

Discrete, Assignment 4

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1. (a) 3
- (b) 3
- (c) 3

2. (a) Shortest path algorithm;

Iteration	$T(v)$	$P(v)$
1		$P(a) = 0$
2	$T(b) = 9, T(d) = 6, T(e) = 3$	$P(e) = 3$
3	$T(d) = 4, T(j) = 7$	$P(d) = 4$
4	$T(g) = 11, T(h) = 5$	$P(c) = 5$
5	$T(f) = 13$	$P(h) = 5$
6	$T(g) = 8, T(z) = 9$	$P(j) = 7$
7		$P(g) = 8$
8		$P(b) = 9$
6	$T(f) = 12$	$P(z) = 9$

- (b) Longest path algorithm (starting by applying $P(a) = 0, P(e) = 3$);

Iteration	$T(v)$	$P(v)$
1	$T(j) = 7, T(d) = 6, T(c) = 5, T(b) = 9$	$P(d) = 6$
2	$T(j) = 10, T(h) = 7, T(g) = 13, T(c) = 11$	$P(j) = 10, P(c) = 11$
3	$T(h) = 13, T(z) = 15, T(g) = 18, T(f) = 19, T(b) = 15$	$P(h) = 13, P(b) = 15$
4	$T(g) = 16, T(f) = 18, T(z) = 17$	$P(g) = 18$
5	$T(f) = 23, T(z) = 20$	$P(f) = 23$
6	$T(z) = 28$	$P(z) = 28$

3. Ranked matching

Round	Offer	Accept
1	$(E_1 \rightarrow C_5), (E_1 \rightarrow C_5), (E_2 \rightarrow C_5), (E_3 \rightarrow C_2), (E_4 \rightarrow C_3), (E_5 \rightarrow C_3)$	$(E_1 \leftrightarrow C_5)$
2	$(E_2 \rightarrow C_4)$	$(E_2 \leftrightarrow C_4)$
3		$(E_3 \leftrightarrow C_2)$
4		$(E_4 \leftrightarrow C_3)$
5	$(E_5 \leftrightarrow C_1)$	$(E_5 \leftrightarrow C_1)$

4. Weighted matching (maximum)

(a) Matrix adjusted for application of Hungarian algorithm

	h	s	w	v	p	w'
A	3	1	7	5	9	5
B	2	0	8	6	4	6
C	9	9	3	4	5	7
D	9	7	5	4	3	5
E	2	4	4	3	3	3
F	5	5	5	2	4	3

Applying Hungarian algorithm. Column and row highlight indicates membership in S_1 .

$$S_1 = \{C, D, E, F, s\}, |S_1| = 5$$

	h	s	w	v	p	w'
A	2	0	6	4	8	3
B	2	0	8	6	4	5
C	6	6	0	1	2	3
D	6	4	2	1	0	1
E	0	2	2	1	1	0
F	3	3	3	0	2	0

Using the table below

	h	s	w	v	p	w'
A	0	0	4	2	6	1
B	0	0	6	4	2	3
C	6	6	0	1	2	3
D	6	4	2	1	0	1
E	0	2	2	1	1	0
F	3	3	3	0	2	0

we can find the set of 6 independent zeros $\{As, Bh, Cw, Dp, Ew', Fv\}$ with maximum suitability, emboldened above. $\{Ah, Bs, Cw, Dp, Ew', Fv\}$ is the other set of 6 independent zeros, but doesn't have maximum suitability.

(b) Weighted matching (minimum)

Applying Hungarian algorithm. Column and row highlight indicates membership in S_1 .

$$S_1 = \{h, s, w, v, p, E\}, |S_1| = 6$$

	<i>h</i>	<i>s</i>	<i>w</i>	<i>v</i>	<i>p</i>	<i>w'</i>
<i>A</i>	6	8	2	3	0	3
<i>B</i>	6	8	0	1	4	1
<i>C</i>	0	0	6	4	4	1
<i>D</i>	0	2	4	4	6	3
<i>E</i>	2	0	0	0	1	0
<i>F</i>	0	0	0	2	1	1

	<i>h</i>	<i>s</i>	<i>w</i>	<i>v</i>	<i>p</i>	<i>w'</i>
<i>A</i>	6	8	2	2	0	2
<i>B</i>	6	8	0	0	4	0
<i>C</i>	0	0	6	3	4	0
<i>D</i>	0	2	4	3	6	2
<i>E</i>	2	0	0	0	1	0
<i>F</i>	0	0	0	1	1	0

Several 6-sets of independent zeros are possible, enumerated below

Set	Weight
$s_1 = \{Ap, Bw, Cs, Dh, Ev, Fw'\}$	13
$s_2 = \{Ap, Bw, Cw', Dh, Ev, Fs\}$	12
$s_3 = \{Ap, Bv, Cw', Dh, Ew, Fs\}$	14
$s_4 = \{Ap, Bv, Cw', Dh, Es, Fw\}$	14
$s_5 = \{Ap, Bv, Cs, Dh, Ew, Fw'\}$	14
$s_6 = \{Ap, Bv, Cs, Dh, Ew', Fw\}$	13
$s_7 = \{Ap, Bw', Cs, Dh, Ev, Fw\}$	13

Any of s_3, s_4, s_5 are suitable.