Solutions to Chapter 3 Exercises

3.11.1 $u_n = 11 \times 2^n - 7 \times 3^n$

3.11.2 $u_n = 2 \times 2^n \cos\left(\frac{n\pi}{3}\right) - \sqrt{3} \times 2^n \sin\left(\frac{n\pi}{3}\right)$

3.11.3 $u_n = 3^{n-1}(6 - 4n)$

3.20.1 $u_n = (2 + n)(n!)^2$

3.20.2 $(6 + \sqrt{3})2^{n-2} \cos\left(\frac{n\pi}{6}\right) + (5 - 4\sqrt{3})2^{n-2} \sin\left(\frac{n\pi}{6}\right) + (4\sqrt{3} - 8)^{-1}2^n$

3.20.3 $1 + (4/3)n - (1/3)n^3$

3.21.1 $u_n = (3/4)3^n + (1/4)(-1)^n$

3.21.2 $u_n = (3/5)3^n + (2/5)(-2)^n$

3.24

$$\begin{aligned} q(x) &= 1 + a_1 x + a_2 x^2 + \cdots + a_k x^k \\ r(x) &= x^k + a_1 x^{k-1} + \cdots + a_k \end{aligned}$$

so

$$\begin{aligned} r(1/x) &= x^{-k} + a_1 x^{1-k} + \cdots + a_k \\ \therefore x^k r(1/x) &= 1 + a_1 x + a_2 x^2 + \cdots + a_k x^k \\ &= q(x) \end{aligned}$$

3.25 see answers to 3.11

3.28

$$\begin{aligned} \binom{2n}{n}/(n+1) &= \frac{(2n)!}{n!n!(n+1)} \\ &= \frac{2n(2n-1)(2n-2)(2n-3)\cdots 3 \cdot 2 \cdot 1}{(n+1)!n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1} \\ &= \frac{2n(2n-1)2(n-1)(2n-3)\cdots 3 \cdot 2 \cdot 1}{(n+1)!n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1} \\ &= \frac{2(2n-1)2(2n-3)2(2n-5)\cdots 2 \cdot 3 \cdot 2 \cdot 1}{(n+1)!} \\ &= \frac{2 \cdot 6 \cdot 10 \cdots 2(2n-1)}{(n+1)!} \end{aligned}$$

3.29

$$\begin{aligned}\binom{2n}{n} - \binom{2n}{n-1} &= \frac{(2n)!}{n!n!} - \frac{(2n)!}{n!n!} \\ &= \frac{(n+1)(2n)!}{n!(n+1)!} - \frac{n(2n)!}{n!(n+1)!} \\ &= \frac{(2n)!}{n!(n+1)!} \\ &= \frac{(n+1)(2n)!}{n!n!} \\ &= (n+1)\binom{2n}{n} \\ &= C_n\end{aligned}$$

3.32.1 the compound interest is a better choice

3.32.2 12 years, 11 years 7 months

3.32.3 358.22

3.33 If $p_{t-1} = \frac{d-b}{c+a}$ then

$$\begin{aligned}p_t &= \frac{a}{c}\frac{d-b}{c+a} + \frac{d-b}{c} \\ &= \frac{a(d-b)}{c(c+a)} + \frac{(c+a)(d-b)}{c(c+a)} \\ &= \frac{c(d-b)}{c(c+a)} \\ &= \frac{d-b}{c+a} \\ &= p_{t-1}.\end{aligned}$$

3.34 1) arithmetic progression 2) grows exponentially 3) stays constant 4) decays to 0 5) not particularly!