Discrete, Assignment 4

BM Corser

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1. (a) $\lambda(G) \geq 3$, G is 3-edge connected.
   (b) $\kappa(G) \geq 2$, G is 2-connected.
   (c) An ae-disconnecting set will be a G-disconnecting set and as such cannot have magnitude < 3. Since $\{\{ab\}, \{ac\}, \{ad\}\}$ is an ea-disconnecting set of size 3, the size of the smallest ea-disconnecting set must also be 3.
   (d) The same logic applies to the size of the smallest be-separating set, where $\{c, d\}$ is a be-separating set with size equal to the smallest value of $\kappa(G)$.

2. (a) Shortest path algorithm;

Iteration	$T(v)$	$P(v)$
1		$P(a) = 0$
2	$T(b) = 9, T(d) = 6, T(e) = 3$	$P(e) = 3$
3	$T(d) = 4, T(j) = 7$	$P(d) = 4$
4	$T(g) = 11, T(h) = 5$	$P(c) = 5$
5	$T(f) = 13$	$P(h) = 5$
6	$T(g) = 8, T(z) = 9$	$P(j) = 7$
7		$P(g) = 8$
8		$P(b) = 9$
6	$T(f) = 12$	$P(z) = 9$

   (b) Longest path algorithm (starting by applying $P(a) = 0$, $P(e) = 3$);

Iteration	$T(v)$	$P(v)$
1	$T(j) = 7, T(d) = 6, T(c) = 5, T(b) = 9$	$P(d) = 6$
2	$T(j) = 10, T(h) = 7, T(g) = 13, T(c) = 11$	$P(j) = 10, P(c) = 11$
3	$T(h) = 13, T(z) = 15, T(g) = 18, T(f) = 19, T(b) = 15$	$P(h) = 13, P(b) = 15$
4	$T(g) = 16, T(f) = 18, T(z) = 17$	$P(g) = 18$
5	$T(f) = 23, T(z) = 20$	$P(f) = 23$
6	$T(z) = 28$	$P(z) = 28$

3. Ranked matching

Round	Offer	Accept
1	$(E_1 \to C_5), (E_1 \to C_5), (E_2 \to C_5), (E_3 \to C_2), (E_4 \to C_3), (E_5 \to C_3)$	$(E_1 \leftrightarrow C_5)$
2	$(E_2 \to C_4)$	$(E_2 \leftrightarrow C_4)$
3		$(E_3 \leftrightarrow C_2)$
4		$(E_4 \leftrightarrow C_3)$
5	$(E_5 \leftrightarrow C_1)$	$(E_5 \leftrightarrow C_1)$

4. Weighted matching (maximum)

(a) Matrix adjusted for application of Hungarian algorithm

	h	s	w	v	p	w'
A	3	1	7	5	9	5
B	2	0	8	6	4	6
C	9	9	3	4	5	7
D	9	7	5	4	3	5
E	2	4	4	3	3	3
F	5	5	5	2	4	3

Applying Hungarian algorithm. Column and row highlight indicates membership in S_1.

$$S_1 = \{C, D, E, F, s\}, |S_1| = 5$$

	h	s	w	v	p	w'
A	2	0	6	4	8	3
B	2	0	8	6	4	5
C	6	6	0	1	2	3
D	6	4	2	1	0	1
E	0	2	2	1	1	0
F	3	3	3	0	2	0

Using the table

	h	s	w	v	p	w'
A	0	**0**	4	2	6	1
B	**0**	0	6	4	2	3
C	6	6	**0**	1	2	3
D	6	4	2	1	**0**	1
E	0	2	2	1	1	**0**
F	3	3	3	**0**	2	0

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we can find the set of 6 independent zeros $\{As, Bh, Cw, Dp, Ew', Fv\}$ with maximum suitability, emboldened above. $\{Ah, Bs, Cw, Dp, Ew', Fv\}$ is the other set of 6 independent zeros, but doesn't have maximum suitability.

(b) Weighted matching (minimum)

Applying Hungarian algorithm. Column and row highlight indicates membership in S_1.

$$S_1 = \{h, s, w, v, p, E\}, |S_1| = 6$$

	h	s	w	v	p	w'
A	6	8	2	3	0	3
B	6	8	0	1	4	1
C	0	0	6	4	4	1
D	0	2	4	4	6	3
E	2	0	0	0	1	0
F	0	0	0	2	1	1

	h	s	w	v	p	w'
A	6	8	2	2	0	2
B	6	8	0	0	4	0
C	0	0	6	3	4	0
D	0	2	4	3	6	2
E	2	0	0	0	1	0
F	0	0	0	1	1	0

Several 6-sets of independent zeros are possible, enumerated below

Set							Weight
$s_1 =$	$\{Ap,$	$Bw,$	$Cs,$	$Dh,$	$Ev,$	$Fw'\}$	13
$s_2 =$	$\{Ap,$	$Bw,$	$Cw',$	$Dh,$	$Ev,$	$Fs\}$	12
$s_3 =$	$\{Ap,$	$Bv,$	$Cw',$	$Dh,$	$Ew,$	$Fs\}$	14
$s_4 =$	$\{Ap,$	$Bv,$	$Cw',$	$Dh,$	$Es,$	$Fw\}$	14
$s_5 =$	$\{Ap,$	$Bv,$	$Cs,$	$Dh,$	$Ew,$	$Fw'\}$	14
$s_6 =$	$\{Ap,$	$Bv,$	$Cs,$	$Dh,$	$Ew',$	$Fw\}$	13
$s_7 =$	$\{Ap,$	$Bw',$	$Cs,$	$Dh,$	$Ev,$	$Fw\}$	13

Any of s_3, s_4, s_5 are suitable.