# Calculus 2, Assignment 4

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1. (a) In this context, $k$ cannot be 0, since $k = 0$ implies there is no relationship between $\frac{\mathrm{d}T}{\mathrm{d}t}$ and $T_s - T$. This gives us $|k| > 0$. However, I don't see a colloquial reason for $k > 0$, since either or both of $T_s$ and $T$ can be negative. I can see it's in some sense meaningless to take a negative factor, since if $a$ is a factor of $b$ then $-a$ is also a factor of $b$.

   ...

   After having found a value for $T$, on the other hand, I can see that a negative value of $k$ would lead to a situation where instead of $T$ approaching $T_s$ as $t$ approaches infinity, $T$ would also go to minus infinity (which breaks the model of reality).

   (b) $\frac{\mathrm{d}T}{\mathrm{d}t} = k(T_s - T)$ is a first order variables separable ordinary differential equation and as such we can write

$$\begin{aligned}
\int \frac{1}{T_s - T}\mathrm{d}T &= k\int \mathrm{d}t \\
k(t + c) &= -\ln(T_s - T) \\
e^{-k(t+c)} &= T_s - T \\
T &= T_s - e^{-k(t+c)}.
\end{aligned}$$

   If $T = T_0$ when $t = 0$ we can write

$$\begin{aligned}
T_0 &= T_s - e^{-kc} \\
e^{-kc} &= T_s - T_0 \\
c &= -\frac{\ln(T_s - T_0)}{k}
\end{aligned}$$

and

$$\begin{aligned} T &= T_s - e^{-k\left(t - \frac{\ln(T_s - T_0)}{k}\right)} \\ &= T_s - e^{\ln(T_s - T_0) - kt} \\ &= T_s - \frac{e^{\ln(T_s - T_0)}}{e^{kt}} \\ &= T_s - \frac{T_s - T_0}{e^{kt}}. \end{aligned}$$

(c) We are given $T(0) = 37$ and $T_s = 24$.

i. Let the amount of time between death and discovery be $A$, now $T(A) = 34$, $T(A + 30) = 32$ and

$$\begin{aligned} e^{kA} &= \frac{13}{10} \\ kA &= \ln\left(\tfrac{13}{10}\right), \\ e^{k(A+30)} &= \frac{13}{8} \\ kA + k30 &= \ln\left(\tfrac{13}{8}\right). \end{aligned}$$

Substituting our value for $kA$ into the second equation

$$\begin{aligned} k &= \frac{\ln\left(\frac{13}{8}\right) - \ln\left(\frac{13}{10}\right)}{30} \\ &= \frac{\ln\left(\frac{5}{4}\right)}{30}. \end{aligned}$$

ii. With a value for $k$, we can write

$$T = T_s - (T_s - T_0) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right)$$

and

$$\begin{aligned} 34 &= 24 - (-13) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right) \\ A &= -\frac{30 \ln\left(\frac{10}{13}\right)}{\ln\left(\frac{5}{4}\right)} \\ &\approx 35.2729347..., \end{aligned}$$

which tells us the time of death was about 35 minutes before high noon.

iii. I was returning some video tapes.

iv. Since the model we have considers surrounding temperature to be constant, I have found in the past that the turning the thermostat up or down before leaving the scene of the crime works nicely. What? Sorry, I have to return some video tapes.

2. (a) Since $f$ is even, $g(f(-x)) = g(f(x))$ and so $g \circ f$ is even. Similarly, $f(g(-x)) = f(-g(x)) = f(g(x))$ so $f \circ g$ is also even.

   (b) $f(-x)g(-x) = f(x)(-g(x)) = -f(x)g(x)$ so $f(x)g(x)$ is odd. Similarly, $\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)}$ and $\frac{f(x)}{g(x)}$ so is also odd.

   (c) Differentiable functions are functions such that for any $u > 0$ there exists $v$ such that $f(x) - f(x+h) < u$ for all $h < v$.