

Calculus 2, Assignment 4

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April 13, 2017

1. (a) In this context, k cannot be 0, since $k = 0$ implies there is no relationship between $\frac{dT}{dt}$ and $T_s - T$. This gives us $|k| > 0$. However, I don't see a colloquial reason for $k > 0$, since either or both of T_s and T can be negative. I can see it's in some sense meaningless to take a negative factor, since if a is a factor of b then $-a$ is also a factor of b .

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After having found a value for T , on the other hand, I can see that a negative value of k would lead to a situation where instead of T approaching T_s as t approaches infinity, T would also go to minus infinity (which breaks the model of reality).

- (b) $\frac{dT}{dt} = k(T_s - T)$ is a first order variables separable ordinary differential equation and as such we can write

$$\begin{aligned}\int \frac{1}{T_s - T} dT &= k \int dt \\ k(t + c) &= -\ln(T_s - T) \\ e^{-k(t+c)} &= T_s - T \\ T &= T_s - e^{-k(t+c)}.\end{aligned}$$

If $T = T_0$ when $t = 0$ we can write

$$\begin{aligned}T_0 &= T_s - e^{-kc} \\ e^{-kc} &= T_s - T_0 \\ c &= -\frac{\ln(T_s - T_0)}{k}\end{aligned}$$

and

$$\begin{aligned}
T &= T_s - e^{-k\left(t - \frac{\ln(T_s - T_0)}{k}\right)} \\
&= T_s - e^{\ln(T_s - T_0) - kt} \\
&= T_s - \frac{e^{\ln(T_s - T_0)}}{e^{kt}} \\
&= T_s - \frac{T_s - T_0}{e^{kt}}.
\end{aligned}$$

(c) We are given $T(0) = 37$ and $T_s = 24$.

- i. Let the amount of time between death and discovery be A , now $T(A) = 34$, $T(A + 30) = 32$ and

$$\begin{aligned}
e^{kA} &= \frac{13}{10} \\
kA &= \ln\left(\frac{13}{10}\right), \\
e^{k(A+30)} &= \frac{13}{8} \\
kA + k30 &= \ln\left(\frac{13}{8}\right).
\end{aligned}$$

Substituting our value for kA into the second equation

$$\begin{aligned}
k &= \frac{\ln\left(\frac{13}{8}\right) - \ln\left(\frac{13}{10}\right)}{30} \\
&= \frac{\ln\left(\frac{5}{4}\right)}{30}.
\end{aligned}$$

- ii. With a value for k , we can write

$$T = T_s - (T_s - T_0) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right)$$

and

$$\begin{aligned}
34 &= 24 - (-13) \cdot \exp\left(-\frac{A \ln\left(\frac{5}{4}\right)}{30}\right) \\
A &= -\frac{30 \ln\left(\frac{10}{13}\right)}{\ln\left(\frac{5}{4}\right)} \\
&\approx 35.2729347...,
\end{aligned}$$

which tells us the time of death was about 35 minutes before high noon.

- iii. I was returning some video tapes.
- iv. Since the model we have considers surrounding temperature to be constant, I have found in the past that the turning the thermostat up or down before leaving the scene of the crime works nicely. What? Sorry, I have to return some video tapes.