(P_1) (2) $A, B \in K \Rightarrow A \cap B \in K,\ A \setminus B \in K$

- $A \cap B = \overline{\bar{A} \cup \bar{B}}$ (L. lui de Morgan)

(Din Def 4 $\Rightarrow$) $A, B \in K \Rightarrow \bar{A}, \bar{B} \in K \Rightarrow \bar{A} \cup \bar{B} \in K \Rightarrow \overline{\bar{A} \cup \bar{B}} \in K \Rightarrow A \cap B \in K$

$A \setminus B = A \cap \bar{B}$

- $A, \bar{B} \in K \Rightarrow A \cap \bar{B} \in K \Rightarrow A \setminus B \in K$.

(3) $\bigcap_{n=1}^{\infty} A_n \in K$ $(\forall)$ $A_n \in K,\ n \in \mathbb{N}^*$

Asemănător ca la (2), folosim $\bigcap_{n=1}^{\infty} A_n = \overline{\bigcup_{n=1}^{\infty} \bar{A}_n}$

(1) $\emptyset, \Omega \in K$

Din Def 4 $\Rightarrow K \neq \emptyset \Rightarrow (\exists)$ $A \in K \Rightarrow \bar{A} \in K$

$A, \bar{A} \in K \Rightarrow A \cup \bar{A} = \boxed{\Omega \in K}$

$\Omega \in K \Rightarrow \bar{\Omega} \in K$, $\bar{\Omega} = \emptyset$ $\Rightarrow$ $\boxed{\emptyset \in K}$

P_2

(1) $P(\bar{A}) = 1 - P(A)$ și $0 \le P(A) \le 1$ $(\forall)\ A \in K$

$A \cap \bar{A} = \emptyset$

$A \cup \bar{A} = \Omega \overset{\text{Def 5.i}}{\Longrightarrow} 1 = P(\Omega) = P(A \cup \bar{A}) \overset{\text{Def 5 iii}}{=} P(\bar{A}) + P(A)$

$\rightarrow P(\bar{A}) = 1 - P(A)$

$0 \le P(A)$ cf. Def 5 ii

$P(\bar{A}) \ge 0 \Rightarrow 1 - P(A) \ge 0 \rightarrow P(A) \le 1$

(2) $P(\emptyset) = 0$

$P(\emptyset) \underset{\uparrow}{=} 1 - P(\bar{\emptyset}) = 1 - P(\Omega) = 1 - 1 = 0$

în (1) luăm $A = \emptyset$

(3) $P(A \setminus B) = P(A) - P(A \cap B)$ $(\forall)\ A, B \in K$

$A \setminus B = A \cap \bar{B}$ $\quad B \cap \bar{B} = \emptyset$

$\Rightarrow (A \setminus B) \cap (A \cap B) = \emptyset$

$(A \setminus B) \cup (A \cap B) = A$ $\Big\} \Rightarrow$

Def 5 iii)

$\Rightarrow P(A) = P(A \setminus B) + P(A \cap B) \Rightarrow P(A \setminus B) = P(A) - P(A \cap B)$

ev. disjuncte

[Dc. $A \cap B = \emptyset \Rightarrow P(A \setminus B) = P(A)$]

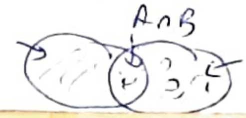
(4) $A \subseteq B \Rightarrow P(A) \le P(B)$ (P este monotonă)

$B = A \cup (B \setminus A)$

$A \cap (B \setminus A) = \emptyset$ $\Big\} \Rightarrow P(B) = P(A) + \underbrace{P(B \setminus A)}_{\ge 0}$

$\Rightarrow P(A) \le P(B)$

(5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$$A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$$

$$\underbrace{(A \cap \bar{B}) \cap (A \cap B)}_{\emptyset} \cap (B \cap \bar{A}) = \emptyset$$

$$\Rightarrow P(A \cup B) = P(A \setminus B) + P(A \cap B) + P(B \setminus A) \overset{(3)}{=} P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(B \cap A)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

<u>Exerciţiu:</u>

$$P(A \cup B \cup C) = P((A \cup B) \cup C) \overset{(5)}{=} P(A \cup B) + P(C) - P((A \cup B) \cap C)$$

$$= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C))$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - (P(A \cap C) + P(B \cap C) - P(A \cap B \cap C))$$

$$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$