Games, Choice and Optimisation Assignment 3

BM Corser

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1. (a) Because the column Colin B dominates the column Colin A, we can rewrite the game

$$\begin{array}{cc|ccc} & & & \text{Colin} & \\ & & B & C & D \\ \hline \text{Rose} & A & -2 & 4 & -4 \\ & B & 3 & 1 & 5 \end{array}.$$

Because Rose has exactly two strategies, we can represent Colin's payoff for each of Rose's pure strategies as vertical lines and draw slopes between his payoffs that represent what his strategy should be in response to Rose's mixed strategies.

Having drawn our picture, we can see that Colin should select a mixed strategy involving B, C and D. Because Rose wants to minimise her loss, she will chose a mixed strategy where Colin's best response is a mixed strategy involving B and C.

Thus, the solution of the game can be realised from the 2×2 subgame

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{Colin}} \\ & & B & C \\ \hline \text{Rose} & A & -2 & 4 \\ & B & 3 & 1 \end{array} .$$

Suppose that Colin plays xB, $(1-x)C$. His expected payoff if Rose plays A is $x(-2)+(1-x)4 = 4-6x$ and if Rose plays B, $x3+(1-x) = 2x+1$. The value for x that satisfies these expressions is $\frac{3}{8}$ and we can write Colin's equalising strategy $\frac{3}{8}B$, $\frac{5}{8}C$.

Suppose Rose plays yA, $(1-y)B$. Her expected payoff when Colin plays B is $y(-2)+(1-y)3 = 3-5y$ and when Colin plays C, and $y4+(1-y) = 3y+1$ when Colin plays D, so Rose's equalising strategy is $\frac{1}{4}A$, $\frac{3}{4}B$.

The value of the game is $3 - 5 \cdot \frac{1}{4} = \frac{7}{4}$.

(b) The saddle points of G are G_{12}, G_{32}, G_{14}, G_{34} and the value of the game is 1.

(c) (i) For Colin C to dominate Colin D, since the values in the matrix represent Colin's loss, all entries in Colin C must be less than or equal to corresponding entries in Colin D, and at least one entry must be strictly less that the corresponding entry in Colin D. This is the case when $-10 \leq x \leq -4$. Notice, since x cannot be both -4 and -10, this is not a strict inequality.

(ii) For H_{31} to be a saddle point, it must be less than or equal to each entry in its row and greater than or equal to each entry in its column. This is true when $-5 \leq x \leq -2$.

2. (a) (i) The movement diagram of G can be written

$$\begin{array}{cc|ccccc} & & & & \text{Colin} & & \\ & & A & B & C & D & E \\ \hline & A & & & & & \\ \text{Rose} & B & & & & & \\ & C & & & & & \\ & D & & & & & \end{array}$$

(ii) Start by "shifting" the game so that all payoffs are positive, we do this by adding 10 to all entries in the matrix. We can then write the linear programme

$$\begin{array}{rrcrcrcrcl} \text{maximise} & x_1 & + & x_2 & + & x_3 & + & x_4, & & \\ \text{subject to} & 14x_1 & + & 15x_2 & + & 8x_3 & + & 6x_4 & \geq & 1, \\ & 8x_1 & + & 13x_2 & + & 7x_3 & + & 18x_4 & \geq & 1, \\ & 17x_1 & + & 14x_2 & + & 4x_3 & + & x_4 & \geq & 1, \\ & 11x_1 & + & 8x_2 & + & 16x_3 & + & 7x_4 & \geq & 1, \\ & 5x_1 & + & 9x_2 & + & 15x_3 & + & 12x_4 & \geq & 1, \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

(iii) By von Neumann's minmax theorem, any $m \times n$ game can be realised by a $k \times k$ subgame where $1 < k \leq \min(m, n)$. For the game G, $\min(m, n) = 4$.

(b) (i) An outcome of a game is a Nash equilibrium when neither player can independently change their strategy to increase their payoff. For the game H, the only entry satisfying this condition is H_{21} (corresponding to Rose A and Colin B).

(ii) To find Colin's prudential strategy, we first write Colin's game

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{Colin}} \\ & & A & B \\ \hline \text{Rose} & A & 1 & 5 \\ & B & 5 & 3 \end{array} .$$

Suppose Colin plays $xA, (1-x)B$ in this game. If Rose plays A, her expected payoff is $5-4x$ and if she plays B it is $2x+3$. The value of x that satisfies both of these expression is $\frac{1}{3}$. Therefore Colin's equalising strategy is $\frac{1}{3}A, \frac{2}{3}B$ with security level $\frac{11}{3}$. When Colin plays his prudential strategy, Rose's expected payoff when she plays A is 5 and $\frac{8}{3}$ when she plays B. As such her counter-prudential strategy is A.