Dr Isabella Gollini

Probability and Statistics

Examples 2

1. If $A \subseteq B$, using the distributive law show that $B = A \cup (B \cap A^c)$ and hence, using the probability axioms, that $\Pr(A) \leq \Pr(B)$.

2. Two events A and B are such that $\Pr(A) = 1/4$, $\Pr(B) = 1/3$ and $\Pr(A \cup B) = 5/12$.

(i) Find $\Pr(A \cap B)$.

(ii) Are the events A and B mutually exclusive? Justify your answer.

(iii) Find $\Pr(B|A)$.

(iv) Are the events A and B independent? Justify your answer.

(v) Find $\Pr(A^c \cap B)$.

3. The events A and B are such that $\Pr(A) = 1/3$, $\Pr(A \cap B^c) = 1/4$ and $\Pr(A^c \cap B) = 1/6$.

(i) Find $\Pr(A \cap B)$.

(ii) Find $\Pr(B)$.

(iii) Are the events A and B independent? Justify your answer.

4. Using the result that $\Pr(A \cap B^c) = \Pr(A) - \Pr(A \cap B)$, prove that if the events A and B are independent then so are A and B^c.

5. Let A, B and C be any events in some sample space S.

(i) From the probability axioms show that

$$\begin{aligned}\Pr(A \cup B \cup C) &= \Pr(A) + \Pr(B) + \Pr(C) - \Pr(B \cap C) - \Pr(A \cap C) \\ &\quad - \Pr(A \cap B) + \Pr(A \cap B \cap C).\end{aligned}$$

(ii) From the definition of conditional probability, assuming that $\Pr(B \cap C) > 0$, show that

$$\Pr(A \cap B \cap C) = \Pr(C)\Pr(B|C)\Pr(A|B \cap C).$$

6. A doctor sees a patient whose symptoms suggest that he may be suffering from any one of three diseases, labelled B_1, B_2 and B_3, respectively. Suppose that the doctor's initial beliefs about what disease the patient is suffering from may be represented by the prior probabilities $\Pr(B_1) = 0.7$, $\Pr(B_2) = 0.2$ and $\Pr(B_3) = 0.1$.

A medical test, which gives a positive response, is carried out on the patient. Denote this event by E. If $\Pr(E|B_1) = 0.1$, $\Pr(E|B_2) = 0.2$ and $\Pr(E|B_3) = 0.9$, what according to Bayes' Theorem are the values of the doctor's posterior probabilities for the three diseases?