Discrete Assignment 3

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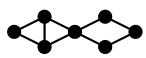
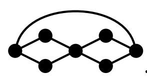
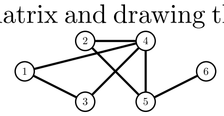
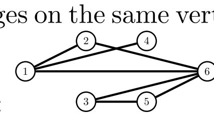
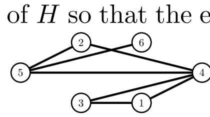
1. (a) Two non-isomorphic graphs with degree sequence $[2, 2, 2, 2, 3, 3, 4]$ are  and .

   (b) The graph G is simple because it has no loops (no non-zero entries on the main diagonal of its adjacency matrix) and all other entries are either 0 or 1. By the handshaking lemma, the number of edges in a simple graph is equal to half the sum of the degrees of its vertices. The sum of entries of a row in an adjacency matrix is the degree of the corresponding vertex. As such, half of the sum of all entries in an adjacency matrix of a simple graph gives the number of edges of that graph. Here, $\frac{0+0+1+1+0+0+0+0+0+1+1+0+1+0+0+1+0+0+1+1+1+0+1+0+0+1+0+1+0+1+0+0+0+0+1+0}{2} = \frac{14}{2} = 7$

   (c) i. H is simple, because all the entries on the main diagonal of its adjacency matrix are zero (the graph has no loops) and all the other entries are either 0 or 1 (there are no multiple edges).

   ii. H is isomorphic to G because $|G| = |H|$, where G has s vertices of degree r, H also has s vertices of degree r (ie. 3 vertices of degree 2 and 1 of degrees 1, 3 and 4) and both G and H are simple. Labelling vertices in G and H according to their position in the adjacency matrix and drawing their edges on the same vertex layout gives G: 

and H: 

. Now, we can (for example) re-label our drawing of H so that the edges joining label pairs match G as follows H: 

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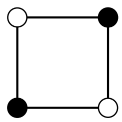
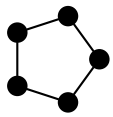
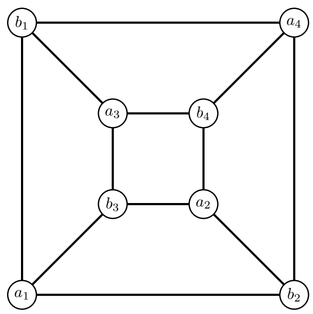
2. (a) 

(b) 

(c) If a graph has no triangles, $m \leq 2n - 4$ where m is the number of edges and n the number of vertices in the graph. By inspection, J has no triangles, 8 vertices and 12 edges. Hence $n = 8$ and $m = 12$ and $m \leq 2n - 4$ is true. It is therefore possible (but not certain) that J is planar. Let's draw J with a different vertex layout to demonstrate its planarity.

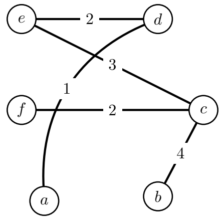
J: 

K is easier to classify, since $m = 20$ and $n = 10$ so observing that K has no triangles and testing against the condition of planarity mentioned above

$$
\begin{aligned}
m &\leq 2n - 4 \\
20 &\leq 2 \cdot 10 - 4 \\
&\leq 20 - 4 \\
20 &\leq 16
\end{aligned}
$$

which is clearly false and as such K is non-planar.

3. Prim's algorithm starting with $T = (\{f\}, \{\})$, edges are added in the following order $\{f, c\}, \{c, e\}, \{e, d\}, \{d, a\}, \{c, b\}$ giving the minimum tree

T: 

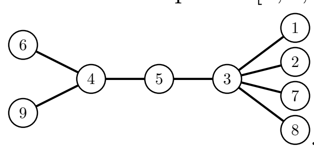
with total weight 12.

4. (a) Dropping and adding vertices in the following sequence

Drop	Add
1	10
2	6
3	10
4	5
7	5
5	10
8	6
9	6
6	10

gives the Prüfer sequence $[10, 6, 10, 5, 5, 10, 6, 6, 10]$.

(b) The Prüfer sequence $[3, 3, 4, 3, 3, 5, 4]$ gives the tree



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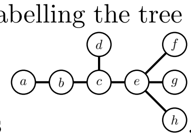
I also implemented an algorithm to draw trees from Prüfer sequences using a computer https://bmcorser.github.io/2017/02/04/prufer.html#fun

(c) n^{n-2} where $n = 8$ is $8^6 = 262144$.

(d) Labelling the tree presented with letters a through h, we can draw it as



. Now, the number of ways of choosing a label for vertex c is 8, the number of ways of choosing labels for those vertices adjacent to c (b, d and e) is $\binom{7}{3}$. The number of ways of choosing labels for a, f, g and h are 4, 3, 2 and 1. So the number of labelling schemes is $8 \cdot \binom{7}{3} \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6720$