Dr Isabella Gollini

Probability and Statistics

Examples 3

In the questions below, where appropriate, calculate the probabilities in two different ways, (a) using the *New Cambridge Statistical Tables* and (b) using `R`, checking that you get the same answers by both methods, apart from the different number of decimal places to which the probabilities are calculated. [Alternatively, calculate the probabilities from the mathematical formulae for the probability distributions or use Microsoft Excel.]

1. A discrete r.v. X has a binomial distribution, $X \sim B(10, 0.3)$. Find

(i) $\Pr(X \leq 4)$,

(ii) $\Pr(X > 4)$,

(iii) $\Pr(X = 4)$,

(iv) $E(X)$,

(v) $\mathrm{var}(X)$.

2. In a large city 1 person in 5 is left-handed. A random sample of n people is taken.

(i) What is the probability distribution of the number of number of left-handed people in the sample?

(ii) Write down an expression in terms of n for the probability that there are no left-handed people in the sample.

(iii) Find the smallest value of n such that the probability that the sample contains at least one left-handed person is greater than 0.95.

(iv) For the value of n obtained in (iii), find the probability that there is at least one left-handed person in the sample.

3. Breakdowns occur on a particular machine at a rate of 2.5 per month. Assuming that the number of breakdowns can be modelled by a Poisson distribution, find the probability that

(i) exactly 3 occur in a particular month,

(ii) at least one occurs in a particular month,

(iii) more than 5 occur in a 4 month period.

4. An electronics manufacturer produces computer monitors. It is found that a randomly selected monitor from this manufacturer has a 2% chance of being defective (due to the existence of one or more "dead" pixels on the screen), independently of all other monitors.

Let X denote the number of defective monitors in a randomly selected batch of 300.

(i) Specify the distribution of X, including the values of any parameters.

(ii) What are the mean and variance of X?

(iii) Specify, including the values of any parameters, what other distribution you could use as an approximation to the distribution of X.

(iv) For comparison, tabulate the values of the cumulative distribution function F_r for $r = 0, 1, 2, \ldots, 10$ both for the exact distribution of (i) and the approximating distribution of (iii).

(v) What is the value of $\Pr(X \geq 5)$ (a) according to the exact distribution and (b) according to the approximating distribution?

5. Some river water contains on average 500 bacteria per litre. The bacteria are assumed to be randomly distributed in the river.

(i) A sample of 1 ml is examined. Write down a suitable distribution to model the number of bacteria in the sample, including the values of any parameters. [Note: 1000 ml = 1 litre.]

(ii) For the sample described in part (i), what is the probability that there are any bacteria at all in the sample?

(iii) If instead a sample of 10 ml is examined, what is the probability that there will be at most 3 bacteria in the sample?

6. Consider the *geometric distribution* with parameter p, where p satisfies $0 < p < 1$. Its probability distribution (p_r) is given by

$$p_r = q^r p \qquad (r = 0, 1, 2, \ldots),$$

where $q = 1-p$. [In a sequence of independent trials like those used to underpin the definition of the binomial distribution, the geometric distribution is the distribution of the number of failures until the first success.]

(i) Check that the above formula does indeed specify a probability distribution. [Hint: Recall the formula for the sum of a geometric series: for $|x| < 1$, $\sum_{r=0}^{\infty} x^r = \frac{1}{1-x}$].

(ii) Show that the p.g.f. $G(t)$ of the geometric distribution is given by

$$G(t) = \frac{p}{1-qt} \qquad (|t| < 1/q).$$

(iii) Deduce that the mean μ and variance σ^2 of the geometric distribution are given by

$$\mu = \frac{q}{p} \quad \text{and} \quad \sigma^2 = \frac{q}{p^2}.$$