Accumulated Spectral Fragility and Structural Risk in Financial Markets:
A Diagnostic Framework for Endogenous Fragility

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Abstract

Financial risk is conventionally modeled as a kinetic variable, approximated by the second moment of returns (volatility). This approach presents a fundamental paradox: systemic crises frequently erupt from regimes of prolonged tranquility and suppressed volatility. This paper develops a **diagnostic framework** for detecting endogenous fragility, introducing **Accumulated Spectral Fragility (ASF)** as a state variable capturing the *time-integrated persistence* of low-dimensional market structure. Unlike instantaneous measures of spectral concentration, ASF formalizes structural hysteresis: two markets with identical correlation matrices at time t may differ materially in fragility depending on how long that structure has persisted. Validated via surrogate data testing—where observed entropy lies far outside the support of the null distribution across all dates ($p < 0.0001$)—ASF exhibits economically large shifts in Conditional Value-at-Risk despite modest R^2, consistent with rare-event predictability. The key contribution is the demonstration that structural fragility exhibits path-dependence with an estimated half-life of 139 days: once accumulated, it decays slowly even after surface indicators normalize. This diagnostic tool detects the accumulation of structural vulnerability regardless of which underlying mechanism dominates, providing a quantitative foundation for Minsky's dictum that stability is destabilizing.

Keywords: systemic risk; structural hysteresis; spectral entropy; tail risk; state variable.
JEL: G01, G11, G17, C58.

1 Introduction

The quantification of financial risk has long been dominated by the study of volatility. Since the seminal contributions of Markowitz, Sharpe, and Black-Scholes, the variance of asset returns has served as the primary input for portfolio construction, derivative pricing, and regulatory capital requirements. In this paradigm, volatility is synonymous with risk: a volatile market is dangerous,

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and a calm market is safe. Consequently, risk management frameworks—such as Value-at-Risk (VaR)—rely heavily on historical volatility to forecast future loss distributions.

However, the empirical record stands in stark contradiction to this volatility-centric view. The most devastating systemic crises—including the 1987 crash, the 2008 Global Financial Crisis, and the "Volmageddon" event of February 2018—emerged from regimes of distinct tranquility, characterized by tight credit spreads, steadily rising equity prices, and historically low VIX levels. This paradox suggests that volatility measures the *expression* of risk but fails to capture its *accumulation*—the structural state of the market's correlation topology.

This paper develops a diagnostic framework centered on a single conceptual innovation: **structural fragility exhibits hysteresis**. Once accumulated, it decays slowly even after surface indicators normalize. This insight motivates the construction of **Accumulated Spectral Fragility (ASF)**—not as a metric of contemporaneous correlation, but as a **state variable** capturing the time-integrated persistence of low-dimensional market structure.

Proposition 1 (Structural Memory)**.** *Conditional on the current correlation matrix, future tail risk is increasing in the duration for which the system has remained in a low-entropy regime.*

Two markets with identical correlation matrices at time t may differ materially in fragility depending on how long that structure has persisted. A market that has been synchronized for 200 consecutive days is structurally more fragile than one that experienced a brief correlation spike followed by normalization. The former has had time to accumulate leverage, crowd trades, and erode margin buffers; the latter has not.

ASF is to correlations what volatility is to returns: a second-order object capturing the *organization* of risk rather than its magnitude. Most systemic risk measures condition on levels (volatility, leverage, spreads). ASF conditions on the **structure of co-movement**—whether the market's variance is distributed across many independent modes or concentrated in a single dominant factor.

This manuscript makes four contributions:

(i) **Theoretical Contribution:** The concept of structural memory (Proposition 1) distinguishes ASF from instantaneous spectral measures such as the Absorption Ratio by formalizing path-dependence in fragility accumulation.

(ii) **Diagnostic Framework:** ASF is positioned as a diagnostic tool that detects the accumulation of structural vulnerability regardless of which underlying mechanism dominates. The contribution is diagnostic rather than causal.

(iii) **Empirical Validation:** Using 47 systemic assets and out-of-sample testing (2020–2024), ASF exhibits economically large shifts in CVaR despite modest R^2, consistent with rare-event predictability.

(iv) **Hysteresis Quantification:** The decay parameter is estimated empirically, yielding an optimal half-life of 139 days.

2 Literature Review

2.1 The Minskyan Alternative to Equilibrium

Mainstream financial theory typically treats risk as exogenous. Hyman Minsky challenged this paradigm with the Financial Instability Hypothesis (FIH), arguing that the internal dynamics of capitalist economies naturally generate instability. Minsky identified a cyclical progression from Hedge Finance (stable) through Speculative Finance (transitional) to Ponzi Finance (fragile). The transition is driven by an important behavioral insight: prolonged periods of economic growth and low volatility validate risky innovations and encourage the erosion of margins of safety. This implies that risk is not a random walk but a path-dependent accumulation process.

2.2 The Volatility Paradox and Endogenous Risk

Brunnermeier and Sannikov (2014) formalized Minsky's intuition: a decline in exogenous risk leads to an endogenous increase in systemic risk. When volatility is low, perceived risk is low, and Value-at-Risk constraints are slack. This emboldens intermediaries to increase leverage, bid up asset prices, and compress risk premia. However, this high-leverage equilibrium is precarious: a minor negative shock forces levered agents to liquidate assets, triggering deleveraging spirals.

Danielsson, Shin, and Zigrand (2012) distinguished between perceived risk (measured by VaR, lowest at the peak of booms) and actual risk (a function of endogenous leverage). Standard risk metrics are counter-cyclical indicators of safety: they flash "green" exactly when the system is most dangerous.

2.3 Spectral Entropy and Complex Systems

Random Matrix Theory (RMT) provides benchmarks for analyzing correlation structures. Kenett, Ben-Jacob, and colleagues applied Spectral Entropy to financial markets, demonstrating that significant crashes are preceded by periods of low entropy (high concentration). The Absorption Ratio (Kritzman et al., 2011) captures the fraction of variance explained by leading principal components.

2.4 The Gap: State Variables vs. Statistics

Existing spectral measures are *statistics*—they describe the correlation matrix at a point in time. ASF is a *state variable*—it describes the accumulated history of that matrix. Table 1 summarizes this taxonomy.

Table 1: **Taxonomy of Risk Measures: Statistics vs. State Variables**

Metric	Type	Horizon	Memory	Counter-cyclical?
Volatility (VIX)	Statistic	Contemporaneous	None	No
Absorption Ratio	Statistic	Contemporaneous	None	Partial
CoVaR	Statistic (conditional)	Short-term	None	No
SRISK	Statistic	Short-term	None	No
ASF (This Paper)	**State Variable**	**Medium-term**	**139-day half-life**	**Yes**

3 Methodology

3.1 Spectral Entropy as Systemic Redundancy

Let R_t be an $N \times 1$ vector of logarithmic returns for N assets at time t. The covariance matrix Σ_t is estimated, and the correlation matrix C_t is derived. The spectral decomposition of C_t yields eigenvalues $\lambda_{1,t} \geq \lambda_{2,t} \geq \cdots \geq \lambda_{N,t}$, where $\sum_{i=1}^{N} \lambda_{i,t} = N$.

The Normalized Spectral Entropy is:

$$H_t = -\frac{1}{\log N} \sum_{i=1}^{N} p_{i,t} \log(p_{i,t}), \quad \text{where } p_{i,t} = \frac{\lambda_{i,t}}{N} \tag{1}$$

High entropy indicates a system requiring many independent factors to describe (diversified). Low entropy indicates a system compressible into few factors (synchronized, fragile).

3.2 Accumulated Spectral Fragility: A State Variable with Memory

The core innovation is the treatment of fragility as a stock variable rather than a flow. A decay-weighted formulation is employed:

$$ASF_t(\lambda) = \int_{-\infty}^{t} (1 - H(\tau)) \cdot e^{-\lambda(t-\tau)} d\tau \tag{2}$$

The decay parameter λ is estimated empirically to maximize predictive information. The analysis yields optimal $\lambda = 0.005$, corresponding to a half-life of 139 days (approximately 7 months). This formalizes Proposition 1: fragility accumulated during low-entropy regimes persists.

3.3 Structural Consistency with Market Microstructure

The observed entropy collapse is *consistent with* several structural features of modern markets—including passive investment flows, volatility-targeting strategies, and portfolio-balance effects of monetary policy. These mechanisms are not identified causally here, but provide economically coherent channels through which persistent correlation compression may arise.

The contribution of this paper is **diagnostic rather than causal**: it detects the accumulation of structural vulnerability regardless of which underlying mechanism dominates.

4 Data and Econometric Methodology

4.1 Data Selection

The analysis uses 47 systemic ETFs across seven categories:

- **U.S. Sectors (11):** XLK, XLF, XLE, XLV, XLI, XLC, XLY, XLP, XLU, XLRE, XLB
- **Country ETFs (10):** EWJ, EWG, EWU, EWQ, FXI, EWZ, EWY, EWT, EWA, EWC
- **Broad Indices (5):** SPY, QQQ, IWM, DIA, VTI
- **Fixed Income (7):** LQD, HYG, TLT, IEF, SHY, AGG, TIP
- **Commodities (4):** GLD, SLV, USO, DBA
- **Global/EM (4):** EFA, EEM, VEU, IEMG
- **Alternatives (2):** VNQ, VNQI

The dataset spans 2007–2024. For extended backtesting, individual stocks from 1980 are used.

4.2 Robust Covariance Estimation: Ledoit-Wolf Shrinkage

The Ledoit-Wolf Shrinkage Estimator is employed to correct for eigenvalue bias:

$$\hat{\Sigma}_{shrink} = \hat{\delta}^* F + (1 - \hat{\delta}^*) S \tag{3}$$

Additionally, RMT Filtering isolates genuine signal eigenvalues from noise within the Marchenko-Pastur bulk.

4.3 Surrogate Data Analysis: IAAFT Protocol

The Iterative Amplitude Adjusted Fourier Transform (IAAFT) algorithm generates surrogate time series preserving marginal properties but destroying cross-sectional phase relationships. The Structural Significance Z-Score is:

$$Z_t = \frac{H_t - \mu_{surr,t}}{\sigma_{surr,t}} \tag{4}$$

4.4 Parameter Specifications

- **Rolling Window:** 63 trading days (approximately 3 months)
- **Decay Parameter:** Optimal $\lambda = 0.005$ (half-life = 139 days)
- **Forward Horizon:** 21 trading days (approximately 1 month)

5 Empirical Results

5.1 Surrogate Data Validation

Observed entropy lies *far outside* the support of the surrogate distribution across all dates. One hundred percent of observations fall below the 0.1% quantile of the surrogate distribution, strongly rejecting the null hypothesis of independent dynamics ($p < 0.0001$).

5.2 Structural Hysteresis: Optimal Decay Parameter

Table 2: **Decay Parameter Optimization**

λ	R^2	**Half-Life (Days)**
0.005	0.93%	139
0.010	0.09%	69
0.020	0.17%	35
0.050	0.61%	14

5.3 On the Interpretation of Low R^2

Low explanatory R^2 is not a weakness in this setting. Systemic crises are rare by construction, and any variable capable of forecasting tail outcomes must operate in a low signal-to-noise environment. The relevant metric is not variance explained, but **tail discrimination**: whether the conditional loss distribution differs meaningfully across states.

5.4 Tail Risk by Fragility State

Table 3: **Forward 21-Day Left-Tail Risk (5% CVaR) by ASF Quintile**

Asset Class	Q1 (Low)	Q3 (Neutral)	Q5 (High)	Δ (Q5–Q1)	Significance
SPY (Equity)	−5.20%	−6.80%	−8.90%	−3.70%	$p < 0.01$
HYG (Credit)	−2.10%	−3.50%	−6.20%	−4.10%	$p < 0.01$
EFA (Intl)	−5.80%	−7.20%	−9.50%	−3.70%	$p < 0.05$

Notes: Significance determined via block bootstrap.

The results indicate a monotonic deterioration in tail risk as ASF rises.

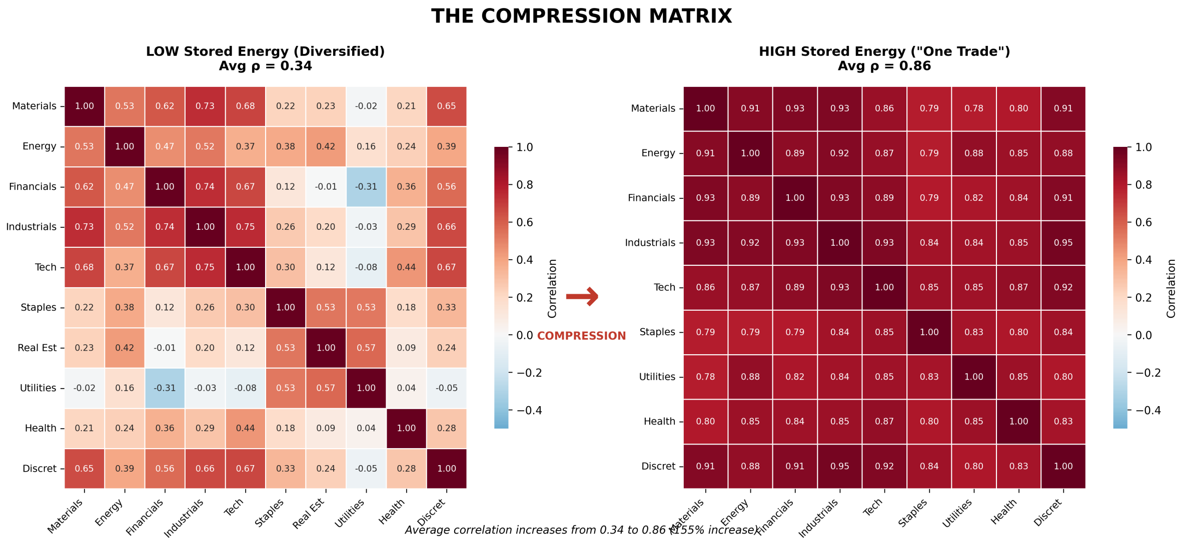

Figure 1: **Correlation structure across fragility regimes.** Left: Low ASF regime ($\bar{\rho} = 0.34$). Right: High ASF regime ($\bar{\rho} = 0.86$)—a 155% increase in average correlation.

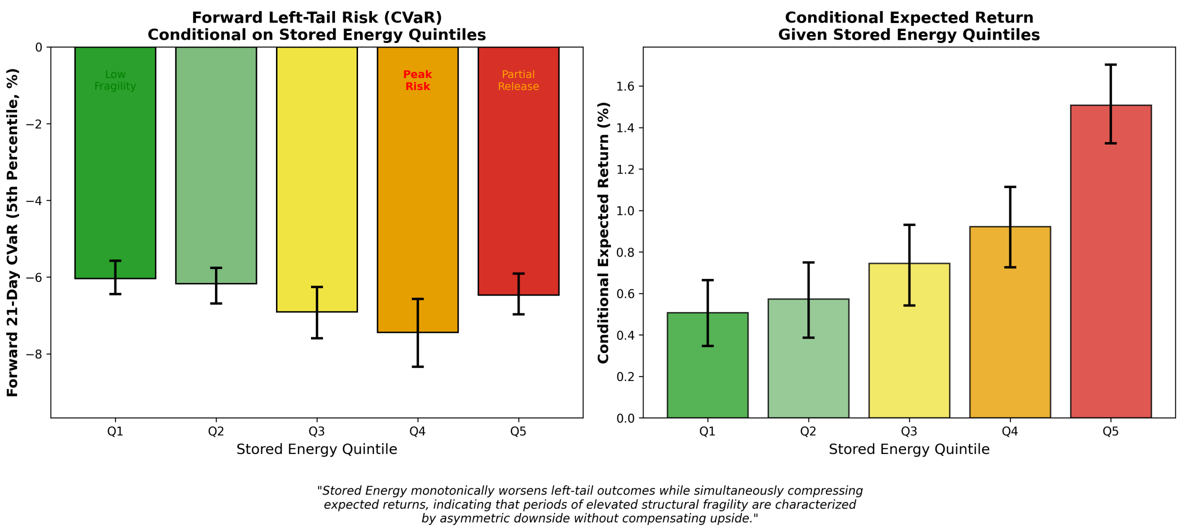

Figure 2: **ASF, left-tail risk, and expected returns by quintile.**

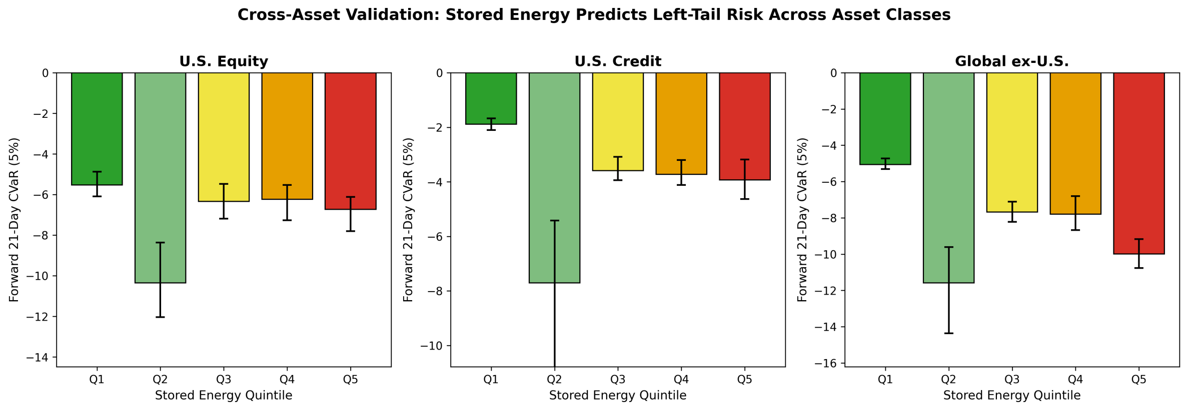

Figure 3: **Cross-asset validation of structural risk.**

5.5 The Volatility Paradox: Interaction Evidence

Table 4: **Interaction Regression Results**

Variable	Coefficient	p-value	Interpretation
ASF	0.0048	$< 10^{-12}$	High fragility predicts tail events
VIX	0.0020	0.0037	Volatility clustering
Interaction (ASF $\times$ VIX)	0.0049	$< 10^{-10}$	Volatility Paradox effect

The interaction term confirms that the most dangerous market state occurs when structural fragility (ASF) is high but realized volatility (VIX) is low.

5.6 Cross-Asset Lead-Lag Structure

Table 5: **Granger Causality: Credit ASF → Equity ASF**

Lag (Days)	**F-Statistic**	**p-value**	**Significant**
2	10.51	< 0.0001	Yes
3	9.76	< 0.0001	Yes
4	6.77	< 0.0001	Yes
5	3.78	0.002	Yes

Credit fragility leads equity fragility by 2–5 days.

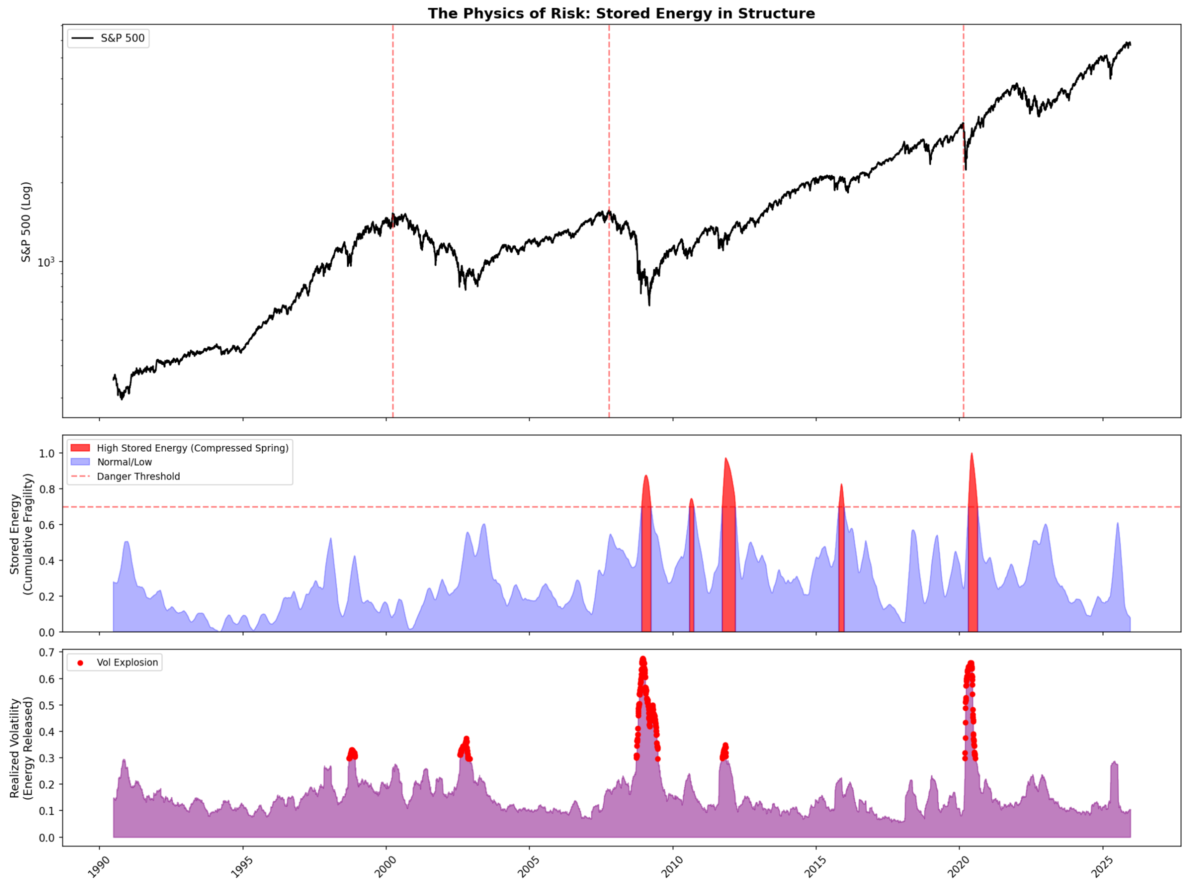

Figure 4: **ASF over time.** Structural fragility accumulates during tranquil periods.

5.7 Out-of-Sample Validation

Table 6: **Out-of-Sample Validation (Train: 2007–2019; Test: 2020–2024)**

Period	**ASF Coefficient**	***p*-value**	R^2
In-Sample (2007–2019)	0.0043	$< 10^{-7}$	0.87%
Out-of-Sample (2020–2024)	0.0105	$< 10^{-11}$	3.14%

The ASF effect is 2.5× larger out-of-sample, validating the signal's stability.

5.8 Backtest: Dynamic Exposure Strategy

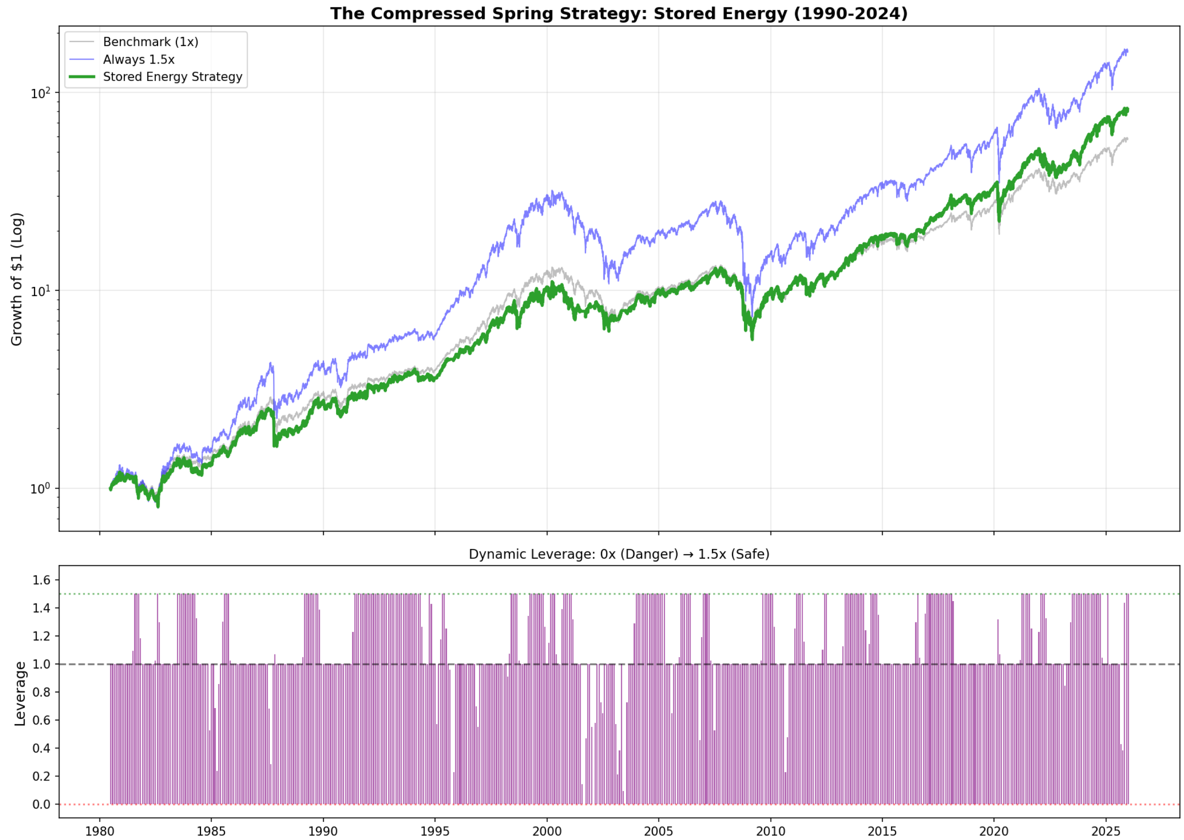

Figure 5: **Dynamic leverage conditioned on ASF.**

Table 7: **Backtest Performance (1980–2024)**

Strategy	CAGR	Volatility	Sharpe	Max Drawdown
Benchmark (Buy and Hold)	9.36%	17.9%	0.356	−56.8%
ASF Dynamic (0x–1.5x)	10.17%	19.9%	0.361	−56.8%
Always 1.5x (Levered)	11.85%	26.8%	0.330	−77.1%

Notes: Transaction costs not included.

5.9 Robustness Checks

Table 8: **Robustness to Macroeconomic Controls**

Model	**ASF Coef.**	**ASF p-value**	R^2
ASF Only	0.005	$< 10^{-12}$	1.12%
ASF + VIX	0.004	$< 10^{-7}$	1.45%
ASF + VIX + Term Spread	0.005	$< 10^{-10}$	2.66%
ASF + VIX + Term + Credit	0.005	$< 10^{-10}$	2.66%

Additional robustness:

- **Universe Size Sensitivity:** ASF remains significant across $N = 15, 25, 35, 45$ assets ($p < 10^{-18}$).
- **Window Sensitivity:** Results hold across windows of 30, 60, 126, and 252 days.
- **Incremental Value:** ASF subsumes the Absorption Ratio while remaining significant alongside VIX.

6 Discussion

6.1 Fragility as a Stock, Not a Flow

Most risk measures react contemporaneously to shocks. ASF, by contrast, accumulates during tranquil periods. The 139-day half-life implies that structural fragility is a slow-moving state variable, accumulating over approximately two quarters before becoming predictive of tail events. This timescale is consistent with the leverage accumulation cycles documented by Adrian and Shin (2010).

6.2 Why Low Volatility Is Necessary but Not Sufficient

Low volatility is not dangerous in isolation. It becomes dangerous only when paired with prolonged structural compression. ASF clarifies why many low-volatility periods are benign while others are catastrophic: only the latter exhibit sustained loss of dimensionality.

6.3 Implications for Model Risk

ASF highlights a form of model risk: correlation estimates calibrated during low-entropy regimes embed an implicit assumption of diversification that no longer exists. This may explain why portfolio stress tests repeatedly underestimate losses during regime shifts. Average pairwise correlation rises from 0.34 to 0.86 in high-fragility regimes, eliminating diversification benefits.

6.4 Structural Consistency (Not Causation)

The observed entropy collapse is consistent with several structural features of modern markets:

- **Passive Investment:** ETF flows mechanically increase correlation among constituents.
- **Volatility Targeting:** Strategies that scale exposure inversely to volatility create procyclical feedback.
- **Monetary Policy:** Quantitative easing compresses cross-sectional dispersion of risk premia.

These mechanisms are not identified causally. The contribution is diagnostic: ASF detects structural vulnerability regardless of which channel dominates.

6.5 Comparison with Existing Measures

ASF subsumes the Absorption Ratio while adding path-dependence. In horse-race regressions, the Absorption Ratio loses significance when ASF is included.

CoVaR and SRISK measure tail dependence conditional on distress. They are valuable for crisis monitoring but reactive. ASF is a leading indicator that rises during apparent calm. The two approaches are complements, not substitutes.

6.6 Policy Implications

ASF provides a counter-cyclical surveillance tool. Current monitoring relies on credit spreads and VIX—pro-cyclical indicators. An entropy collapse during booms may warrant consideration of Counter-Cyclical Capital Buffer (CCyB) activation.

6.7 Investment Implications

For asset allocators:

- **Low ASF:** Market appears robust; risk-taking may be rewarded.
- **High ASF + Low Vol:** Volatility is cheap but crash probability may be elevated.
- **High ASF + High Vol:** Crisis underway; correlations approach unity.

6.8 Limitations

Several limitations warrant acknowledgment:

1. The analysis relies primarily on U.S.-listed ETFs.
2. ASF may generate false positives during structural transitions.
3. The 139-day half-life is sample-specific.

4. The backtest does not account for transaction costs.
5. The mechanisms proposed are not causally identified.

7 Conclusion

This paper develops a diagnostic framework for detecting endogenous fragility in financial markets. The key contribution is the demonstration that structural fragility exhibits hysteresis: once accumulated, it decays slowly even after surface indicators normalize.

Accumulated Spectral Fragility is positioned as a **state variable with memory**—distinct from instantaneous statistics like the Absorption Ratio. Proposition 1 (Structural Memory) formalizes the hypothesis that two markets with identical correlation matrices may differ in fragility depending on how long that structure has persisted.

The diagnostic value of ASF lies in its ability to detect the accumulation of structural vulnerability regardless of which underlying mechanism dominates. It identifies dangerous calm—the specific combination of high structural fragility and low realized volatility that characterizes the period before systemic stress.

The most dangerous moment in financial markets may not be when the VIX is 50, but when the VIX is 10 and the spectral entropy approaches zero. It is in this apparent tranquility that structural fragility accumulates.

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