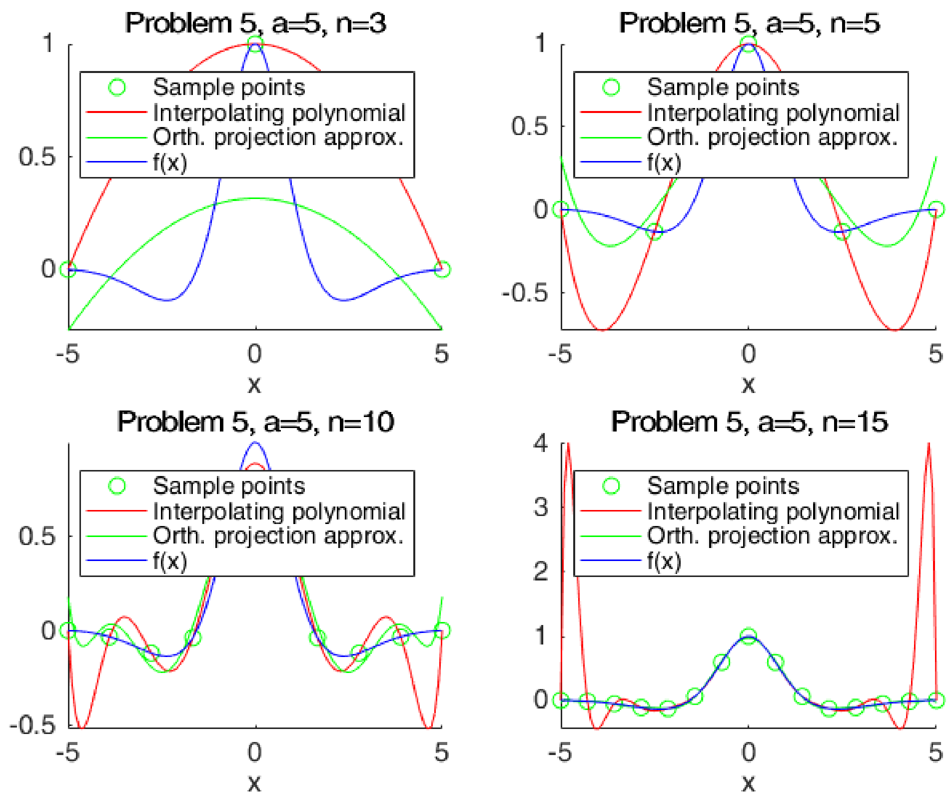
```
% Problem 1

f=@(x) cos(x)./(cosh(x));
a = 5;
n = [3, 5, 10, 15];

for j=1:length(n)
    lpCoords = zeros(n(j), 1);
    for i=1:n(j)
        tlp = @(y) legendreP(i - 1, y / a);
        num = @(z) f(z) .* tlp(z);
        den = @(z) tlp(z) .* tlp(z);
        lpCoords(i) = integral(num, -a, a) / integral(den, -a, a);
    end

    % data for fitting
    tfit = linspace(-a, a, n(j));
    yfit = f(tfit);

    p = polyfit(tfit,yfit,n(j)-1);
    tval = -a:0.1:a;

    % y values for orthogonal projection approximation
    lpval = zeros(length(tval), 1);
    for i=1:length(lpval)
        val = 0;
        for k=1:n(j)
            xval = tval(i) / a;
            val = val + lpCoords(k) * legendreP(k - 1, xval);
        end
        lpval(i) = val;
    end

    % plotting
    yval = polyval(p,tval);
    subplot(2,2,sub2ind([2,2],j));
    hold on;
    plot(tfit,yfit,'og');
    plot(tval,yval,'r');
    plot(tval,lpval,'g');
    h = ezplot(f,[-a,a]);
    set(h, 'Color', 'blue');
    legend('Sample points', 'Interpolating polynomial', 'Orth.
 projection approx.', 'f(x)');
    axis tight;
    title(strcat(strcat('Problem 5, a=', int2str(a)), strcat(', n=',
 int2str(n(j)))));
end
```

Warning: Polynomial is badly conditioned. Add points with distinct X values,
reduce the degree of the polynomial, or try centering and scaling as described

`in HELP POLYFIT.`



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ACM 104 Homework 5

① Attached

② Let $h_{n-1}(x)$ be the n^{th} Hermite polynomial.

$$h_0(x) = 1$$

$$h_1(x) = x - \frac{\langle x, 1\rangle}{\|1\|^2} = x$$

$$h_2(x) = x^2 - \frac{\langle x^2, 1\rangle}{\|1\|^2} - \frac{\langle x^2, x\rangle}{\|x\|^2}\cdot x = x^2 - \frac{\sqrt{2\pi}}{\sqrt{2\pi}} - 0 = x^2 - 1$$

$$h_3(x) = x^3 - \frac{\langle x^3, 1\rangle}{\|1\|^2} - \frac{\langle x^3, x\rangle}{\|x\|^2}\cdot x - \frac{\langle x^3, x^2-1\rangle}{\|x^2-1\|^2}\cdot(x^2-1)$$

$$= x^3 - 0 - \frac{3\sqrt{2\pi}}{\sqrt{2\pi}}\cdot x = x^3 - 3x$$

$$h_4(x) = x^4 - \frac{\langle x^4, 1\rangle}{\|1\|^2} - \frac{\langle x^4, x\rangle}{\|x\|^2}\cdot x - \frac{\langle x^4, x^2-1\rangle}{\|x^2-1\|^2}\cdot(x^2-1) - \frac{\langle x^4, x^3-3x\rangle}{\|x^3-3x\|^2}\cdot(x^3-3x)$$

$$= x^4 - \frac{3\sqrt{2\pi}}{\sqrt{2\pi}} - 0 - \frac{12\sqrt{2\pi}}{2\sqrt{2\pi}}\cdot(x^2-1) - 0 = x^4 - 3 - 6(x^2-1)$$

$$= x^4 - 6x^2 + 3$$

(3) For $\langle x,y \rangle_1$ we can find the cross product of v_1 and v_2; the resultant vector will be orthogonal to both and will also be orthogonal to any linear combination of the two. The span of the cross product will then be our $W_1^\perp$

$$u_1 = v_1 \times v_2 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} \rightsquigarrow W_1^\perp = \text{span}\left(\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}\right)$$

For $\langle x,y \rangle_2$ we can define 2 equations using the weighted inner product and the 2 vectors:

$$\langle u, v_1 \rangle = 0 \rightsquigarrow 1 \cdot x + 2 \cdot 2 \cdot y + 3 \cdot 3 \cdot z = x + 4y + 9z = 0$$

$$\langle u, v_2 \rangle = 0 \rightsquigarrow 2 \cdot x + 0y + 3 \cdot 1 \cdot z = 2x + 3z = 0$$

Solving the system gives: $x = -\frac{3}{2}z$

$$-\frac{3}{2}z + 4y + 9z = 0$$

$$y = -\frac{15}{8}z$$

$z = 1 \longrightarrow x = -\frac{3}{2}, \ y = -\frac{15}{8}$

$$\text{So} \quad W_2^\perp = \text{span}\left(\begin{bmatrix} -3/2 \\ -15/8 \\ 1 \end{bmatrix}\right)$$

(4) $\det \begin{bmatrix} -\lambda & 0 & -1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{bmatrix} = 0$

$$\downarrow$$

$$-\lambda(-\lambda)(1-\lambda) - 1(-1+\lambda) = 0$$

$$-\lambda^3 + \lambda^2 - \lambda + 1 = 0$$

$\lambda = 1$ so $(\lambda - 1)$ is a factor.

$$(\lambda-1)(-\lambda^2-1) = 0$$

$\lambda = 1$ or $-\lambda^2 - 1 = 0$

$$\lambda^2 = -1$$

$$\lambda = \pm i$$

Eigenvalues: $-1, i, -i$ (multiplicity 1 for each)

Eigenvectors: $Av = \lambda v \longrightarrow \begin{aligned} -z &= \lambda x \\ y &= \lambda y \\ x &= \lambda z \end{aligned}$

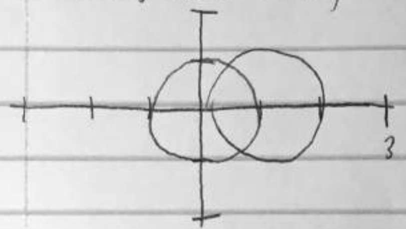
$\lambda = 1 \longrightarrow \begin{aligned} -z &= x \\ y &= y \\ x &= z \end{aligned} \rightarrow \begin{aligned} x &= 0 \\ y &= \alpha \\ z &= 0 \end{aligned}$

$\lambda = i \rightarrow \begin{aligned} -z &= xi \\ y &= yi \\ x &= zi \end{aligned} \rightarrow \begin{aligned} x &= zi \\ y &= 0 \end{aligned}$

$\lambda = -i \rightarrow \begin{aligned} -z &= -xi \\ y &= -yi \\ x &= -zi \end{aligned} \rightarrow \begin{aligned} x &= -zi \\ y &= 0 \end{aligned}$

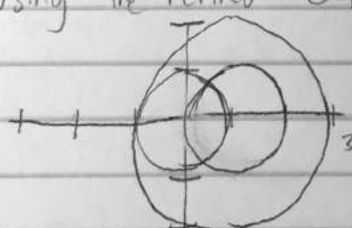
From the general equations of the vectors corresponding to the eigenvalues, we conclude each eigenvalue has multiplicity 1, so all eigenvalues are complete. This implies A is complete. We can see the generated eigenvectors span $\mathbb{C}^3$.

⑤ Ⓐ By simple calculation, we know the radii of all Gerschgorin disks are 1. The centers for the 2 possible disks are $(0,0)$ and $(1,0)$



Ⓑ By Gershgorin theorem, we know that $\mathrm{spec}(A) \subset D_A$ and $\mathrm{spec}(A^T) \subset D_{A^T}$. We also know the eigenvalues of A are the solutions to $\det(A-\lambda I)=0$ and the eigenvalues of A^T satisfy $\det(A^T-\lambda I)=0$. However, $\det(A^T-\lambda I) = \det((A-\lambda I)^T) = \det(A-\lambda I)$ since the determinant of a matrix is equal to the determinant of its transpose, so A and A^T have the same eigenvalues. This means $\mathrm{spec}(A)=\mathrm{spec}(A^T)$ so both $\mathrm{spec}(A) \subset D_A$ and $\mathrm{spec}(A) \subset D_{A^T}$ hold. From this we conclude $\mathrm{spec}(A) \subset D_A \cap D_{A^T}$

Ⓒ Using the refined Gerschgorin domain, we have one circle centered at $(1,0)$:



Ⓓ $$\det \begin{bmatrix} -\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = 0 \rightsquigarrow -\lambda((1-\lambda)^2+1) - 0 + 0 = 0$$

$$-\lambda(\lambda-1+i)(\lambda-1-i)=0$$

$$\downarrow$$

$$\lambda=0 \text{ OR } \lambda=1-i \text{ OR } \lambda=1+i$$

The eigenvalues of A are $0, 1-i$, and $1+i$, which lie in the refined Gerschgorin domain