Calculus 2 Assignment 1

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1. Because $f(0) = 0$, there can be no constant component to the function. Because the domain and codomain are $\mathbb{R}$, $f(x)$ cannot be $ax^{\frac{1}{b}}$ where $b > 1$ because $f(x) \notin \mathbb{R}$ when x is negative. Also the function cannot be of the form $\frac{a}{bx}$ because $f(0) \notin \mathbb{R}$. The function f must then be of the form $f(x) = ax^b$ where $b > 1$, $f(x) = \frac{x^a}{b}$ where $a > 1$ or the null case $f(x) = 0$.

   (a) To meet the condition $f(x) \leq x \ \forall \ x \in \mathbb{R}$, then the only possible forms $f(x)$ can take are $f(x) = x$ for which $\frac{df}{dx} = 1$ and $f(x) = 0$ for which $\frac{df}{dx} = 0$. In both these cases $\frac{df}{dx} \leq 1$ and as such the statement is true.

   (b) Let $\frac{df}{dx} = \frac{1}{3}$, then $\int \frac{df}{dx} = f(x) = \frac{x}{3} + c$. Since we have established our f cannot have a constant component, $c = 0$. Now let $x = -1$, then $f(x) = -\frac{1}{3} > x$ which is a counterexample, proving the statement false.

2. Let $f(x) = x + 2$ and $g(x) = 2x + 1$, then

$$
\begin{aligned}
f(g(x)) &\neq g(f(x)) \\
(2x+1) + 2 &\neq 2(x+2) + 1 \\
2x + 3 &\neq 2x + 5
\end{aligned}
$$

also

$$
\begin{aligned}
\frac{d}{dx}\Big(f\big(g(x)\big)\Big) &= \frac{d}{dx}\Big(g\big(f(x)\big)\Big) \\
\frac{d}{dx}(2x+3) &= \frac{d}{dx}(2x+5) \\
2 &= 2
\end{aligned}
$$

3. Given

$$\lim_{x\to 0}\frac{f(x)}{x}=7$$

and since when

$$\lim_{x\to a} g(x)=l$$

and

$$\lim_{x\to a} h(x)=m$$

then

$$lim_{x\to a}h(x)\cdot\lim_{x\to a}\frac{g(x)}{h(x)}=m\cdot\frac{l}{m}=l$$

and

$$\begin{aligned}\lim_{x\to 0}\frac{f(x)}{x}&=7\\ \lim_{x\to 0}f(x)&=7\cdot\lim_{x\to 0}x\\ \lim_{x\to 0}f(x)&=0\end{aligned}$$

and since for continuous functions $\lim_{x\to a}p(x)=p(a)$

$$f(0)=0$$

and since for $x=0$

$$\begin{aligned}\frac{df}{dx}&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ \frac{df}{dx}&=\lim_{h\to 0}\frac{f(h)}{h}\end{aligned}$$

now let $h=x$ then clearly $f'(0)=7$. Another function that satisfies the conditions presented here is $f(x)=x^{9000}+7x$

4. Let $b=\frac{1}{3}$ and $b=9$, then

   (a)

$$\begin{aligned}\lim_{x\to\infty}\frac{x+9}{\frac{1}{3}x+1}&=\frac{x}{\frac{1}{3}x}\\ &=\frac{3x}{x}\\ &=3\end{aligned}$$

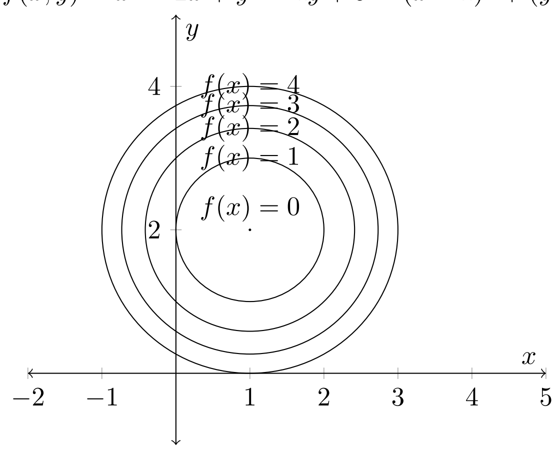
and

$$\lim_{x\to 0} \frac{x+9}{\frac{1}{3}x+1} = \frac{9}{1}$$
$$= 9$$

also

(b) $U = \mathbb{R} \setminus \{3\}$

5. (a) $f(x,y) = x^2 - 2x + y^2 - 4y + 5 = (x-1)^2 + (y-2)^2$



It doesn't make sense to plot contours for values of $a < 0$ because there is no circle with a smaller radius than 0.

(b)

$$f(x,y) = x^2 - 2x + y^2 - 4y + 5$$
$$f_x = 2x - 2$$
$$f_y = 2y - 4$$

Let $a = 0$ and $b = 2$. The equation of tangent plane passing through the point $P = (a, b, f(a, b))$ is

$$f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b) - z = 0$$

$$f_x(a,b) = -2$$
$$f_y(a,b) = 0$$
$$f(a,b) = 1$$

$$
\begin{aligned}
-2x+1-z&=0\\
z&=1-2x
\end{aligned}
$$

(c)

$$\nabla f = \begin{pmatrix} 2x-2 \\ 2y-4 \end{pmatrix}$$

6.

$$
\begin{aligned}
\Pi = 2x+y+3z&=6\\
z&=2-\frac{2x}{3}-\frac{y}{3}
\end{aligned}
$$

setting $z=0$

$$
\begin{aligned}
2-\frac{2x}{3}-\frac{y}{3}&=0\\
y&=6-2x
\end{aligned}
$$

bounds are

$$
\begin{aligned}
0\leq x&\leq 3\\
0\leq y&\leq 6-2x
\end{aligned}
$$

let D be the bounded area

$$
\begin{aligned}
\int\int_D \Pi dA &= \int_0^3\int_0^{6-2x} 2-\frac{2x}{3}-\frac{y}{3}dydx\\
&=\int_0^3\left[2y-\frac{2xy}{3}-\frac{y^2}{6}\right]_0^{6-2x}dx\\
&=\int_0^3 2(6-2x)-\frac{2x(6-2x)}{3}-\frac{(6-2x)^2}{6}dx\\
&=6
\end{aligned}
$$

setting bounds for dx inner

$$
\begin{aligned}
0\leq y&\leq 6\\
0\leq x&\leq 3-\frac{y}{2}
\end{aligned}
$$

$$
\begin{aligned}
\int\int_D \Pi dA &= \int_0^6 \int_0^{3-\frac{y}{2}} 2 - \frac{2x}{3} - \frac{y}{3} dx dy \\
&= \int_0^6 \left[2x - \frac{2x^2}{6} - \frac{xy}{3}\right]_0^{3-\frac{y}{2}} dy \\
&= \int_0^6 2(3 - \frac{y}{2}) - \frac{2(3-\frac{y}{2})^2}{6} - \frac{(3-\frac{y}{2})y}{3} dy \\
&= 6
\end{aligned}
$$