Probability and Statistics

Examples 5

In the examples below carry out the analyses (i) using a hand calculator together with a book of statistical tables, and (ii) using R. Where appropriate, check that, apart from any differences due to rounding, the two methods yield the same results.

1. A random sample of forty adult dogs of a certain breed were weighed. Their weights in ounces are listed below.

```
66.2
      59.2
             70.8
                    58.0
                           64.3
                                  50.7
                                         62.5
                                               58.4
                                                      48.7
                                                             52.4
      35.7
                    52.3
                           41.2
                                         52.9
51.0
             62.6
                                  61.1
                                               58.8
                                                      64.1
                                                             48.9
74.3
      50.3
             55.7
                    55.5
                           51.8
                                  55.8
                                         48.9
                                               51.8
                                                      63.1
                                                             44.6
47.0
      49.0
             62.5
                    45.0
                           78.6
                                  54.2
                                         72.2
                                               52.4
                                                      60.5
                                                             46.8
```

The Kennel Club states that the mean weight of adult dogs of this breed is 58 ounces, but it has been claimed by an expert that the mean weight is less than 58 ounces.

- (i) Set up a statistical model for the data, carefully stating your assumptions. In terms of the model parameters, formulate appropriate null and alternative hypotheses to be tested.
- (ii) Carry out a formal test of the Kennel Club's claim against that of the expert and draw conclusions.
- (iii) Calculate an estimate of the mean weight for this breed together with a 95% confidence interval for the mean and a 99% confidence interval for the mean.
- (iv) Calculate an estimate of the variance of weights within the breed together with a 95% confidence interval for the variance.

2. Measurements of the wing span of two varieties of sparrow in millimetres were made. A random sample of ten was taken from each variety, and the data are given below.

It is being investigated whether the mean wingspans for sparrows from the two varieties are different.

- (i) Set up a statistical model for the data, assuming that the variance of the wingspans for the two varieties is the same. In terms of the model parameters, formulate appropriate null and alternative hypotheses to be tested.
- (ii) Carry out a formal test of the hypothesis that the mean wingspan for the two varieties is the same and draw conclusions.
- (iii) Calculate an estimate of the difference of the mean wingspans between the two varieties together with a 95% confidence interval.
- (iv) On closer examination of the data, it is found that an error has been made in the recording of one of the wingspans for Variety 1. The number 128 should be replaced by 158. Make this replacement and then repeat the analysis.
- **3.** Eight athletes ran a 400 metre race at sea level and, at a later meeting, ran another 400 metre race at high altitude. Their times in seconds are presented below.

Runner	1	2	3	4	5	6	7	8
Sea level time								
High altitude time	50.4	47.3	50.8	52.3	47.7	54.5	48.9	49.9

The point at issue is whether these data provide evidence that athletes perform better at sea level.

- (i) Set up a statistical model for the data, carefully stating your assumptions. In terms of the model parameters, formulate appropriate null and alternative hypotheses to be tested.
- (ii) Carry out a formal test of the hypothesis that there is no difference between performance at sea level and at high altitude. Draw conclusions.
- (iii) Calculate an estimate of the mean difference between performance times at the sea level location and the high altitude location, together with a 95% confidence interval.