Calculus 2 - Assignment 3

Due 13^{th} March 2017

ANSWER ALL QUESTIONS

Please **show all your working.** You will lose marks if it is not clear how you get from one line to the next.

1. Consider the differential equation: [5]

$$x^2 + y^2 \frac{dy}{dx} = 0.$$

   (a) Solve the equation using any valid method.

   (b) Solve the equation using a different method than you used in part (a).

   (c) Solve the equation using a different method than you used in parts (a) or (b).

2. Consider the differential equation: [3]

$$y^2 + x^2 \frac{dy}{dx} = 0.$$

   (a) Solve the equation using any valid method.

   (b) Solve the equation using a different method than you used in part (a).

3. Suppose we have a first order ODE of the form:

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0,$$

   and suppose further that by doing some algebraic manipulation that we can rearrange the equation into the form:

$$f_1(x) \cdot g_1(y) + f_2(x) \cdot g_2(y)\frac{dy}{dx} = 0. \quad (1)$$

   Where f_1 and f_2 are functions of x alone and g_1 and g_2 are functions of y alone.

   (a) Show that the function $\mu(x, y) = \frac{1}{g_1(y)f_2(x)}$ is an integrating factor for the differential equation (1). [1]

   (b) Use this method to transform the differential equation given in question 2 into an exact equation and hence solve it via a third method, different from those you used in 2(a) and 2(b). [3]

4. Consider the differential equation:

$$x^4 \frac{d^2y}{dx^2} + 2x^3 \frac{dy}{dx} - 4y = 4.$$

Use the substitution $x = t^{-1}$ to transform it into the equation: [2]

$$\frac{d^2y}{dt^2} - 4y = 4.$$

Hence find the general solution to the original equation. (Hint: make repeated use of the chain rule $\frac{d}{dx}(\cdot) = \frac{d}{dt}(\cdot)\frac{dt}{dx}$). [2]

5. Just for fun, a Calculus 2 student solves a second order, linear, non-homogeneous differential equation of the form $P\frac{d^2y}{dx^2} + Q\frac{dy}{dx} + Ry = S(x)$ and correctly gets the solution:

$$y = e^{2x}(A\cos(3x) + B\sin(3x)) + 4x^2 + x - 1.$$

Which equation did the student solve? [4]