Discrete Mathematics

Lecturer: Maura Paterson

Module Code: BUEM002S5 Level: 5 Credit Value: 30 Credits

http://www.ems.bbk.ac.uk/for_students/bsc_maths/coursematerials/discretemaths

Aims

This aim of this module is to provide exposure to some fundamental counting principles and techniques, and to introduce some important branches of discrete mathematics in which such techniques are widely used, namely graph theory and the theory of difference equations. These have widespread applications in fields such as economics and computer science, as well as being studied in their own right. Elementary counting techniques, such as the pigeonhole principle and the inclusion-exclusion theorem will be introduced, as will techniques based on the used of generating functions. You will learn how to solve several different types of difference equations, and you will apply these methods to explore various examples of how difference equations can be used to model real-world problems. You will be introduced to the basic concepts and definitions of graph theory, as well as problems involving finding cycles and paths in graphs, and problems relating to matching pairs of vertices in a graph.

Resources for the Module

Printed lecture notes will be handed out in lectures, and will be available on the web. They will contain all the material needed for the course; however, it is likely to be presented in more detail during the lectures.

How to contact the lecturer

I can be reached by email at m.paterson@bbk.ac.uk; this is probably the most effective way of reaching me. My office phone is 020 7631 6440. I am also happy to meet you in person during term time to discuss any questions you may have. There are two ways of arranging this:

- **Office hours:** Every Tuesday from 4:00 to 5:30 during term time I will be in my office, and you are free to drop in to see me during that time. Throughout most of the year these sessions tend not to be busy.
- **By appointment:** If you cannot make my office hours, I am happy to arrange an appointment to see you outside those times. Send me an email with a selection of times that would work for you and I will reply to fix a time.

- Please be aware that I am unlikely to be able to help you if you simply turn up at my office without an appointment.

Teaching and Assessment

This course will be taught through eight lectures in each of the Autumn and Spring terms, with two additional revision lectures in the Summer term. The final three-hour exam contributes 80% of the marks for the module, with the remaining 20% coming from coursework. The coursework for this module consists of four problem sets, each worth 5% of the final mark, which will be assigned after each four-week teaching block. You will have at least three weeks to complete each assignement.

Exercises in the Notes

The lecture notes include a range of practice exercises. Numerical solutions to these are posted on Moodle. I do not hand out worked solutions, but I am very happy to give feedback on your attempts at solving them. The best way to obtain this feedback is to post your attempts at problems or any questions you might have on the Moodle forum for the relevant chapter; I am usually able to respond very promptly to such posts. Alternatively I am happy to discuss these exercises during office hours.

Syllabus

1. **Sequences and Counting:** Arithmetic and geometric progressions, the sequences $u_n = Ru_{n-1} + d$ and $u_n = n^k$ $(k = 1, 2, 3, 4)$, the rule of sum, the rule of product, r-sequences, r-permutations, r-combinations, r-multisets, distribution problems, binomial coefficients, the pigeonhole principle, the inclusion-exclusion theorem.

2. **Generating Functions:** Power series, finding the generating function of a problem, finding the coefficients of a generating function, application of generating functions to sequences.

3. **Difference Equations:** Linear difference equations, homogeneous difference equations, first order difference equations, second order linear difference equations with constant coefficients, modelling simple problems in finance and economics using difference equations, population models, cobweb diagrams, the discrete logistic equation, Catalan numbers, using generating functions to solve difference equations.

4. **Basic Graph Theory:** Vertices and edges, simple graphs, the degree of a vertex, the degree sequence of a graph, the handshaking lemma, subgraphs, isomorphic graphs, null and complete graphs, cycles and paths, platonic graphs, connectedness, planar graphs, trees, weighted graphs, digraphs, networks.

5. **Trees:** Trees and forests, characterisations of a tree, spanning trees, minimum connector, Kruskal's and Prim's algorithms, labelled trees, Cayley's Theorem for labelled trees.

6. **Traversability:** Cycles, Eulerian graphs, Fleury's algorithm, semi-Eulerian graphs, Hamiltonian cycles, Ore's Theorem, Dirac's Theorem, the travelling salesman problem

7. **Path Problems:** Paths, connectedness, vertex and edge-disjoint paths between two vertices, separating sets, Menger's Theorem, networks, shortest path problem, longest path problem, scheduling problems, flows on a network, cuts in a network, the maximum flow-minimum cut theorem.

8. **Matching Problems:** Pairings, matchings in graphs, bipartite graphs, perfect, complete, maximal and maximum matchings in a graph, Hall's Theorem, stable matchings, optimal assignments, the Hungarian algorithm, the bottleneck problem.

Learning Outcomes

The overall learning outcome for this course can be found on the course website. At the end of each chapter of the lecture notes you will find a list of learning outcomes specific to that chapter.

Recommended Books

Everything you need to know for this course will be presented through the lectures and assignments. However, if you are seeking a different perspective on some of the material or some additional problems to practice, you may wish to consult some of the following books, which are all available in the library.

Unfortunately there is no single textbook that covers all the material in the syllabus, though the books recommended below are each useful for various parts of it. I am frequently asked which of these is the 'best' book for the course. This is largely a matter of personal taste, depending on which material you want to see covered, and how you respond to the authors' particular style. I always recommend visiting the library to browse these books (and the various other discrete mathematics books on the shelf) to get a feel for which ones seem useful to you.

- R.P. Grimaldi, Discrete and Combinatorial Mathematics: An Applied Introduction, Addison Wesley.
- B.W. Jackson, D. Thoro, Applied Combinatorics with Problem Solving, Addison Wesley.
- A. Tucker, Applied Combinatorics, Wiley.
- M Townsend, Discrete Mathematics: Applied Combinatorics and Graph Theory, Benjamin-Cummings.
- R.J. Wilson, J.J. Watkins, Graphs: An Introductory Approach, Wiley.
- N.L. Biggs, Discrete Mathematics, Oxford.
- V.K. Balakrishnan, Introductory Discrete Mathematics, Prentice Hall.