Discrete Mathematics

Assignment 1 Due date: Friday 25th November 2016

Answer all questions. A total mark out of 20 will be awarded, with individual marks for each question being given in square brackets. This work is worth 5% of the marks for this module. Late submissions will be awarded at most 8/20; work that is more than 14 days late will receive 0.

1. Find the value of the following sums:
   (a) $\sum_{i=1}^{1001} \binom{1000}{i-1} 2^i$; [2]
   (b) $\sum_{i=1}^{n} \sum_{j=1}^{i} j$. [3]

2. (a) A youth club is selecting a programme of three films to show at their annual movie marathon. The selection of titles they have to choose from include ten comedy films, fifteen drama films and eight action films. How many possible programmes are there if:
      i. there are no further restrictions on the selection? [1]
      ii. the first film showed must be a comedy? [2]
   (b) The club needs to select a team to play in a ping-pong tournament. If the club has 30 members, and the team consists of a captain plus 7 further players, how many ways are there to choose the team? [1]
   (c) Of the 8 players on the ping-pong team, two pairs of players will compete in doubles matches, and the remaining players will all compete in singles matches. How many ways are there to select the pairs who will play in doubles matches? [2]

3. Alice, Bob and Charlie each wrote down fifteen distinct whole numbers between 1 and 50 inclusive. Alice and Bob find they wrote eight numbers in common, Alice and Charlie had six numbers in common, and Bob and Charlie had seven numbers in common. In total there were 29 distinct numbers written down. Determine (showing all working) whether it is possible to find numbers x and y such that Alice, Bob and Charlie all had written x and y, and for which $x - y$ is even. [4]

4. A company delivers food hampers to customers. It employs two motorcycle couriers who can each deliver up to three hampers at once, and also possesses a Smart car that can fit up to five hampers, as well as a delivery van that can fit up to fifteen hampers, and which is always used to carry at least five hampers. If the company wishes to deliver fifteen hampers, how many ways are there in which it can distribute the hampers between its vehicles? [5]