Games, Choice and Optimisation Assignment 2

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January 18, 2018

1. (a) The dual of $\mathcal{L}$ is

$$\begin{array}{rrcrcrl} \text{maximise} & 80y_1 & + & 165y_2 & + & 105y_3, & \\ \text{subject to} & 2y_1 & + & 3y_2 & - & y_3 & \leq 6, \\ & 4y_1 & + & y_2 & - & 2y_3 & \leq 2, \\ & -3y_1 & - & 2y_2 & + & 2y_3 & \leq -2, \end{array}$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0.$$

(b) (i) $x_1 = 135$, $x_2 = 68$, $x_3 = 154$ is an optimal solution of $\mathcal{L}$ with value 638.

(ii) The specified change to the objective function of $\mathcal{L}$ would yield the following tableau

x_1	x_2	x_3	x_4	x_5	x_6	
1	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	135
0	0	1	$\frac{1}{5}$	$\frac{1}{5}$	1	154
0	1	0	$\frac{2}{5}$	$-\frac{1}{10}$	$\frac{1}{2}$	68
0	-4	0	$\frac{2}{5}$	$\frac{12}{5}$	2	638

All that remains is to send $r_4 \to r_4 + 4r_3$ to return x_2 to basis,

x_1	x_2	x_3	x_4	x_5	x_6	
1	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	135
0	0	1	$\frac{1}{5}$	$\frac{1}{5}$	1	154
0	1	0	$\frac{2}{5}$	$-\frac{1}{10}$	$\frac{1}{2}$	68
0	0	0	2	2	4	910

this doesn't change the fact that this tableau shows an optimal solution, but changes the value of the linear programme to 910.

(iii) We first note that this linear programme is $\mathcal{L}$ with the addition of the variable x_0 as represented by the column $\begin{array}{r} x_0 \\ \hline 3 \\ -3 \\ 1 \\ \hline -1 \end{array}$ in the initial tableau. In the final tableau for $\mathcal{L}$, $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and from the initial tableau $c = \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix}$, so $c^* = \begin{pmatrix} 6 \\ -2 \\ 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{5} & \frac{1}{5} & 1 \\ \frac{2}{5} & -\frac{1}{10} & \frac{1}{2} \end{pmatrix}$.

We can now compute the new column corresponding to x_0 in the final tableau.

$$\frac{B\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}}{(c^*)^T B \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} - 2} = \frac{\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 6 & -2 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} - 2}$$
$$= \frac{\begin{matrix} -1 \\ 1 \\ 2 \end{matrix}}{24}.$$

We can see the tableau still presents an optimal solution.

x_0	x_1	x_2	x_3	x_4	x_5	x_6	
-1	1	0	0	0	0.5	0.5	135
1	0	0	1	0.2	0.2	1	154
2	0	1	0	0.4	-0.1	0.5	68
24	0	0	0	0.4	2.4	2	638

This final tableau shows an optimal solution with $x_1 = 135$, $x_2 = 68$, $x_3 = 154$ and a value of 638.

2. (a) (i) (I) Rose's choice BC from M does not satisfy the contraction condition because his choice from AC does not contain C.

(II) Rose's choice BC from M does not satisfy the expansion condition because he chooses D from all size 2 submenus containing D but his choice from M does not contain D.

(ii) By the contraction condition, Colin did not choose A because her choice BD from ABD does not contain A, she did not choose C because her choice B from BC doesn't contain C, similarly for D because her choice AC from ACD doesn't contain D. For her choice to the satisfy expansion condition, the only element Colin must have chosen from M is B. Hence the only reasonable element to be in X is B.

(b) (i) N is not a preference ordering because $A_1 \geq A_2$, $A_2 \geq A_3$, but $A_1 \not\geq A_3$.

(ii) (I) The restriction of $\geq$ to N_1 is
$A_1 \geq A_1, A_7$
$A_3 \geq A_1, A_3$
$A_7 \geq A_3, A_7$.
The restriction of $\geq$ to N_2 is
$A_3 \geq A_3, A_4$
$A_4 \geq A_4, A_5$
$A_5 \geq A_3, A_5$
$A_7 \geq A_3, A_4, A_5, A_7$.

(II) Mary chooses $A_{\geq} = \{A_2, A_6\}$ from M.
(A) No, since $N_{1\geq} = \{\}$.
(B) Yes, since $N_{2\geq} = \{A_7\}$. Mary chooses A_7.

(iii) Because of the fact that Najma is indifferent between X and $\frac{17}{6}W$, $\frac{17}{6}W$ we know $U(X) = \frac{7}{16} \cdot U(W) + \frac{9}{16} \cdot U(W)$ and $U(W) = 20$, $U(Z) = 4$ we can write $U(X) = \frac{7}{16} \cdot 20 + \frac{9}{16} \cdot 4 = 11$. Similarly, since Najma is indifferent between Y and $\frac{5}{8}X$, $\frac{3}{8}Z$ we know $U(Y) = \frac{5}{8} \cdot U(X) + \frac{3}{8} \cdot U(Z) = \frac{67}{8}$, to determine whether or not Najma prefers $\frac{1}{5}W, \frac{4}{5}Y$ over $\frac{4}{5}X, \frac{1}{5}Z$ we evaluate the truth of the inequality

$$\frac{1}{5}W, \frac{4}{5}Y \geq \frac{4}{5}X, \frac{1}{5}Z$$
$$\frac{1}{5} \cdot U(W) + \frac{4}{5} \cdot U(Y) \geq \frac{4}{5} \cdot U(X) + \frac{1}{5} \cdot U(Z)$$
$$\frac{1}{5} \cdot 26 + \frac{4}{5} \cdot \frac{67}{8} \geq \frac{4}{5} \cdot 11 + \frac{1}{5} \cdot 4$$
$$\frac{107}{10} \geq \frac{96}{10},$$

which is true, so Najma does prefer $\frac{1}{5}W, \frac{4}{5}Y$ over $\frac{4}{5}X, \frac{1}{5}Z$.