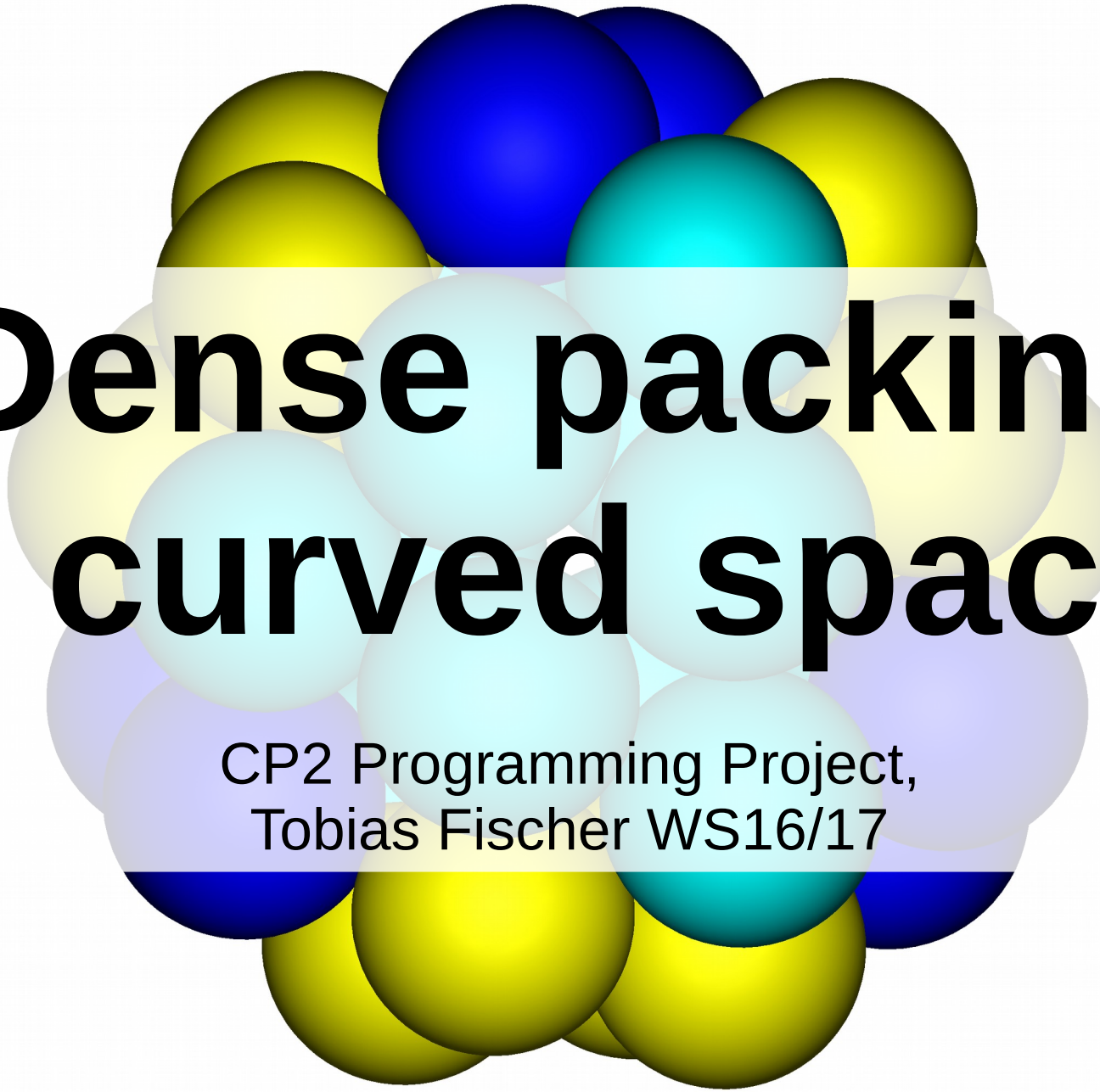
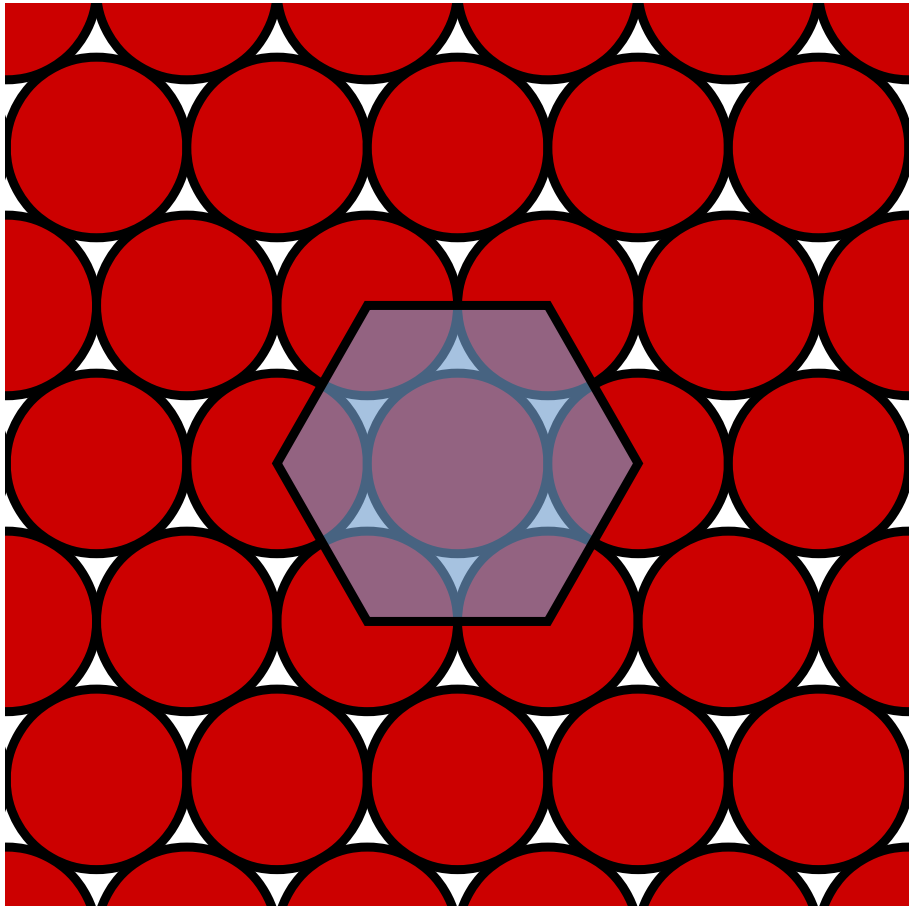
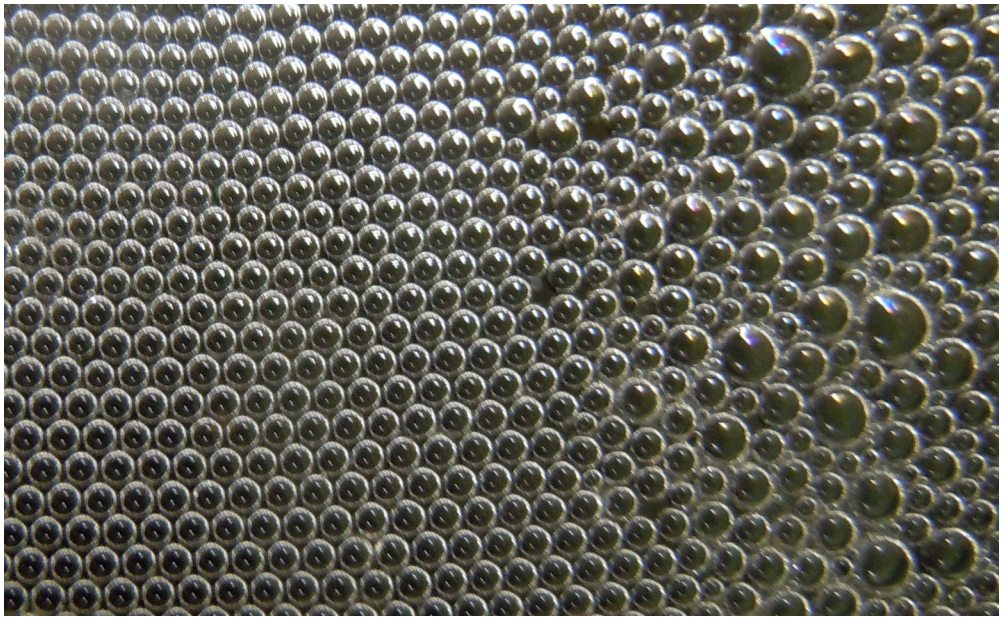


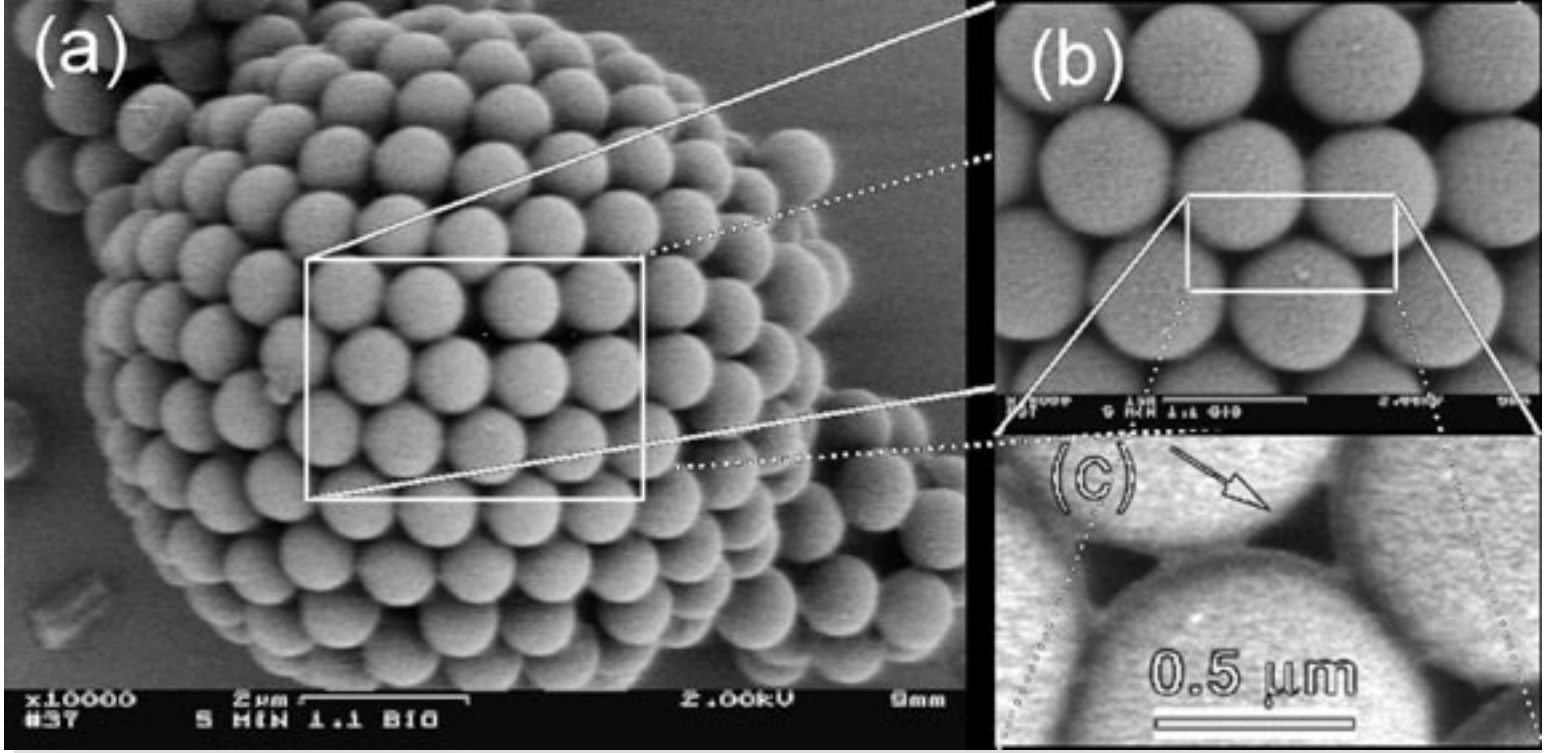
Dense packing in curved spaces

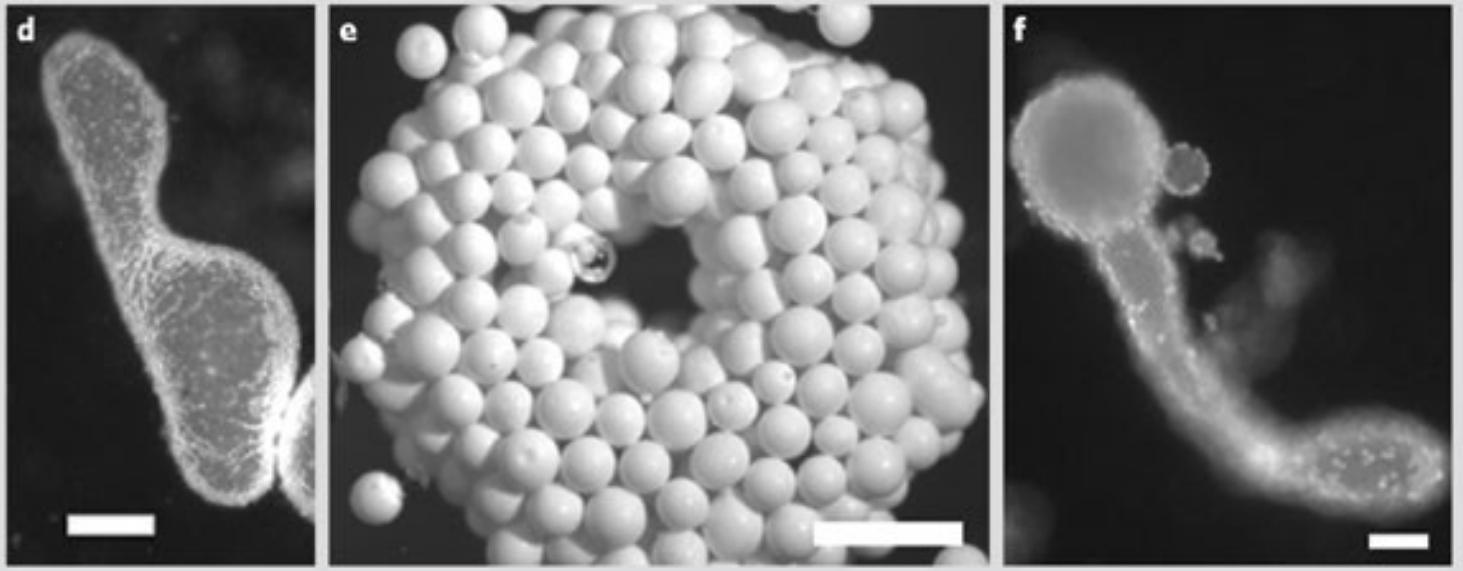
CP2 Programming Project,
Tobias Fischer WS16/17

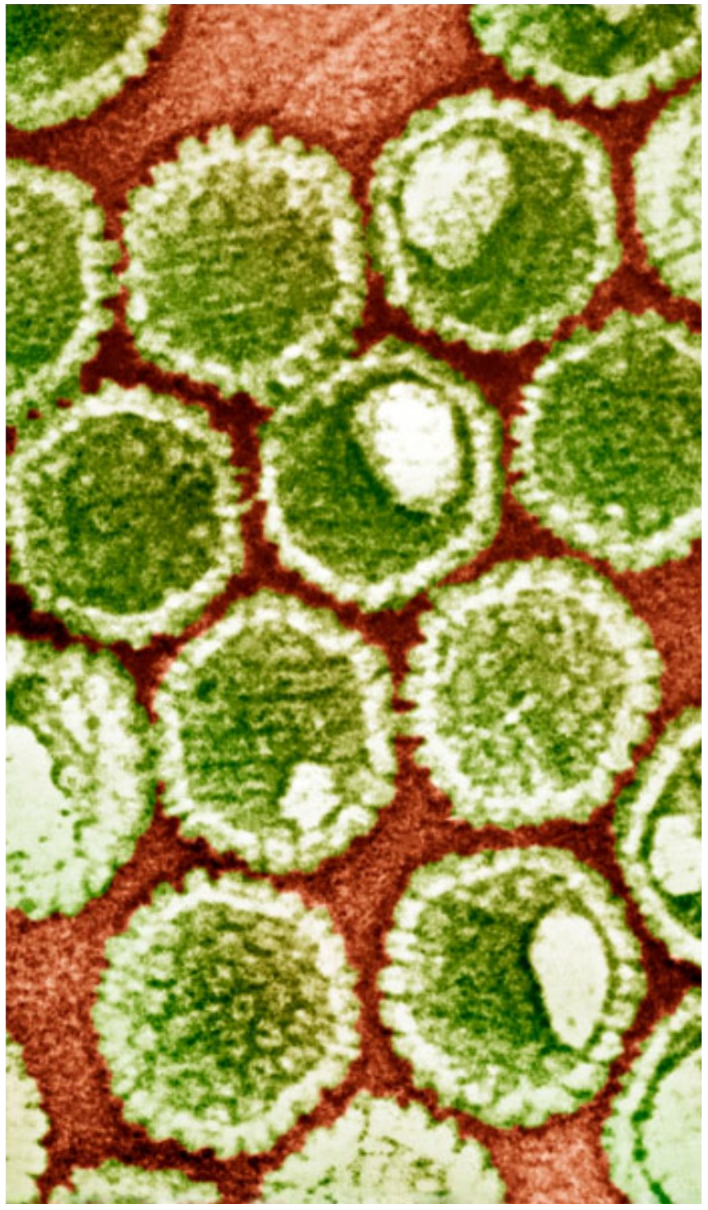


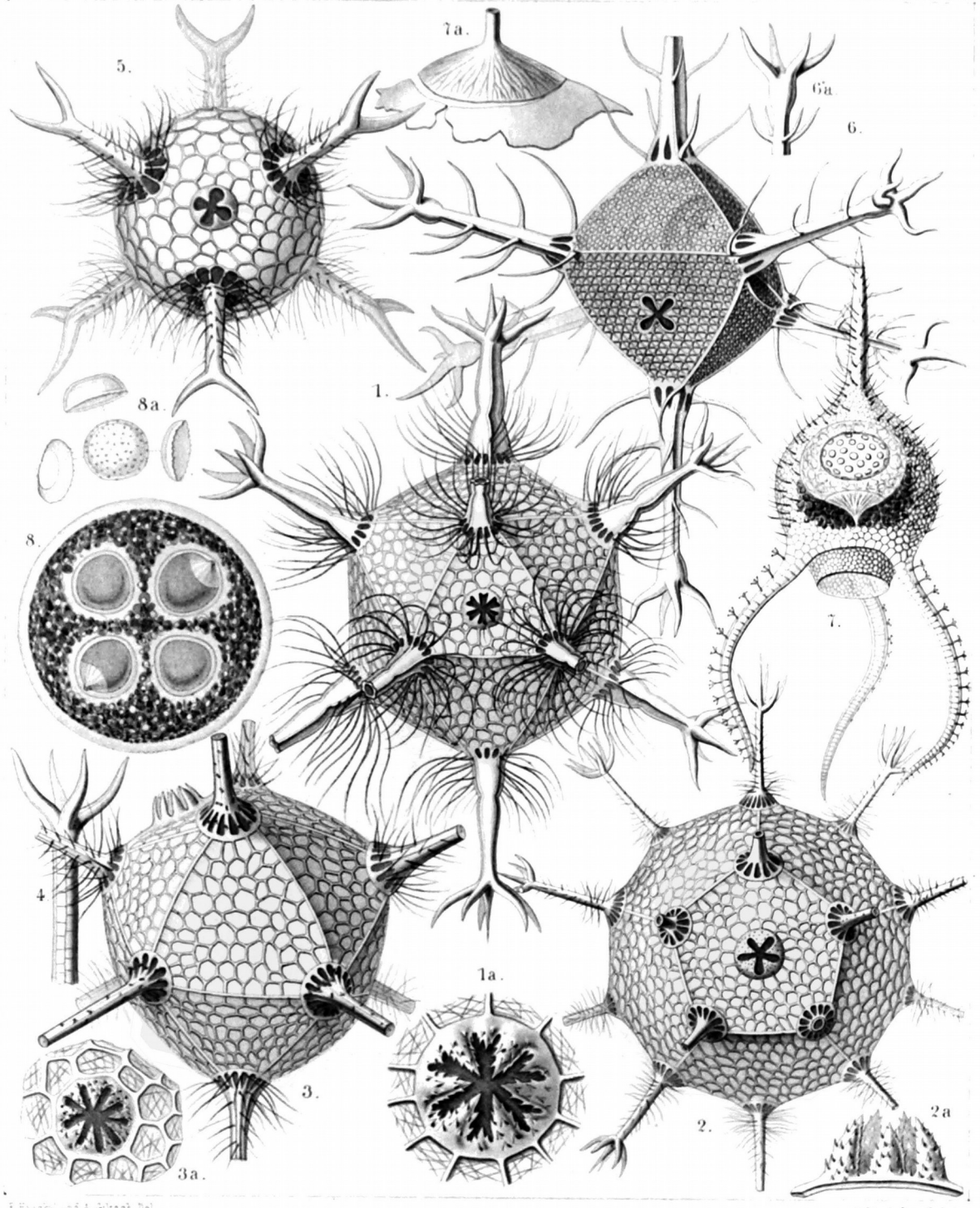


$$\frac{\pi}{2\sqrt{3}} \approx 0.9069$$


(a)
(b)
(c)
0.5 µm
x10000
837
2µm
5 MIN 1.1 BIO
2.00kV
9mm


d
e
f





E. Haeckel, ad. d. Nat. del. A. Giltsch Jena Lith. Anst.

1. CIRCOGONIA. 2. CIRCORRHEGMA. 3. CIRCOSPATHIS.
4–6. CIRCOPORUS. 7. CORTINETTA. 8. CATINULUS.

Pseudocode

Initialize randomly: φ_0, θ_0 $r_{particle,0}=0$

iterate n times:

  try_Move

  try_Inrease_Radius

try_Move:

  p = random particle

  $(\varphi,\theta)(p)$+= rnd_gauss

  if overlap: reset

  if $\left|\frac{det(Df((\varphi,\theta)_{new}))}{det(Df((\varphi,\theta)_{old}))}\right|$<rnd:

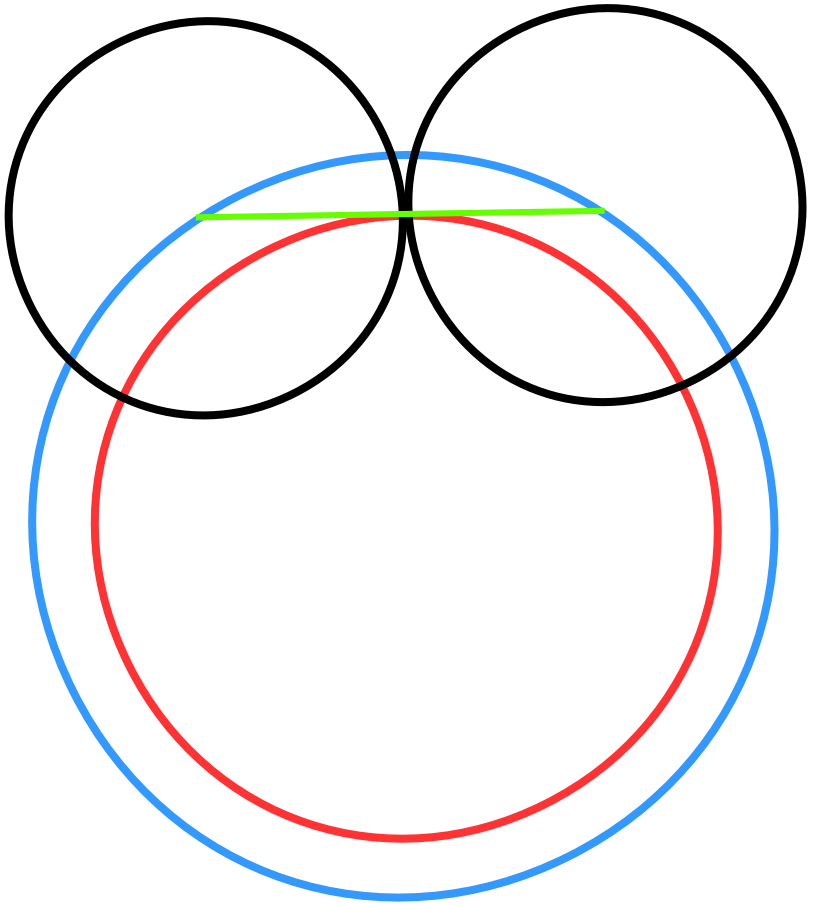
    reset

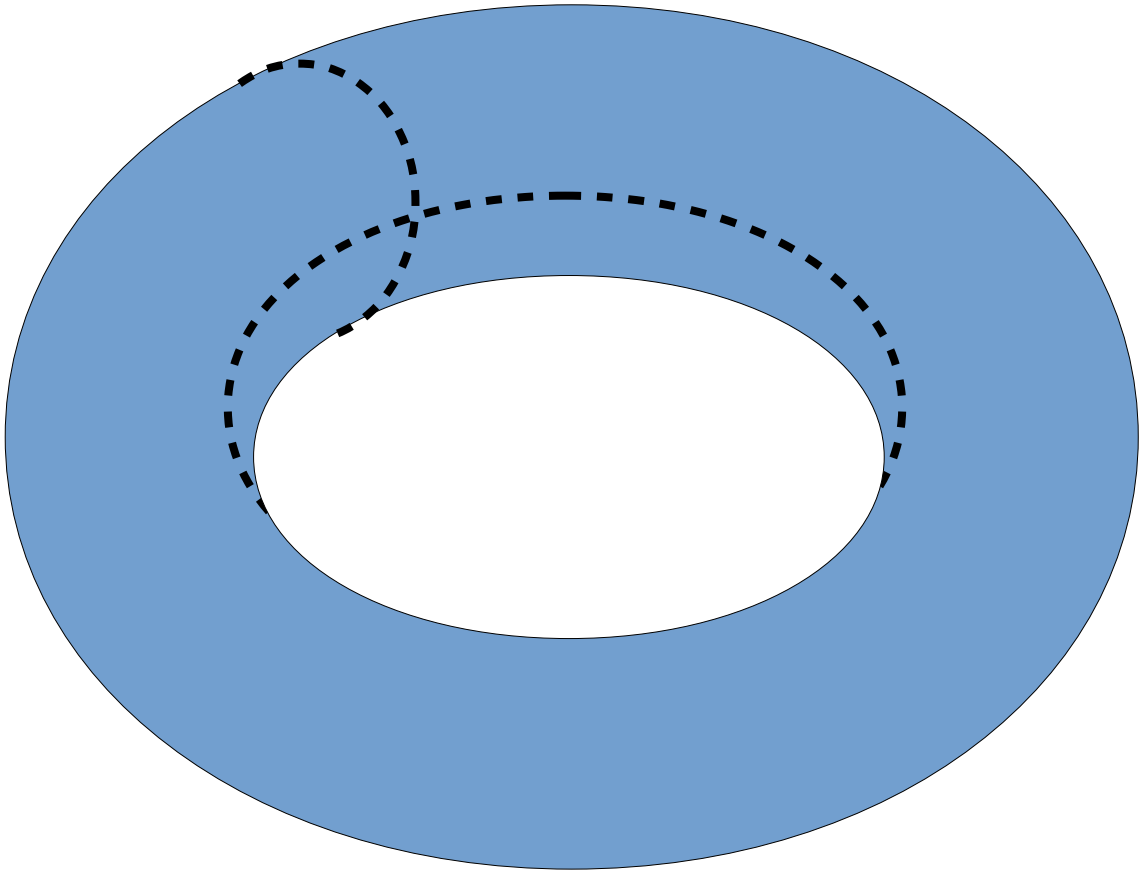
try_Inrease_Radius:

  $r_{particle}$ += rnd_exp

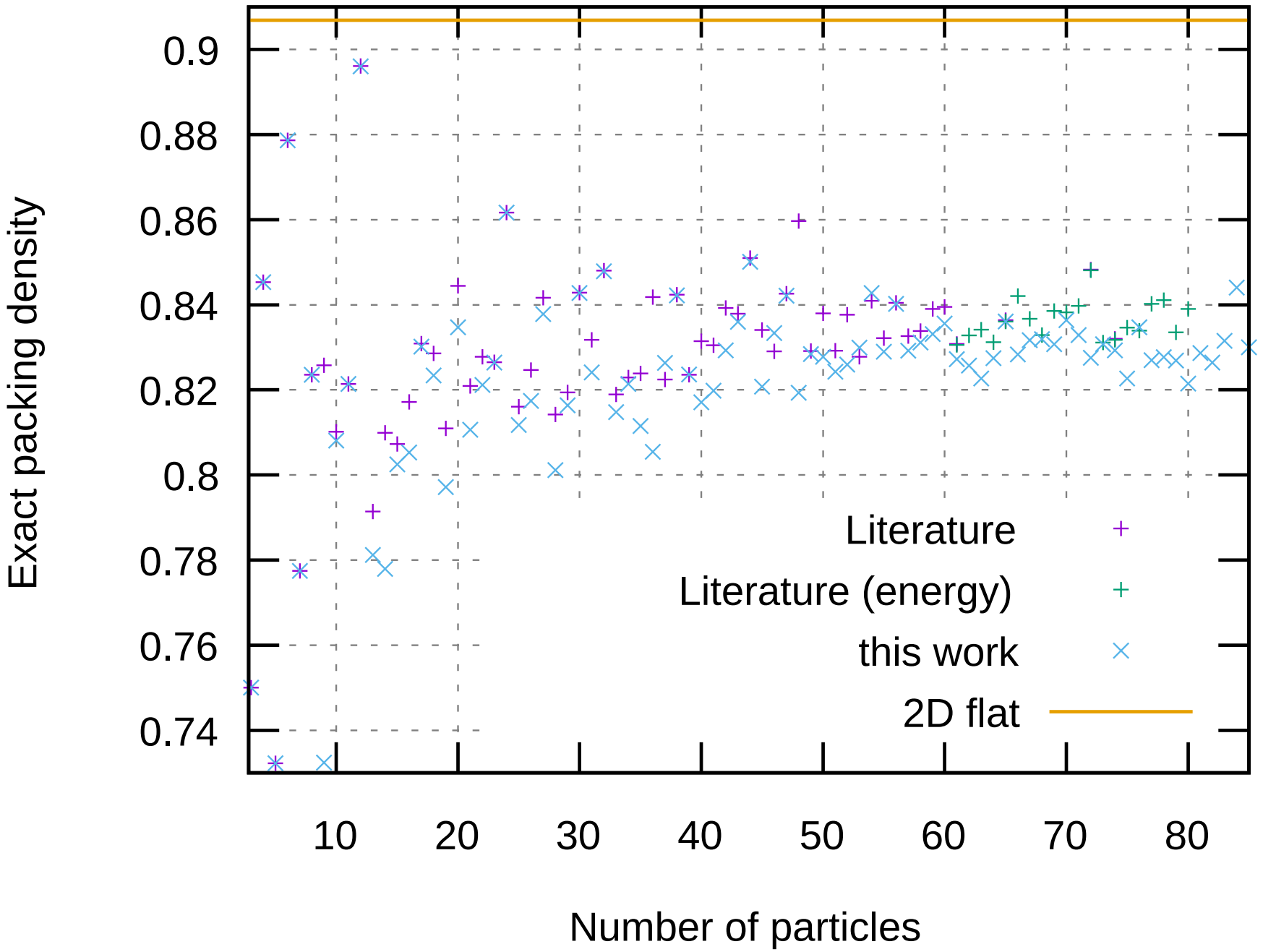
  if overlap: reset

Global properties:
Packing density, topology

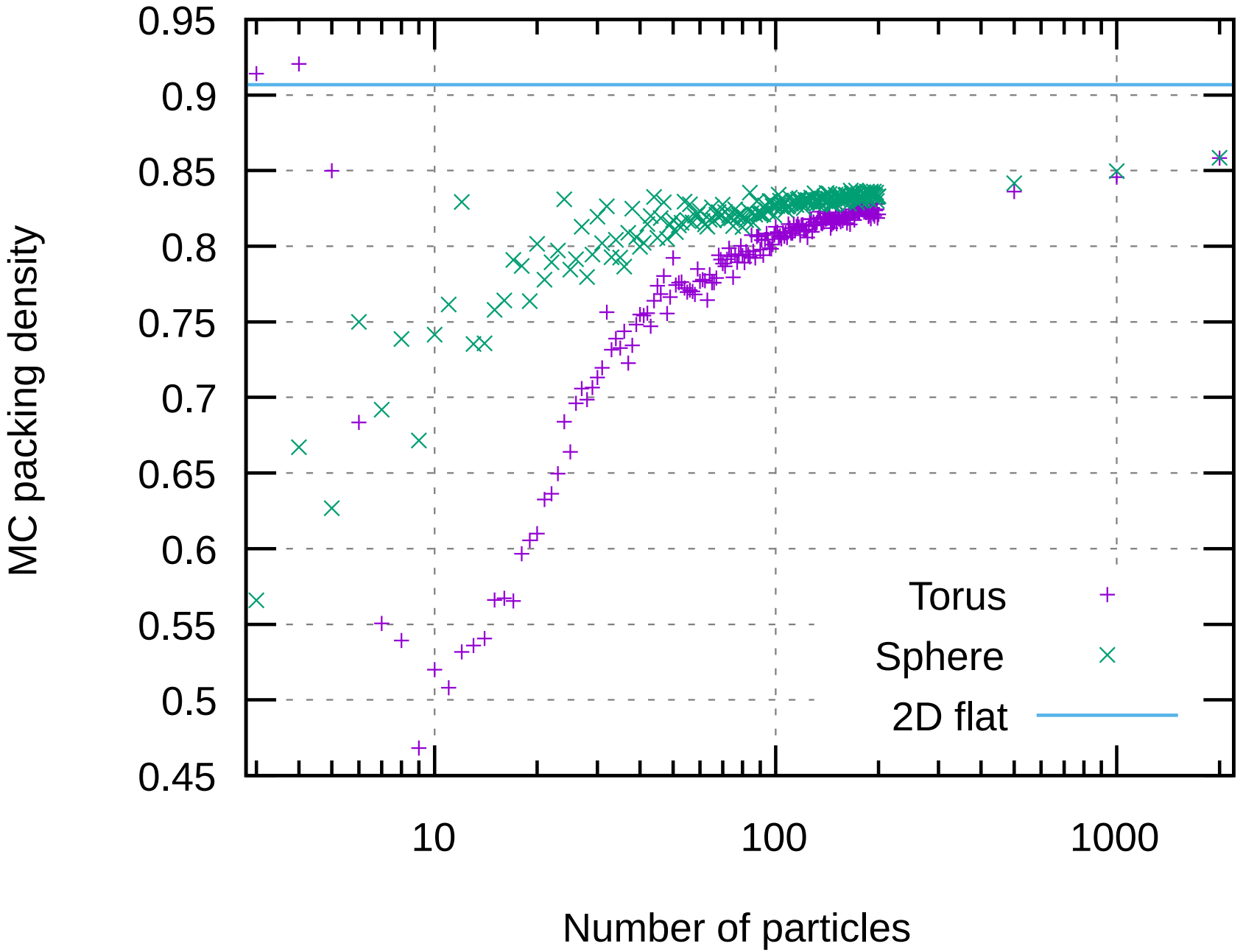




Comparison to literature



Sphere vs Torus

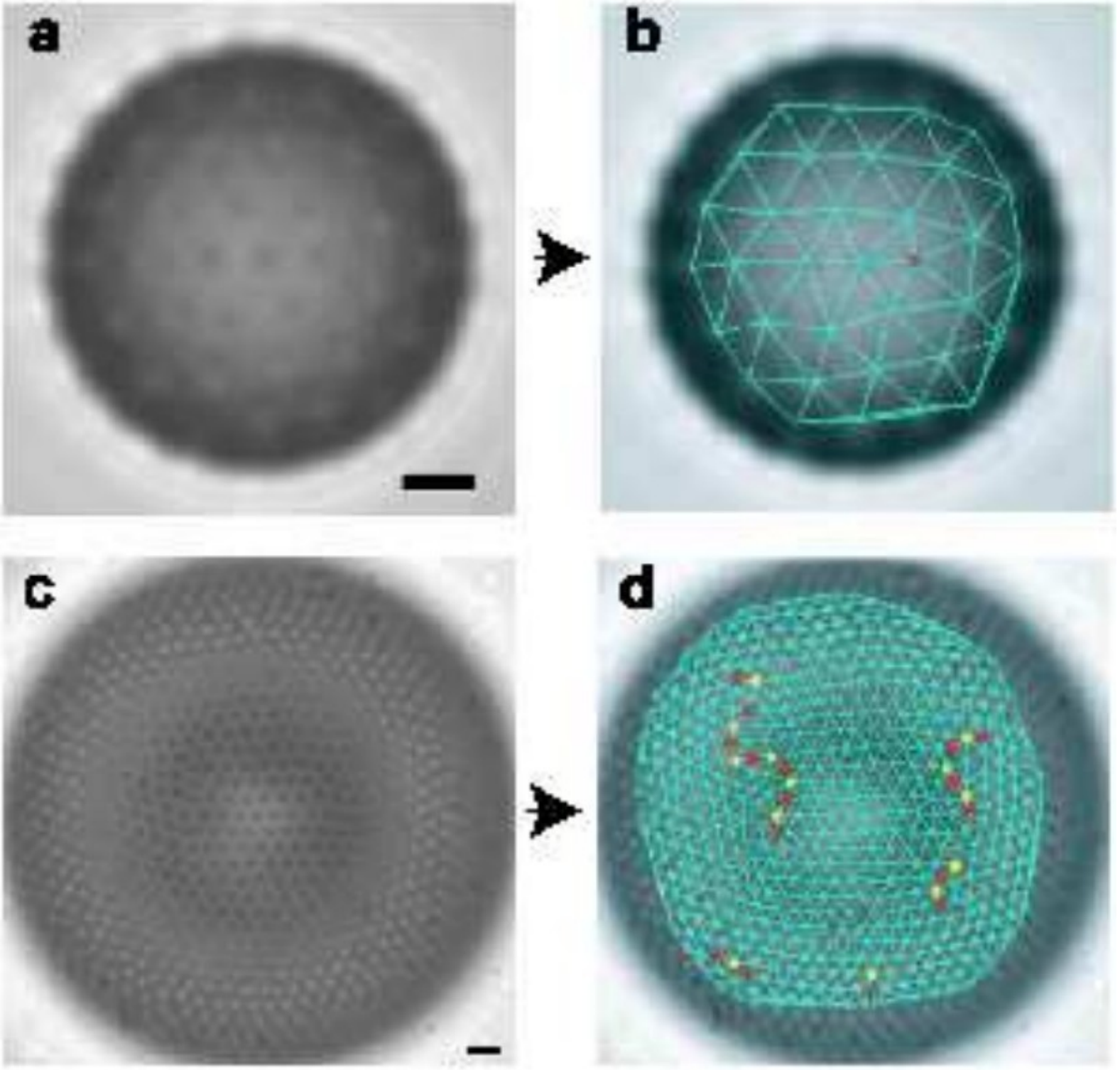


Topology

Euler characteristic: $\chi = V - E + F$

- 2D-flat $\chi = 0$
- Torus $\chi = 0$
- Sphere $\chi = 2$

=> Defects must appear on Sphere!


a
b
c
d

Conclusions

- Simulation in agreement with literature and reality (scars)

$$\lim_{N_{Particles} \to \infty} : \rho_{sphere} \approx \rho_{torus} \overset{?}{\approx} \rho_{flat}$$

- Runtime optimization possible

- Challenging nearest neighbor count for torus

Questions?