Response to reviewer #1

- **Question/remark:**
  Cold electrons are in local thermal equilibrium → not true that cold electron assumption means thermal equilibrium

  Response:
  We agree to this statement: thermal equilibrium would actually imply a Maxwellian distribution for the electrons which is contradictory to the cold electron assumption. The latter means, as suggested, a delta distribution in velocity space which yields a fluid closure when taking the first two moments of the Vlasov equation in velocity space. We clarified this in **Sec. 2.1, page 2, lines 53 – 61**.

- **Question/remark:**
  What about divergence constraints div(B) = 0 and div(E) = rho_c/eps0 in the discrete cases?

  Response:
  Div(B) = 0 is satisfied for the two schemes due to the assumption of variations in one spatial direction only (z-direction). From Faraday‘s law (1g) it then follows that B_z does not change it time. If we take it initially to be zero, it will remain zero for all times and since B_x and B_y only depend on z, the divergence-free constraint is satisfied trivially.
  Gauss law div(E) = rho/eps0 is not satisfied exactly by both schemes. We are aware that even for a uniform distribution function in space on the continuous level, the random particle initialization leads to local charge and current densities (also in z-direction). According to Gauss law, the former would generate a „noisy“ electric field in z-direction which would enter the Vlasov equation, thus leading to an acceleration of particles in z-direction. However, in this work we are only interested in electromagnetic phenomena with pure transverse field components and no perturbations of charge and current densities in z-direction are considered. By doing this, we are aware that we cannot use the schemes in this form to simulate electrostatic phenomena like electron Landau damping. We added this statement in **Sec. 3, page 6, lines 151 – 154**.

- **Question/remark:**
  In Sec. 3.1: Is a global matrix inversion needed to update the FE coefficients? Does this impact the efficiency of the scheme?

  Response:
  Since we are using a Crank-Nicolson implicit time stepping scheme, we need to invert a global matrix (the matrix on the left-hand side in equation (24)). In practice, we do this by performing a LU decomposition of the matrix at the beginning of a simulation (the matrix is constant in time!). This allows us to solve for the new FE coefficients in each time step very quickly by forward and backward substitution. All this happens on a time scale well below the one needed for pushing the particles and accumulating the current densities. This means that this does not impact the efficiency of the scheme. We clarified this in **Sec. 3.1, page 8, lines 188 – 192.**

- **Question/remark:**
  How hard is it to extend the schemes to fully nonlinear problems?

  Response:
  Extending the standard scheme to also treat nonlinear fluid equations would in principle not be too hard. However, if one still wanted to use an implicit time stepping scheme (what we do here) one would have to solve a nonlinear system of equations in each time step e.g. by Picard iteration or Newton's method. However, extending the structure-preserving scheme in way that it still preserves the noncanonical Hamiltonian structure would be much harder because nonlinear terms in the fluid equations like the (u_c*nabla)*u_c term are not in the right polynomial spaces V_0, respectively V_1, when using the discrete version for u_c. How to apply the projectors (see Fig. 3) in a way that the noncanonical Hamiltonian structure is still preserved is actually an ongoing work.
  As already mentioned, in the present work, we only deal with transverse components with respect to the direction of wave propagation (z-direction). This has the consequence that the nonlinear term (u_c*nabla)*u_c term vanishes and we do not have to deal with it. This is why we decided not to mention it in the text.

- **Question/remark:**
  Run-times of the two schemes? Do more particles in the standard PIC while having the same run-time as the structure-preserving PIC lead to same results?

  Response:
  It is quite hard to compare the run times for the two schemes quantitatively because the two schemes were coded independently from each other and differ quite a lot in the coding style. In principle, however, one can state that there is a slight increase of computational costs for the structure-preserving scheme when computing the line integrals along the particle trajectories as shown in the appendix for the sub-step 6 (equations (C.12h) and (C.12i)). We added this statement in **Sec. 3.1, page 18, lines 480 – 482**. Qualitatively, we claim that the standard PIC can never reach the good properties of the structure-preserving one no matter how many particles are used or how much the mesh is refined because the strength of the structure-preserving scheme is that it preserves the noncanonical Hamiltonian structure independently of the mentioned parameters.

Response to reviewer #2

- **Question/remark:**
  Additional run with high k-mode

  Response:
  We have chosen this particular k (see Tab. 2) for the single mode test case because from the dispersion relation an instability is expected. Going to significantly higher wave numbers would mean that there is no instability anymore which can be seen by looking at the growth rates in Fig. 1b with respect to the wavenumber. It is true that it would be interesting to see if we get a mode with nearly zero real frequency if we choose a high k-mode for the standard scheme with same finite element spaces for all quantities. But in our opinion this is already clear from the wave spectrum in Fig. 8.

- **Question/remark:**
  Typographical corrections:
  - Replace the word “proof” with the verb “prove” throughout the paper
  - Replace “in” with “is” in the caption of Fig. 3
  - Replace “e_j” with “e_i” in equation (38)

  Response:
  We corrected these typos:
  - We replaced the noun “proof” with the verb “prove” **(Sec. 3.2, page 16, lines 358 and 371)**
  - We replaced “in” with “is” in the caption of Fig. 3 **(Sec. 3.2, page 11)**
  - We replaced “e_j” with “e_i” in equation (38) **(Sec. 3.2, page 13)**

Response to reviewer #3

- **Question/remark:**
  Typographical corrections:
  - Replace “v” with “u_c” in equation (2a)
  - Remove tilde over B in equation (7)
  - References in Sec. 3.1 should be for equations (8) instead of equations (6)

  Response:
  We corrected the first typo and replaced “v” with “u_c” in equation (2a) **(Sec. 2.2, page 3).** However, we think that the second and third raised points are correct in the current manuscript. The given energy (7) is the one of the wave, excluding the background magnetic field, and one can show that this energy is conserved by equations (6) (as explained in the text) without assuming transverse components only. The references in Sec. 3.1 must be for equations (6) because there we keep the full Vlasov equation (6b) with the total magnetic field B=B_0 + B_tilde. That is the model we discretize and simulate. In equations (8) the Vlasov equation (8b) is linearized as well which is just for the purpose of showing how the linear dispersion relation (9) can be obtained.

- **Question/remark:**
  Is there any challenge in using higher order shape functions within this type of scheme?

  Response:
  No, higher order shape functions can be used in both schemes without any additional difficulties. We added one sentence to emphasize that **(Sec. 4.1, page 19, lines 442-443)**. However, in this work we chose linear basis functions for both schemes for two reasons: First, B-splines and Lagrange basis functions coincide in this case which makes it more sensible to compare the two schemes to each other. Second, we wanted to demonstrate the differences in the schemes for a rather simple setup since using a very fine mesh and/or higher order basis functions would of course improve the standard scheme meaning that the differences would not be so visible.

- **Question/remark:**
  Does the standard PIC algorithm conserve charge and thus Gauss law?

  Response:
  Gauss’s law div(E) = rho/eps0 is not satisfied exactly by both schemes. We are aware that even for an initial, uniform distribution function in space on the continuous level, the random particle initialization leads to local charge and current densities (also in z-direction). According to Gauss law, the former would generate a „noisy“ electric field in z-direction which would enter the Vlasov equation, thus leading to an acceleration of particles in z-direction. However, in this work we are only interested in electromagnetic phenomena with pure transverse field components and no perturbations of charge and current densities in z-direction are considered. By doing this, we are aware that we cannot use the schemes in

this form to simulate electrostatic phenomena like electron Landau damping. We added this statement in **Sec. 3, page 6, lines 151 – 154**.

- **Question/remark:**
  Conserved Casimirs in the discrete FEEC scheme (Gauss law)?

  Response:
  See question above: Gauss's law is not satisfied exactly and therefore it is not a conserved Casimir of the system. The same is true for momentum as explained in ref. [2], equation (2.26), page 8.

- **Question/remark:**
  Is the slow in increase of the error in Strang splitting due to choice of explicit advance?

  Response:
  We would deny that. The first-order Lie-Trotter splitting scheme (67) is fully explicit as well and we do not see an increase in the error of the conservation of energy in Fig. 7. Interestingly, this slow drift is not present when using higher order shape functions which can be seen in the plot below where we used cubic polynomials (p = 3). However, for the reasons we stated in one of the above questions, we would like to keep the comparison for linear basis functions in the paper. Why the Strang splitting seems to be more sensitive to the choice of the polynomial degree than the Lie-Trotter splitting, is a question we cannot answer at the moment and which needs to be investigated further.