BSc Games, choice & optimization
MSc Linear programming & game theory

Assignment 1

Due in Monday 27 November 2017

There are four assignments for the BSc module (five for the MSc module) each of which contributes 5% (4% for the MSc module) to the overall mark for the module.

The solutions for each assignment will be posted on the Moodle page for the module two weeks after the due date. Marked assignments will be returned within four weeks of the due date, unless this period lies outside of term time, in which case they will be returned in the first lecture of the next term.

All questions are for assessment. Each question is worth 10 *marks, and a total mark out of* 20 *will be given.*

Question 1

(a) A property developer has 600 units of land on which he plans to build a new housing estate consisting of two storey executive homes, three storey blocks of flats, single storey bungalows and two storey social housing blocks. Each executive home requires 4 units of land, each block of flats requires 3 units of land, each bungalow requires 1 unit of land and each social housing block requires 1.5 units of land. The developer makes a profit of \$70 000 on each executive home, \$30 000 on each block of flats, \$25 000 on each bungalow and \$5 000 on each social housing block.

The work on two individual properties cannot take place at the same time. Each executive home takes 3 weeks to complete, each block of flats takes 2 weeks to complete and each bungalow and social housing unit take 1 week to complete, and the property developer wants to complete the new estate in 140 weeks, or less.

To satisfy local planning laws, the mean (average) number of storeys over all of the properties built cannot exceed 1.8, and the number of social housing blocks built must be at least $\frac{1}{4}$ of the total number of properties built.

Given that the property developer wants to maximize the profit from the sale of the properties built, find a linear programme that can be used to determine the number of each type of property built, explaining clearly what each variable and each constraint represents. (You **do not** have to solve the linear programme.) [4]

(b) Express the following linear programme in standard form.

$$\begin{array}{ll} \text{minimize} & 2x_1 - x_2 - 3x_3, \\ \text{subject to} & \begin{array}{rcl} x_1 + 2x_2 + 3x_3 & \leq & 25, \\ 2x_1 - 3x_2 - x_3 & \geq & 17, \end{array} \\ & x_1 \geq 0,\ x_3 \geq 0. \end{array}$$ [2]

(c) Consider the following linear programme $\mathcal{L}$.

$$\begin{array}{ll} \text{maximize} & z = 3x_1 - 7x_2 + 10x_3, \\ \text{subject to} & \begin{array}{rcr} x_1 - 6x_2 + 4x_3 & \leq & 136, \\ 2x_1 + 3x_2 - 8x_3 & \leq & 44, \\ 4x_1 - 2x_2 + 4x_3 & \leq & 56, \end{array} \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{array}$$

(i) Introduce the appropriate number of slack variables for $\mathcal{L}$. Starting with the slack variables as the initial basic variables of $\mathcal{L}$, determine the pivot variable and which of the current basic variables should be made nonbasic. [1]

(ii) Express each of the new nonbasic variables, and z, in terms of the new basic variables, and write down the basic feasible solution of $\mathcal{L}$ corresponding to the new basic variables. [3]

Question 2

(a) Use the simplex method to solve the following linear programme.

$$\begin{array}{ll} \text{maximize} & 5x_1 + 4x_2 - 7x_3, \\ \text{subject to} & \begin{array}{rcl} 2x_1 + 3x_2 - 3x_3 & \leq & 21, \\ 4x_1 + 9x_2 - 4x_3 & \leq & 72, \end{array} \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{array}$$ [4]

(b) Applying the simplex method to a linear programme $\mathcal{L}$ results in the following final tableau.

x_1	x_2	x_3	x_4	x_5	x_6	
0	-1	4	1	$\frac{5}{2}$	2	97
1	$-\frac{3}{2}$	2	0	1	$-\frac{1}{2}$	36
0	$\frac{5}{2}$	0	0	$\frac{9}{2}$	$\frac{3}{2}$	427

If possible, find the final tableau corresponding to an alternative solution of $\mathcal{L}$, and state this solution of $\mathcal{L}$. [1]

(c) Consider the following linear programme $\mathcal{M}$:

$$\begin{array}{lrcl} \text{maximize} & 12x_1 + 2x_2 - 9x_3 + 5x_4, & & \\ \text{subject to} & 5x_1 - 4x_2 - 6x_3 - 2x_4 & \leq & -68, \\ & 3x_1 + \ \ x_2 - 2x_3 + 4x_4 & \leq & -32, \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0. & & \end{array}$$

(i) Write down the auxiliary linear programme of $\mathcal{M}$. [1]

(ii) By solving the auxiliary linear programme of $\mathcal{M}$, determine whether $\mathcal{M}$ is feasible. [4]