Computer-Aided Analysis Across the Tonal Divide: Cross-Stylistic Applications of the Discrete Fourier Transform

The discrete Fourier transform (DFT) is a mathematical tool that can provide insight into large-scale harmonic motions in music of disparate styles. The DFT can be used to quantify differences in harmonic language by analyzing what Tymoczko refers to as *Macroharmonies*, or "the total collection of notes heard over moderate spans of time." Building on work by Quinn, Yust, Amiot, and Chiu, I apply the DFT to music by Mozart and Messiaen to illustrate how this single analytical methodology can describe the macroharmonic motions used by two vastly different composers.

When the DFT is applied to pitch classes, the resultant data are divided into six non-trivial components (indicated as f_1, f_2, f_3, etc.), each analogous to a different qualia or "flavor" of harmony: chromaticity (f_1), dyadicity (f_2), hexatonicity (f_3), octatonicity (f_4), diatonicity (f_5), and whole-tone quality (f_6). With the exception of the f_2 component (dyadicity), which refers to chords built from stacked tritones, all of these components correspond to readily recognized harmonic properties. The DFT calculate two features—magnitude (amount) and phase (direction)—for each component. If we imagine each component as a dinner plate balanced in the center on a single finger, when we add a pitch class to its prescribed position on the plate, the plate begins to tilt more (magnitude) in that direction (phase). The magnitude of each component corresponds to its salience, while phase correlates with specific pitch-class collections. For example, in the case of the f_5 component (diatonicity), the specific phase indicates a particular diatonic collection.

To demonstrate this methodology, I will examine two passages of music: the exposition from the first movement of Mozart's String Quartet in C Major, K. 157, and the theme from Messiaen's *Theme and Variations for Violin and Piano*. I create a sliding window that passes over the score encoded as XML, utilizing the music21 Python library. My program performs the DFT on the pitch classes within each window before it slides over by a beat to repeat the process. The numerical data can be conveniently represented in visualizations that capture both the magnitude and phase fluctuations for each Fourier component.

The results of the DFT macroharmonic analysis of the Mozart excerpt show that, as expected the f_5 component (diatonicity) overwhelmingly has the highest magnitude throughout. An analysis of the phase of the f_5 component shows that the exposition begins with the C major diatonic collection and shifts to the G major diatonic collection roughly a third of the way through. A wide, 16-beat sliding window shows these key areas as phase values and the shift between them clearly and with almost now deviation from these collections. Running the same analysis but with a smaller, 4-beat sliding window returns a much more granular view of the exposition. With this more refined analysis, the phase begins to move in ways that indicate changes in harmonic function. In other words, adjusting the size of the sliding window allows the analysis to shift from examining large-scale, macroharmonic motions to more localized harmonic activity as desired.

Turning to Messiaen's *Theme and Variations*, we have a very different harmonic landscape. The theme is divided into three formal sections, each dominated by a different set of Fourier components. The first and third sections are characterized by the f_3 (hexatonicity) and f_6 (whole-tone quality) components, which together correspond with

Messiaen's third mode of limited transposition. Characteristically of this mode, the phase of the f_3 component fluctuates as pitch classes are added or removed while the phase of the f_6 component remains constant. The middle section exclusively uses the octatonic collection, Messiaen's second mode of limited transposition, and is thus dominated exclusively by the f_4 component (octatonicity). Here, the phase shows that this passage specifically uses the OCT01 collection, and only fluctuates as members from one of the constituent diminished-seventh chords are emphasized over the other.

This single analytical methodology can be effectively applied to music with vastly different harmonic languages, and from different eras, genres, and styles. These analyses serve as a proof of concept that my computational methodology confirms and aligns with well-established music-theoretical practices within exhaustively studied repertoire. The program is adaptable to any piece of music encoded as in XML, which means there are tremendous implications for future research, including corpus studies, comparisons of different styles, and adaptations for understudied quarter-tone and other microtonal repertoire.