R(k,S) as a Regime Response Surface in Energy-Flow Cosmology

A Unified Framework for Structure-Dependent Gravitational Response

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Scope: This is a theoretical framework paper. It introduces a coordinate system for gravitational response, derives testable structure, and specifies falsification criteria. It does not perform quantitative fits, MCMC analysis, or Boltzmann-code implementation. Those are subsequent steps that this framework enables.

Abstract

We extend Energy-Flow Cosmology (EFC) phenomenology from time-dependent modified gravity parameters $\mu(a)$ and $\mu(k, z)$ to a state-dependent response surface $R(k, S)$, where S is a derived, model-dependent state variable quantifying nonlinear structure accumulation. This synthesizes two prior results: the regime-dependent transition estimator (Fugaku–DESI) and the operational $\mu(k, z)$ formalism (weak lensing phenomenology). We derive falsification criteria against DES Y6 data. The core prediction: the L1→L2 transition (CMB to late-universe) explains the S_8 tension, not probe-dependent variation within L2. This paper presents theoretical infrastructure for testing, not claimed validation.

Core Postulate: The gravitational response μ in late-universe structure formation depends on the accumulated structural state S, not merely on cosmic time.

Formally: $\mu = \mu(k, S)$ where $S = S(z)$ is a derived measure of nonlinear maturity.

This implies: (1) $\mu \approx 1$ when $S \approx 0$ (linear regime, CMB epoch), and (2) $\mu \neq 1$ when $S > 0$ (nonlinear regime, late universe).

The framework is falsifiable: a single $R(k, S)$ surface must describe all late-universe probes consistently.

Regime Coordinate Principle: *Different cosmological probes do not measure different physics—they sample different coordinates (k, S) on a single gravitational response surface.*

1 Introduction: What Are We Modeling?

Cosmological observations present a consistent pattern:

Regime	Observation	Gravitational behavior
L1 (CMB, $z \sim 1100$)	Planck temperature + polarization	$G_{\text{eff}} \approx G$ (standard GR)
L2 (Late universe, $z < 2$)	Weak lensing, RSD, cluster counts	$G_{\text{eff}} > G$ (enhanced growth)

The S_8 tension—where late-universe probes consistently measure $S_8 \approx 0.78$–0.79 while CMB implies $S_8 \approx 0.83$—can be interpreted as evidence that **gravitational response depends on structural state**.

This interpretation does not require new gravity everywhere. It requires a **response function** that activates when the universe transitions from linear to nonlinear structure formation.

Two prior EFC papers establish the foundation:

1. **Fugaku–DESI Transition Metric** [1]: Demonstrates that a single $\mu(a)$ function satisfying $\mu \approx 1$ at recombination and $\mu > 1$ at late times can reinterpret apparent matter density offsets as regime-dependent gravitational coupling.

2. **EFC Weak Lensing Phenomenology** [2]: Introduces an operational closure (Postulate A) yielding $\mu(k, z)$ in standard cosmological perturbation form, enabling direct comparison with survey data.

The present work extends these results by replacing **cosmic time** (z) with **structural state** (S) as the fundamental variable controlling gravitational response.

2 Existing EFC Foundation

2.1 $\mu(a)$: Regime-Dependent Coupling (Fugaku–DESI)

The Fugaku–DESI analysis defines a transition estimator:

$$\Delta_F \equiv \int W(a)\, [\mu(a) - 1]\, d \ln a \tag{1}$$

where $W(a)$ represents the observational sensitivity window. Key results:

- $\mu \approx 1$ at recombination: Preserves CMB constraints
- $\mu > 1$ at late times: Consistent with enhanced structure growth
- $\Delta_F \approx 0.1$: Empirical constraint on integrated transition strength

This establishes that **one transition function can connect multiple regimes**.

2.2 $\mu(k, z)$: Operational Field Closure (Weak Lensing)

The weak lensing phenomenology paper derives:

$$k^2 \Phi = -4\pi G a^2 \mu(k, z)\, \bar{\rho}\, \delta \tag{2}$$

where $\mu(k, z)$ emerges from Postulate A coupling entropy production to matter density. This places EFC in **standard cosmological perturbation formalism**.

3 S as a Structural State Variable

3.1 Physical Motivation

Cosmic time z is a **proxy** for what physically matters: the degree of nonlinear structure accumulation. We therefore define a structural maturity parameter:

$$S(z) \equiv \ln \left[\frac{\sigma^2(R_8, z)}{\sigma^2_{\rm lin}(R_8, z)} \right] \tag{3}$$

where $R_8 = 8\, h^{-1}$Mpc is the standard smoothing scale and $\sigma^2_{\rm lin}$ is the linear-theory prediction.

3.2 Operational Definition

For practical implementation:

$$S(z) = \ln\left[\frac{\sigma_8^2(z)}{\sigma_{8,\mathrm{lin}}^2(z)}\right] = \ln\left[\frac{D^2(z)\cdot\sigma_8^2(z=0)}{D_{\mathrm{lin}}^2(z)\cdot\sigma_{8,\mathrm{lin}}^2(z=0)}\right] \tag{4}$$

This is deterministic given a background cosmology—S **is not a free parameter per data point**.

3.3 Regime Interpretation

Regime	S **value**	**Physical meaning**
L1 (CMB)	$S \approx 0$	Linear universe, $\sigma \approx \sigma_{\mathrm{lin}}$
L1→L2 transition	$0 < S < 1$	Nonlinear growth initiating
Late L2	$S > 1$	Structure-dominated dynamics

This makes the model **regime-based** (EFC-consistent), **operationally defined** (computable under a specified nonlinear mapping), and **physically grounded** (measures accumulated nonlinearity).

Important caveat: S depends on the choice of smoothing scale (R_8), nonlinear model (halofit, emulator, or N-body calibration), and reference linear prediction. It is therefore a *derived* state variable, not a direct observable.

Robustness: Different nonlinear prescriptions yield monotonic transformations of S, preserving regime ordering even when the exact numerical scale varies. The physically meaningful content is the *ordering* of states ($S_1 < S_2$ implies less nonlinear structure), not the absolute value.

4 The $R(k, S)$ Response Surface

4.1 Definition

We generalize the modified gravity parameter to:

$$\mu(k, S) = 1 + R(k, S) \tag{5}$$

where $R(k, S)$ is the **regime response function** encoding how gravitational coupling deviates from GR as a function of both scale k and structural state S.

Critical constraint: $R(k, S)$ is not a free function per probe; it is a single global surface that must simultaneously describe all late-universe observables. This is what makes the framework falsifiable.

4.2 Minimal Parametrization

A first-order Taylor expansion around a reference point (k_0, S_0):

$$R(k, S) = R_0\left[1 + a\ln\left(\frac{k}{k_0}\right) + b(S - S_0) + c\ln\left(\frac{k}{k_0}\right)(S - S_0)\right] \tag{6}$$

This expansion is phenomenological and valid only locally in (k, S)-space.

Parameters:

- R_0: Overall response amplitude

- a: Scale dependence (positive = stronger response at small scales)
- b: State dependence (positive = stronger response at higher S)
- c: Scale–state interaction

Limiting case: Setting $c = 0$ reduces the model to separable scale and state dependence, providing a controlled null hypothesis.

Theoretical justification: Locally, any smooth response surface can be expanded to first order; this represents the most general phenomenological form near a reference regime point. The logarithmic scale dependence $\ln(k/k_0)$ is standard in modified gravity phenomenology.

Reference point choice: (k_0, S_0) should be chosen where data are most constraining—typically $k_0 \sim 0.1\, h/\text{Mpc}$ and S_0 corresponding to $z \sim 0.5$. This is a coordinate choice, not an additional free parameter.

4.3 Effect on Structure Growth

In linear perturbation theory, $\mu > 1$ enhances the gravitational source term, modifying the growth equation. To first order, positive $R(k, S)$ leads to *enhanced* gravitational response relative to GR, with magnitude depending on the integrated response history.

Note: The precise mapping from $R(k, S)$ to observable quantities (S_8, $f\sigma_8$, etc.) requires solving the modified perturbation equations. The relation is not a simple multiplicative correction.

5 DES Y6 Consistency and the L1↔L2 Test

5.1 Key Observational Fact

Recent DES Year 6 analyses report:

- 3×2pt $S_8 \approx 0.79$ (with uncertainty ∼0.01)
- CMB (Planck+ACT+SPT) $S_8 \sim 0.83$
- Tension: $\sim 2\sigma$ in full parameter space

Critically, DES Y6 shows **good internal consistency** between cosmic shear, galaxy clustering, and galaxy-galaxy lensing. The tension is not between L2 probes—it is between L1 (CMB) and L2 (late universe).

5.2 Model Prediction

Epoch	S **value**	$R(k, S)$	**Observed** S_8
CMB ($z \sim 1100$)	$S \approx 0$	$R \approx 0$	∼0.83 (Planck)
DES ($z \sim 0.3$–1.0)	$S > 0$	$R > 0$	∼0.79 (suppressed)

The sign of R determines whether inferred S_8 is enhanced or suppressed relative to the ΛCDM baseline. In this framework, $R > 0$ (enhanced μ) leads to *lower* inferred S_8 from late-universe probes. The physical intuition: a stronger effective gravitational coupling allows observed clustering to arise from a lower primordial fluctuation amplitude—gravity does more work, so less initial "seed" is needed.

The **gradient in** S explains the L1→L2 offset without requiring different physics for different L2 probes.

5.3 Why L2 Probes Are Internally Consistent

Different late-universe probes (WL, RSD, clusters) sample different (k, S) windows on the same response surface. If the c coefficient (k×S interaction) is small, they see similar R values. This explains:

- WL and galaxy clustering agreeing within L2
- Both disagreeing with CMB (different S regime)

6 Visualization: The $R(k, S)$ Response Surface

Figure 1 illustrates the regime response surface and probe coverage.

7 Falsification Protocol

7.1 Test 1: Tomographic Consistency

DES Y6 provides tomographic bins across redshift. **All bins must follow a single global** $R(k, S)$ with fixed (R_0, a, b, c).

Failure criterion: Fitting requires different parameters per bin.

7.2 Test 2: Trend Sign Matching

When two probes are matched to the same k-window AND the same S/z-window, they must show the same trend sign in any residuals from ΛCDM.

Failure criterion: Probes with overlapping (k, S) coverage show opposite deviations.

7.3 Test 3: Predicted S-Gradient

Given $R(k, S)$, compute the predicted S_8 offset as a function of effective redshift:

$$\Delta S_8(z_{\text{eff}}) \propto R(k_{\text{eff}}, S(z_{\text{eff}})) \tag{7}$$

This predicted gradient must be consistent with observed redshift trends in tomographic analyses.

Failure criterion: Predicted gradient disagrees with observed redshift evolution.

8 Epistemic Status

Component	**Status**
Regime-dependent μ	✓ Supported (Fugaku–DESI)
$\mu(k, z)$ formalism	✓ Supported (Weak Lensing paper)
S as structural state	△ New, physically motivated extension
$R(k, S)$ response surface	△ New model structure
$k \times S$ interaction	△ Hypothesis, requires testing
L1↔L2 as primary tension locus	✓ Supported by DES Y6 internal consistency

This framework does not claim validation. It provides the **minimal theoretical infrastructure** required to test whether EFC can explain structure growth phenomenology.

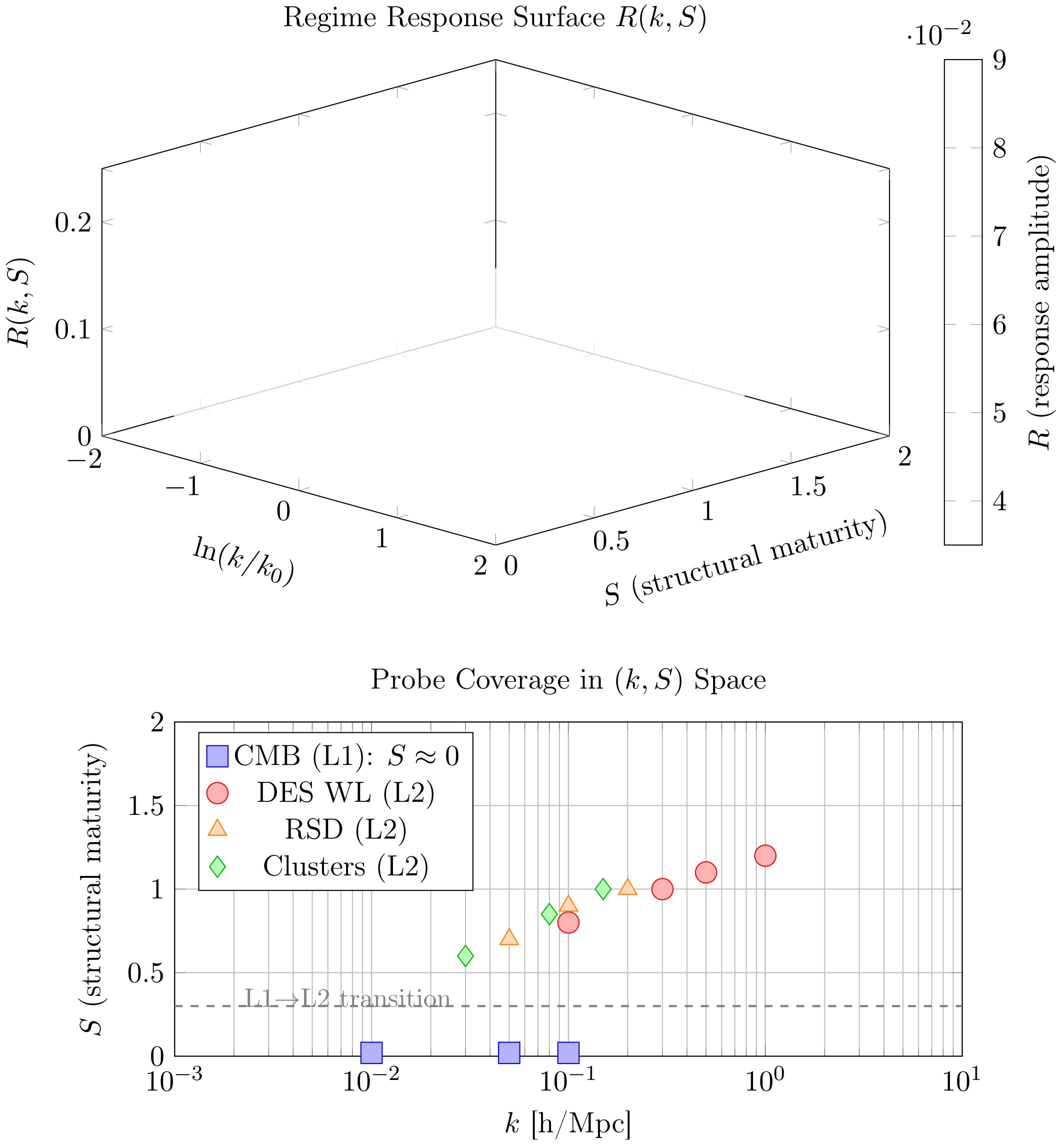

Figure 1: **Top:** The regime response surface $R(k,S)$ with illustrative parameters ($R_0 = 0.05$, $a = 0.1$, $b = 0.3$, $c = 0.05$). Response increases with both k and S. **Bottom:** Schematic probe coverage in (k,S) space. CMB (L1) probes low-S regime where $R \approx 0$. Late-universe probes (WL, RSD, clusters) sample higher-S regime where $R > 0$, but overlap significantly in (k,S) space, explaining their internal consistency. The L1→L2 transition (dashed line) marks where gravitational response activates.

9 Conclusion

The $R(k, S)$ framework unifies prior EFC phenomenology into a single response surface:

1. $\mu(a)$ from Fugaku–DESI $\rightarrow$ absorbed into S-dependence
2. $\mu(k, z)$ from Weak Lensing $\rightarrow$ absorbed into k-dependence
3. **New**: S as state variable, making response physically grounded

The key prediction is that the S_8 tension reflects an **L1→L2 regime transition**, not systematic errors or probe-dependent physics. A single $R(k, S)$ surface should explain:

- $\mu \approx 1$ at CMB ($S \approx 0$)
- $\mu > 1$ at late times ($S > 0$)
- Internal consistency within L2 (probes sample similar S values)

This is testable with current data. Failure of the tomographic consistency test or the S-gradient prediction would falsify this formulation of EFC structure response.

References

[1] Magnusson, M. (2026). "Regime-Dependent Growth Enhancement: A Transition Metric Interpretation of the Fugaku–DESI Matter Density Offset." Figshare. doi:10.6084/m9.figshare.31144030.v1

[2] Magnusson, M. (2026). "EFC Weak Lensing Phenomenology." Figshare. doi:10.6084/m9.figshare.31188193.v1

[3] DES Collaboration (2026). "Dark Energy Survey Year 6 Results: Cosmological Constraints from the Analysis of Cosmic Shear, Galaxy–Galaxy Lensing, and Galaxy Clustering." arXiv:2601.14559

[4] Planck Collaboration (2020). "Planck 2018 results. VI. Cosmological parameters." A&A 641, A6.

A Relation to Standard MG Parametrization

In the standard modified gravity literature, perturbations are characterized by:

$$k^2\Psi = -4\pi G a^2 \mu(k, a)\,\bar{\rho}\,\delta \tag{8}$$

$$\frac{\Phi}{\Psi} = \gamma(k, a) \tag{9}$$

The $R(k, S)$ framework sets:

$$\mu(k, a) = 1 + R(k, S(a)) \tag{10}$$

$$\gamma(k, a) = 1 \quad \text{(no gravitational slip in minimal EFC)} \tag{11}$$

Note on γ: The choice $\gamma = 1$ is a simplifying assumption in this minimal framework, not a fundamental requirement. Future extensions may allow $\gamma \neq 1$ to capture additional degrees of freedom. The present framework is modular: $R(k, S)$ can be constrained independently before relaxing the slip assumption.

This is directly implementable in CLASS/CAMB with appropriate modifications.

B Computing S from Observables

For a given background cosmology:

1. Compute linear growth factor $D_{\mathrm{lin}}(z)$ from perturbation equations
2. Compute nonlinear growth from halofit or N-body calibration
3. $S(z) = \ln[\sigma_8^2(z)/\sigma_{8,\mathrm{lin}}^2(z)]$

For DES Y6 tomographic bins:

- Each bin has effective redshift z_{eff}
- Compute $S(z_{\mathrm{eff}})$ for each bin
- This provides ∼10 points along the S-axis without free parameters