

Linear MHD using discrete differential forms

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$$\tilde{\mathbf{J}}_h(\boldsymbol{\xi}) = qDF^\top(\boldsymbol{\xi}) \int \tilde{f}_h(\boldsymbol{\xi}, \mathbf{v}, t) \mathbf{v} \, d\mathbf{v} = qDF^\top(\boldsymbol{\xi}) \sum_{p=1}^{N_p} \omega_p \frac{\delta(\boldsymbol{\xi} - \boldsymbol{\xi}_p)}{|J_F(\boldsymbol{\xi}_p)|} \mathbf{v}_p$$

For Ampere's law, we insert (??) and (??) into (??) and choose the basis functions $\{\tilde{\Lambda}_i^1\}_{i=1..3}$ as test-functions

$$\begin{aligned} & \frac{d}{dt} \int_{\tilde{\Omega}} \left(N(\boldsymbol{\xi}) \tilde{\Lambda}^1(\boldsymbol{\xi}) \right)^\top N(\boldsymbol{\xi}) \tilde{\Lambda}^1(\boldsymbol{\xi}) |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi} = \\ & \int_{\tilde{\Omega}} \left(\frac{DF(\boldsymbol{\xi})}{J_F(\boldsymbol{\xi})} \nabla_{\boldsymbol{\xi}} \times \tilde{\Lambda}^1(\boldsymbol{\xi}) \right)^\top \frac{DF(\boldsymbol{\xi})}{J_F(\boldsymbol{\xi})} \tilde{\Lambda}^2(\boldsymbol{\xi}) |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi} - \int_{\tilde{\Omega}} \left(N(\boldsymbol{\xi}) \tilde{\Lambda}^1(\boldsymbol{\xi}) \right)^\top N(\boldsymbol{\xi}) \tilde{\mathbf{J}}_h(\boldsymbol{\xi}) |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi}. \end{aligned}$$

Next, we use the relation (??) for the $\nabla \times$ and insert the transformed current (??)

$$\begin{aligned} & \int_{\tilde{\Omega}} \tilde{\Lambda}^1(\boldsymbol{\xi})^\top N(\boldsymbol{\xi})^\top N(\boldsymbol{\xi}) \tilde{\Lambda}^1(\boldsymbol{\xi}) |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi} = \int_{\tilde{\Omega}} (\tilde{\Lambda}^2(\boldsymbol{\xi}))^\top DF(\boldsymbol{\xi})^\top DF(\boldsymbol{\xi}) \tilde{\Lambda}^2(\boldsymbol{\xi}) \frac{1}{|J_F(\boldsymbol{\xi})|} d\boldsymbol{\xi} \\ & \quad - \int_{\tilde{\Omega}} \tilde{\Lambda}^1(\boldsymbol{\xi})^\top N(\boldsymbol{\xi})^\top q \sum_{p=1}^{N_p} \omega_p \frac{\delta(\boldsymbol{\xi} - \boldsymbol{\xi}_p)}{|J_F(\boldsymbol{\xi}_p)|} \mathbf{v}_p |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi} \\ & \Leftrightarrow \int_{\tilde{\Omega}} \tilde{\Lambda}^1(\boldsymbol{\xi})^\top N(\boldsymbol{\xi})^\top N(\boldsymbol{\xi}) \tilde{\Lambda}^1(\boldsymbol{\xi}) |J_F(\boldsymbol{\xi})| d\boldsymbol{\xi} = \int_{\tilde{\Omega}} \tilde{\Lambda}^2(\boldsymbol{\xi})^\top DF(\boldsymbol{\xi})^\top DF(\boldsymbol{\xi}) \tilde{\Lambda}^2(\boldsymbol{\xi}) \frac{1}{|J_F(\boldsymbol{\xi})|} d\boldsymbol{\xi} \\ & \quad - \sum_{p=1}^{N_p} q \omega_p \tilde{\Lambda}^1(\boldsymbol{\xi}_p)^\top N(\boldsymbol{\xi}_p)^\top \mathbf{v}_p. \end{aligned}$$