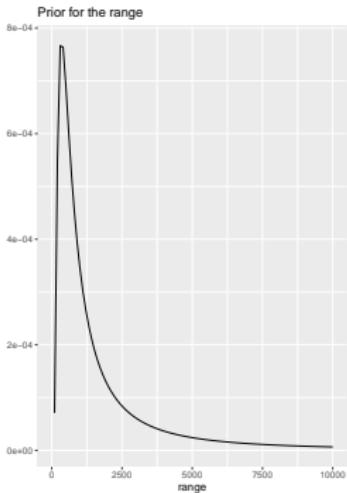
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Prior for range ρ and sd σ



Run the model inlabru

```
# create a spatial object
coordinates(meuse) = c("x","y")
# covariate values
dist_SPDE = SpatialPixelsDataFrame(data$dist_raster[,c(1,2)],
                                     data = data.frame(dist = data$dist_raster[,3]))
# model components
cmp = ~ Intercept(1) + dist(dist_SPDE, model = "linear") +
      spde(coordinates, model = meuse.spde)
# define likelihood
lik = like(formula = Y ~ Intercept + dist + spde,
           family = "gaussian",
           data = meuse)
#fit the model
fit <- bru(cmp, lik)
# define prediction area
pix <- pixels(mesh, nx = 200, ny = 200, mask = boundary)
# generate predictions
pred = predict(fit, pix, ~ data.frame(
  spde = spde,
  logscale = Intercept + dist + spde,
  naturalscale = exp(Intercept + dist + spde)))
```

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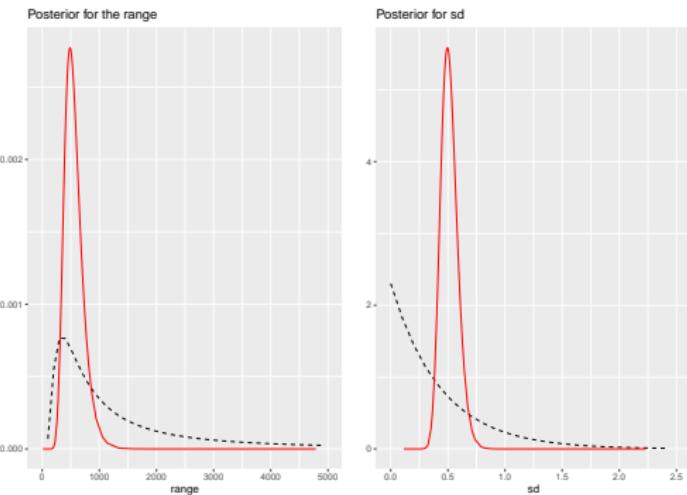
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Notes!

- The data are a spatial object!
- For prediction, the covariates are stored in a `SpatialPixelsDataFrame` and need to cover all the mesh nodes

Estimation: Posterior for the hyperparameters



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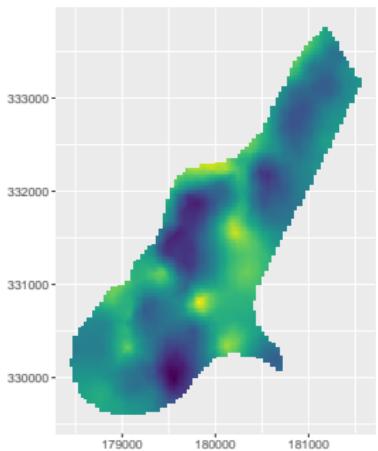
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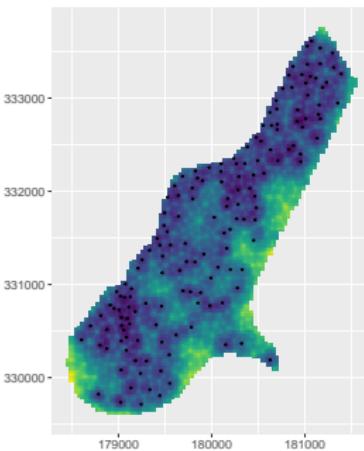
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Predictions: The SPDE field



mean

0.5
0.0
-0.5
-1.0



sd

0.5
0.4
0.3
0.2

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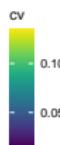
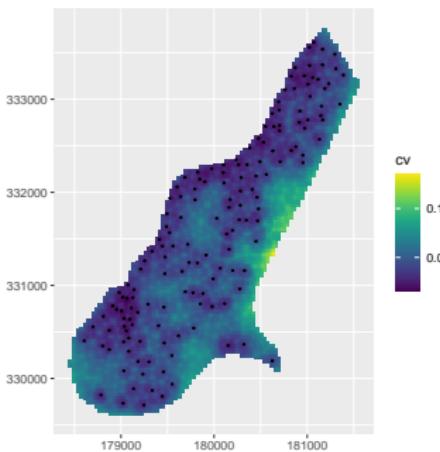
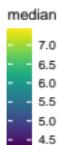
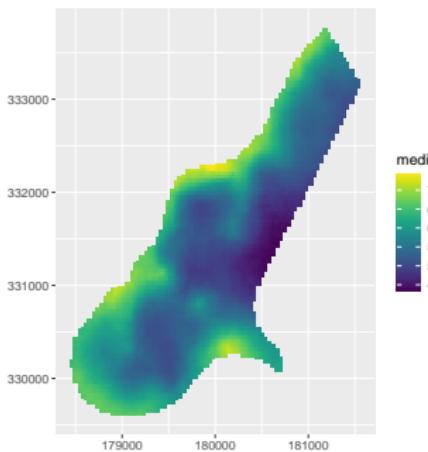
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Predictions: The log concentrations



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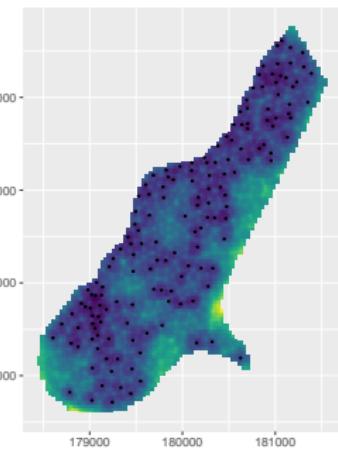
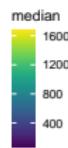
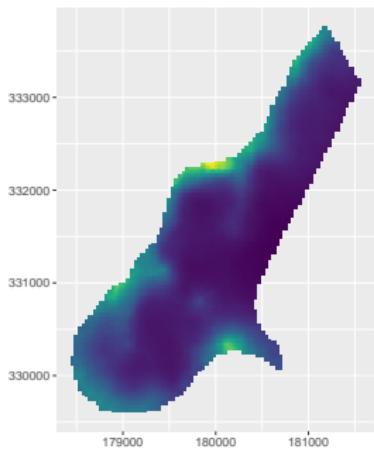
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Predictions: The concentrations



Same in plain INLA (1)

```
A.meuse <- inla.spde.make.A(mesh = mesh, loc = coordinates(meuse))
s.index <- inla.spde.make.index(name = "spatial.field",
  n.spde = meuse.spde$n.spde)

#Create data structure
meuse.stack <- inla.stack(data = list(zinc = meuse$zinc),
  A = list(A.meuse, 1),
  effects = list(c(s.index, list(Intercept = 1)),
    list(dist = meuse$dist)),
  tag = "meuse.data")

data(meuse.grid)
coordinates(meuse.grid) = ~x+y
gridded(meuse.grid) = TRUE

#Create data structure for prediction
A.pred <- inla.spde.make.A(mesh = mesh, loc = coordinates(meuse.grid))
meuse.stack.pred <- inla.stack(data = list(zinc = NA),
  A = list(A.pred, 1),
  effects = list(c(s.index, list (Intercept = 1)),
    list(dist = meuse.grid$dist)),
  tag = "meuse.pred")

#Join stack
join.stack <- inla.stack(meuse.stack, meuse.stack.pred)
```

Same in plain INLA (2)

```
#Fit model
form <- log(zinc) ~ -1 + Intercept + dist + f(spatial.field, model = spde)

m1 <- inla(form, data = inla.stack.data(join.stack, spde = meuse.spde),
family = "gaussian",
control.predictor = list(A = inla.stack.A(join.stack), compute = TRUE))
```

Note: We still have not compute predictions... and this is not too easy in plain INLA!!

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When is `inlabru` easier to use

- spatial modeling.
- point processes.
- multiple likelihoods
- when interested in spatial predictions

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- spatial modeling.
- point processes.
- multiple likelihoods
- when interested in spatial predictions
- `inlabru` is also useful if one has non-linearities in the predictor η
 - born for ecological models (for example transect sampling) but used also in other fields

When is **inlabru** easier to use

- spatial modeling.
- point processes.
- multiple likelihoods
- when interested in spatial predictions
- **inlabru** is also useful if one has non-linearities in the predictor η
 - born for ecological models (for example transect sampling) but used also in other fields

What cannot (at the moment) be done with **inlabru**

- Survival models

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Space-time Modeling

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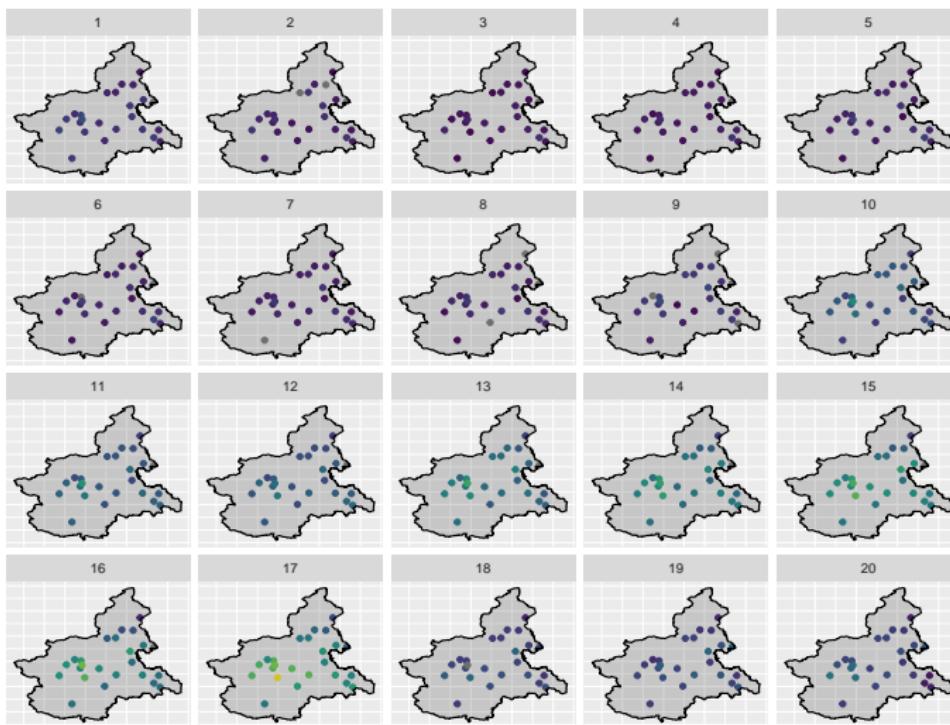
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Modeling PM10 concentration in Piemonte (Italy)



Modeling PM10 concentration in Piemonte (Italy)

Example from the Blangiardo & Cameletti book but simplified!

```
##   Station.ID      Date    dem      x      y    WS    temp   HMIX PREC    EMI PM10
## 1           1 2005-10-01  95.2 469.45 4972.85 0.90 288.81 1294.6     0 26.05   28
## 2           2 2005-10-01 164.1 423.48 4950.69 0.82 288.67 1139.8     0 18.74   22
## 3           3 2005-10-01 242.9 490.71 4948.86 0.96 287.44 1404.0     0  6.28   17
## 4           4 2005-10-01 149.9 437.36 4973.34 1.17 288.63 1042.4     0 29.35   25
## 5           5 2005-10-01 405.0 426.44 5045.66 0.60 287.63 1038.7     0 32.19   20
##   time logPM10
## 1     1 3.332205
## 2     1 3.091042
## 3     1 2.833213
## 4     1 3.218876
## 5     1 2.995732
```

The model

We model the log PM10 concentration:

$$y_{it} \sim \mathcal{N}(\eta_{it}, \sigma_e^2)$$

$$\eta_{it} = \alpha + \beta_1 \text{dem}_i + \beta_2 \text{temp}_{it} + \omega_{it}$$

- y_{it} is the log-concentration at location i in time t
- α is an intercept
- β_1 and β_2 parameters of altitude and temperature
- ω_{it} is the space-time residual

Space time residual model

A first order autoregressive process with spatially colored innovations

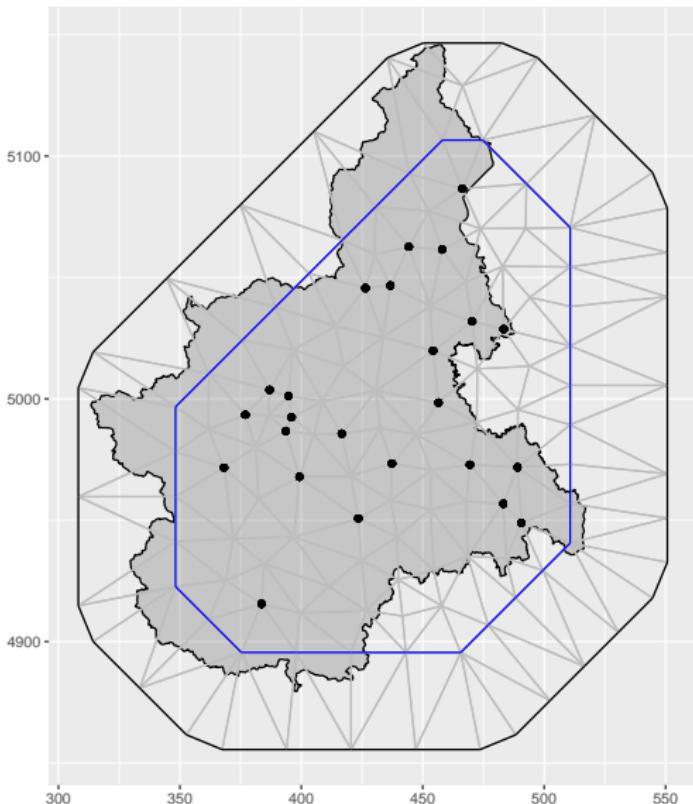
$$\omega_{it} = a\omega_{i(t-1)} + \xi_{it}$$

- $|a| < 1$ parameter of the AR1 process
- ξ_{it} is a zero mean, temporally independent, Gaussian field with

$$\text{Cov}(\xi_{it}, \xi_{ju}) = \begin{cases} 0, & \text{if } t \neq u \\ \mathcal{C}(h), & \text{if } t = u \end{cases}$$

- h distance between locations i and j - $\mathcal{C}(h)$ is a Matern correlation function

Building the model: mesh



Building the model: Prior for the spde model

```
spde = inla.spde2.pcmatern(mesh = mesh,  
                           prior.range=c(100,0.5),  
                           prior.sigma=c(1,0.1))
```

- $\text{Prob}(\rho < 100 \text{ Km}) = 0.5$
- $\text{Prob}(\sigma > 1) = 0.1$

Implement the model

Model Parameters

Fixed effects:

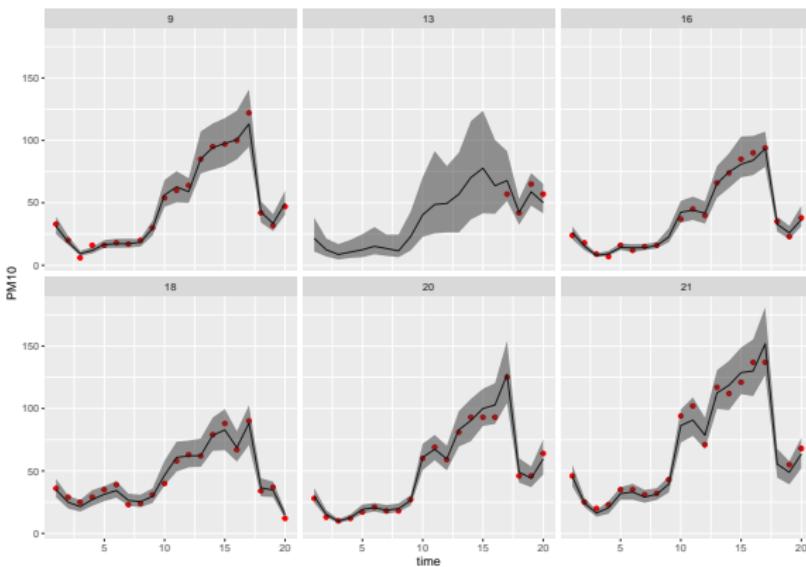
```
##               mean        sd 0.025quant 0.975quant
## Intercept -1.438103e+01 9.2183351517 -32.139029373 4.0476156436
## dem       -7.283307e-04 0.0008432533 -0.002398673 0.0009354501
## temp      6.250902e-02 0.0320760645 -0.001679595 0.1243716481
```

Hyperparameters of the random field:

```
##                               mean        sd 0.025quant 0.975quant
## Precision for the Gaussian observations 41.8579575 7.23518895 29.3185249
## Range for SPDE                      129.1799687 16.96397904 101.0510932
## Stdev for SPDE                      0.7511733 0.09963733 0.5913784
## GroupRho for SPDE                   0.9105294 0.02594638 0.85555876
##                                         0.975quant
## Precision for the Gaussian observations 57.6625010
## Range for SPDE                      167.2757328
## Stdev for SPDE                      0.9796968
## GroupRho for SPDE                   0.9557794
```

Prediction at station locations

```
pred_at_station = predict(fit, df,
                           ~exp(Intercept + SPDE + dem + temp ))
```

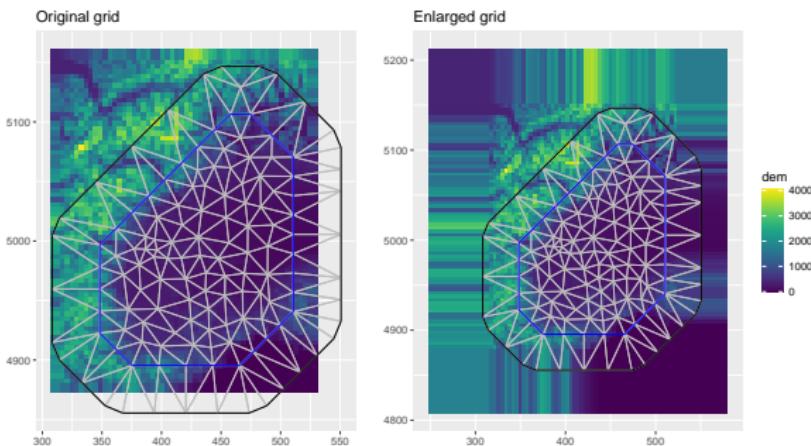


Prediction is space

- To do this we need to have the covariates over the space of interest.
- They should cover the whole mesh

Expanding the covariates: the `bru` `fill` `missing`

Expanding the covariates: the bru_fill_missing



Note: You can do this with your favourite method!

What have we learned

oooooooo

Spatial models

ooooo

Gaussian Random Field models

ooooo

The SPDE approach

oooo

Fitting spatial

oooooooooooo

Prediction in space

Prediction in space

```
# Create a space time grid
pxl = pixels(mesh, nx = 200, ny = 200, mask = shape)
ips2 <- ipoints(domain = c(1:5), name = "time")
pxl_time <- cbind(cprod(pxl,ips2), data.frame(temp = 0))

pred_space = predict(fit, p xl_time,
                     ~ Intercept + SPDE +
                     dem_eval(inlabru:::eval_SpatialDF(dem_large,
                     .data.)) +
                     temp_eval(inlabru:::eval_SpatialDF(temp_large,
                     .data.,
                     selector = "time")))
```

What have we learned
oooooooo

Spatial models
oooooo

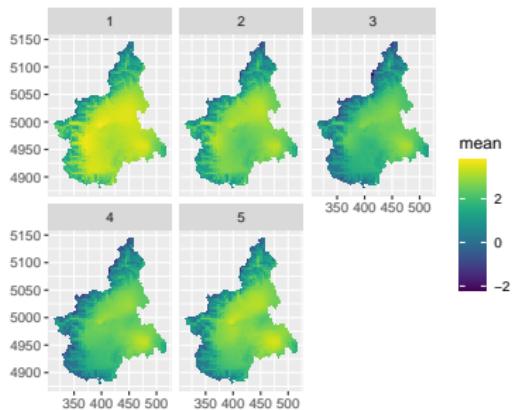
Gaussian Random Field models
oooooo

The SPDE approach
oooo

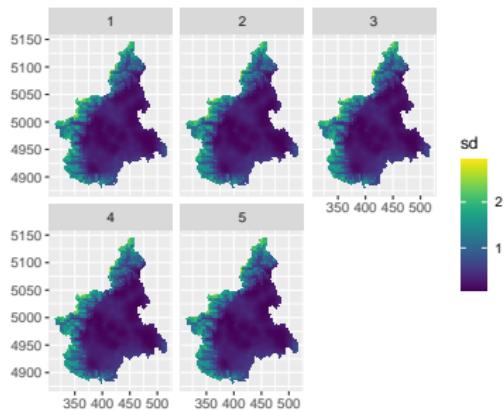
Fitting spatial
oooooooooooooooooooo

Prediction in space

Posterior mean



Posterior sd



What have we learned
oooooooo

Spatial models
ooooo

Gaussian Random Field models
ooooo

The SPDE approach
oooo

Fitting spatial
oooooooooooo

Samples form the fitted model

It is also possible to generate from the fitted model

```
sim_space = generate(fit, pxi_time,
                     ~ exp(Intercept + SPDE +
                           dem_eval(inlabru:::eval_SpatialDF(dem_large,
                                                               .data.)) +
                           temp_eval(inlabru:::eval_SpatialDF(temp_large,
                                                               .data.,
                                                               selector = "time"))),
                     n.samples = 2)
```

This can be useful as the posterior means are always smoother than the “real” field

What have we learned
oooooooo

Spatial models
oooooo

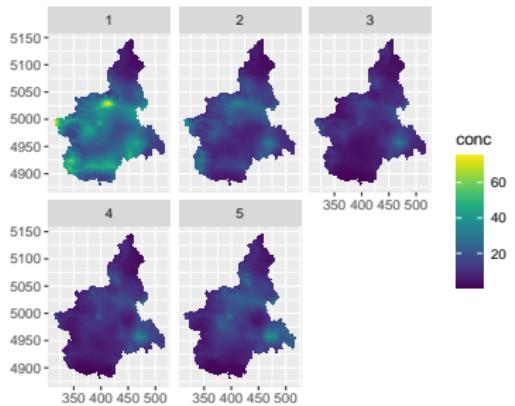
Gaussian Random Field models
oooooo

The SPDE approach
oooo

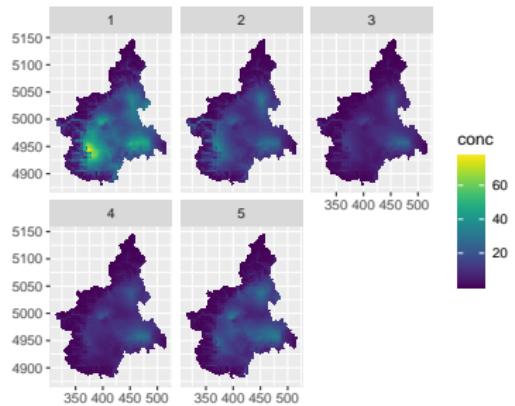
Fitting spatial
oooooooooooooooooooo

Samples form the fitted model

Sample 1



Sample 1



Some concluding remark

- The functions `predict()` and `generate()` in `inlabru` use the INLA function `inla.posterior.sample()` internally.
- `inlabru` is a wrapper around INLA so all the internal computations are identical!
- `inlabru` makes handling of spatial object a lot easier
- Transition from `sp` to `sf/terra` some changing can be expected

Thank you for your attention!

If you have any doubts or questions, please write :
sara.martino@math.ntnu.no

