Calculus 2 - Assignment 4

Due 14^{th} April 2017

ANSWER ALL QUESTIONS

1. Let $T(t)$ be the temperature of an object at time t. It is hypothesised that the rate of change of T is proportional to the difference between the temperature of the object and the temperature of its surroundings, T_s. We assume T_s to be constant. Thus we have:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(T_s - T),$$

where $k \in \mathbb{R}$ is some constant of proportionality.

(a) Explain why we must have $k > 0$. [1]

(b) Assuming that at time $t = 0$ we have $T = T_0$, solve this differential equation to find the temperature at time t in terms of T_0, T_s, k and t. [2]

(c) Now let us assume that T is measured in ^{o}C, t is measured in minutes and recall that the body temperature of a healthy adult is 37^{o}C. Let's do some CSI trickery:

A coroner is called after the discovery of a body and arrives on the scene at 12 noon. The temperature of the body is taken immediately and found to be 34^{o}C. Half an hour later the temperature is taken again and found to be 32^{o}C. The thermostat on the wall is set to 24^{o}C.

i. Use this information and your answer to part (b) to determine the value of k. [2]

ii. What was the time of death? [2]
(Hint: it will make life simpler to assume death happened at time $t = 0$ and that 12 noon corresponds to time $t = A$. Then finding the time of death boils down to finding the value of A).

iii. Where were *YOU* at that time? (Only joking). [0]

iv. How could a potential murderer use their knowledge of differential equations to stop the coroner from estimating time of death by this method? [1]
("Kill all coroners" is not a viable option).

2. Recall that a function $f : \mathbb{R} \to \mathbb{R}$ is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$ and *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$.

   (a) Suppose that $f(x)$ is even and $g(x)$ is odd and both are nice differentiable functions, defined on all of $\mathbb{R}$. Decide if the following statements are true or false. Give a reason if true or a counterexample if false:

      i. both $g(f(x))$ and $f(g(x))$ are even. [0.5]

      ii. both $f(x)g(x)$ and $\frac{f(x)}{g(x)}$ are odd. [0.5]

      iii. $f'(x)$ is odd and $g'(x)$ is even. [1]

   (b) Let $h(x)$ be any other nice function but not necessarily even or odd. Define two new functions $h_e(x)$ and $h_o(x)$ like this:

$$h_e(x) = \frac{1}{2}\left(h(x) + h(-x)\right)$$
$$h_o(x) = \frac{1}{2}\left(h(x) - h(-x)\right)$$

      i. Show that $h_e(x)$ is even and $h_o(x)$ is odd and hence deduce that any function can be written as the sum of an odd part and an even part. [1]

      ii. Illustrate this by giving an example of a function that is neither odd nor even and write it explicitly as the sum of its odd and even parts. [1]

3. A particle is released from rest at time $t = 0$. The acceleration of the particle is given by the equation $\ddot{x} = t^2 + bt + c$, where $b, c \in \mathbb{R}$.

   (a) Find expressions for the velocity $\dot{x}(t)$ and the displacement $x(t)$ in terms of b, c and t. [2]

   (b) Choose any values for b and c that ensure that the particle returns to its starting position at some time $t > 0$ *and prove that these values work.* [1]

   (c) What is the behaviour of the particle in the long-run? [1]

4. This question is based on Example 5.6 on page 124 of the notes. In that example we solved the differential equation,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy, \text{ and } y(0) = 0.5,$$

numerically using the higher derivatives Euler method and produced Table 3 on the same page. The purpose was to estimate the value of y and $x = 0.3$. Note that this table actually contains *two* solutions, one using three terms and the other using four terms.
In this question I want you to do the same thing for this differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3xy, \text{ and } y(0) = 0.5.$$

Again, use a step size of $h = 0.1$ to estimate y when $x = 0.3$. Just find the solution using four terms. [4]
You have two choices:

(a) Do the calculuations by hand (well, calculator) on paper showing each step as you build up the table.

OR

(b) Use Excel to do the actual calculations. If you choose this option you must:

i. make it very clear to me on paper what formula you are putting into which cell in Excel.
ii. email me your Excel file to d.mcveagh@bbk.ac.uk (same deadline).
iii. name your Excel file “calculus2_your_full_name”

This is a tedious boring question, but I've already told you I won't make you do this sort of thing in the exam and I am giving you the option of using Excel. So looked at that way, this is me being nice to you.