Discrete Mathematics Assignment 3

Due date: Friday 3rd March 2016

Answer all questions. A total mark out of 20 will be awarded, with individual marks for each question being given in square brackets. This work is worth 5% of the marks for this module. Late submissions will be awarded at most 8/20; work that is more than 14 days late will receive 0.

1. (a) Draw two non-isomorphic graphs with the degree sequence $[2, 2, 2, 2, 3, 3, 4]$. [1]

   (b) The graph G has the following adjacency matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

   Determine (showing your working) how many edges G has. [1]

   (c) The graph H has the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

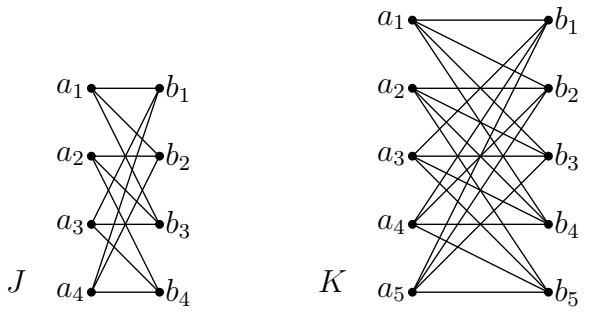
   i. Determine (giving full justification) whether H is simple. [1]

   ii. Determine (giving full justification) whether H is isomorphic to G. [1]

2. (a) Give an example of a bipartite planar graph. [1]

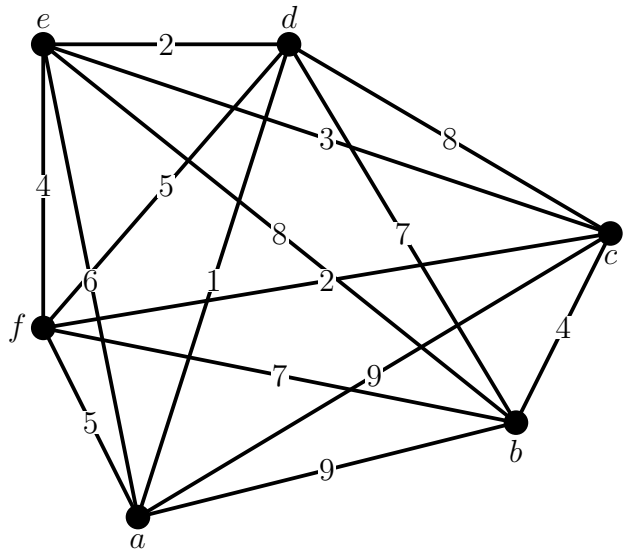
   (b) Give an example of a planar graph that is not bipartite. [1]

   (c) Determine (giving full justification) whether the graphs J and K shown below are planar:



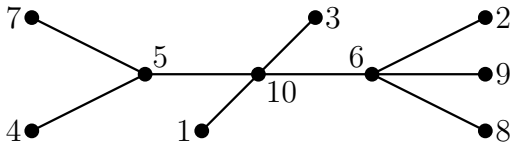
   [4]

3. Use Prim's algorithm to find a minimum spanning tree for the following graph:



[3]

4. (a) Find the Prüfer sequence corresponding to the following tree:

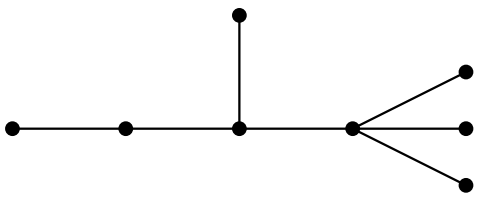


[2]

(b) Find the labelled tree corresponding to the Prüfer sequence $[3, 3, 4, 3, 3, 5, 4]$. [2]

(c) How many labelled trees are there with eight vertices? [1]

(d) Determine the number of ways of labelling the following tree such that the resulting labelled trees are not isomophic as labelled trees.



[2]