# WP3: First Empirical Slice Through the Regime Response Surface $R(k, S)$

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**Abstract**

We present the first empirical mapping of the regime response surface $R(k, S)$ along the redshift-space distortion (RSD) trajectory. Using $f\sigma_8(z)$ measurements from BOSS DR12, eBOSS DR16, and DESI Y1 ($N = 12$), we constrain one coordinate on the gravitational response surface: $R(k \approx 0.13\,h/\mathrm{Mpc}, S \approx 0.30) \approx +0.30$. Under Akaike Information Criterion (AIC), $\Lambda$CDM ($R = 0$) is preferred by $\Delta\mathrm{AIC} = -3.8$, indicating that the data are consistent with zero response within complexity penalty. This work is *cartography*—a measurement in a new coordinate system—not a model comparison or validation claim. WP3 provides the first entry in a planned response atlas mapping gravitational behavior across structural regimes.

**Keywords:** Energy-Flow Cosmology, modified gravity, regime response surface, growth rate, $f\sigma_8$, RSD
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## 1 Introduction

Standard approaches to testing modified gravity theories typically parameterize deviations from General Relativity using time-dependent functions such as $\mu(a)$ or scale-and-time-dependent functions $\mu(k, z)$. These parameterizations treat modifications as perturbations to be constrained, leading naturally to model comparison frameworks where modified gravity "competes" with $\Lambda$CDM.

The $R(k, S)$ framework (Magnusson, 2026a) introduces a fundamentally different perspective. Rather than parameterizing deviations from GR, it establishes a *coordinate system* for gravitational response based on:

- $k$: spatial scale (wavenumber)
- $S$: structural maturity (a state variable encoding entropy/structure evolution)

The gravitational response is then:

$$\mu = 1 + R(k, S) \tag{1}$$

where $R(k, S)$ is a response surface that can take positive, negative, or zero values depending on the structural regime.

This reframing transforms the question from "Does modified gravity fit better than $\Lambda$CDM?" to "Where does this observation sit in regime-response space?" The result is *cartography*, not competition.

## 2 The $R(k, S)$ Framework

### 2.1 From Time to Structural Maturity

In standard cosmology, evolution is parameterized by cosmic time or equivalently scale factor $a$ and redshift $z$. The $R(k, S)$ framework replaces this with *structural maturity* $S$, defined as a state variable that tracks the universe's progression from singularity ($S = 0$) to altularity ($S = 1$).

For this WP3 feasibility slice, we adopt a monotonic proxy mapping:

$$S(z) = \frac{1}{1 + (z/z_{\text{mid}})^2} \tag{2}$$

where $z_{\text{mid}} \approx 1.5$ corresponds to peak structure formation.

**Important caveat:** This $S(z)$ is a convenient monotonic proxy for structural maturity, not yet a physically derived observable. In the full $R(k, S)$ framework, $S$ should be computed from nonlinear growth statistics (e.g., $\sigma/\sigma_{\text{lin}}$, halo mass function evolution, or entropy production rates). The present work demonstrates the methodology using this proxy; future work will replace it with observationally grounded $S$ definitions. Without this replacement, $S$ remains a reparameterized redshift.

### 2.2 Scale Dependence

Different observational probes are sensitive to different spatial scales $k$. Redshift-space distortions (RSD) primarily probe the quasi-linear regime at $k \sim 0.1\,h/\text{Mpc}$. The effective wavenumber varies mildly with redshift:

$$k_{\text{eff}}(z) \approx 0.1 \times (1 + 0.15z) \quad [h/\text{Mpc}] \tag{3}$$

### 2.3 WP3 as a Trajectory

WP3 does not test the entire $R(k, S)$ surface. It tests *one trajectory* through this space:

$$\text{WP3 trajectory:} \quad (k_{\text{eff}}(z), S(z)) \quad \text{for } z \in [0.3, 1.5] \tag{4}$$

This corresponds to:

- $k \in [0.1, 0.13]\,h/\text{Mpc}$
- $S \in [0.3, 0.7]$

## 3 Data and Method

### 3.1 Dataset

We use 12 $f\sigma_8(z)$ measurements from three surveys:

**Limitation:** We use diagonal errors only; covariance matrices are not included. Data hash: `c76720490b99dbd6`.

**Statistical caveat:** Because no covariance matrices are used, the $\chi^2$, AIC, and BIC values reported here are approximate and should be interpreted as indicative only. A full analysis would require the published covariance matrices from each survey.

### 3.2 Models

**$\Lambda$CDM ($R = 0$):**

$$f\sigma_8(z) = \Omega_m(z)^\gamma \times \sigma_8 \times D(z) \tag{5}$$

with 3 free parameters: $\sigma_8$, $\Omega_m$, $\gamma$.

Table 1: $f\sigma_8(z)$ data used in this analysis

| Survey | $z$ | $f\sigma_8$ | $\sigma$ | Reference |
|---|---|---|---|---|
| BOSS DR12 | 0.38 | 0.497 | 0.045 | Alam et al. (2017) |
| BOSS DR12 | 0.51 | 0.458 | 0.038 | Alam et al. (2017) |
| BOSS DR12 | 0.61 | 0.436 | 0.034 | Alam et al. (2017) |
| eBOSS DR16 | 0.70 | 0.473 | 0.041 | eBOSS (2021) |
| eBOSS DR16 | 0.85 | 0.315 | 0.095 | eBOSS (2021) |
| eBOSS DR16 | 1.48 | 0.462 | 0.045 | eBOSS (2021) |
| DESI Y1 | 0.295 | 0.447 | 0.028 | DESI (2024) |
| DESI Y1 | 0.510 | 0.470 | 0.025 | DESI (2024) |
| DESI Y1 | 0.706 | 0.447 | 0.022 | DESI (2024) |
| DESI Y1 | 0.930 | 0.437 | 0.024 | DESI (2024) |
| DESI Y1 | 1.184 | 0.389 | 0.032 | DESI (2024) |
| DESI Y1 | 1.491 | 0.297 | 0.055 | DESI (2024) |

$R(k, S)$ **slice:**

$$f\sigma_8(z) = \Omega_m(z)^\gamma \times \sqrt{\mu(z)} \times \sigma_8 \times D(z) \tag{6}$$

where $\mu(z)$ is parameterized using a local Gaussian basis function to probe response amplitude along the trajectory:

$$\mu(z) = 1 + \Delta R \exp\left[-\frac{(z - z_{\text{peak}})^2}{2 \times 0.6^2}\right] \tag{7}$$

**Important caveat:** This Gaussian profile is a phenomenological basis function chosen for mathematical convenience, not an EFC prediction. It allows us to measure whether the data prefer non-zero response amplitude ($\Delta R \neq 0$) at some characteristic redshift ($z_{\text{peak}}$). The true $R(k, S)$ surface shape must ultimately be derived from first principles or reconstructed non-parametrically.

This model has 5 free parameters: $\sigma_8$, $\Omega_m$, $\Delta R$, $z_{\text{peak}}$, $\gamma$.

### 3.3 Optimization

We use differential evolution (seed=42, maxiter=1000, tol=$10^{-8}$) to minimize $\chi^2$.

## 4 Results

Table 2: Model comparison results

| Model | $k$ (params) | $\chi^2$ | dof | AIC | BIC |
|---|---|---|---|---|---|
| ΛCDM | 3 | 9.36 | 9 | 15.4 | 16.8 |
| $R(k, S)$ slice | 5 | 9.12 | 7 | 19.1 | 21.4 |

**Best-fit parameters:**

- ΛCDM: $\sigma_8 = 0.775$, $\Omega_m = 0.250$, $\gamma = 0.450$
- $R(k, S)$: $\sigma_8 = 0.797$, $\Omega_m = 0.276$, $\Delta R = +0.30$, $z_{\text{peak}} = 2.30$, $\gamma = 0.61$

**Model comparison:**

- $\Delta\chi^2 = +0.24$ (marginal improvement for $R(k, S)$)
- $\Delta\text{AIC} = -3.8$ ($\Lambda$CDM preferred)
- $\Delta\text{BIC} = -4.6$ ($\Lambda$CDM preferred)

**Coordinate on response surface:**

$$R(k \approx 0.13\,h/\text{Mpc}, S \approx 0.30) \approx +0.30 \tag{8}$$

# 5 Interpretation

## 5.1 What WP3 Shows

WP3 demonstrates that:

1. A non-zero response $R \approx +0.30$ is *allowed* by the data at this trajectory
2. However, $R = 0$ ($\Lambda$CDM) is *preferred* when complexity is penalized
3. The measurement is *prior-sensitive*: the location on the surface is not tightly constrained

## 5.2 What WP3 Does Not Show

WP3 does *not*:

- Validate the $R(k, S)$ framework
- Falsify the $R(k, S)$ framework
- Claim that EFC is better than $\Lambda$CDM
- Provide a complete map of the response surface

## 5.3 Epistemic Status

This is **cartography**: a measurement in a new coordinate system. The observable ($f\sigma_8$) becomes a coordinate; the theory becomes a geometry.

WP3 provides *one noisy point* on the response surface—exactly what early cartography looks like.

# 6 The Response Atlas

WP3 is the first entry in a planned response atlas:

Table 3: Response Atlas: planned trajectories through $R(k, S)$

| Probe | $k$ window | $S$ range | Status |
|---|---|---|---|
| RSD / $f\sigma_8$ | $\sim 0.1\,h/\text{Mpc}$ | $[0.3, 0.7]$ | **WP3 (this work)** |
| Weak lensing | different $k$ | overlapping $S$ | WP4 (ready) |
| Clusters | smaller $k$ | higher $S$ | Planned |
| CMB | large scales | $S \approx 0$ | Boundary condition |

Each probe maps a different trajectory through $(k, S)$ space. Together, they will constrain the shape of the response surface.

## 7 Conclusion

WP3 provides the first empirical coordinate on the regime response surface $R(k, S)$:

$$R(k \approx 0.13, S \approx 0.30) \approx +0.30 \quad \text{(prior-sensitive, AIC prefers } R = 0) \tag{9}$$

This is not a validation or falsification of Energy-Flow Cosmology. It is a measurement that locates one class of observations within a new coordinate system for gravitational response.

The shift from "Does modified gravity fit better?" to "Where does this observation sit in regime-response space?" represents a move from curve fitting to state-space mapping—from parameter tuning to theory-driven cartography.

## Data Availability

Manifest and analysis code: `wp3_rks_manifest.json`
Data hash: `c76720490b99dbd6`
Run ID: `wp3_rks_v2.0`

## Acknowledgments

This work made use of public data from BOSS, eBOSS, and DESI collaborations.

## References

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