

Probability and Statistics

Assignment 1

- **Submission Date: Monday 30th January 2017**
 - **Five questions are given and 30 points are accounted in total**
 - **You must answer to all the five questions**
1. A sample of 20 high-powered copper coils yields the following tensile strength values (in units of kilogram per square centimetre):
- 280, 155, 329, 140, 307, 116, 202, 262, 130, 131,
187, 187, 292, 83, 207, 197, 134, 294, 163, 217
- (i) Calculate the sample mean. [1]
 - (ii) Calculate the sample variance. [2]
 - (iii) Now use R to calculate the sample mean and the sample variance. Write down the command used and the output as returned by R. [3]
2. Given two events A and B for which $P(A) = 0.3$, $P(B) = 0.15$ and $P(A|B) = 0.67$. Find:
- (i) $P(B^C)$ [1]
 - (ii) $P(A \cap B)$ [1]
 - (iii) $P(A \cap B^C)$ [1]
 - (iv) $P(A|B^C)$ [1]
 - (v) Are the events A and B independent? Justify your answer. [1]
 - (vi) Are the events A and B mutually exclusive? Justify your answer. [1]
3. The probability that a dog is affected by a certain rare disease is 0.002. We have collected a random sample of 3000 dogs. We are interested in the probability that there is at most one dog affected by that disease.
- (i) Calculate the exact probability. Give also the R commands. [1 + 1]
 - (ii) Calculate the probability by using the Poisson approximation. Give also the R commands. [1 + 1]
 - (iii) Calculate the probability by using the Normal approximation. Give also the R commands. [1 + 1]
 - (iv) Suppose there is a blood test to detect this disease. This test is very accurate: in fact, the probability of the test resulting positive given that the dog has the disease is 0.99, and the probability of the test resulting negative given that the dog does not have the disease is 0.95.
Find the probability that the test results positive given that the dog does not have the disease. [3]

Please turn over

4. A salesman assumes that the demand for a certain product is distributed according to a Poisson distribution with mean of two items per day.

How many items does he have to buy in order to have 95% probability of not having a demand higher than the supply for 7 days? [5]

5. In a science test the marks were normally distributed with mean 68 and standard deviation 10.

(i) What is the probability that a student scored 75 or more? [2]

(ii) What is the probability that a student scored between 70 and 75? [2]