Calculus 2, Assignment 4

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1. (a) In this context, k cannot be 0, since $k = 0$ implies there is no relationship between $\frac{\mathrm{d}T}{\mathrm{d}t}$ and $T_s - T$. This gives us $|k| > 0$. However, I don't see a "colloquial" reason for $k > 0$, since either or both of T_s and T can be negative. I can see it's in some sense meaningless to take a negative factor, since if a is a factor of b then $-a$ is also a factor of b.

   ...

   After having found a value for T, on the other hand, I can see that a negative value of k would lead to a situation where instead of T approaching T_s as t approaches infinity, T would also go to minus infinity (which breaks the model of reality).

   (b) $\frac{\mathrm{d}T}{\mathrm{d}t} = k(T_s - T)$ is a first order variables separable ordinary differential equation and as such we can write

$$\begin{aligned}
\int \frac{1}{T_s - T}\mathrm{d}T &= k \int \mathrm{d}t \\
-\ln(T_s - T) &= k(t + c) \\
T_s - T &= e^{-k(t+c)} \\
T &= T_s - e^{-k(t+c)}.
\end{aligned}$$

   If $T = T_0$ when $t = 0$ we can write

$$\begin{aligned}
T_0 &= T_s - e^{-kc} \\
e^{-kc} &= T_s - T_0 \\
c &= -\frac{\ln(T_s - T_0)}{k}
\end{aligned}$$

and

$$
\begin{aligned}
T &= T_s - e^{-k\left(t - \frac{\ln(T_s - T_0)}{k}\right)} \\
&= T_s - e^{\ln(T_s - T_0) - kt} \\
&= T_s - \frac{e^{\ln(T_s - T_0)}}{e^{kt}} \\
&= T_s - \frac{T_s - T_0}{e^{kt}}.
\end{aligned}
$$

(c) We are given $T(0) = 37$ and $T_s = 24$.

i. Let the amount of time between death and discovery be A, now $T(A) = 34$, $T(A + 30) = 32$ and

$$
\begin{aligned}
e^{kA} &= \frac{13}{10} \\
kA &= \ln\left(\tfrac{13}{10}\right), \\
e^{k(A+30)} &= \frac{13}{8} \\
kA + k30 &= \ln\left(\tfrac{13}{8}\right).
\end{aligned}
$$

Substituting our value for kA into the second equation

$$
\begin{aligned}
k &= \frac{\ln\left(\frac{13}{8}\right) - \ln\left(\frac{13}{10}\right)}{30} \\
&= \frac{\ln\left(\frac{5}{4}\right)}{30}.
\end{aligned}
$$

ii. With a value for k, we can write

$$
T = T_s - (T_s - T_0) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right)
$$

and

$$
\begin{aligned}
34 &= 24 - (-13) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right) \\
A &= -\frac{30 \ln\left(\frac{10}{13}\right)}{\ln\left(\frac{5}{4}\right)} \\
&\approx 35.2729347...,
\end{aligned}
$$

which tells us the time of death was about 35 minutes before high noon.

iii. I was returning some video tapes.

iv. Since the model we have considers surrounding temperature to be constant, I have found in the past that the turning the thermostat up or down before leaving the scene of the crime works nicely. What? Sorry, I have to return some video tapes.

2. (a) Since f is even, $g(f(-x)) = g(f(x))$ and so $g \circ f$ is even. Similarly, $f(g(-x)) = f(-g(x)) = f(g(x))$ so $f \circ g$ is also even.

   (b) $f(-x)g(-x) = f(x)(-g(x)) = -f(x)g(x)$ so $f(x)g(x)$ is odd. Similarly, $\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)}$ and so $\frac{f(x)}{g(x)}$ is also odd.

   (c) Functions of the form $a_1x^{b_1} + ... + a_nx^{b_n}$ are odd when $b_i \in 2\mathbb{N}+1$ and even when $b_i \in 2\mathbb{N}$ and since $\frac{\mathrm{d}(x^b)}{\mathrm{d}x} = bx^{b-1}$, in this case differentiating takes a funciton from even to odd and vice-versa. Similarly, where sin is odd and cos is even and $\frac{\mathrm{d}\sin}{\mathrm{d}} = \cos$ and $\frac{\mathrm{d}\cos}{\mathrm{d}} = -\sin$ and there is the same relationship between derivative and oddness.

   (d) i. $h_e(-x) = \frac{1}{2}(h(-x)+h(x)) = h_e(x)$ and h_e is even, also $h_o(-x) = \frac{1}{2}(h(-x) - h(x)) = -h_o(x)$ and h_o is odd.

   ii. $f(x) = x^2 + x$, $h_e = x^2$, $h_o = x$, h_e is even, h_o is odd, $f(x) = h_e(x) + h_o(x)$.