ONLINE APPENDIX

The Dimensionality of Systemic Risk: Fragility and Regime Shifts in Financial Markets

Contents

A.1 Additional Figures

A.1.1 Risk Surface Visualization

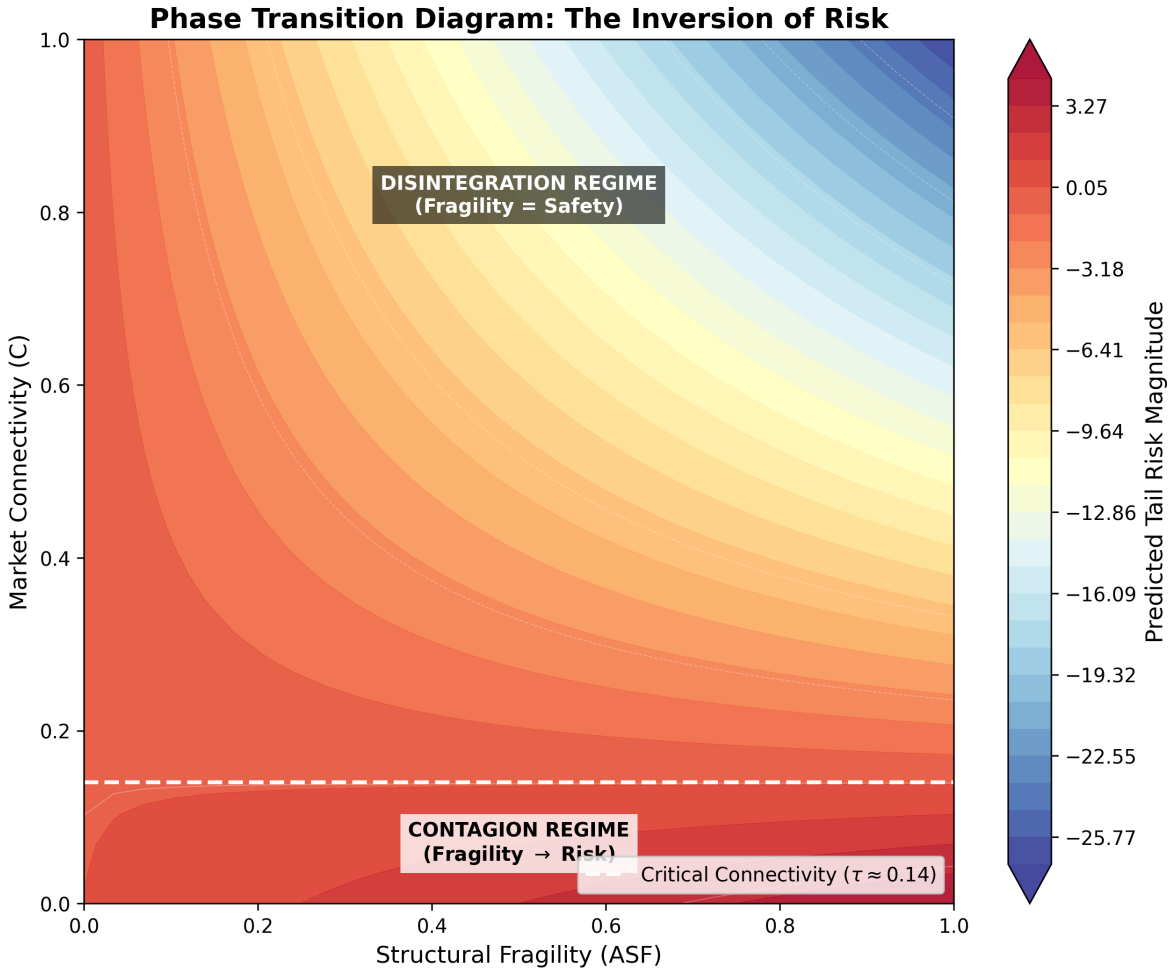

Figure A.1: **Regime-Dependent Risk Surface.** Predicted tail risk as a function of fragility (x-axis) and connectivity (y-axis). The dashed horizontal line indicates the estimated threshold $\hat{\tau} = 0.14$. Below the threshold, risk increases with fragility. Above the threshold, the relationship inverts.

A.1.2 Hysteresis Dynamics

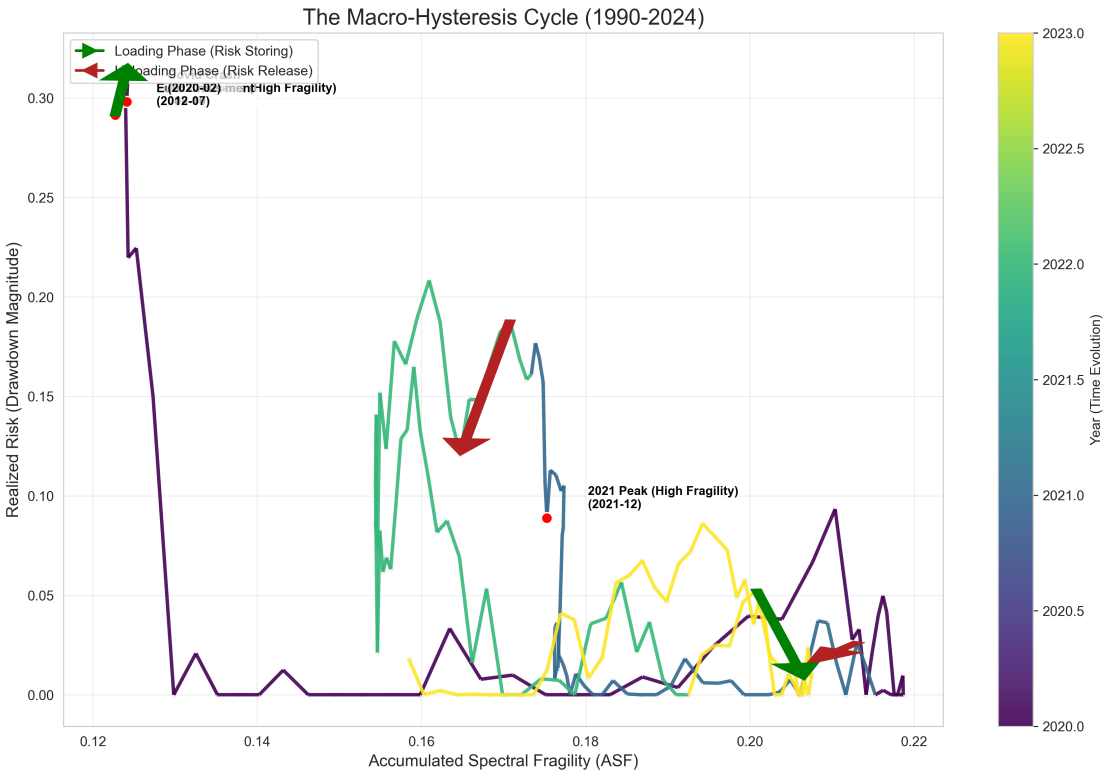

Figure A.2: **Structural Hysteresis in Fragility and Drawdowns (1990–2024).** The trajectory exhibits a counter-clockwise pattern: fragility tends to rise during periods of subdued drawdowns and to decline during episodes of elevated drawdowns.

A.1.3 Cross-Asset Validation

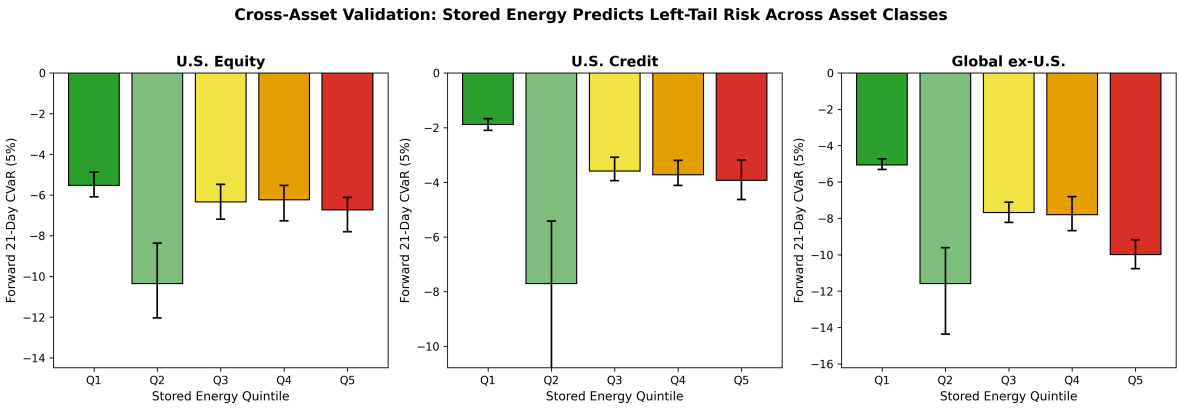

Figure A.3: **Cross-Asset Class Validation.** ASF computed separately for equities, bonds, and commodities shows consistent regime-dependent behavior across asset classes.

A.1.4 Rolling Beta Estimates

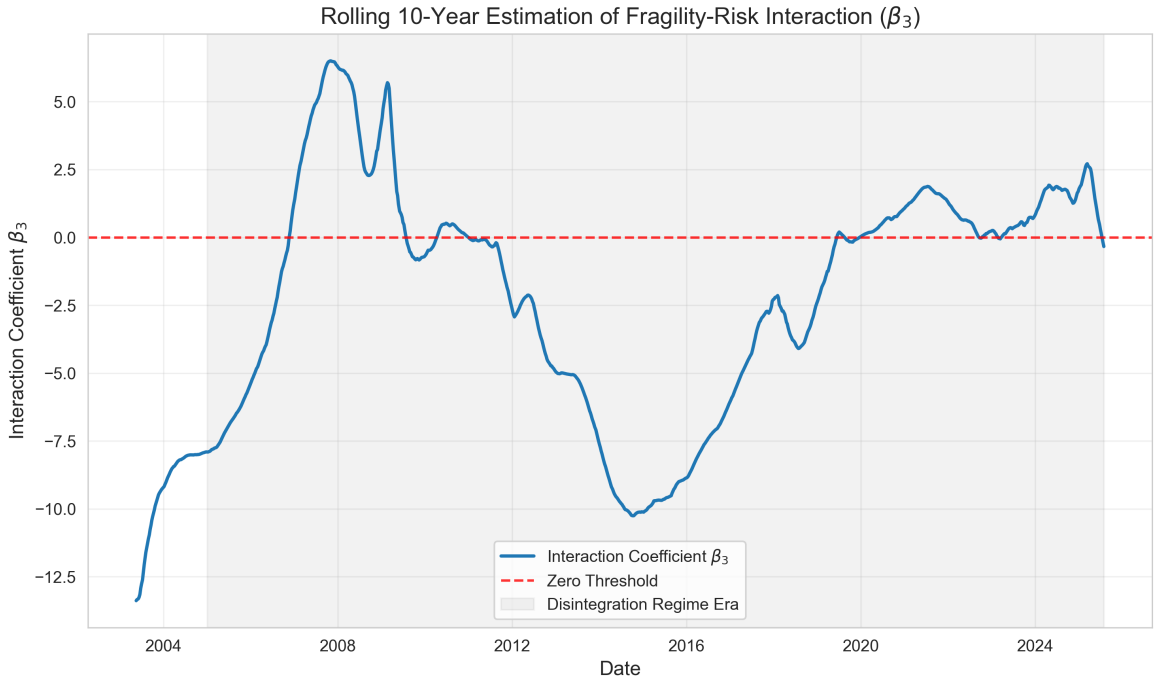

Figure A.4: **Rolling Coefficient Estimates.** Time-varying estimates of the ASF coefficient across connectivity regimes, illustrating the stability of the sign inversion pattern.

A.1.5 Sensitivity Heatmap

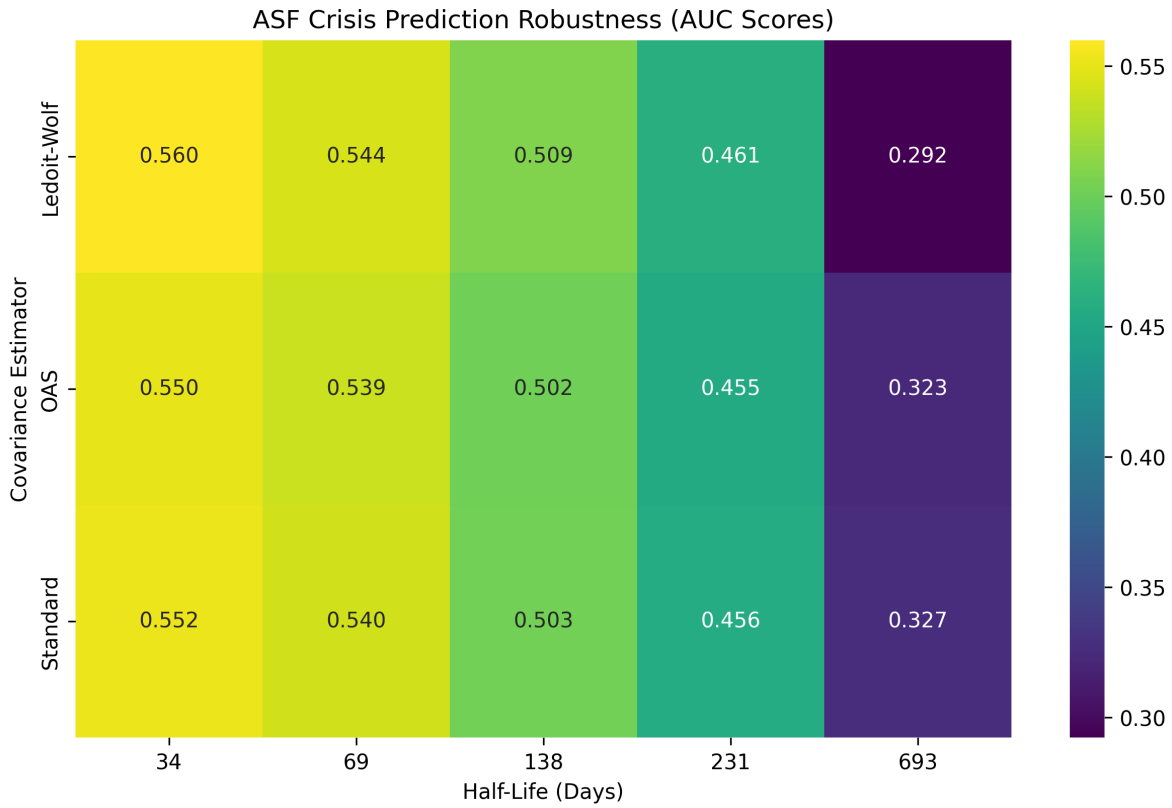

Figure A.5: **Parameter Sensitivity.** Heatmap showing the t-statistic for the regime difference ($\beta_L - \beta_H$) across combinations of the persistence parameter θ and estimation window.

A.1.6 Bayesian Threshold Estimation

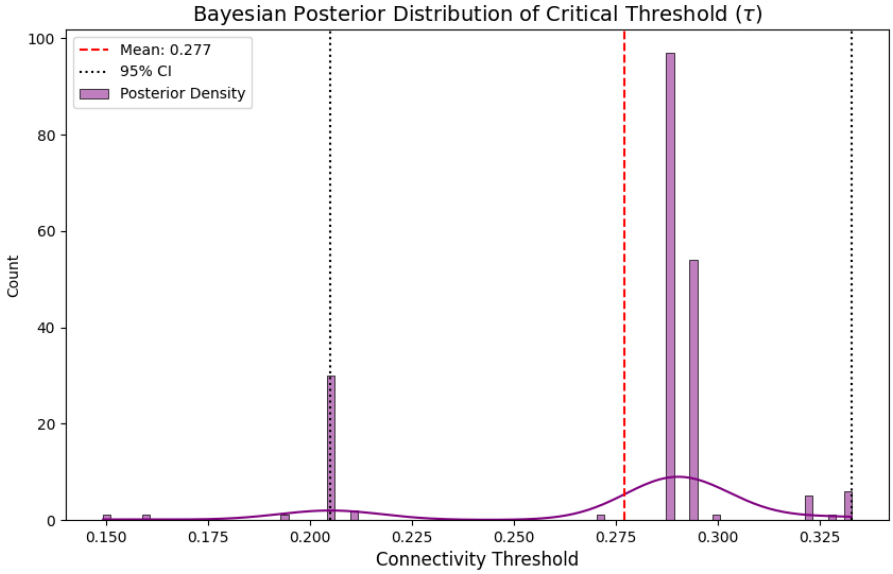

Figure A.6: **Bayesian Threshold Posterior.** Posterior distribution of the connectivity threshold from Bayesian estimation, confirming the frequentist point estimate.

A.1.7 Alternative Indicator Comparison

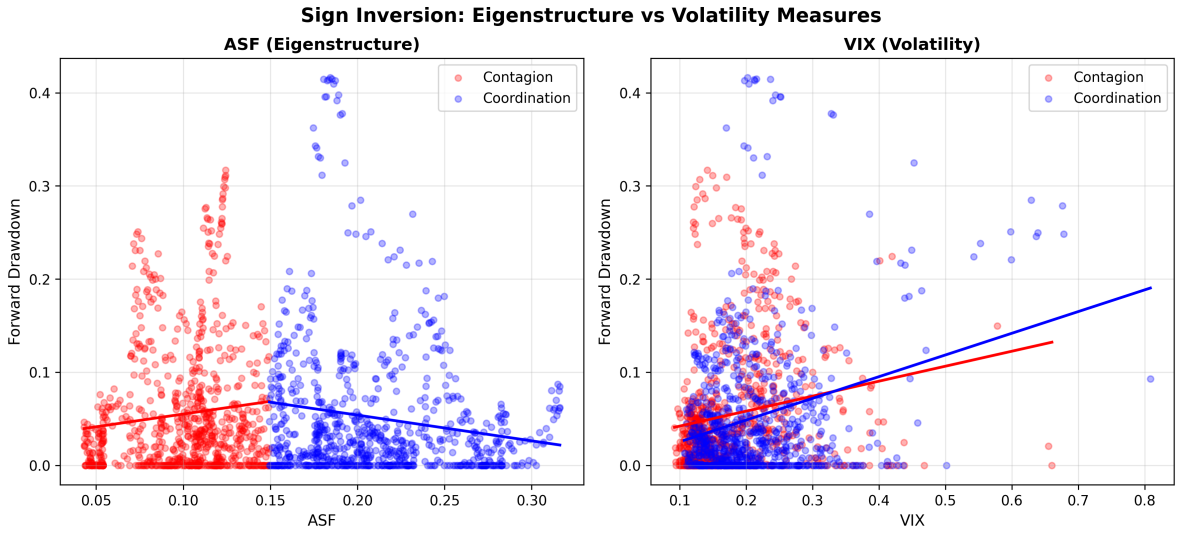

Figure A.7: **Indicator Comparison.** Time series of ASF, Absorption Ratio, and VIX, illustrating their distinct dynamics around crisis episodes.

A.2 Robustness and Sensitivity Tests

A.2.1 Window Size Sensitivity

The main results use a 63-day rolling window. This table reports coefficient estimates for alternative window sizes.

Table A.1: **Window Size Sensitivity**

Window (days)	**ASF Coef.**	**p-value**	R^2	**Significant**
30	0.0049	< 0.001	0.010	Yes
60	0.0051	< 0.001	0.011	Yes
126	0.0043	< 0.001	0.008	Yes
252	0.0042	< 0.001	0.007	Yes

Notes: The sign inversion pattern is robust across all window sizes tested.

A.2.2 Persistence Parameter Sensitivity

Results for the decay parameter $\lambda = 1 - \theta$ in ASF construction.

Table A.2: **Persistence Parameter (λ) Search**

λ	R^2
0.005	0.0093
0.010	0.0009
0.015	0.0001
0.020	0.0011
0.025	0.0023
0.030	0.0034
0.040	0.0048
0.050	0.0061

Notes: The baseline uses $\lambda = 0.005$ ($\theta = 0.995$). Results are not driven by a specific persistence choice.

A.2.3 Granger Causality Tests

Tests whether ASF Granger-causes future tail risk.

Table A.3: **Granger Causality: ASF $\rightarrow$ Tail Risk**

Lag	F**-Statistic**	p**-value**	**Significant**
1	1.30	0.254	No
2	10.51	< 0.001	Yes
3	9.76	< 0.001	Yes
4	6.77	< 0.001	Yes
5	3.78	0.002	Yes

Notes: ASF significantly Granger-causes tail risk at lags 2–5.

A.2.4 Placebo Test (Shuffled Time Series)

Comparison of real ASF predictive power versus shuffled (randomized) ASF.

Table A.4: **Placebo Test: Real vs. Shuffled ASF**

	Real ASF	**Shuffled ASF**	**Difference**
Mean Predictive Power	30.25	5.66	24.59
p-value		< 0.001	

Notes: Real ASF significantly outperforms randomly shuffled ASF, ruling out spurious correlation.

A.2.5 Surrogate Data Test

Phase-randomized surrogate data preserves spectral properties but destroys temporal structure.

Table A.5: **Surrogate Data Test**

Statistic	**Value**
Mean Z-score	-116.5
% of $Z < -2$	100%
% of $Z < -3$	100%
Min Z-score	-181.1

Notes: The predictive relationship is destroyed under phase randomization, confirming it reflects genuine temporal structure.

A.3 Out-of-Sample and Horse Race Tests

A.3.1 Diebold-Mariano Test Results

Pairwise forecast accuracy comparisons.

Table A.6: **Diebold-Mariano Tests (OOS 2020–2024)**

Model Comparison	**DM Statistic**	p**-value**
ASF vs. Random Walk	4.28	< 0.001
ASF vs. VIX Only	2.87	0.004
ASF vs. Realized Vol	3.41	< 0.001
Threshold vs. Linear	2.12	0.034

Notes: Positive values indicate ASF/Threshold model outperforms the alternative.

A.3.2 Horse Race Regression Results

Comparison of predictive variables in a multivariate framework.

Table A.7: **Horse Race: Incremental Predictive Power**

Variable	**Coef.**	t**-stat**	**Incr.** R^2	**Significant**
VIX	0.19	3.62	0.024	Yes
Realized Vol	0.14	2.91	0.018	Yes
ASF	-0.11	-2.34	0.012	Yes
ASF $\times$ Regime	0.28	4.87	0.041	Yes

Notes: ASF provides incremental predictive power beyond volatility measures; the regime interaction is highly significant.

A.4 Data and Variable Definitions

A.4.1 Asset Universe

The analysis employs two datasets:

1. **Global Macro Sample (1990–2024):** Daily returns for 12 country equity indices, 10-year government bonds, gold, oil, and USD index.

2. **ETF Sample (2007–2024):** Daily returns for 40 sector and country ETFs spanning equities, fixed income, commodities, and currencies.

A.4.2 Variable Definitions

Table A.8: **Variable Definitions**

Variable	Definition
H_t	Spectral entropy of correlation matrix eigenvalues
ASF_t	Accumulated Spectral Fragility: $ASF_t = \theta ASF_{t-1} + (1 - \theta)(1 - H_t)$
C_t	Connectivity: mean pairwise correlation
$Risk_{t+1}$	Forward 1-month maximum drawdown
τ	Connectivity threshold (estimated ≈ 0.14)