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Probability and Statistics

Very short summary (a lot is missing!)

Discrete and Continuous Random Variables

	If X is a discrete r.v. $x = 0, 1, 2, \ldots$	If X is a continuous r.v. $-\infty < x < \infty$
p.d.f.:	p_x	$f(x)$
Properties:	$p_x \geq 0$	$f(x) \geq 0$
	$\sum_{x=0}^{\infty} p_x = 1$	$\int_{-\infty}^{\infty} f(x)\, dx = 1$
$\Pr(X = x)$	p_x	0
c.d.f. $\Pr(X \leq x)$	$F_x = \sum_{r=0}^{x} p_r$	$F(x) = \int_{-\infty}^{x} f(u)\, du$
$\Pr(a \leq X \leq b)$ for $a \leq b$	$F_b - F_{a-1}$	$F(b) - F(a)$
$\mu \equiv E(X)$	$\sum_{x=0}^{\infty} x\ p_x$	$\int_{-\infty}^{\infty} x\ f(x)\, dx$
$E(g(X))$	$\sum_{x=0}^{\infty} g(x)\ p_x$	$\int_{-\infty}^{\infty} g(x)\ f(x)\, dx$
$\sigma^2 \equiv \mathrm{var}(X) = E(X^2) - \mu^2$	$\sum_{x=0}^{\infty} x^2\ p_x - \mu^2$	$\int_{-\infty}^{\infty} x^2\ f(x)\, dx - \mu^2$

`R` functions for p.d.f, c.d.f. and percentiles

Distribution	$\Pr(X = x)$	$\Pr(X \leq x)$	a such that $\Pr(X \leq a) = q$
Binomial $Bin(n, p)$	`dbinom`(x, n, p)	`pbinom`(x, n, p)	`qbinom`(q, n, p)
Poisson $P(\mu)$	`dpois`(x, μ)	`ppois`(x, μ)	`qpois`(q, μ)
Standard Normal $N(0, 1)$	0	`pnorm`(x)	`qnorm`(q)
Normal $N(\mu, \sigma^2)$	0	`pnorm`(x, μ, σ)	`qnorm`(q, μ, σ)
χ^2 with ν df	0	`pchisq`(x, ν)	`qchisq`(q, ν)
t with ν df	0	`pt`(x, ν)	`qt`(q, ν)

Discrete Distributions

Binomial Distribution $B(n,p)$

The probability of getting r success in n independent trials.

$$\Pr(X=r)=p_r=\binom{n}{r}p^r q^{n-r} \quad \text{for} \quad r=0,1,\ldots,n$$

where p is the probability of success, and $q=1-p$.

$\mu \equiv E(X)=np$, and $\sigma^2 \equiv \text{var}(X)=npq$.

Poisson Distribution $P(\mu)$

The number of events that occurs in a period of time or space, during which an average of μ events are expected to occur.

$$\Pr(X=r)=p_r=\frac{\mu^r e^{-\mu}}{r!} \quad \text{for} \quad r=0,1,\ldots,n$$

$\mu \equiv E(X)=\mu$, and $\sigma^2 \equiv \text{var}(X)=\mu$.

Continuous Distributions

Standard Normal Distribution $N(0,1)$

The *standard normal distribution* has p.d.f. $f(z)$ given by

$$f(z)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right) \qquad (-\infty<z<\infty)$$

$\mu \equiv E(X)=0$, and $\sigma^2 \equiv \text{var}(X)=1$.
The c.d.f. is given by:

$$\Phi(z)\equiv \Pr(Z\le z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left(-\frac{1}{2}u^2\right)du \qquad (-\infty<z<\infty).$$

$$\Phi(-z)=1-\Phi(z)$$

The $P\%$ *percentage point* $x(P)$ of $Z\sim N(0,1)$ is such that $\Pr(Z>x(P))=P\%=P/100$:

$$\Phi(x(P))=\Pr(Z\le x(P))=1-\frac{P}{100}$$

and hence

$$x(P)=\Phi^{-1}\left(1-\frac{P}{100}\right)$$

Normal Distribution $N(\mu, \sigma^2)$

The $N(\mu, \sigma^2)$ distribution has p.d.f

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad (-\infty < x < \infty).$$

$\mu \equiv E(X) = \mu$, and $\sigma^2 \equiv \mathrm{var}(X) = \sigma^2$.

If $X \sim N(\mu, \sigma^2)$ then

$$Z \equiv \frac{X-\mu}{\sigma} \sim N(0,1).$$

and

$$\Pr(X \leq x) = \Pr\left(Z \leq \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

The sample mean

$X_1, X_2, \ldots, X_n$ are a random sample of size n, i.e., independently and identically distributed r.v.s, with mean μ and variance σ^2. The sample mean $\bar{X}$ is

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \ .$$

$E(\bar{X}) = \mu$ and $\mathrm{var}(\bar{X}) = \dfrac{\sigma^2}{n}$

The Central Limit Theorem

Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independently and identically distributed r.v.s, each having a distribution with mean μ and variance σ^2. Define

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (n = 1, 2, 3, \ldots).$$

As $n \to \infty$,

$$\frac{(\bar{X}-\mu)}{\left(\dfrac{\sigma}{\sqrt{n}}\right)} \xrightarrow{D} Z,$$

where $Z \sim N(0,1)$.

Approximations of Distributions