Chapter 3

Modeling Choices:

From Simple Graphs to Advanced Models

RECAP:
Network Science and Hypotheses formulation

- Research Objectives
- Data Availability
- Testability
- Complexity and Scope
- Network Characteristics
- Ethical Considerations

Goal

- Matching real-world problems with appropriate models;
- Research Objectives are shaped by the choice of the **network model**

Only nodes and edges?

- **Directed Networks:**
  Edges have direction
- **Weighted Networks:**
  Edges have values assessing the interaction strength
- **Signed Networks:**
  Edge are either positive (+) or negative (-)
- **Multilayer/Multiplex Networks:**
  Networks have multiple layers of interactions
- **Temporal Networks:**
  Networks have a dynamic nature
- **Bipartite Networks**:
  There are two classes of nodes
- **Heterogeneous Networks:**
  There are *n* classes of nodes
- **Higher-order Networks:**
  Interactions are not pairwise

How to choose the right model?

General criteria for model selection:

- Generality vs. specificity trade-off (do I need a weighted-directed-attributed-multiplex network?)
- Nature of data (e.g., static vs. dynamic);
- Focus on the specific research objectives.

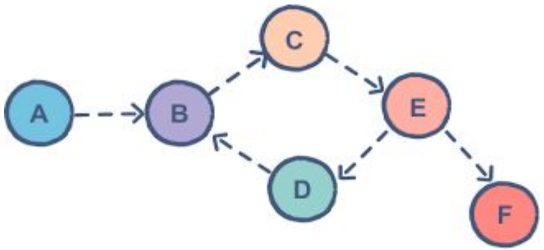
Directed Networks

Overview
Directed Networks

Networks in which edges have a direction, indicating a one-way relationship from one node to another.

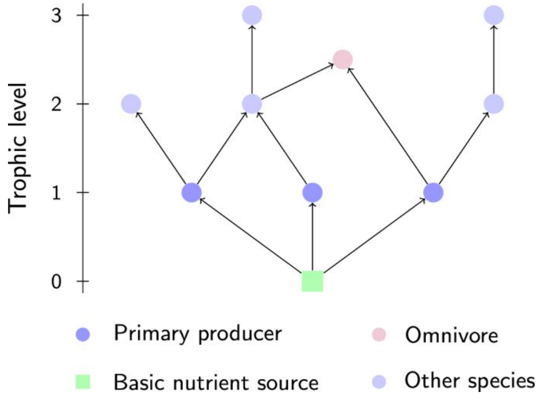
Real Examples

- **Citation Networks**: Scientific papers cite other papers, forming directed links.
- **Social Networks (Twitter)**: Users follow other users, creating directed connections.
- **Gene Regulation Networks**: Genes regulate the expression of other genes through directed interactions.

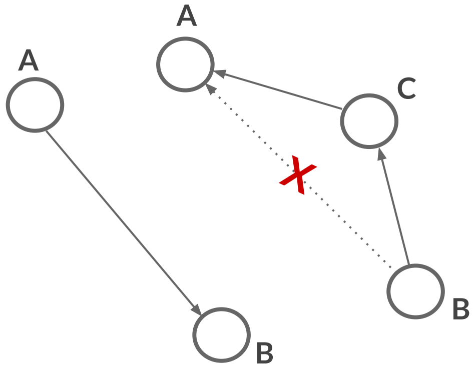


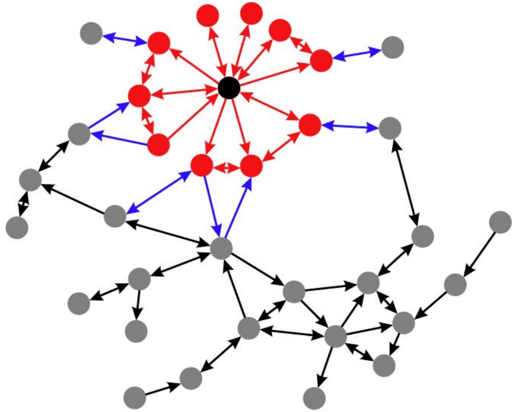
Directions can represent *precedence*, *influence*, *causality*, etc...

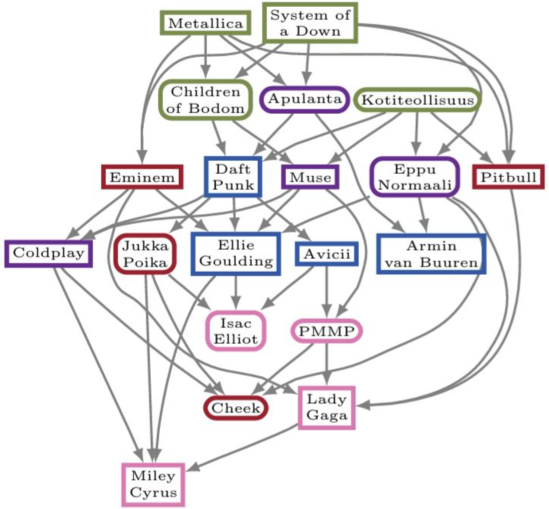
Directions can represent *hierarchies*



Do I need a Directed Network?







E.g., without directions, *distances* flatten

E.g., without directions, *influence*, *hierarchies*, etc., flatten

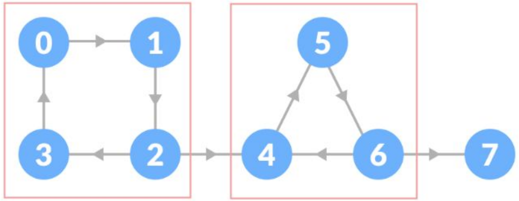
Overview
Directed Networks

Networks in which edges have a direction, indicating a one-way relationship from one node to another.

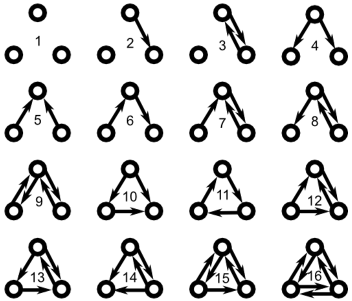
Pros / Applications

- Model the asymmetry in many real-world interactions, such as influence, citation, and control.
- Allow for the analysis of cause-and-effect relationships.
- Can represent hierarchical systems or trophic levels.
- Model flows, such as information dissemination, traffic, and financial transactions.

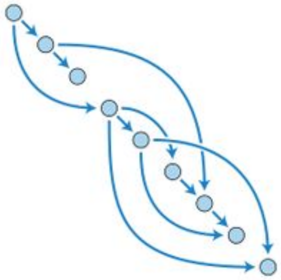
Some useful structures
(specific only for directed graphs)



Strongly **C**onnected **C**omponents (**SCCs**) are subgraphs where every node is reachable from every other node



Motifs (involving three nodes)



Directed **A**cyclic **G**raphs (**DAG**) have **no cycles**

Weighted Networks

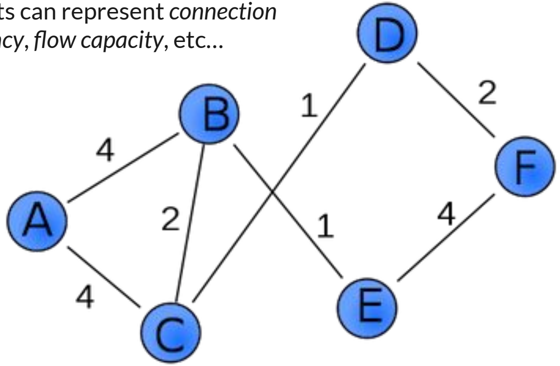
Overview
Weighted Networks

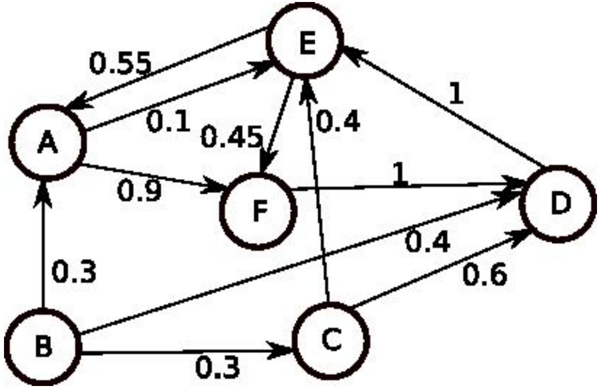
Networks in which edges have associated weights, representing the strength or capacity of the connection

Real Examples

- **Transportation Networks:** Roads where weights indicate distance or capacity.
- **Social Networks:** Relationships with weights reflecting the strength or frequency of interactions.
- **Collaboration Networks:** Co-authorships with weights showing the number of joint publications.

Weights can represent *connection frequency*, *flow capacity*, etc...

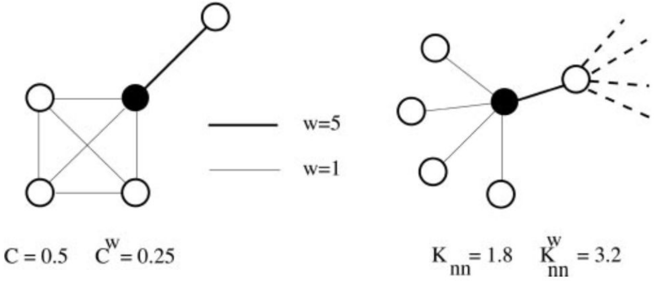




Directions can be represented too

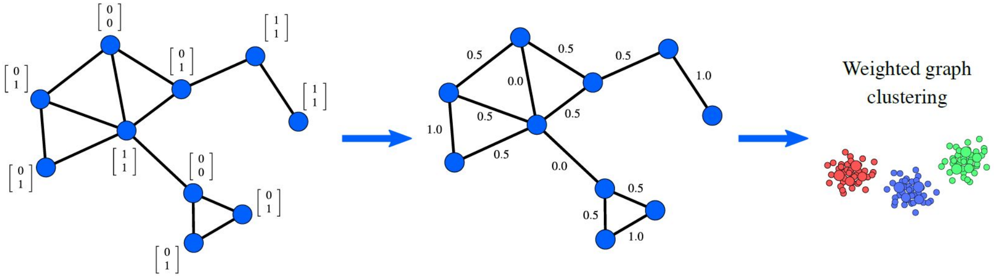
Do I need a Weighted Network?

Image from *Barrat, A., Barthelemy, M., Pastor-Satorras, R., & Vespignani, A. (2004). The architecture of complex weighted networks. Proceedings of the national academy of sciences, 101(11), 3747-3752.*



E.g., without weights, centrality measures provide worse descriptions

Image from *Chunaev, P. (2020). Community detection in node-attributed social networks: a survey. Computer Science Review, 37, 100286.*



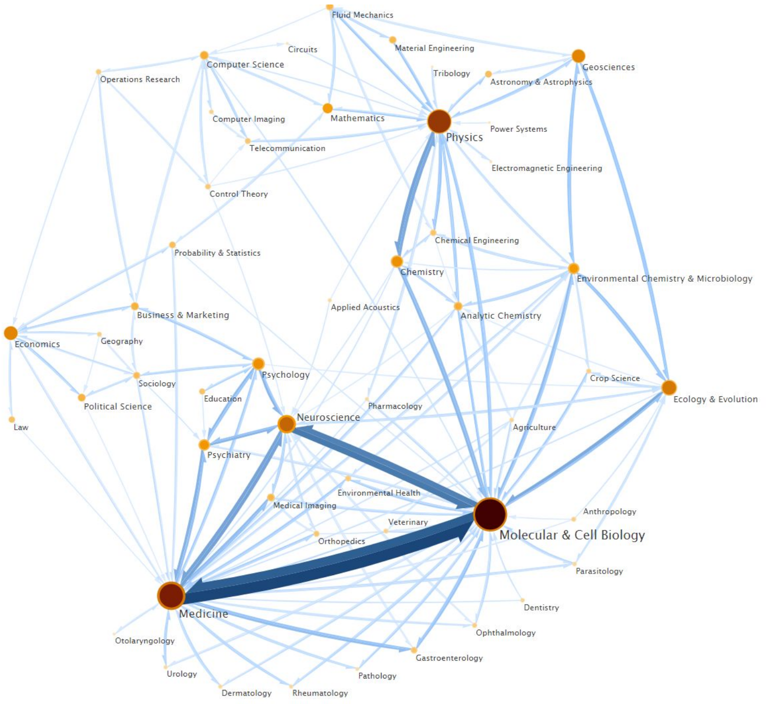
Weighted networks can be used to “simplify” other models (cf. later, attributed networks) and improve tasks such as community detection

Overview
Weighted Networks

Networks in which edges have associated weights, representing the strength or capacity of the connection

Pros / Applications

- Optimize flows in logistics and transportation based on weighted paths.
- Identify stronger connections between nodes enhancing tasks such as community detection or attack/failure
- Many optimization algorithms for weighted networks, e..g, Dijkstra, Bellman-Ford for finding shortest paths



from the *Infomap* Community Detection algorithm

Signed Networks

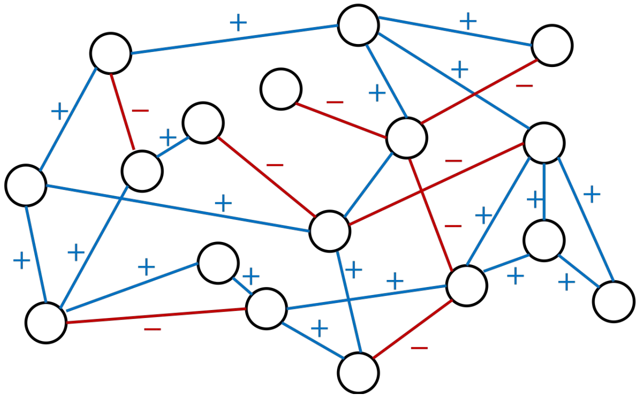
Overview

Signed Networks

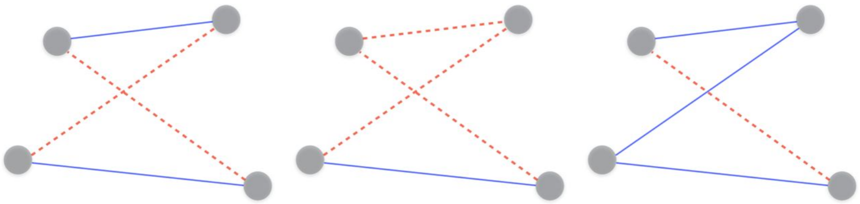
Networks in which edges have positive or negative signs, indicating the nature of the relationship (e.g., friendly vs. antagonistic).

Real Examples

- **Political Networks:** Alliances (positive) vs. opposition (negative) among political entities.
- **Recommendation Systems:** Positive reviews (friendly) vs. negative reviews (critical) between users and products.
- **Market Networks:** Trust (positive) vs. distrust (negative) between companies or stakeholders.
- **Email Networks:** Positive interactions (helpful emails) vs. negative (spam or conflicts).

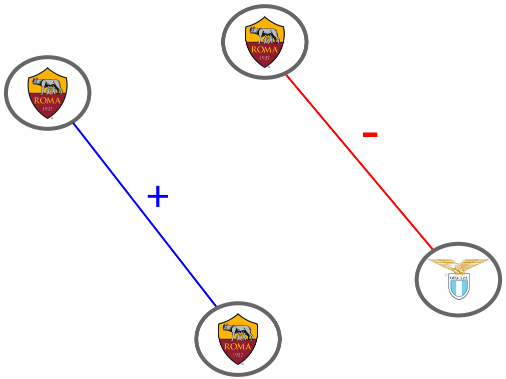


The enemy of my enemy is my friend



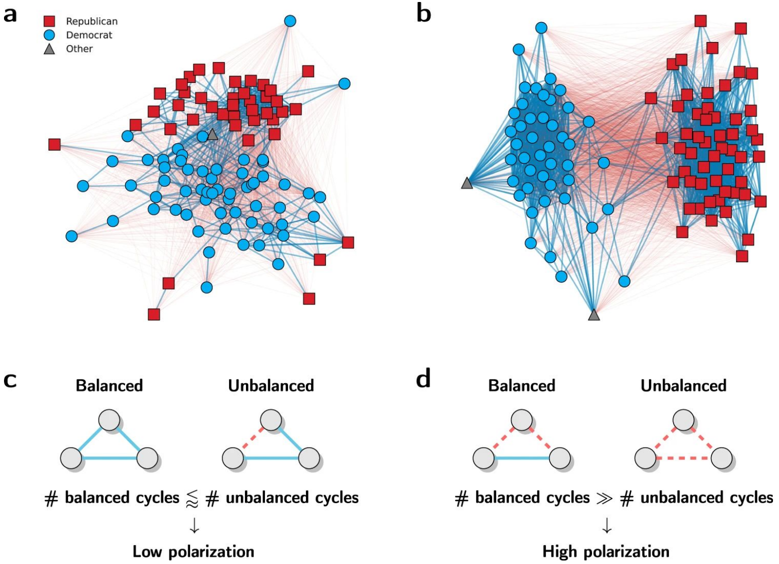
(a) Strong balance. (b) Weak balance. (c) Unbalance.

Do I need a Signed Network?



E.g., without signs, I can not say whether interactions (e.g., on social networks) are positive/negative or supportive/conflictual

Image from *Talaga, S., Stella, M., Swanson, T. J., & Teixeira, A. S. (2023). Polarization and multiscale structural balance in signed networks. Communications Physics, 6(1), 349.*



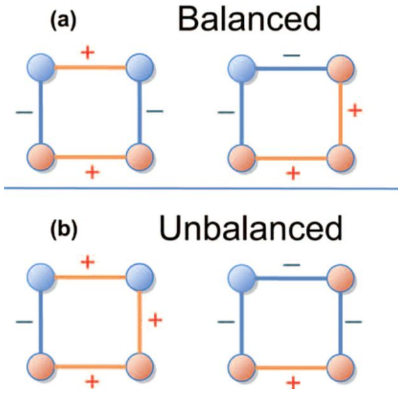
E.g, with signs on links I can provide better descriptions of dynamics on social networks

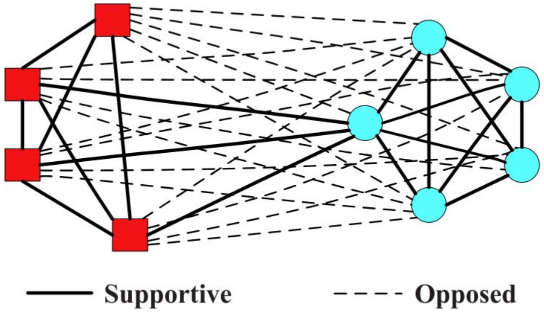
Overview
Signed Networks

Networks in which edges have positive or negative signs, indicating the nature of the relationship (e.g., friendly vs. antagonistic).

Pros / Applications

- Reflect both positive and negative influences in social dynamics.
- Understand the balance of relationships within a network.
- Discover cohesive groups versus conflicting factions.





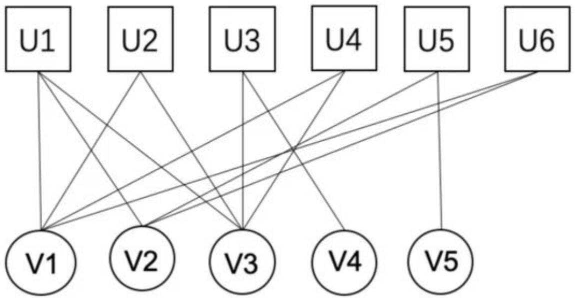
Bipartite / Heterogeneous Networks

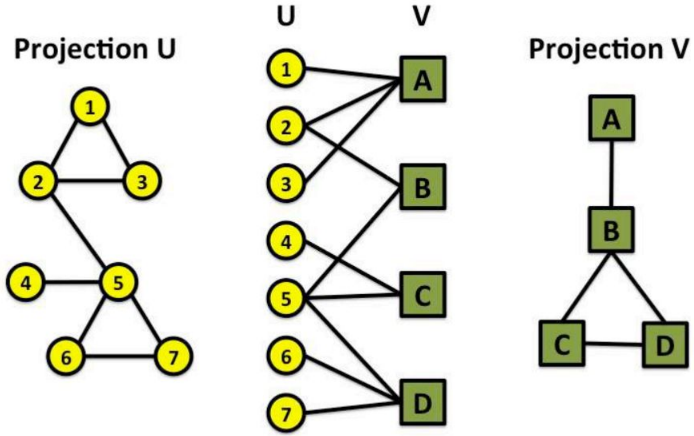
Overview
Bipartite Networks

Networks consisting of two distinct sets of nodes, with edges only connecting nodes from different sets.

Real Examples

- **Recommendation Systems:** Users and products, where edges indicate user preferences.
- **Co-authorship Networks:** Authors and papers, connecting authors to their published works.
- **Species-Interaction Networks:** Species and habitats, showing which species inhabit which environments.





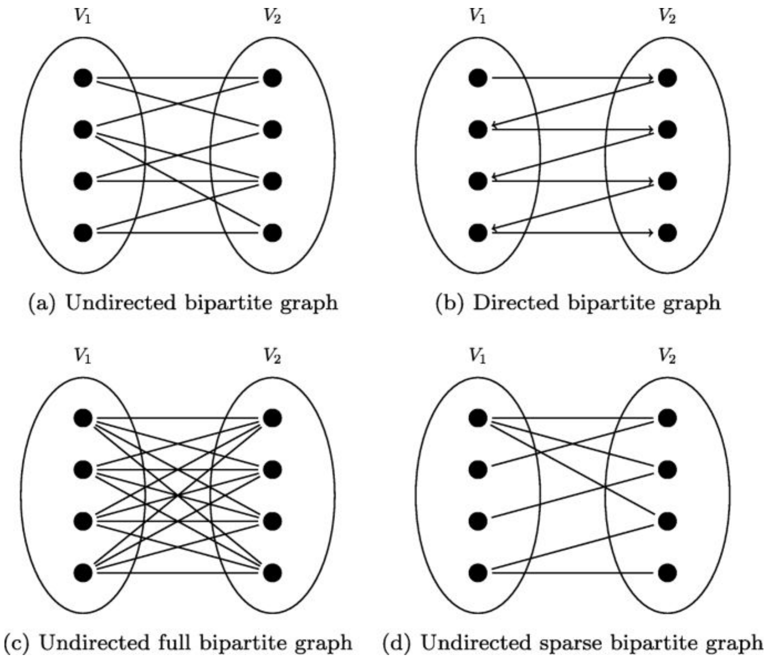
Overview

Bipartite Networks

Networks consisting of two distinct sets of nodes, with edges only connecting nodes from different sets.

Pros / Applications

- **R**epresenting complex structures such as hypergraphs.
- **A**nalyzing "diseasomes", i.e., disease networks of genes (proj u) and diseases (proj v).
- **C**o-Clustering: simultaneously grouping the two distinct sets of nodes based on their interactions.

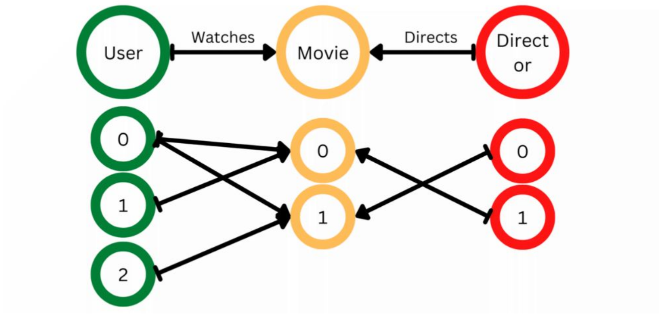


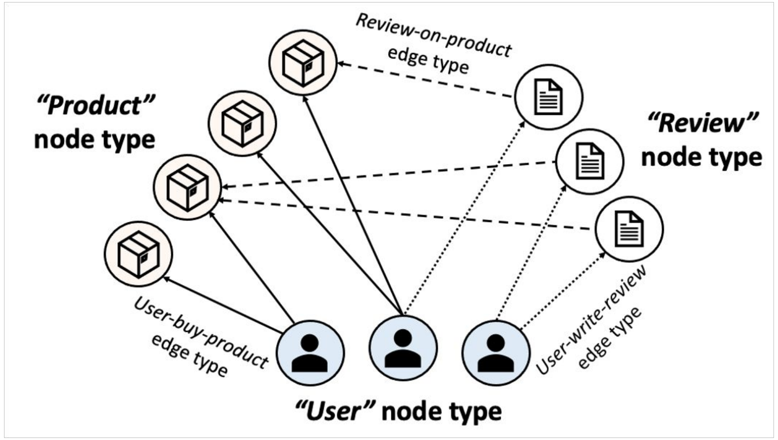
Overview
Heterogeneous Networks

Networks that contain multiple types of nodes and edges, capturing a variety of relationships and interactions.

Real Examples

- **Knowledge Graphs:** Entities and relationships of various kinds, such as people, places, and events.
- **Biological/Ecological Networks:** Entities and relationships of various kinds, such as pollinators, plants, herbivores.
- **Citation Networks**: Authors, papers, journals, and institutions, connected through citations, affiliations, and collaborations.



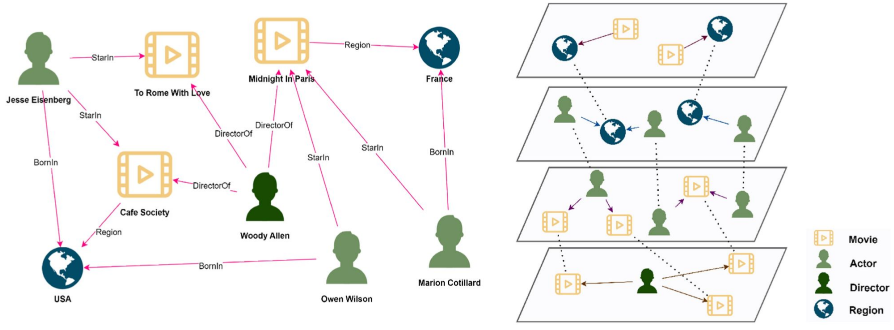


Overview
Heterogeneous Networks

Networks that contain multiple types of nodes and edges, capturing a variety of relationships and interactions.

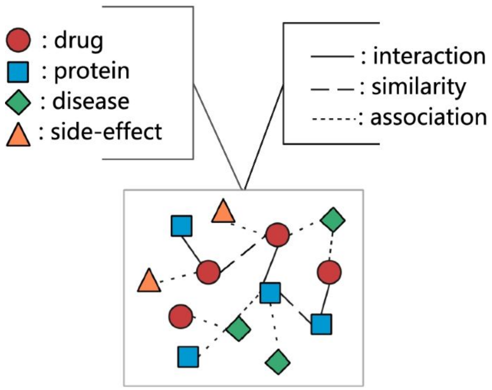
Pros / Applications

- Tailor recommendations based on varied node types.
- Entity/Matching resolution, e.g., matching users across multiple social media platforms.
- Cross-domain recommendations and multi-relational data/network mining.



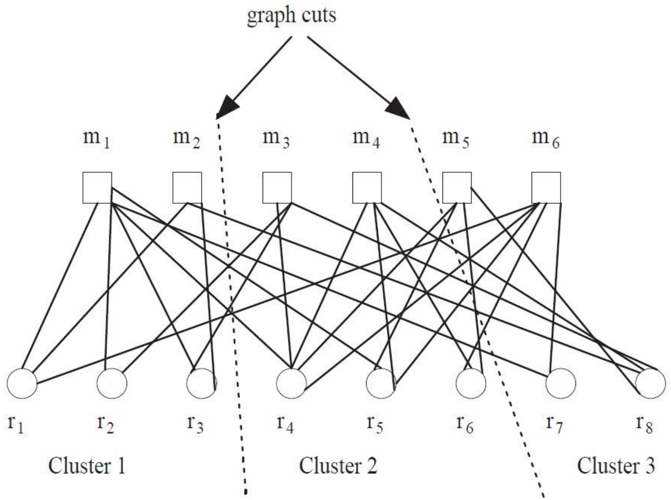
(a) Knowledge graph.

(b) Knowledge heterogeneous graph.

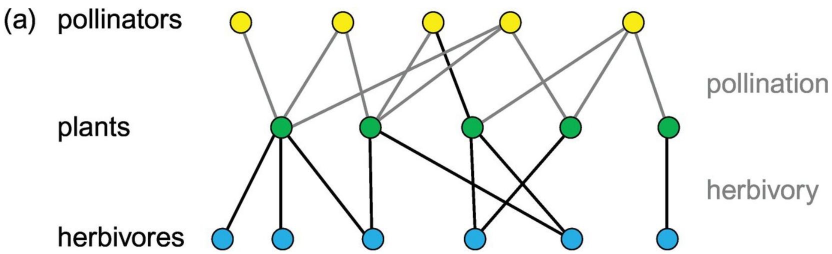


heterogeneous graph

Do I need a Bipartite/Heterogeneous Network?



E.g., I want to find subgraph patterns
according to the relationships between two classes of nodes



E.g., improve the representation of complex systems
like biological networks

Multiplex / Multilayer Networks

Overview

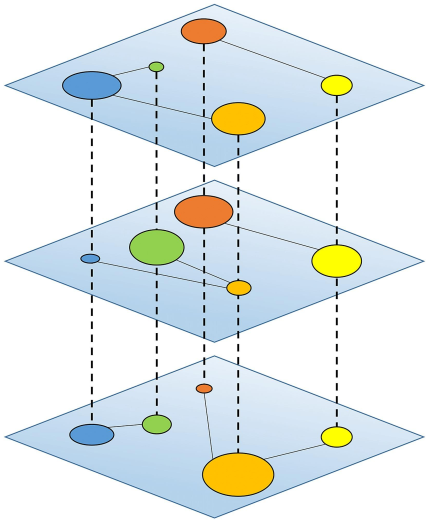
Multiplex Networks

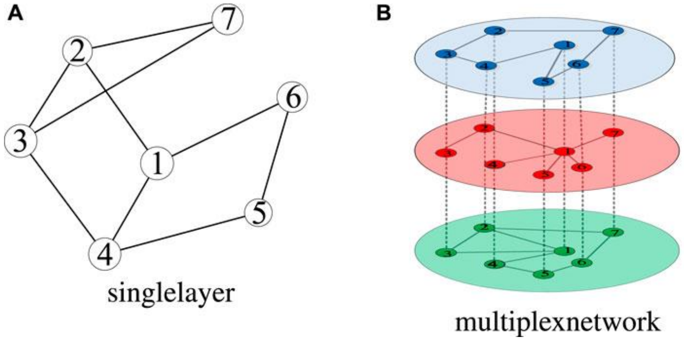
Networks where multiple types of edges exist between the same set of nodes. Each edge type represents a different kind of relationship or interaction.

Real Examples

- **Social Networks**: Incorporating different interaction types (e.g., friendship, collaboration) between the same set of individuals
- **Transportation Networks**: Modes of transport (e.g., road, rail, air) connecting the same locations.
- **Information Networks**: Different channels (e.g., email, phone calls, social media) connecting individuals or entities.
- **Cognitive/Linguistic Networks:** Linguistic similarities between the same set of words (e.g., phonological distances, semantic associations)

Nodes are *replicas* across all layers with different structural characteristics (different links) and also node properties (node size)



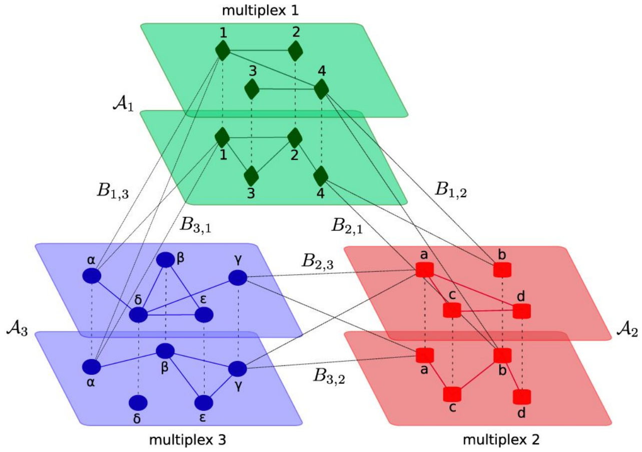


Overview
Multilayer Networks

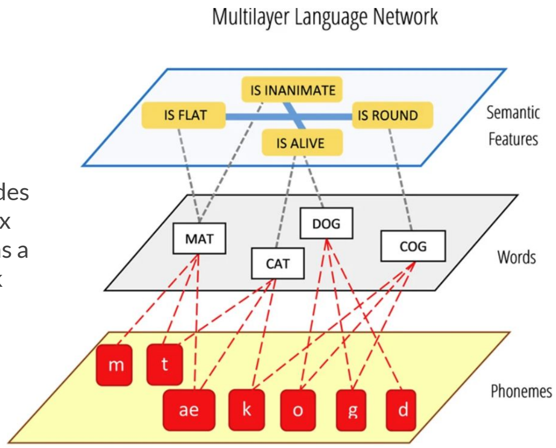
Networks where nodes can have different sets of neighbors across different layers. It can also refer to networks where nodes interact through multiple types of edges, each represented as a separate layer.

Real Examples

- **Financial Networks**: Interconnected stock markets, banks, insurance markets, etc.
- **Cognitive/Linguistic Networks:** Linguistic similarities between different layers of linguistic units (e.g., syllables, morphemes, words, sentences...)

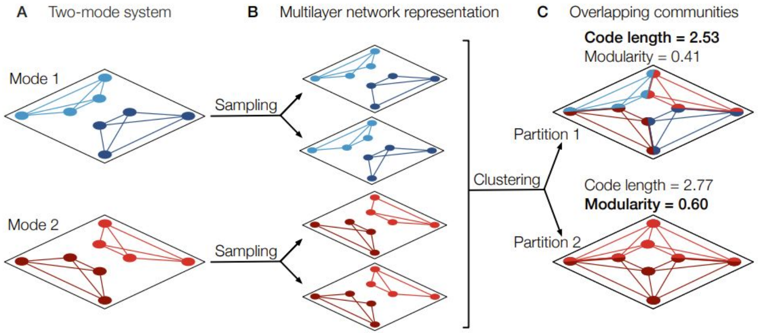


Different types of nodes constitute a complex system represented as a multilayer network

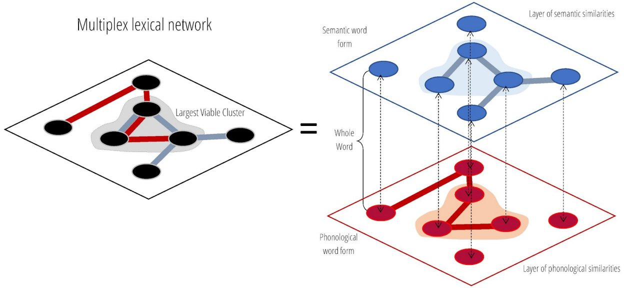


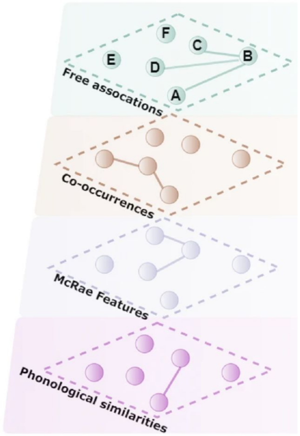
Do I need a Multiplex/Multilayer Network?

Image from *De Domenico, M., Lancichinetti, A., Arenas, A., & Rosvall, M. (2015). Identifying modular flows on multilayer networks reveals highly overlapping organization in interconnected systems. Physical Review X, 5(1), 011027.*



E.g., improve the community detection task





E.g., modeling human mental lexicon, semantic memory, cognitive structures

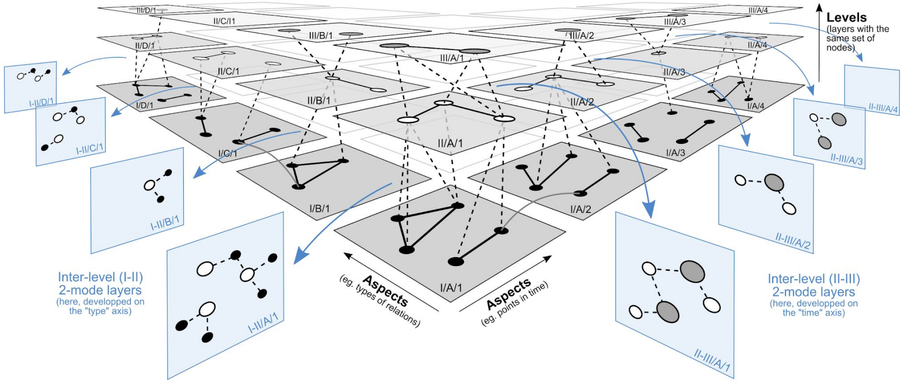
Overview

Multiplex/Multilayer Networks

(Multiplex) networks where multiple types of edges exist between the same set of nodes. (Multilayer) networks where nodes can have different sets of neighbors across different layer.

Pros / Applications

- **Cross-Layer Influence:** Enables the study of how interactions in one layer affect dynamics and behaviors in other layers.
- **Community Detection**: Enhances community detection algorithms by considering multiple types of interactions that define community structures.
- **Robustness and Resilience**: Analyzes network robustness against failures or disruptions across multiple interaction types.



Temporal Networks

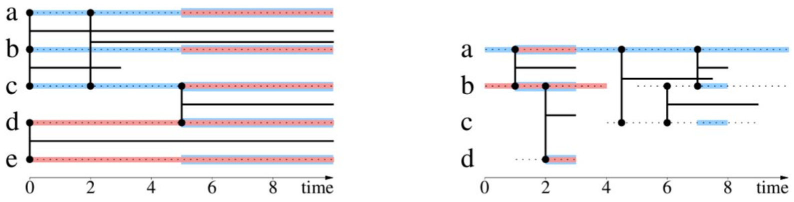
Overview
Temporal Networks

Temporal networks are networks where nodes and edges change over time.

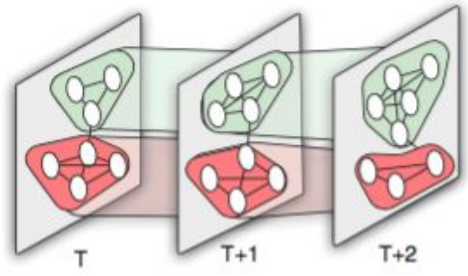
Real Examples

- **Face-to-face interactions:** Each connection represents a conversation/contact at a specific time.
- **Online Social Networks:** Mentions have a timestamp, they can be aggregated every day/month/year.

NB: Relations vs interactions, stability vs. instability, instantaneous vs. duration, etc



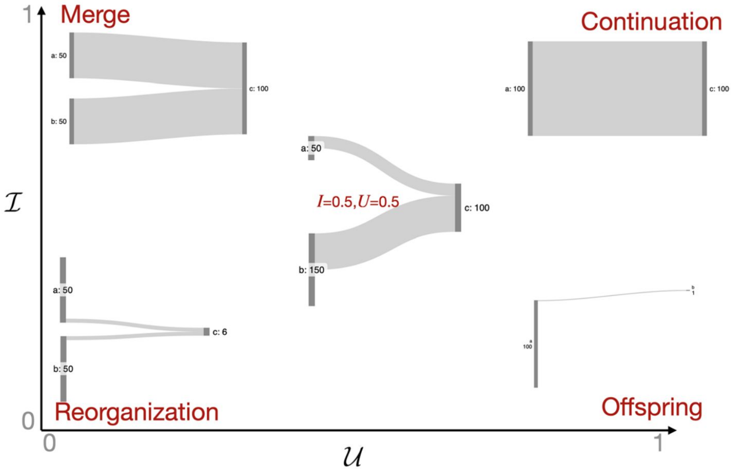
A temporal network as a **stream graph**



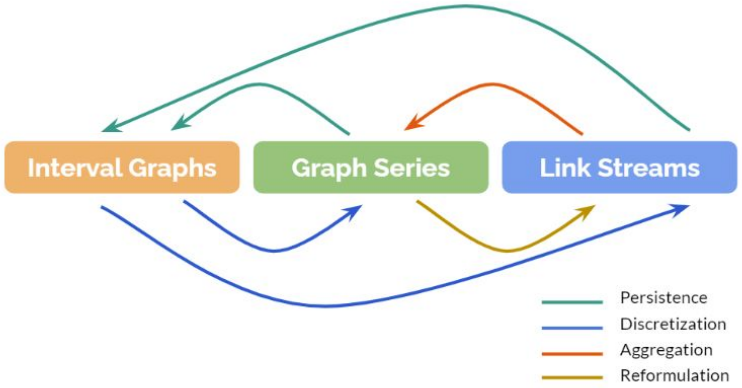
A temporal network as a **sequence of snapshots** (with communities)

Do I need a Temporal Network?

Image from *Failla, A., Cazabet, R., Rossetti, G., & Citraro, S. (2024). Describing group evolution in temporal data using multi-faceted events. Machine Learning, 1-25.*



E.g., the evolution of communities over time allows to define **meso-scale events**



It depends on how I want to **represent** time…

Overview

Temporal Networks

Temporal networks are networks where nodes and edges change over time.

Pros / Applications

- **Dynamics of Networks:** Link formation, prediction, spatio-temporal dynamics, etc.
- **Activity Driven Models:** Agent-based models of temporal interactions, from SIR/SIS to information cascades;
- **Temporal Community Detection** for detecting meso-scale events;

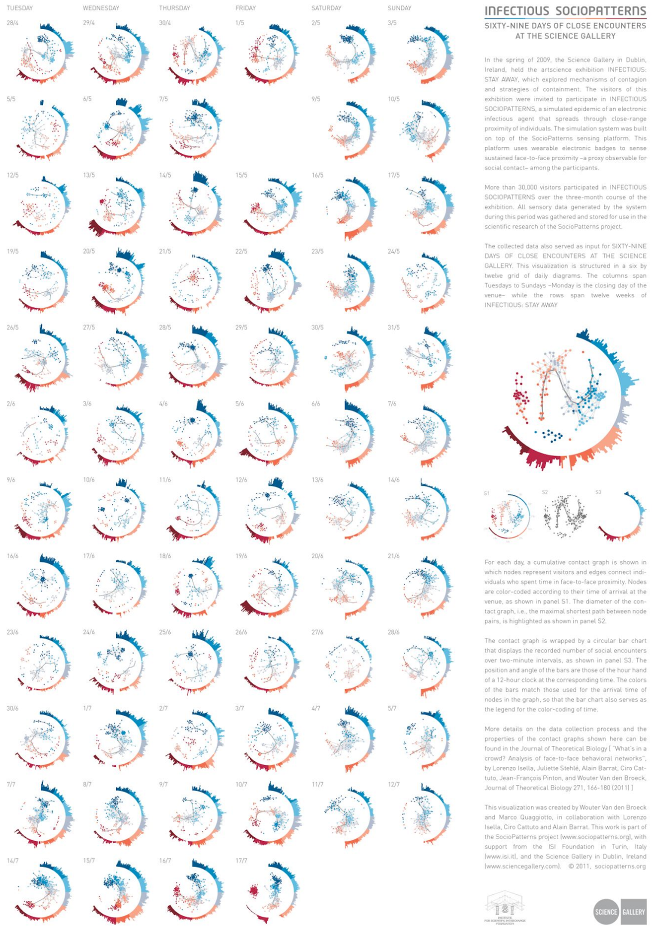


Image from the **SocioPatterns** Project

Higher-order Networks

Overview

Higher-Order Networks

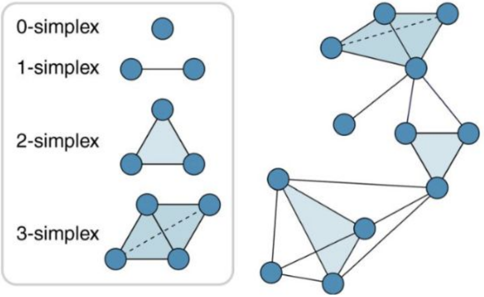
Higher-order networks extend traditional graphs by capturing multi-node interactions.

Real Examples

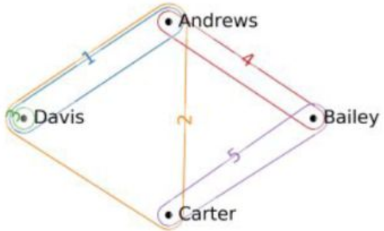
- **Hypergraphs**: In online debates, connections (hyperlinks) are group discussions;
- **Simplicial Complexes**: In scientific collaborations, simplicial complexes represent groups of co-authors, with shared subsets indicating overlapping collaborations within the larger teams.

In general, co-authoring, co-locations, joint discussions, co-firing, etc...

Simplicial complexes, multi-node interactions with overlapping group subsets



Hypergraphs, generalized graphs connecting any number of nodes directly



Do I need a Higher-Order Networks?

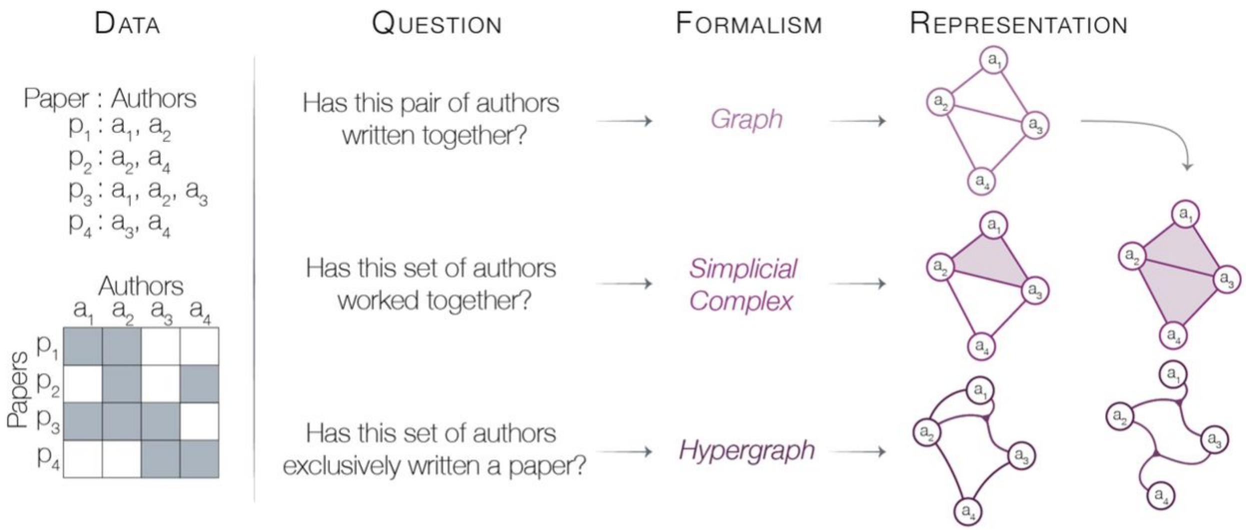


Image from *Torres, L., Blevins, A. S., Bassett, D., & Eliassi-Rad, T. (2021). The why, how, and when of representations for complex systems. SIAM Review, 63(3), 435-485.*

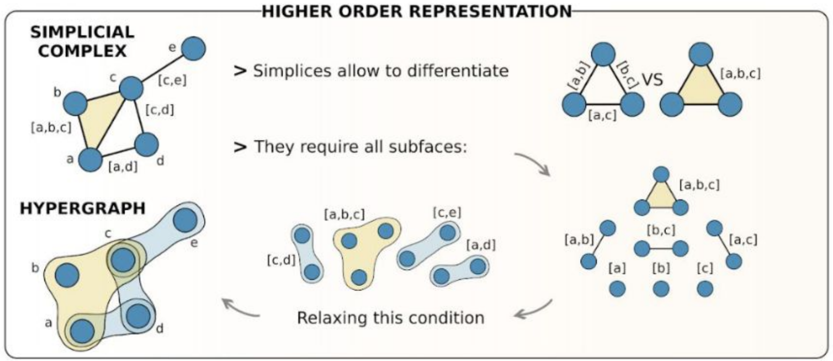
Overview
Higher-Order Networks

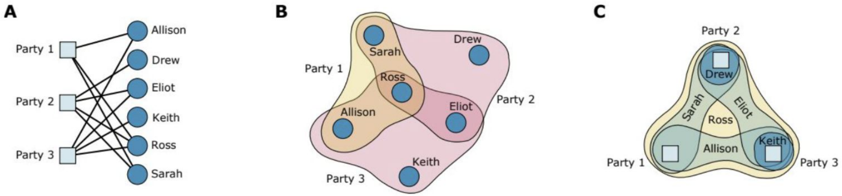
Higher-order networks extend traditional graphs by capturing multi-node interactions.

Pros / Applications

- **Transformations** for hypergraph modeling, e.g., transforming hypergraphs into bipartite graphs, line graphs (**s-analysis framework**), etc…
- **Co-Firing Patterns**: Modeling simultaneous activity among multiple neurons to capture complex neural interactions.
- **Linguistic Networks**: Capturing complex relationships among words or phrases in natural language processing (e.g., **free association** games).

Image from *Battiston, F., Cencetti, G., Iacopini, I., Latora, V., Lucas, M., Patania, A., ... & Petri, G. (2020). Networks beyond pairwise interactions: Structure and dynamics. Physics reports, 874, 1-92.*





Relevant transformations for hypergraphs

Chapter 3

Conclusion

Take Away Messages

1. Different research questions, different network modeling;
2. Enriched representations reflect detailed relationships;
3. Generality vs. specificity trade-off.

What's Next

Chapter 4: Network Sampling