Structural Constraints on Entropy-Gradient Gravity from Cluster Merger Geometry

Falsification of Local Couplings and Predictive Tests of Non-Local Formulations

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`Reproducible Technical Constraint Study`

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Abstract

We test whether Energy-Flow Cosmology (EFC) entropy-gradient formulations can reproduce the Bullet Cluster (1E 0657-56) gravitational lensing geometry without invoking particle dark matter. Using reconstructed baryonic distributions, we find that **local gradient-based couplings are definitively ruled out** (0/42 parameter combinations pass geometric criteria), while **non-local, component-sensitive formulations can reproduce the observed spatial separation** under specific structural requirements. We derive a minimal $w(M,t)$ model with one global parameter ($\eta = 66.2$) and generate *a priori* predictions for MACS J0025 and Abell 520. These predictions are falsifiable and were not tuned to their lensing maps. This work establishes structural constraints on EFC-type theories rather than empirical validation, which requires real shear catalog data.

Epistemological Status: This is a *Reproducible Technical Constraint Study*, not an observational validation. All κ-maps are synthetic reconstructions from published parameters. The value lies in falsifying local formulations and establishing necessary structural requirements for any viable entropy-gradient gravity theory.

Contents

1 Introduction

The Bullet Cluster provides a critical test for modified gravity theories: the gravitational lensing mass (κ-map) is spatially offset from the dominant baryonic component (X-ray gas) by $\sim$150 kpc at 8σ significance [1]. Any theory claiming to explain gravitational phenomena without dark matter particles must reproduce this geometry.

Test objective: Determine whether EFC entropy-gradient gravity can produce κ-peaks at galaxy positions (not gas positions) using only baryonic inputs.

2 Tested Postulates

2.1 Postulate v1.0 (Local Gradient)

$$G_{\text{eff}} = 1 + \alpha \times L_0 \times |\nabla \ln \Sigma_b| \tag{1}$$

$$\kappa_{\text{EFC}} = \frac{\Sigma_b \times G_{\text{eff}}}{\Sigma_c} \tag{2}$$

Free parameters: α (coupling), L_0 (length scale)

2.2 Postulate v1.1 (Non-local + Component-sensitive)

$$q_\nabla = |\nabla \ln \Sigma_b| \quad \text{(edge signal)} \tag{3}$$

$$q_\Delta = -\nabla^2 \ln \Sigma_b \quad \text{(center signal)} \tag{4}$$

$$q = (1-\beta) q_\Delta + \beta q_\nabla \quad (\beta = 0.3 \text{ fixed}) \tag{5}$$

$$\bar{q} = \mathcal{G}(L_0) * q \quad \text{(non-local smoothing)} \tag{6}$$

$$\Sigma_{\text{eff}} = \Sigma_{\text{gas}}(1 + \alpha\bar{q}) + w\,\Sigma_{\text{stellar}}(1 + \alpha\bar{q}) \tag{7}$$

Free parameters: α, L_0, w **Fixed:** $\beta = 0.3$

3 Pass/Fail Criteria

Criterion	Requirement	Physical meaning
Peak count	$= 2$	Two distinct mass concentrations
Gas offset	> 100 kpc	Mass NOT at gas position
Galaxy proximity	< 50 kpc	Mass AT galaxy position
Peak ratio	$0.9 - 1.2$	Comparable main/sub masses

Table 1: Geometric criteria for κ-map validation

4 Results

4.1 v1.0 Local Gradient: FALSIFIED

Mechanism: The gradient operator $|\nabla \ln \Sigma|$ acts as an edge detector. For any radial density profile, the maximum gradient occurs at the inflection point (ring/edge), not at the center. This places κ-peaks at density *transitions* rather than at density *peaks*.

Conclusion: Local gradient-based G_{eff} is **definitively ruled out** for cluster-scale lensing regardless of parameter choice.

Metric	Result
Combinations tested	42
Passing	**0**
Best χ^2	10.3
Failure mode	Peaks at 222–847 kpc from galaxies

4.2 v1.1 Non-local: CONDITIONAL PASS

Metric	Result
Combinations tested	75
Passing	**14**
Best χ^2	3.42
Best parameters	$\alpha = 1.5$, $L_0 = 200$ kpc, $w = 20$

Critical caveat: v1.1 passes *only when the input stellar distribution already has the correct peak ratio* matching the observed κ-map. This means the test verifies that EFC can *preserve* a mass distribution geometry, not that it *predicts* the mass distribution from first principles.

5 Result Claims

BC-001 (Constraint): A purely local entropy-gradient coupling ($G_{\rm eff} \propto |\nabla \ln \rho_b|$) fails to reproduce the Bullet Cluster lensing geometry. The operator acts as an edge-enhancer and displaces κ-peaks away from galaxy centers. This version of EFC is therefore **ruled out** at cluster-merger scales.

BC-002 (Structural Requirement): Reproducing the observed mass–gas separation requires non-local response and stronger coupling to the collisionless component than to shocked plasma. This is a *necessary condition* for EFC-type gravity models in merger regimes.

BC-003 (Working Hypothesis): A minimal non-local, component-sensitive formulation with $w(M,t) = r(M) \times (1 + t/\tau_{\rm mix})$ and $L_0 \sim R_{\rm core}$ can reproduce the Bullet geometry without dark matter particles, provided these parameters are set by independent cluster dynamics.

BC-004 (Predictive Status): Using a single global mixing parameter η, the model yields *a priori* predictions for w and L_0 in other mergers (MACS J0025, Abell 520). These predictions are falsifiable and were not tuned to their lensing maps.

BC-005 (Current Limitation): All κ-maps used here are reconstructed from published parameters rather than raw shear catalogs. The results therefore constitute theoretical and structural constraints, not a definitive empirical validation.

BC-006 (Next Empirical Test): The hypothesis will be considered supported only if the predicted (w, L_0) reproduce κ-peak geometry in at least one independent merger without parameter retuning.

6 The $w(M,t)$ Model

6.1 Physical Motivation

The component weighting w reflects entropy-gradient *preservation*:

- Collisionless matter (galaxies/stars): gradients preserved ($f_* \approx 1$)

- Collisional matter (ICM gas): gradients destroyed by shock + mixing ($f_{\text{gas}} \ll 1$)

6.2 Model Equations

Compression ratio (Rankine-Hugoniot):

$$r(M) = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2} \quad [\gamma = 5/3] \tag{8}$$

Mixing timescale:

$$\tau_{\text{mix}} = \eta \times L_0/\sigma_v \tag{9}$$

Saturating model (recommended):

$$\boxed{w(M,t) = r(M) \times \left(1 + \frac{t}{\tau_{\text{mix}}}\right)} \tag{10}$$

Global parameter: $\eta = 66.2$ (calibrated on Bullet Cluster)

6.3 Why Saturating, Not Exponential

An exponential mixing law $w \propto e^{t/\tau}$ is *ill-conditioned*: ±50 Myr uncertainty in merger age gives ±100–200% change in w. The saturating form gives only ±25% sensitivity, making predictions robust.

7 Predictions for Other Mergers

7.1 Locked Parameters

Parameter	Value
η	66.2 (global, from Bullet)
c	0.8 ($L_0 = c \times R_{\text{core}}$)
σ_v	500 km/s (fixed)
α	1.5
β	0.3 (fixed)

Table 2: Locked parameters – NO per-cluster fitting

7.2 Predicted w and L_0

Cluster	M	t (Myr)	R_{core}	w_{pred}	L_0 (kpc)	Uncertainty
Bullet	3.0	150	250	20.0	200	calibration
MACS J0025	2.0	150	150	**23.9**	**120**	±30%
Abell 520	2.5	200	200	**28.2**	**160**	±23%

Table 3: Predictions using locked $\eta = 66.2$

Cluster	Peaks	Gas offset	Galaxy prox.	Ratio	Result
MACS J0025	2 ✓	119 kpc ✓	8.5 kpc ✓	1.07 ✓	**PASS**
Abell 520	2 ✓	126 kpc ✓	8.5 kpc ✓	1.21 ×	**PASS 3/4**

Table 4: Prediction tests on synthetic κ-maps (no tuning)

7.3 Validation Results (Synthetic Data)

8 Discussion

8.1 What We Have Shown

1. **Robust falsification of v1.0:** Local entropy-gradient couplings cannot reproduce Bullet Cluster geometry under any reasonable parameterization.
2. **Structural requirements identified:** To match observations, EFC must have non-local response ($L_0 \sim 200$ kpc) and strong preference for collisionless component ($w \sim 20$).
3. **Predictive framework:** Using locked parameters, the model predicts w and L_0 for independent clusters.

8.2 What We Have NOT Shown

- That EFC *predicts* the correct mass ratio from baryonic physics alone
- That the $w \sim 20$ weighting has fully independent physical derivation
- That these parameters work on *real* κ-maps from shear catalogs

8.3 Path Forward

The hypothesis (BC-003) will be considered supported if:

1. Predicted (w, L_0) reproduce κ geometry on real FITS data
2. No per-cluster parameter adjustment is required
3. The model works for at least one "clean" merger (MACS J0025 preferred)

9 Conclusion

The Bullet Cluster test **constrains** EFC rather than confirms it. We have:

✓ Ruled out local gradient formulations

✓ Identified the structural form required for compatibility

✓ Mapped the viable parameter space

✓ Generated falsifiable predictions for other mergers

○ NOT demonstrated predictive power on real observational data

This represents genuine progress in theory-building: converting a qualitative challenge ("Bullet Cluster disproves modified gravity") into a quantitative constraint on the theory space.

Data and Code Availability

All code, synthetic data, and figures are available at:
`https://doi.org/10.6084/m9.figshare.31173850`

- `code/` – Complete Python pipeline
- `data/` – Synthetic Σ and κ maps
- `figures/` – All generated visualizations

The analysis is fully reproducible. Real κ-map validation requires access to original shear catalogs (contact Clowe, Bradač, or Cha et al.).

Acknowledgments

This work builds on the pioneering analyses of Clowe et al. (2006), Bradač et al. (2006), and Brownstein & Moffat (2007). We thank the weak lensing community for making cluster data progressively more accessible.

References

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