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# Introduction to microwave modeling

## Motivations for SMRT

# Context

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Many snow microwave emission models have been developed over the last 30-40 years. The most “generic” ones are widely used by the PM community: HUT, MEMLS, DMRT-QMS, DMRT-ML, ...

A few snow radar backscatter models have been developed by the AM community.

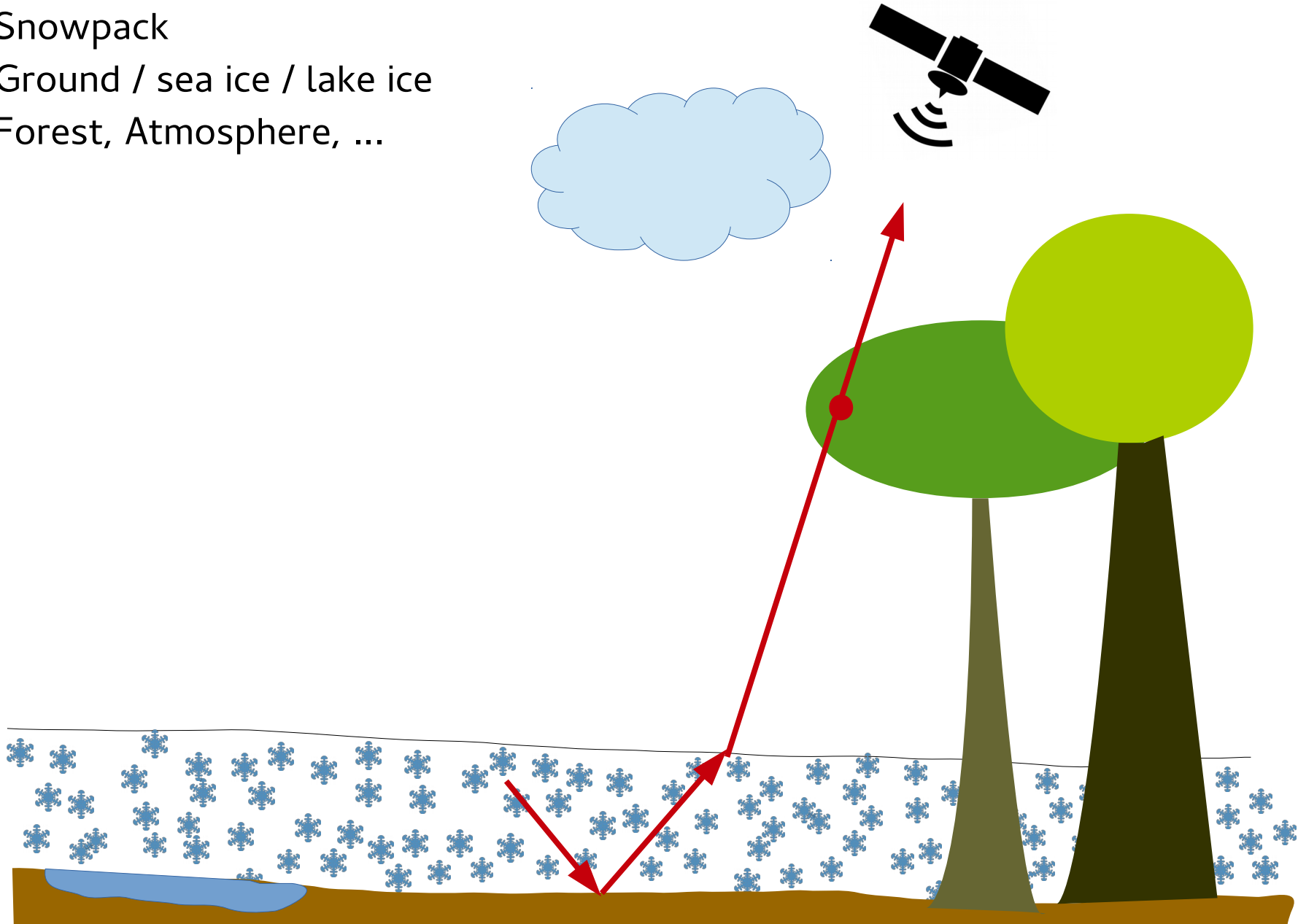
- several in specific studies
- DMRT-QMS (L. Tsang's group) is dual mode
- MEMLS has recently been extended to active mode

In this introduction lecture:

- Why such a diversity ?
- Is this diversity apparent or profound ?
- Is this diversity beneficial or counter-productive for the community ?
- What about the dual mode ? Good or bad ?
  
- Why a new model ?

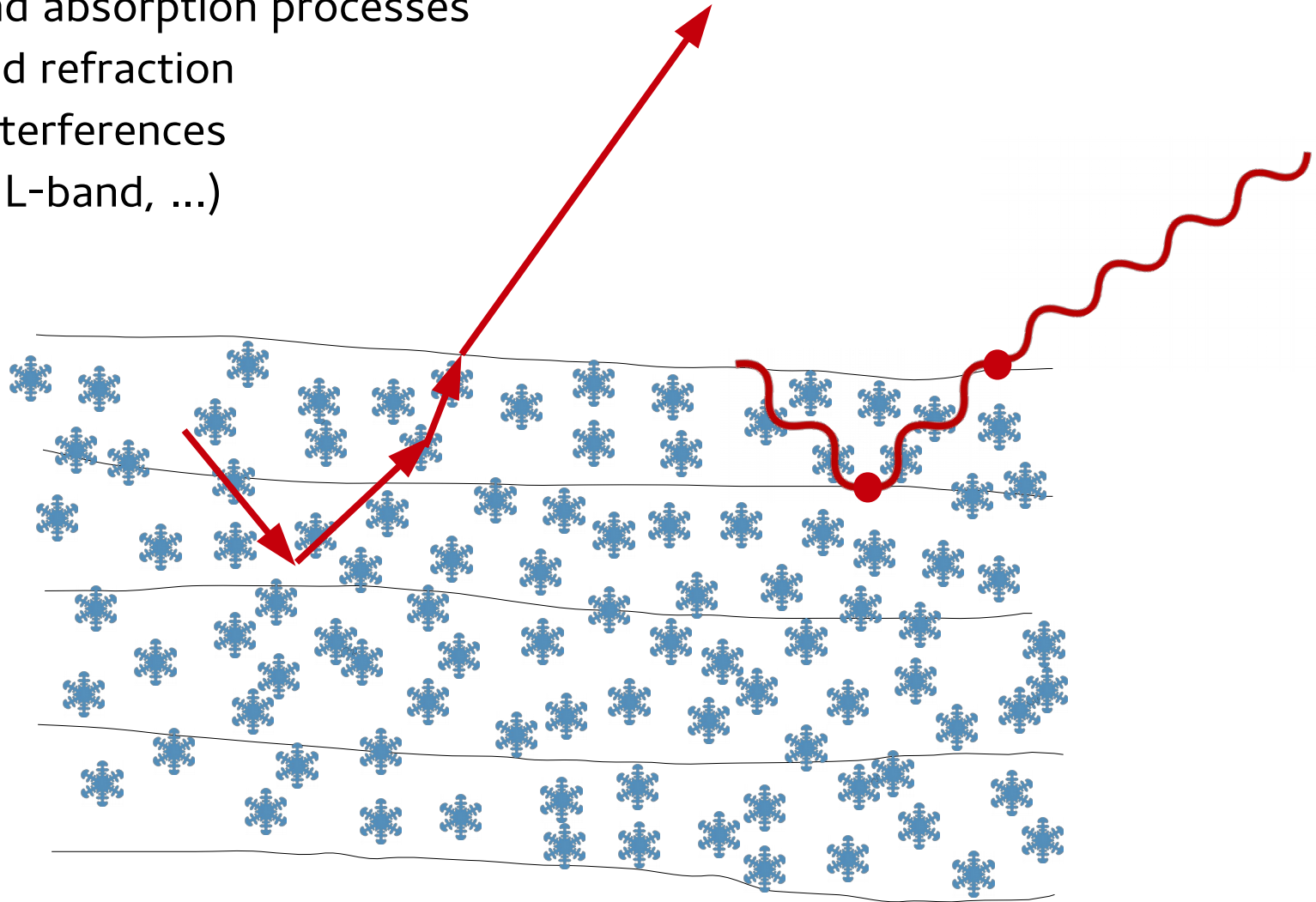
# Microwave model ingredients

- Snowpack
- Ground / sea ice / lake ice
- Forest, Atmosphere, ...



# Microwave model ingredients

- Thermal Emission
- Scattering and absorption processes
- Reflection and refraction
- Inter-layer interferences  
(e.g. ice crust, L-band, ...)

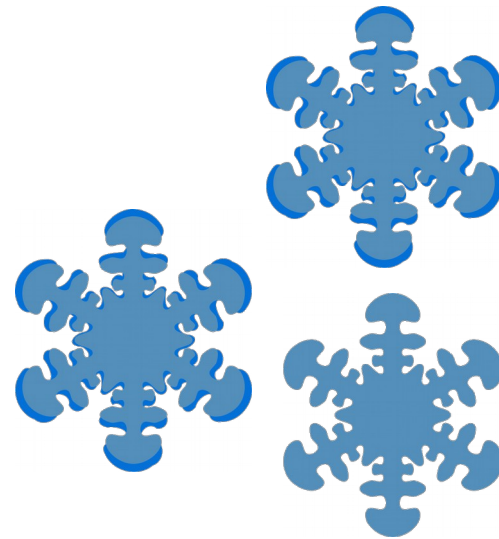
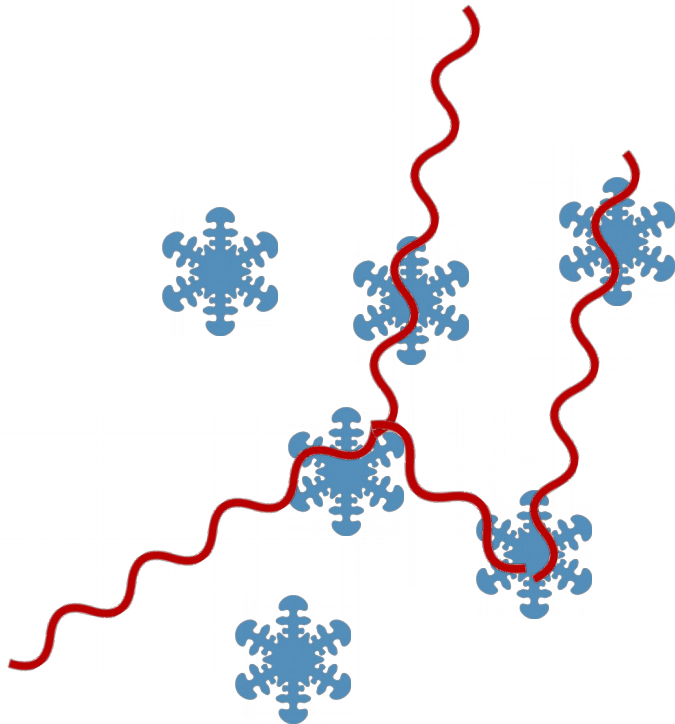


# Microwave model ingredients

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Snow is a **dense media** from the perspective of EM wave:

- Scattered by many particles → change effective incident field
  - Multiple scattering between particles
- Concept of effective permittivity and Born approximation(s)



- Multi-species (e.g. wet snow)

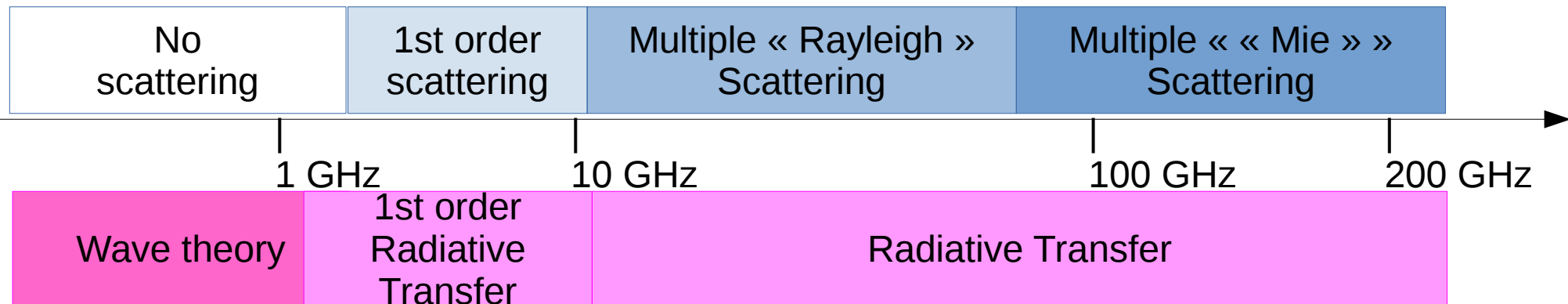
# Microwave model ingredients

Models differ in the ingredients and how detailed is each component described  
e.g. HUT (Snow + atmosphere) versus DMRT-ML (snow) + RTTOV (atmosphere)

Other constraints:

- frequency range

For typical snowpack:



SMRT is definitely a RT model. The following is about RT models

# Microwave model ingredients

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## Other constraints (cont.):

- Application context → performance, adjoint needed, ...
- language
- license
- ecosystem around the model, documentation, support, ...
- collaboration network, institutional constraints, community

There are many good reasons for different models.

But: our community is not so 'big'. Question: are the differences profound or superficial ?

The following is mostly based on « Are existing snow microwave emission models so different ? », Picard et al. AGU 2015

# Microwave model ingredients

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Radiative transfer models in general:

The radiative transfer equation

$$\mu \frac{\partial \mathbf{I}(\mu, \phi, z)}{\partial z} = -\kappa_e(\mu, \phi, z) \mathbf{I}(\mu, \phi, z) + \frac{1}{4\pi} \iint_{4\pi} \mathbf{P}(\mu, \phi; \mu', \phi', z) \mathbf{I}(\mu', \phi', z) d\Omega' + \kappa_a(\mu, \phi, z) \alpha T(z) \mathbf{1}$$

accompanied with boundary conditions:

$$\begin{aligned} \mathbf{I}^{(l)}(\mu < 0, \phi, z_{l-1}) &= \mathbf{R}^{\text{spec, top}, (l)}(\mu) \mathbf{I}^{(l)}(-\mu, \phi, z_{l-1}) + \frac{1}{2\pi} \iint_{2\pi, \mu' > 0} \mathbf{R}^{\text{diff, top}, (l)}(\mu, \mu', \phi - \phi') \mathbf{I}^{(l)}(\mu', \phi', z_{l-1}) d\Omega' \\ &+ \mathbf{T}^{\text{spec, bottom}, (l-1)}(\mu) \mathbf{I}^{(l-1)}(\mu, \phi, z_{l-1}) + \frac{1}{2\pi} \iint_{2\pi, \mu' < 0} \mathbf{T}^{\text{diff, bottom}, (l-1)}(\mu, \mu', \phi - \phi') \mathbf{I}^{(l-1)}(\mu', \phi', z_{l-1}) d\Omega' \end{aligned}$$



# Microwave model ingredients

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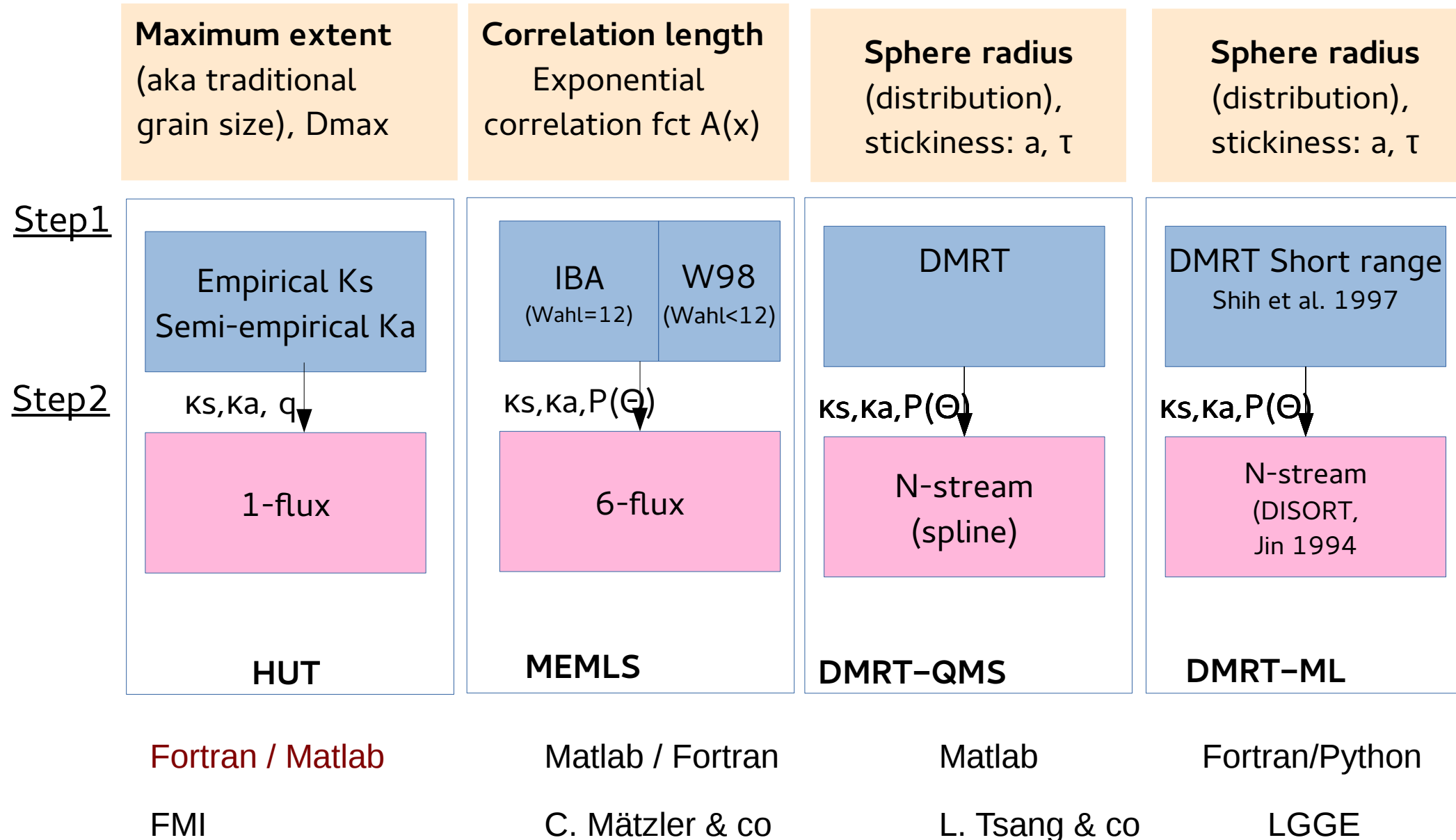
Computation:

Step 1 a - compute layer electromagnetic intrinsic properties (Ke, Ks, Ka, P, eps)  
b - compute interfaces electromagnetic intrinsic properties (R, T)

Step 2 solve the radiative transfer equation

# Comparison of models

It's incredible how different they look:



# Comparison of models

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**Numerical comparisons** showed that none of the models is significantly/always better than the others.

Tedesco et al. 2006, « Intercomparison of Electromagnetic Models for Passive Microwave Remote Sensing of Snow »

Tian, B. « Quantifying inter-comparison of the microwave emission model of layered snowpacks (MEMLS) and the multilayer dense media radiative transfer theory (DMRT) in modeling snow microwave radiance (IGARSS) », 2010

L. Brucker et al. 2011, thesis and « Modeling time series of microwave brightness temperature at Dome C, Antarctica, using vertically resolved snow temperature and microstructure measurements »

Roy et al. 2013, « Brightness temperature simulations of the Canadian seasonal snowpack driven by measurements of snow specific surface area »

Kwon, Y, « Error Characterization of Coupled Land Surface-Radiative Transfer Models for Snow Microwave Radiance Assimilation », 2015

Roy, A., A. Royer, O. St-Jean-Rondeau, B. Montpetit, G. Picard, A. Mavrovic, N. Marchand, and A. Langlois, Microwave snow emission modeling uncertainties in boreal and subarctic environments, *The Cryosphere* 10, 623-638, doi:10.5194/tc-10-623-2016, 2016

Sandells, M., Essery, R., Rutter, N., Wake, L., Leppänen, L., and Lemmetyinen, J.: Microstructure representation of snow in coupled snowpack and microwave emission models, *The Cryosphere*, 11, 229-246, tc-11-229-2017, 2017

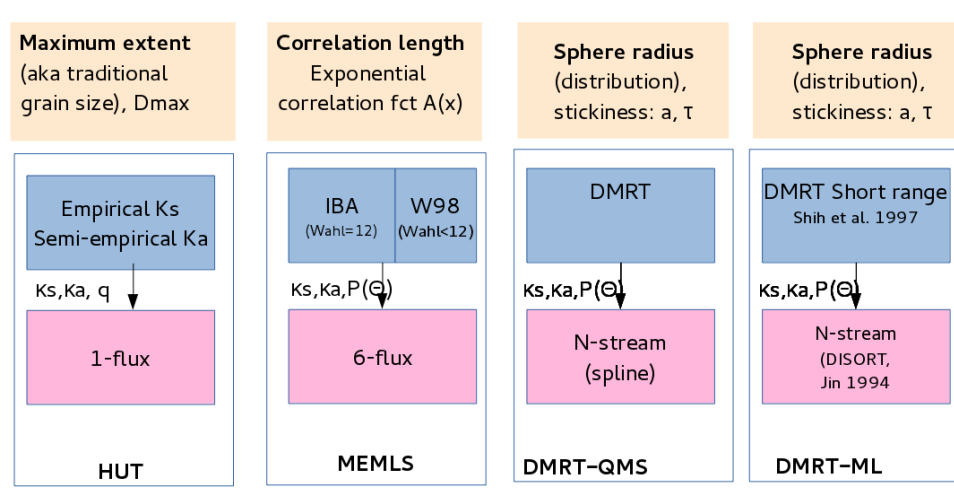
Royer A., A. Roy, B. Montpetit, O. Saint-Jean-Rondeau, G. Picard, L. Brucker, and A. Langlois, Comparison of commonly-used microwave radiative transfer models for snow remote sensing. *Remote Sensing of Environment*, 190, 247–259, doi:10.1016/j.rse.2016.12.020, 2017

Performing a fair comparison is challenging because of the many different components and the different « grain size » metrics (microstructure).

# Comparison of models

This talk:

~~Are~~ existing snow microwave  
emission models ~~so different~~ ?  
*aren't*



Reconcilate:

- the different **electromagnetic theories**
- the different **micro-structure representation** used by these models
- the different **solutions** of the radiative transfer equation

Recent studies: Löwe and Picard (TC, 2015) and Pan et al. (2016)

Guess who ?

$$K^2 = k^2 + \frac{f_v(k_s^2 - k^2)}{1 + \frac{k_s^2 - k^2}{3K^2}(1 - f_v)} \left\{ 1 + i \frac{2(k_s^2 - k^2)Ka^3}{9 \left[ 1 + \frac{(k_s^2 - k^2)}{3K^2}(1 - f_v) \right]} \times \right. \\ \left. \left[ 1 + 4\pi n \int_0^\infty dr r^2 (g(r) - 1) \right] \right\}$$

$$\tilde{\omega} = \frac{2a^3 f_v}{9 \kappa_e} \left| \frac{k_s^2 - k^2}{1 + \frac{k_s^2 - k^2}{3K^2}(1 - f_v)} \right|^2 \left[ 1 + 4\pi n \int_0^\infty dr r^2 (g(r) - 1) \right]$$

$$\gamma^{\text{bi}}(\hat{\mathbf{o}}, \hat{\mathbf{i}}) = \frac{k_{\text{eff}}^4}{4\pi V} |\mathbf{F}_f|^2 \sin^2 \chi \\ = \nu(1 - \nu)(\epsilon_2 - \epsilon_1)^2 K^2 I \cdot k^4 \sin^2 \chi \quad I = \frac{1}{\alpha} \int_0^\infty A(x) x \sin(\alpha x) dx$$

$$\epsilon_{\text{eff}} = \frac{2\epsilon_1 - \epsilon_2 + 3\nu(\epsilon_2 - \epsilon_1) + \sqrt{(2\epsilon_1 - \epsilon_2 + 3\nu(\epsilon_2 - \epsilon_1))^2 + 8\epsilon_1\epsilon_2}}{4}$$

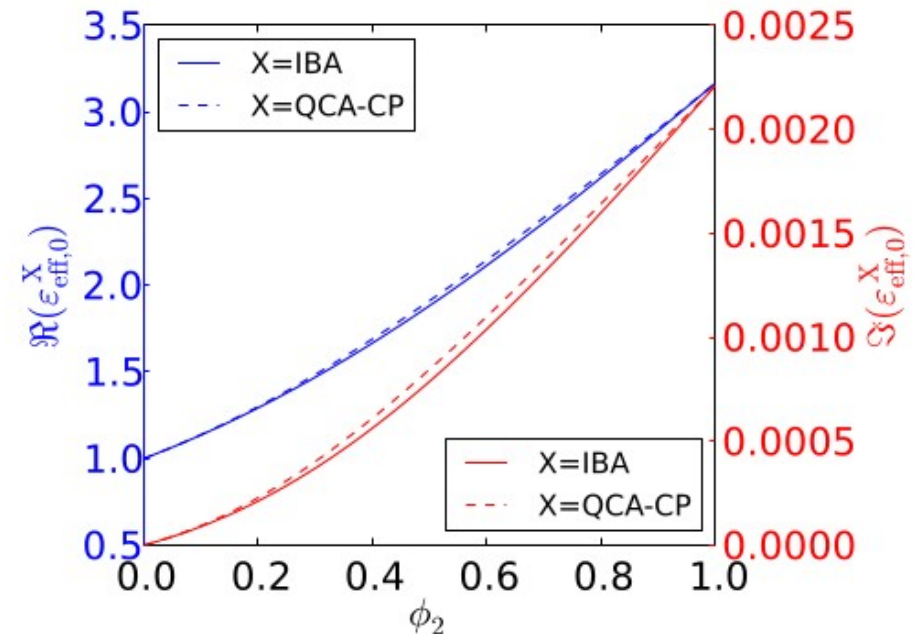
# "IBA" and "DMRT QCA-CP" theories in MEMLS and DMRT\*

Löwe and Picard, 2015, in the low frequency limit ( $\leq 37$  GHz for most snow) for spherical scatters

- Effective medium permittivity/wavenumber:

$$\varepsilon_{\text{eff},0}^{\text{IBA}} = \frac{2\varepsilon_1 - \varepsilon_2 + 3\phi_2(\varepsilon_2 - \varepsilon_1)}{4} + \frac{\sqrt{(2\varepsilon_1 - \varepsilon_2 + 3\phi_2(\varepsilon_2 - \varepsilon_1))^2 + 8\varepsilon_1\varepsilon_2}}{4}$$

$$\varepsilon_{\text{eff},0}^{\text{QCA-CP}} = \frac{\varepsilon_1 - \frac{(\varepsilon_2 - \varepsilon_1)}{3}(1 - 4\phi_2)}{2} + \frac{\sqrt{\left(\varepsilon_1 - \frac{(\varepsilon_2 - \varepsilon_1)}{3}(1 - 4\phi_2)\right)^2 + 4\varepsilon_1 \frac{(\varepsilon_2 - \varepsilon_1)}{3}(1 - \phi_2)}}{2}$$



- Absorption formulations are identical

- Scattering coefficients:

$$\kappa_s^{\text{IBA}} = \frac{2}{9}k_0^4a^3\phi_2 \left| \frac{(\varepsilon_2 - \varepsilon_1)(2\varepsilon_{\text{eff},0}^{\text{IBA}} + \varepsilon_1)}{(2\varepsilon_{\text{eff},0}^{\text{IBA}} + \varepsilon_2)} \right|^2 S(0)$$

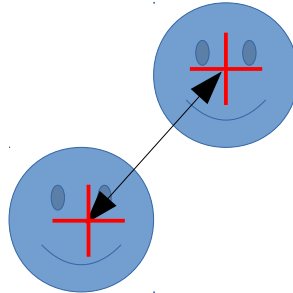
$$\kappa_s^{\text{QCA-CP}} = \frac{2}{9}k_0^4a^3\phi_2 \left| \frac{3\varepsilon_{\text{eff},0}^{\text{QCA-CP}}(\varepsilon_2 - \varepsilon_1)}{3\varepsilon_{\text{eff},0}^{\text{QCA-CP}} + (\varepsilon_2 - \varepsilon_1)(1 - \phi_2)} \right|^2 S(0).$$

# Micro-structure in the models

$S(0)$  = snow micro-structure

## DMRT-QCA

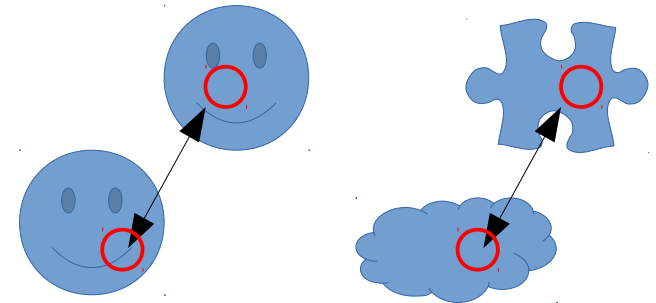
Position of  
the scatterers



Tied to the concept of scatterers.  
Shape, size and position are not  
coupled

Pair-correlation  $g(\mathbf{r}) \sim$  Probability of  
distance between centres of the  
scatterers

## IBA



Micro-structure can be any  
discrete biphase medium

Autocorrelation of the indicator  
function  $C(\mathbf{r}) \sim$  Probability of the  
distance between masses

Distribution      Sticky hard sphere

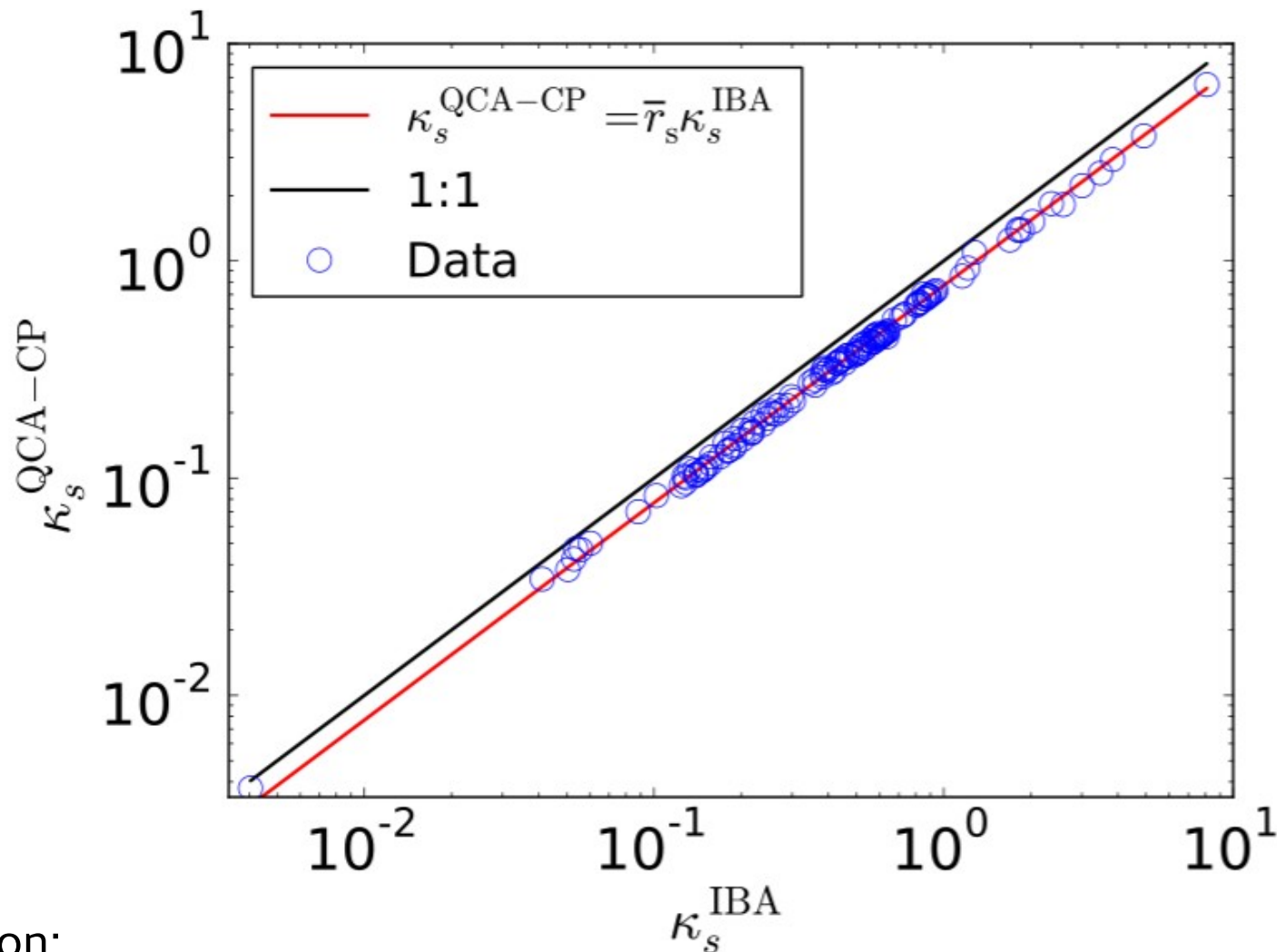
Parameters      **Radius, stickiness**

Exponential autocorrelation  
function

**Correlation length ( $p_{ex}$ )**

## Micro-structure in the models

When IBA uses Sticky Hard Sphere like DMRT instead of exponential autocorrelation:



Conclusion:

The main difference between MEMLS and DMRT family is the microstructure



## Micro-structure in the models

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HUT has semi-empirical formulation of scattering/extinction coefficient

Grain size  $d_0$

$$\kappa_e = 0.0018 f^{2.8} d_0^{2.0} \quad \text{where } d_0 = 1.5 (1 - \exp(-1.5 D_{\max}))$$

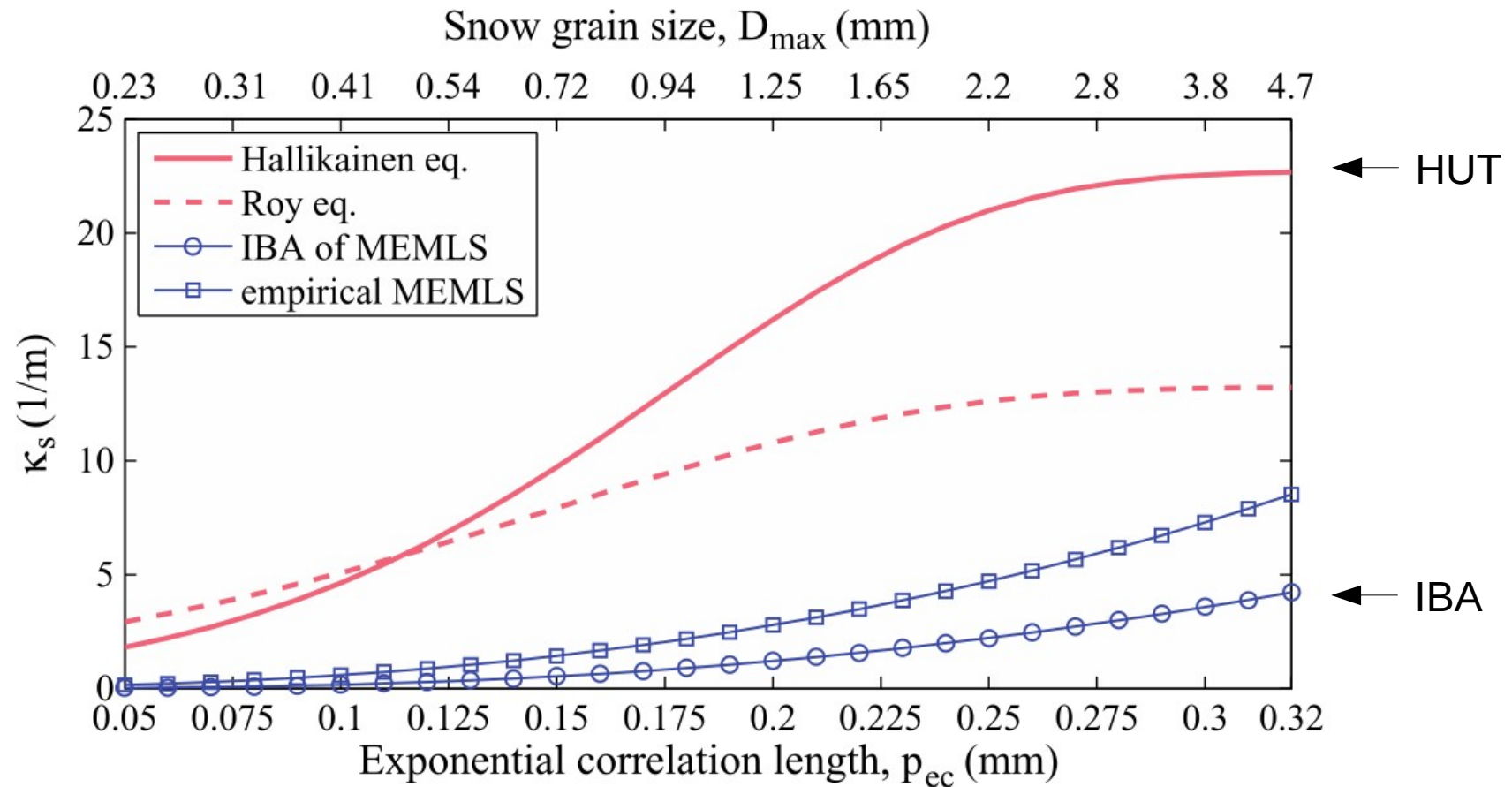
$$\kappa_s^{\text{IBA}} = \frac{2}{9} k_0^4 a^3 \phi_2 \left| \frac{(\varepsilon_2 - \varepsilon_1) (2\varepsilon_{\text{eff},0}^{\text{IBA}} + \varepsilon_1)}{(2\varepsilon_{\text{eff},0}^{\text{IBA}} + \varepsilon_2)} \right|^2 S(0)$$

$$\kappa_s^{\text{QCA-CP}} = \frac{2}{9} k_0^4 a^3 \phi_2 \left| \frac{3\varepsilon_{\text{eff},0}^{\text{QCA-CP}} (\varepsilon_2 - \varepsilon_1)}{3\varepsilon_{\text{eff},0}^{\text{QCA-CP}} + (\varepsilon_2 - \varepsilon_1) (1 - \phi_2)} \right|^2 S(0).$$

Using micro-structure images  $\rightarrow$  a geometrically-based relationship between  $D_{\max}$  and  $p_{\text{ex}}$

# Micro-structure in the models

Pan, Durand and co-authors, 2016



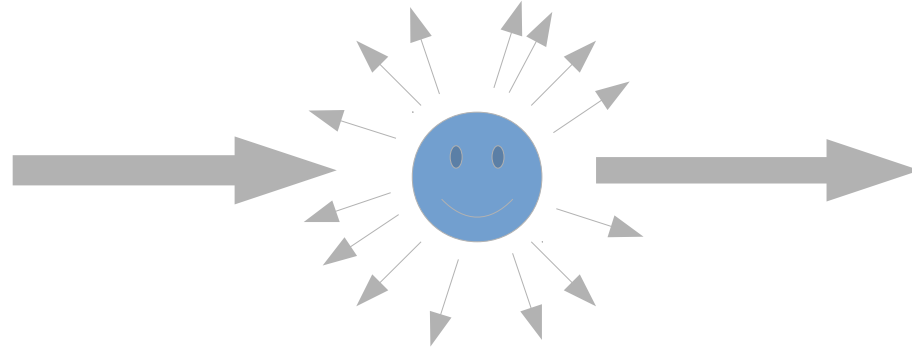
HUT and IBA have very different scattering coefficients !

Surprising because HUT and MEMLS are known to have good performance...

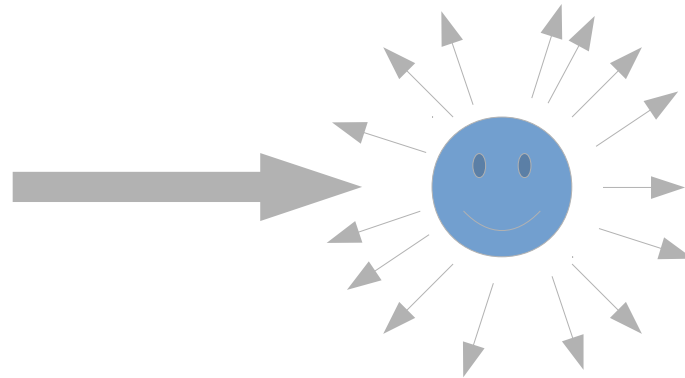
## Micro-structure in the models

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HUT : snow is a strongly forward scattering :  $q=0.96$



IBA and QCA-CP : snow scattering is almost isotropic (Rayleigh or moderate Mie)



# Similarity theory

Radiative transfer equation:

$$\mu \frac{\partial \mathbf{I}(\mu, \phi, z)}{\partial z} = -\kappa_e(\mu, \phi, z) \cdot \mathbf{I}(\mu, \phi, z) + \frac{1}{4\pi} \iint_{4\pi} \mathbf{P}(\mu, \phi; \mu', \phi', z) \cdot \mathbf{I}(\mu', \phi', z) d\Omega' + \kappa_a(\mu, \phi, z) T(z)$$

Different formulations of  $\kappa_e$  and  $\mathbf{P}$  may lead to exactly the same RT equation (and exactly the same solution)

e.g.

C. Mitrescu, , G.L. Stephens, On similarity and scaling of the radiative transfer equation, Journal of Quantitative Spectroscopy and Radiative Transfer 86, 4, 387–394, 2004

H.C. van de Hulst, Multiple light scattering, Academic Press, New York, 1980

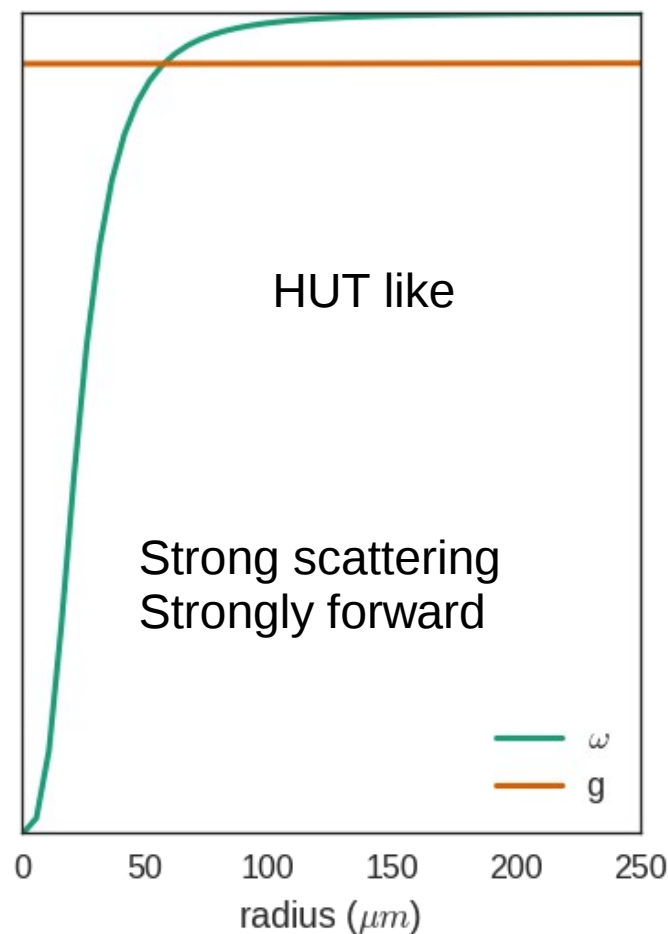
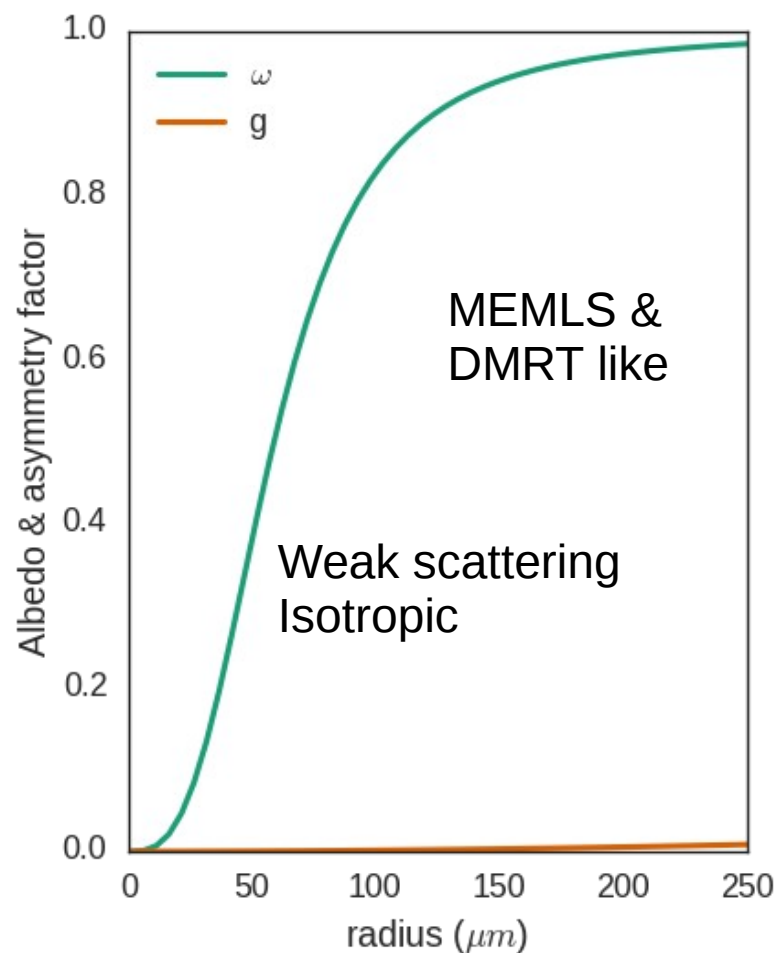
Joseph, Wiscombe, Weiman. The Delta-Eddington Approximation for Radiative Flux Transfer. Journal of the Atmospheric Sciences, 1976, 33, 2452-2459.

# Similarity theory

Visible in the two-flux theory: single scattering albedo  $\omega$  ( $\sim K_s$ ) and asymmetry factor  $g$  ( $\sim P$ ):

$$\omega, g \leftrightarrow \omega', g' \quad \omega' = \frac{(1-f)\omega}{1-f\omega} \quad g' = \frac{g-f}{1-f} \quad \text{for any } f$$

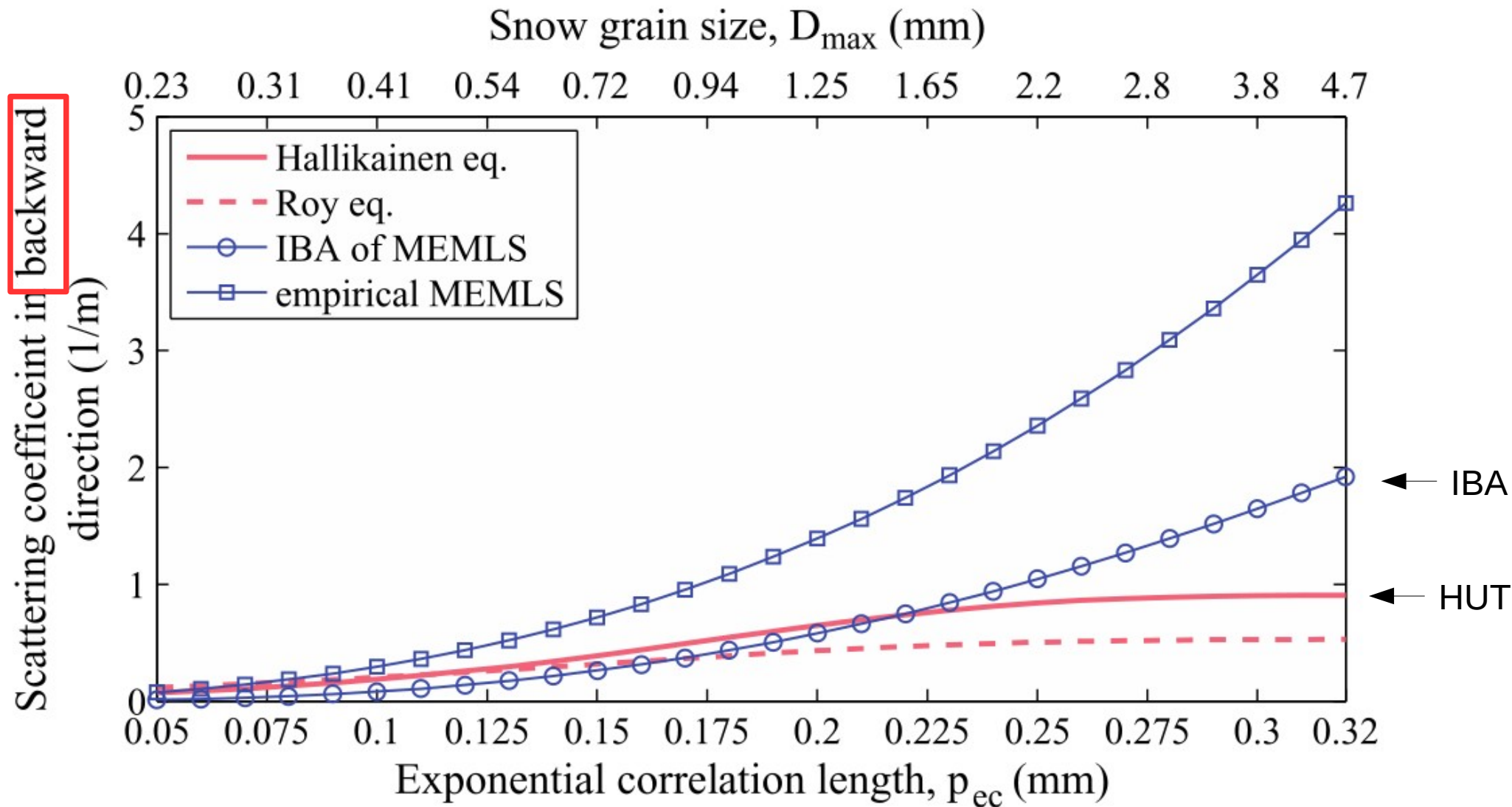
M-delta approximation: choose  $f$  to reduce the forward peak ( $0 < f < g$ )



Mie calculation

# Similarity theory

Pan, Durand and co-authors, 2016



Broad agreement once the comparison takes into account the different phase function shape → **HUT has similar behavior as MEMLS despite huge apparent differences**

# Conclusion

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Back to the introductive questions, my opiniated response:

**– Is this diversity apparent or profound ?**

- overall apparent
- all models converge to the “right” snow behaviour and give reasonable results (not always for physically correct reasons)

**– Why such a diversity ?**

- historical
- different focus/approach

**– Is this diversity beneficial or counter-productive for the community ?**

- it has been beneficial until many users started to be spend more time performing numerical inter-comparisons (incl. myself) than really using models to develop useful algorithms for end-users.

**– What about the dual mode ? Good or bad ?**

- it's time to merge both because of dual mode missions and in-situ datasets

**– Why a new model ?**

# Conclusion

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## – Why a new model ?

We don't need a new model (yet) but we need:

a repository of microwave community knowledge

= merge all RT models / theories in one code base, one framework

with extended capabilities to explore the micro-structure

with multi mode capabilities (passive, radar, altimeter)

with easier access for beginners and non-specialists

using modern and more efficient languages and programming techniques



# Conclusion

