Discrete Mathematics

Assignment 2 Due date: Friday 13 January 2017

Answer all questions. A total mark out of 20 will be awarded, with individual marks for each question being given in square brackets. This work is worth 5% of the marks for this module. Late submissions will be awarded at most 8/20; work that is more than 14 days late will receive 0.

1. Solve the following difference equations:

   (a) $u_n = 16^{n^3} u_{n-1} + 2^{n^2(n+1)^2}, \qquad u_0 = 1.$ [3]

   (b) $a_{n+2} = 4a_n + 10 \cdot 3^n, \qquad a_0 = 9,\ a_1 = 4.$ [3]

   (c) $b_n = 6b_{n-1} - 5b_{n-2} + 120n - 33, \qquad b_0 = 9,\ b_1 = 30.$ [3]

2. Express the solution to the following counting problem in terms of a difference equation, then use a generating function to solve the difference equation to give an explicit solution:

   An intergalactic botanist discovers a new type of plant on the planet Zod. She observes that the plant reproduces in a very particular way: specimens of the plant that are at least two years old drop exactly six branches at the start of each year, which each then immediately become a new plant. If she collects three one-year-old specimens for her greenhouse, how many of the plants will she have after n years? (Assume that the plants reproduce in her greenhouse in the same way they do in the wild, and that none of her specimens die.) [4]

3. Consider the difference equation $u_n = \frac{1}{5}(u_{n-1}^3 + u_{n-1})$.

   (a) Describe how the resulting sequence behaves as $n \to \infty$ in the cases where $u_0 = 0$ or $u_0 = 2$. [1]

   (b) If $u_0 = 3$, how does the resulting sequence behave as $n \to \infty$? Draw a cobweb diagram to illustrate your answer. [3]

   (c) If $u_0 = 0.5$ how does the resulting sequence behave as $n \to \infty$? Draw a cobweb diagram to illustrate your answer. [3]