# Discrete Mathematics Assignment 4
## Due date: Friday 28th April 2017

Answer all questions. A total mark out of 20 will be awarded, with individual marks for each question being given in square brackets. This work is worth 5% of the marks for this module. Late submissions will be awarded at most 8/20; work that is more than 14 days late will receive 0.

1. Let G be the following graph:
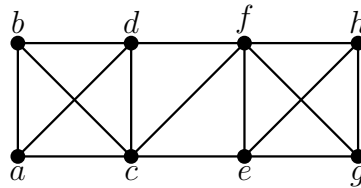
    (a) What is the edge-connectivity of G?   [1]

    (b) What is the connectivity of G?   [1]

    (c) What is the size of the smallest ea-disconnecting set? (Justify your answer.)   [2]

    (d) What is the size of the smallest be-separating set? (Justify your answer.)   [2]

2. Consider the following network:
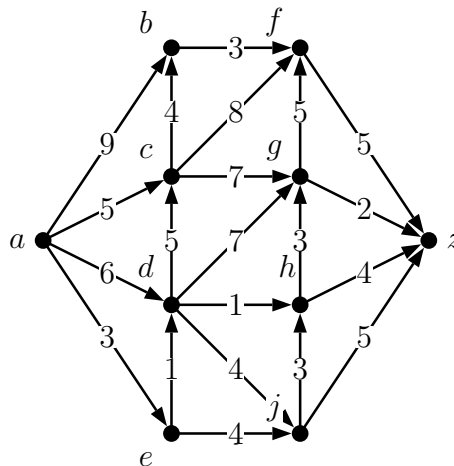
    (a) Use the shortest path algorithm to find the shortest path from vertex a to vertex z, displaying all working.   [2]

    (b) Use the longest path algorithm to find the longest path from vertex a to vertex z, displaying all working.   [2]

3. Five employers E_1, E_2, E_3, E_4 and E_5 are each offering one apprenticeship, and there are five candidates C_1, C_2, C_3, C_4 and C_5 applying for these positions. Suppose the employers rank the candidates in order of preference, and similarly the candidates rank the employers, as shown in the following table:

$$E_1{:}C_5, C_4, C_1, C_3, C_2 \quad C_1{:}E_1, E_2, E_3, E_4, E_5$$
$$E_2{:}C_5, C_4, C_3, C_2, C_1 \quad C_2{:}E_1, E_2, E_3, E_4, E_5$$
$$E_3{:}C_2, C_3, C_4, C_1, C_5 \quad C_3{:}E_1, E_2, E_3, E_4, E_5$$
$$E_4{:}C_3, C_1, C_5, C_4, C_2 \quad C_4{:}E_1, E_2, E_3, E_4, E_5$$
$$E_5{:}C_3, C_1, C_4, C_5, C_2 \quad C_5{:}E_1, E_2, E_3, E_4, E_5$$

Find a stable matching between the employers and candidates that is optimal for the employers. [4]

4. Alice, Barbara, Curt, Danielle, Elmer and Fred work as carwashers. Their suitability (as a score out of 10) for performing the tasks of hosing, scrubbing, window washing, vacuuming, polishing, waxing and are given in the following table.

	hosing	scrubbing	window washing	vacuuming	polishing	waxing
Alice	6	8	2	4	0	4
Barbara	7	9	1	3	5	3
Curt	0	0	6	5	4	2
Danielle	0	2	4	5	6	4
Elmer	7	5	5	6	6	6
Fred	4	4	4	7	5	6

(a) Find a way of assigning the carwashers to the tasks so that the total suitability is maximised. [4]

(b) Find an assignment of carwashers to tasks in which the minimum suitability over all six carwashers is as large as possible. [2]