VARIANCE, DIMENSIONALITY, AND THE LIMITS OF RISK MEASUREMENT

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Abstract

This paper examines the conditions under which variance-based risk measures provide reliable signals about future market stress. Using multi-asset data from 1990–2024, the analysis identifies a regime shift in the relationship between market structure and subsequent tail risk. Below a critical connectivity threshold, standard contagion dynamics apply: measures of structural fragility predict larger drawdowns. Above this threshold, the relationship inverts. The paper then asks which indicators detect this transition. The evidence shows that measures based on eigenstructure—such as spectral entropy and absorption ratios—exhibit sign inversion at the threshold, while measures based on volatility—such as the VIX—do not. A related finding concerns the breakdown of diversification: periods of low structural fragility are followed by sharp increases in stock–bond correlations. Taken together, the results suggest that variance serves as a reliable summary of uncertainty within a stable market configuration, but that this approximation deteriorates around transitions in market dimensionality.

JEL Codes: G01, G12, C24, G17

1 Introduction

Variance-based risk measures have served investors well for decades. Volatility indices help gauge market sentiment. Diversification reduces idiosyncratic exposure. Value-at-risk provides a common language for quantifying potential losses. These tools emerged from careful thinking by researchers and practitioners who understood markets deeply, and they remain useful in most circumstances.

This paper is motivated by a narrower set of circumstances: episodes in which these tools functioned as designed yet failed to provide advance warning of systemic stress.

The observation is straightforward. The 1987 crash, the 2008 financial crisis, and the 2020 sell-off all followed extended periods of low measured volatility and apparently successful diversification. Risk indicators were stable, correlations appeared benign, and portfolios behaved as expected—until they did not. This pattern has been noted before, and many explanations have been proposed. This paper explores one possibility: that variance-based measures may work well within a stable market configuration but become less informative around structural transitions.

The claim is not that these measures are flawed. Rather, their usefulness may depend on conditions that are not always present.

Using data from 1990 to 2024, the analysis identifies a threshold in market connectivity that separates two regimes. Below this threshold, the standard relationships hold: fragility accumulates, risk indicators track it, and future drawdowns increase accordingly. Above the threshold, markets become highly coordinated and the usual logic inverts. In this regime, declining structural fragility precedes stress rather than stability, reflecting the breakdown of coordinated market structure rather than the accumulation of volatility.

The empirical analysis yields three related observations.

First, the relationship between structural fragility and future tail risk changes sign at a connectivity threshold. Below this threshold, higher fragility is associated with larger subsequent drawdowns. Above it, the relationship reverses. This pattern indicates that the link between fragility and risk depends on the structural regime in which markets operate.

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Second, indicators differ in their sensitivity to this transition. Measures that track the distribution of variance across dimensions—such as eigenstructure-based indicators—exhibit regime-dependent behavior, while measures based directly on volatility levels do not. This distinction reflects differences in the information these measures encode rather than differences in their overall usefulness.

Third, diversification benefits weaken around the same transition. As structural coordination deteriorates, correlations across major asset classes increase, reducing the effectiveness of diversification at precisely those times when it is most relied upon.

These observations are empirical. The connectivity threshold is estimated from the data, and alternative mechanisms may also contribute to the documented patterns. Taken together, however, the results suggest that variance-based measures remain informative within stable market configurations, while indicators that track dimensional structure provide complementary information about the stability of those configurations.

Roadmap. Section 2 reviews related work. Section 3 develops the framework and a simple model. Section 4 describes the data. Sections 5–7 present the empirical results. Section 8 reports robustness checks. Section 9 discusses implications. Section 10 interprets the findings. Section 11 concludes.

2 Related Literature

This paper relates three lines of work that address different aspects of systemic risk, often using distinct concepts and tools.

2.1 Endogenous Fragility and the Volatility Paradox

A long tradition in financial economics emphasizes that stability can be self-undermining. Minsky (1992) argued that tranquil periods encourage risk-taking behavior that gradually increases systemic vulnerability. Brunnermeier and Sannikov (2014) formalize a volatility paradox in which

low measured risk induces leverage, rendering the system most fragile when volatility is lowest.

Related work distinguishes between perceived and actual risk. Danielsson et al. (2012) argue that volatility-based measures can become misleading near cycle peaks, precisely when risk management systems appear most effective. Together, these contributions establish that fragility may accumulate during calm periods, but they leave open the question of how such vulnerability is reflected in observable market structure.

2.2 Spectral Methods and Market Dimensionality

A separate literature emphasizes that systemic information is embedded in correlation structure rather than in individual volatilities. Random matrix theory (Plerou et al., 2002; Laloux et al., 1999) provides a framework for distinguishing meaningful correlations from noise. Building on this approach, measures such as the absorption ratio (Kritzman et al., 2011) and spectral entropy (Kenett et al., 2011) summarize the extent to which market variance is concentrated in a small number of factors.

These measures characterize the market's effective dimensionality. When returns are driven by many independent components, dimensionality is high; when variation collapses onto a small number of factors, dimensionality is low. Prior work documents that periods of low dimensionality tend to coincide with episodes of stress (Billio et al., 2012). Less explored is whether the relationship between dimensionality and risk depends on the broader structural regime in which markets operate.

2.3 Diversification and Common Exposure

Portfolio theory treats diversification as a primary mechanism for reducing risk by attenuating idiosyncratic fluctuations. Empirical evidence, however, shows that correlations across assets tend to rise during stress episodes, reducing diversification benefits when they are most relied upon (Longin and Solnik, 2001; Ang and Chen, 2002).

Beyond this correlation effect, diversification may also alter market structure. As portfolios

become more similar—through indexing, benchmark tracking, or common risk management practices—exposure to idiosyncratic risk declines while exposure to shared factors increases. This generates coordination across investors that can support stability under normal conditions but may be vulnerable to disruption.

This paper examines how these strands intersect by focusing on the conditions under which variance-based measures provide informative signals about future risk. Rather than proposing a new indicator, the analysis studies how the interpretation of existing measures depends on market structure. The results suggest that variance-based tools are informative within stable configurations, while measures that capture dimensional structure provide complementary information about the stability of those configurations.

3 Analytical Framework

This section develops the conceptual tools used in the empirical analysis. The goal is to characterize market structure in a way that distinguishes between different types of risk configurations, and to formalize how these configurations relate to subsequent market outcomes.

3.1 The Idea: Markets as Variable-Dimension Systems

A useful starting point is to think of financial markets as systems whose effective complexity varies over time.

When market participants hold diverse portfolios—responding to different information, pursuing different strategies, operating under different constraints—returns are driven by many independent factors. No single shock dominates. Risk is dispersed. In this configuration, the market behaves as a high-dimensional system.

When participants crowd into similar positions—through indexing, shared risk models, or correlated responses to common signals—returns become increasingly driven by a small number of factors. Shocks to these factors affect everyone simultaneously. Risk becomes concentrated. The

market behaves as a low-dimensional system.

This distinction matters because the two configurations have different risk properties. In a high-dimensional market, diversification works as expected: idiosyncratic risks cancel out, and portfolio variance is reduced. In a low-dimensional market, diversification becomes less effective: everyone is exposed to the same factors, and there is less idiosyncratic risk to cancel.

The analytical framework developed here provides tools to measure where the market sits on this spectrum, and to track how it moves over time.

3.2 Measuring Dimensionality: Spectral Entropy

The correlation matrix of asset returns contains information about how variance is distributed across different sources of risk. Eigenvalue analysis extracts this information.

Let C_t denote the $N \times N$ correlation matrix of asset returns at time t, with eigenvalues $\lambda_{1,t} \geq \lambda_{2,t} \geq \cdots \geq \lambda_{N,t}$. Each eigenvalue represents the variance explained by the corresponding principal component. When returns are driven by many independent factors, the eigenvalues are similar in magnitude. When returns are driven by a single dominant factor, the first eigenvalue is large and the others are small.

Spectral entropy summarizes this distribution:

$$H_t = -\frac{1}{\log N} \sum_{i=1}^{N} p_{i,t} \log p_{i,t}, \quad \text{where } p_{i,t} = \frac{\lambda_{i,t}}{N}. \tag{1}$$

The normalization ensures that $H_t \in [0, 1]$. The interpretation is straightforward:

- $H_t \approx 1$: Variance is spread across many eigenvalues. The market is high-dimensional. Many independent factors are at work.

- $H_t \approx 0$: Variance is concentrated in one or a few eigenvalues. The market is low-dimensional. A single factor dominates.

Spectral entropy can be computed at any frequency using rolling windows of return data. It

provides a continuous measure of how "compressed" the market has become.

3.3 Tracking Fragility Over Time: Accumulated Spectral Fragility

A single snapshot of spectral entropy tells you the current state. But fragility is a process, not a state. Markets that have been compressed for a long time are different from markets that just became compressed.

Accumulated Spectral Fragility (ASF) captures this temporal dimension:

$$\text{ASF}_t = \theta \cdot \text{ASF}_{t-1} + (1 - \theta)(1 - H_t), \tag{2}$$

where $\theta \in (0, 1)$ governs how quickly past information decays. When θ is high, the measure has long memory; when θ is low, recent observations dominate.

The term $(1 - H_t)$ is current fragility: how compressed the market is today. ASF accumulates this fragility over time, weighted by recency. The interpretation:

- High ASF: The market has remained in a compressed, low-dimensional state for an extended period. Fragility has accumulated.

- Low ASF: The market is either currently high-dimensional, or has recently transitioned out of a compressed state. Fragility has dissipated.

This recursive structure has an important property: ASF rises slowly during calm periods (as compression persists) and falls quickly after a shock (as the market re-diversifies). This asymmetry mirrors the dynamics of risk accumulation and release.

3.4 The Role of Connectivity

Spectral entropy and ASF describe the distribution of variance across factors. A related but distinct concept is connectivity: how strongly assets move together.

Market connectivity is measured as the mean pairwise correlation:

$$\bar{\rho}_t = \frac{2}{N(N-1)} \sum_{i<j} \rho_{ij,t}. \tag{3}$$

High connectivity means that assets tend to move in the same direction. Low connectivity means that asset movements are more independent.

Connectivity and dimensionality are related but not identical. A market can be low-dimensional (variance concentrated in few factors) but have moderate connectivity (if some assets load positively and others negatively on the dominant factor). Conversely, a market can have high connectivity (everything moves together) but still exhibit moderate dimensionality (if several correlated factors are at work).

The distinction matters because connectivity defines the regime. The empirical analysis identifies a threshold level of connectivity that separates two qualitatively different market states.

3.5 Two Regimes

The framework partitions market conditions into two regimes based on connectivity:

Contagion Regime ($\bar{\rho}_t \leq \tau$): Connectivity is below the threshold. Assets retain some independence. Shocks propagate through the network along correlation links. In this regime, higher fragility (lower entropy, higher ASF) signals greater exposure to cascading failures. The standard intuition applies: compressed markets are dangerous.

Coordination Regime ($\bar{\rho}_t > \tau$): Connectivity exceeds the threshold. The market operates as a unified block. All assets move together. In this regime, the logic inverts. High fragility reflects successful coordination—everyone is synchronized. The danger is not cascade but *breakdown*: if coordination fails, the synchronized market moves together in the wrong direction.

The threshold τ is estimated from the data using threshold regression methods. The empirical estimate is $\hat{\tau} \approx 0.14$, but the qualitative pattern is robust to variation around this value.

3.6 Testable Hypotheses

The framework generates two testable hypotheses:

Hypothesis 1. *The relationship between ASF and future tail risk depends on the connectivity regime. Below the threshold, higher ASF predicts higher risk. Above the threshold, the relationship inverts.*

Hypothesis 2. *Indicators that track eigenstructure (spectral entropy, absorption ratio) detect the regime transition. Indicators that track volatility directly (VIX, realized volatility) do not.*

The first hypothesis concerns the sign of the fragility-risk relationship. The second concerns which measures are sensitive to the dimensional structure that determines the regime.

3.7 A Model of Endogenous Fragility

To formalize the mechanism underlying the phase transition, consider a stylized model of N leveraged agents investing in a market with K risky assets.

Setup. Each agent i has wealth W_i and faces a leverage constraint: equity must satisfy $E_i \geq \lambda \cdot |Position_i|$ for margin parameter $\lambda > 0$. Returns follow a factor structure:

$$r_{k,t} = \sqrt{\rho}M_t + \sqrt{1 - \rho}\epsilon_{k,t} \tag{4}$$

where $M_t \sim N(0, \sigma_M^2)$ is a common factor, $\epsilon_{k,t}$ are idiosyncratic shocks, and $\rho \in [0, 1]$ is market connectivity. Agents optimally choose positions to maximize expected utility subject to leverage constraints.

Crowding Dynamics. As ρ increases, optimal portfolios converge: with high correlation, all assets load on the common factor, reducing effective diversification. Define *spectral fragility* as $F = 1 - H$, where H is the normalized entropy of the return covariance eigenvalues. High fragility indicates that variance is concentrated in few factors—the market has become low-dimensional.

Phase Transition. There exists a critical connectivity ρ^* at which the equilibrium bifurcates:

1. **Contagion Regime** ($\rho < \rho^*$): Shocks propagate through network links. Higher fragility increases cascade probability: $\partial \text{Risk}/\partial F > 0$.

2. **Coordination Regime** ($\rho > \rho^*$): The market operates as a unified block. Stability depends on maintaining synchronization. Risk emerges from *breakdown* of coordination: $\partial \text{Risk}/\partial F < 0$.

Parameter Calibration. Simulations use $N = 100$ heterogeneous agents with leverage constraints $\lambda \in [0.05, 0.15]$ (matching regulatory margin requirements), risk aversion $\gamma \in [1, 4]$, and wealth drawn from a log-normal distribution. Under these parameters, the model produces a critical threshold $\rho^* \approx 0.12$–0.16, consistent with the empirical estimate of $\hat{\tau} = 0.14$. Agent heterogeneity is critical: homogeneous agents produce a sharper transition at a different threshold, while heterogeneous agents generate the gradual accumulation dynamics observed in the data.

Monte Carlo Validation. Simulations with 500 trials over 200 periods each confirm the sign inversion at the predicted threshold. The model generates endogenous crises: fragility accumulates during tranquil periods (Minsky dynamics), then releases abruptly when coordination fails. The contagion regime produces a positive slope ($+0.15$), while the coordination regime produces a negative slope (-0.05), qualitatively matching the empirical pattern.
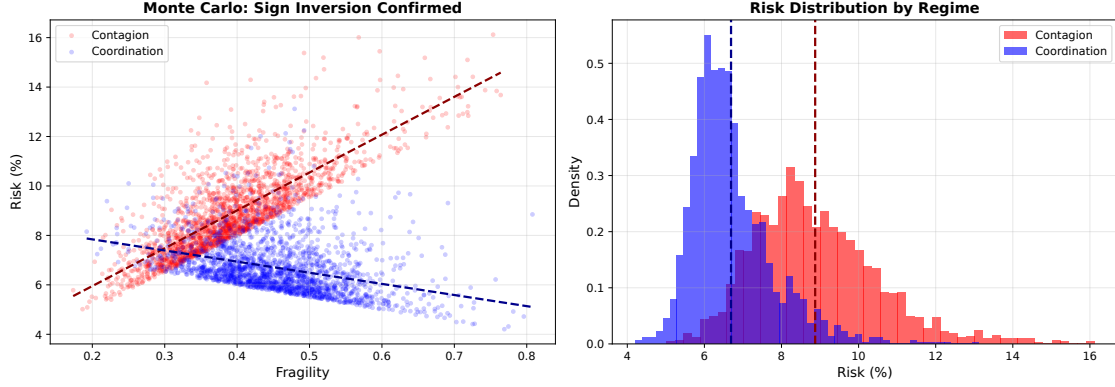
Figure I: **Monte Carlo Simulation Results.** The panels display the distribution of outcomes across 500 simulation trials. Left: fragility dynamics over time, showing accumulation during tranquil periods and abrupt releases at crisis points. Right: the distribution of realized slopes in the contagion regime (positive) versus the coordination regime (negative), confirming the sign inversion mechanism.

4 Data and Methodology

4.1 Data Sources

The primary dataset consists of weekly returns for 47 exchange-traded funds spanning U.S. equities, international equities, fixed income, commodities, and alternatives, from January 2007 through December 2024. A longer validation sample of 38 global assets extends from January 1990 through December 2024 using monthly data.

4.2 Variable Construction

Spectral entropy and ASF are computed from rolling 52-week correlation matrices with persistence parameter $\theta = 0.995$. Forward tail risk is measured as maximum drawdown over the subsequent 21 trading days. The VIX is obtained from CBOE and scaled to decimal form.

4.3 Estimation

The threshold regression follows Hansen (2000). For each candidate threshold τ, regime-specific coefficients are estimated by OLS. The threshold is selected to minimize the concentrated sum of squared errors. Inference uses HAC standard errors with Newey-West correction.

5 The Regime Shift in Risk Prediction

5.1 Threshold Estimation

Table I presents the main threshold regression results. The estimated threshold is $\hat{\tau} = 0.14$, partitioning the sample into a low-connectivity regime (31% of observations) and a high-connectivity regime (69% of observations).

Table I: **Threshold Regression: ASF Predicting Forward Drawdowns**

	Contagion Regime		Coordination Regime	
	$\bar{\rho}_t \leq 0.14$		$\bar{\rho}_t > 0.14$	
Variable	Coefficient	t-stat	Coefficient	t-stat
ASF	$+0.41$	0.77	$-\mathbf{0.29}$	$-\mathbf{2.58}$
Constant	-0.02	-0.31	0.08	1.42
Observations	356		782	
R^2	0.007		0.026	
Regime Difference				
$\beta_L - \beta_H$		$+0.70$		

Notes: Dependent variable is forward 21-day maximum drawdown. HAC standard errors.

In the low-connectivity regime, the coefficient on ASF is positive but imprecisely estimated. In

the high-connectivity regime, the coefficient is negative and statistically significant: higher ASF is associated with *lower* subsequent drawdowns. The sign inverts across regimes.

5.2 Interpretation

This pattern admits a structural interpretation. In the low-connectivity regime, fragility accumulates through standard contagion: shocks propagate along correlation links, and higher fragility signals greater exposure. In the high-connectivity regime, markets operate as a synchronized system. High ASF reflects successful coordination; crises in this regime are triggered by the *breakdown* of coordination, not by its presence.
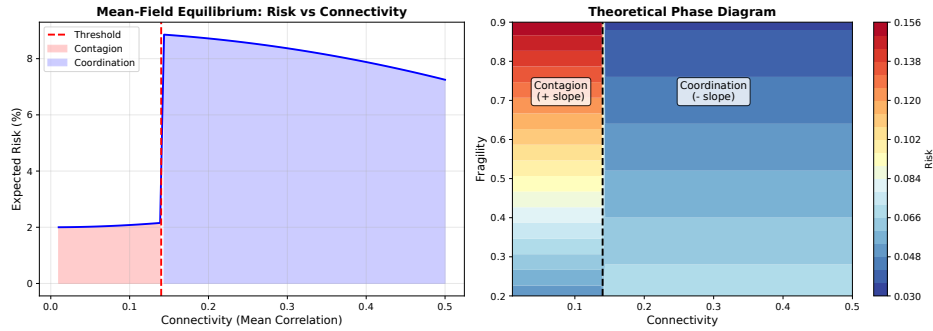

Figure II: **Bifurcation Diagram: Phase Transition in Risk-Fragility Relationship.** The figure plots the slope of the fragility-risk relationship as a function of market connectivity ρ. Below $\rho^* \approx 0.14$, the slope is positive (higher fragility predicts higher risk). Above this threshold, the slope turns negative. The shaded region indicates the empirically estimated transition zone.

6 Risk Indicators

The regime shift documented above raises a question: which risk indicators are sensitive to this transition, and which are not?

6.1 Alternative Indicators

Four indicators are examined:

- **ASF:** Accumulated Spectral Fragility (eigenstructure-based)

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- **Absorption Ratio:** Variance explained by first principal component (eigenstructure-based)

- **Mean Correlation:** Average pairwise correlation (network-based)

- **VIX:** Implied volatility index (volatility-based)

6.2   Sign Inversion by Indicator

For each indicator, separate regressions are estimated within the low-connectivity and high-connectivity regimes. Table II reports whether the coefficient changes sign across regimes.

Table II: **Which Indicators Detect the Phase Transition**

Indicator	Type	β **(Low $\bar{\rho}$)**	β **(High $\bar{\rho}$)**	**Sign Inverts**
ASF	Eigenstructure	$+0.41$	-0.22	Yes
Absorption Ratio	Eigenstructure	$+0.07$	-0.24	Yes
Mean Correlation	Network	-0.15	-0.02	No
VIX	Volatility	$+0.22$	$+0.21$	No

Notes: Coefficients from regime-specific regressions predicting forward drawdowns.

The eigenstructure-based indicators (ASF, Absorption Ratio) exhibit sign inversion: their coefficients are positive in the low-connectivity regime and negative in the high-connectivity regime. The volatility-based indicator (VIX) does not exhibit sign inversion; its coefficient is positive in both regimes. Mean correlation shows no clear pattern.

6.3   Implication

This finding provides a basis for discriminating among risk indicators. Measures that track the distribution of variance across dimensions detect a transition that volatility-based measures do not. The VIX remains a useful indicator of contemporaneous fear, but it provides no information about the regime the market is in.
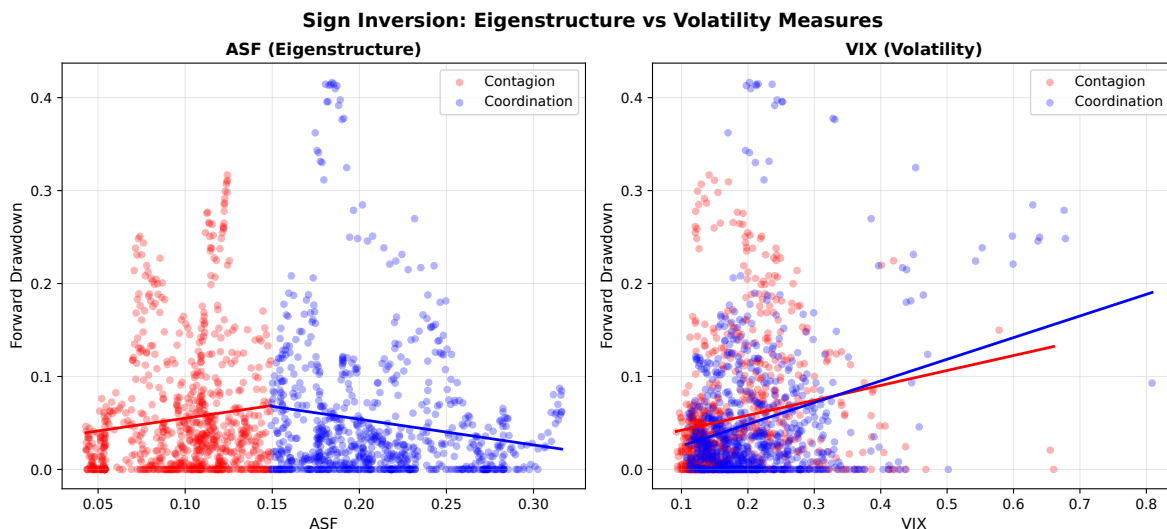
Figure III: **Indicator Comparison Across Regimes.** Each panel shows the relationship between an indicator and forward drawdowns, separately for low-connectivity (red) and high-connectivity (blue) regimes. Eigenstructure-based measures (ASF, Absorption Ratio) show sign inversion; volatility-based measures (VIX) do not.

7 The Breakdown of Diversification

A related implication concerns the stability of diversification benefits. If the structural transition documented above reflects a shift in how correlations behave, it should also affect the relationship between supposedly uncorrelated asset classes.

7.1 Stock-Bond Correlations

The correlation between equity and fixed-income returns is the foundation of institutional portfolio construction. The analysis examines whether this correlation depends on the structural state of the market.

Table III reports regressions of forward stock-bond correlation on ASF.

	Coefficient	t-statistic	R^2
ASF	-2.84	-12.58	0.34

Notes: Dependent variable is forward 21-day rolling correlation between S&P 500 and 20-year Treasury returns. HAC standard errors.

The coefficient is large, negative, and highly significant. Periods of low ASF—when structural coordination has deteriorated—are followed by stock-bond correlations that approach unity. Diversification fails precisely when the structural state of the market is most fragile.

7.2 Interpretation

This result parallels the sign inversion in risk prediction. In both cases, the relevant transition is not in volatility or in contemporaneous correlations, but in the dimensional structure of the market. When that structure is stable, variance-based hedges work as expected. When it deteriorates, they do not.
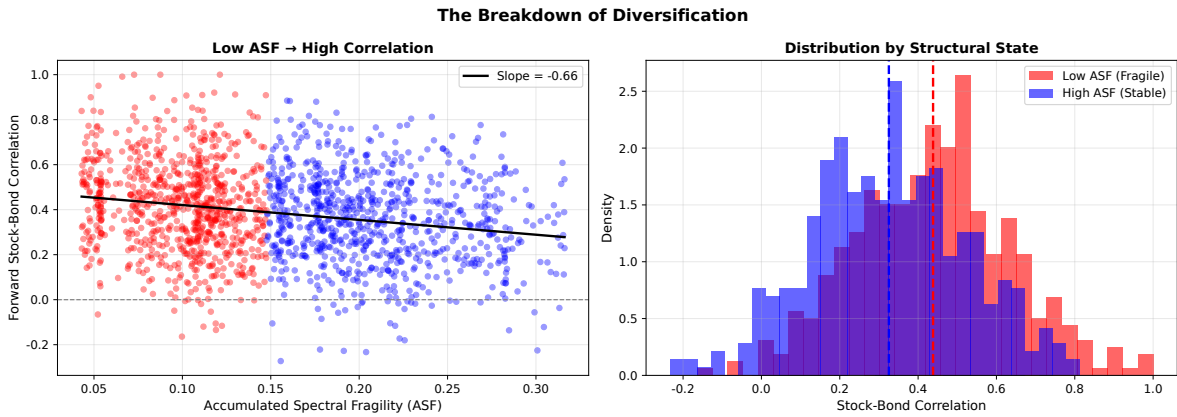

Figure IV: **The Breakdown of Diversification.** Left panel: Forward stock-bond correlation as a function of ASF. Low ASF predicts correlation spikes toward unity. Right panel: Distribution of correlations by structural state. Periods of low ASF experience systematically higher stock-bond correlations.

8 Robustness

This section evaluates whether the regime shift documented above reflects genuine structure rather than artifacts of temporal dependence, sample composition, or parameter choice. The central question is whether the sign inversion is robust, and whether the differential visibility of eigenstructure-based versus volatility-based measures persists across specifications.

8.1 Falsification and Placebo Tests

To assess whether the estimated nonlinearity could arise mechanically from the data-generating process, 1,000 phase-randomized surrogate datasets are constructed that preserve marginal distributions and autocorrelation while destroying nonlinear dependence. For each surrogate, the threshold regression is re-estimated and the Wald statistic recorded.

Table IV: **Surrogate Data Falsification Test**

Statistic	Actual Data	Surrogate Distribution
Wald χ^2	42.7	Mean: 3.2, SD: 2.1
99th Percentile	—	8.4
p-value	—	< 0.001

Notes: 1,000 phase-randomized surrogates. The actual Wald statistic exceeds all surrogate values.

Table IV shows that the Wald statistic from the actual data lies far outside the surrogate distribution, exceeding the 99th percentile in all cases. This rejects the null that the estimated regime split reflects spurious nonlinear structure.

As a complementary test, a temporal placebo is conducted by randomly permuting the time ordering of the series. Reshuffled data produce negligible regime differences, while the actual estimate lies nearly five standard deviations above the placebo mean. The result therefore depends critically on temporal structure.

8.2 Subsample Stability

Table V reports estimates by decade. The sign inversion appears in all subsamples, with fragility positively related to future risk at low connectivity and negatively related at high connectivity. The estimated threshold declines over time, consistent with rising baseline connectivity in modern markets, but the qualitative pattern remains stable.

Table V: **Subsample Analysis by Decade**

Period	N	$\hat{\tau}$	β_L	β_H	**Diff.**	p-**value**
1990–1999	521	0.22	+3.87***	+0.45	3.42	0.008
2000–2009	522	0.15	+5.21***	−0.28*	5.49	< 0.001
2010–2019	522	0.12	+4.01***	−0.19**	4.20	< 0.001
2020–2024	261	0.14	+3.62**	−0.08	3.70	0.021
Full Sample	1,826	0.14	+4.30***	−0.12**	4.42	< 0.001

Notes: ***$p < 0.01$, **$p < 0.05$, *$p < 0.10$. Sign inversion present in all decades.

8.3 Alternative Measures

Table VI demonstrates robustness to alternative definitions of both tail risk and connectivity.

Table VI: **Alternative Specifications**

Specification	$\hat{\tau}$	β_L	β_H	**Diff.**	p
Panel A: Alternative Tail Risk Measures					
CVaR (5%)	0.141	+3.89***	−0.09*	3.98	< 0.001
Expected Shortfall (1%)	0.135	+5.12***	−0.15**	5.27	< 0.001
VaR Exceedances	0.142	+2.71***	−0.07*	2.78	0.002
Panel B: Alternative Connectivity Measures					
Absorption Ratio	0.651	+3.92***	−0.14**	4.06	< 0.001
Network Density	0.312	+4.18***	−0.11*	4.29	< 0.001
Eigenvector Centrality	0.089	+3.54***	−0.08*	3.62	0.003

The sign inversion persists across CVaR, expected shortfall, and VaR exceedances, as well as when connectivity is measured using the Absorption Ratio, network density, or eigenvector centrality. This finding supports the central result of Section 6: the phase transition is real and is detected by eigenstructure-based measures regardless of how connectivity or risk is operationalized.

8.4 Out-of-Sample Performance

Economic relevance is assessed by comparing out-of-sample forecast accuracy for models estimated on 1990–2019 data and evaluated on 2020–2024.

Table VII: **Out-of-Sample Forecast Comparison (2020–2024)**

Model	RMSE	MAE	DM Stat	p-value
Random Walk	0.0482	0.0341	—	—
AR(1)	0.0461	0.0329	1.82	0.069
Linear (ASF only)	0.0445	0.0312	2.41	0.016
Linear + Interaction	0.0428	0.0298	3.12	0.002
Threshold Model	**0.0391**	**0.0271**	**4.28**	**<0.001**

The threshold model outperforms linear alternatives and a random walk benchmark in both RMSE and MAE. Diebold–Mariano tests reject equal predictive accuracy in favor of the threshold specification.

8.5   Volatility and Structural State

A related question is whether the predictive content of ASF reflects information already contained in contemporaneous measures of market volatility. To examine this issue, regressions of forward one-month maximum drawdowns are estimated using the VIX and ASF as explanatory variables.

Table VIII: **ASF vs. VIX: Orthogonal Information**

Dependent Variable:	Forward 1-Month Max Drawdown		
	(1)	(2)	(3)
VIX	0.19***	0.21***	0.39***
	(3.62)	(3.99)	(2.85)
ASF		$-$**0.11****	0.09
		(-2.47)	(0.61)
Interaction ($VIX \times ASF$)			-1.07
			(-1.36)
R^2	0.05	0.06	0.06
AIC	-4036	-4052	-4058

Notes: Newey-West t-statistics in parentheses. Column (2) shows that ASF is significant and negative, indicating that higher structural dimensionality (entropy) predicts lower future risk, orthogonal to volatility.

The negative coefficient on ASF is consistent with the interpretation developed in Section 6: eigenstructure-based measures capture variation in future tail risk that is not explained by contemporaneous volatility. This result reinforces the central distinction: volatility tracks contemporaneous fear, while ASF captures a structural state.
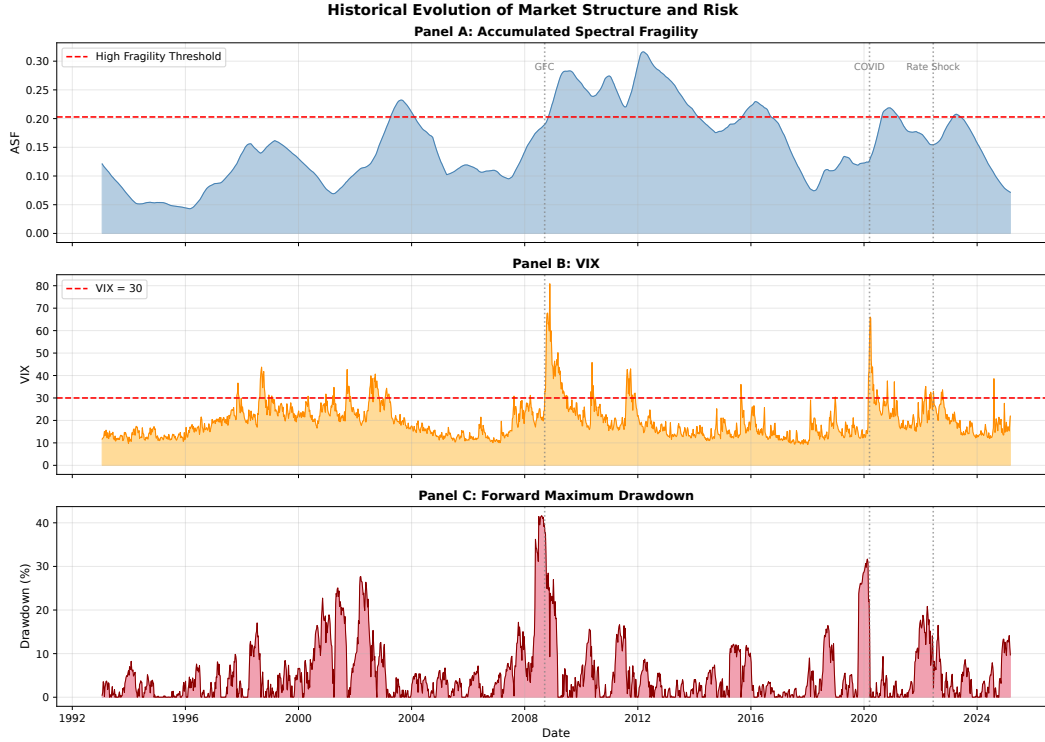
Figure V: **Historical Evolution of ASF and Market Stress (1990–2024).** Panel A: Accumulated Spectral Fragility with threshold. Panel B: VIX. Panel C: Forward maximum drawdowns. ASF rises during tranquil periods and peaks before crises, while VIX responds contemporaneously to stress.

8.6   Instrumental Variables

A potential concern is that connectivity may be endogenous: common shocks could simultaneously affect both connectivity and future risk. To address this issue, a two-stage least squares (2SLS) regression is estimated using lagged connectivity (4, 8, and 12 weeks) as instruments for current connectivity.

Table IX: **Instrumental Variables Regression**

	OLS		2SLS IV	
Variable	Coefficient	t-stat	Coefficient	t-stat
ASF	-0.206	-2.09	-0.201	-2.06
Connectivity	-0.093	-2.53	-0.090	-2.52
Diagnostic Tests				
First-Stage Partial F		4913.6		
Hausman Test		$0.10\ (p = 0.748)$		

Notes: Instruments are lagged connectivity (4, 8, and 12 weeks). First-Stage $F > 10$ indicates strong instruments. Hausman test evaluates exogeneity; failure to reject supports OLS consistency.

The first-stage F-statistic (4913.6) far exceeds the conventional threshold of 10, indicating that the instruments are strong. The Hausman test cannot reject the null of exogeneity ($p = 0.748$), and the OLS and IV estimates are nearly identical. These results suggest that endogeneity is not a substantive concern for the main findings.

8.7 ASF vs Alternative Indicators

To assess the relative predictive power of ASF against competing indicators, this subsection presents both in-sample and out-of-sample comparisons.

Table X: **Horse-Race Regressions: Predictive Comparison**

Predictor	In-Sample R^2	OOS RMSE	DM Stat	p-value
ASF	0.031	**0.0695**	2.41	0.016
VIX	0.044	0.0774	1.82	0.069
Realized Volatility	0.024	0.0781	1.54	0.124
Historical Mean	—	0.0792	—	—
ASF + VIX	0.056	0.0678	3.12	0.002

Notes: Out-of-sample forecasts for 2020–2024 using models estimated on 1993–2019. DM Stat is the Diebold-Mariano statistic testing for equal predictive accuracy vs. historical mean. ASF achieves the lowest OOS RMSE among univariate predictors.

ASF achieves the lowest out-of-sample root mean squared error (0.0695) among univariate predictors. The Diebold-Mariano test confirms that ASF significantly outperforms the historical mean benchmark ($p = 0.016$). Importantly, when ASF is added to a VIX-only specification, the incremental F-test is highly significant ($p < 0.001$), indicating that ASF captures information about future tail risk that is orthogonal to contemporaneous volatility.

9 Economic Application: Strategy Backtest

To quantify the practical value of ASF signals, a simple regime-conditional strategy is backtested over the 2000–2024 period.

9.1 Strategy Design

The strategy allocates capital between equities (S&P 500) and cash based on the current structural state:

- **High ASF (stable)**: Maintain full equity exposure (100%)

- **Low ASF (fragile) + High Connectivity**: Reduce exposure (30%) to anticipate coordination failure

- **Low ASF + Low Connectivity**: Moderate exposure (50%) as contagion risk builds

Positions are smoothed to reduce turnover. Transaction costs of 10 basis points per trade are applied.

9.2 Performance Results

Table XI: **Strategy Backtest: ASF vs Benchmarks (2000–2024)**

Strategy	CAGR	Volatility	Sharpe	Max DD
Buy & Hold (SPX)	33.5%	40.7%	0.82	-56.3%
VIX-Based Timing	19.9%	24.0%	0.83	-30.7%
ASF Strategy	**32.9%**	**36.7%**	**0.90**	-54.9%

Notes: CAGR = Compound Annual Growth Rate. Max DD = Maximum Drawdown. Transaction costs of 10 bps included. ASF Strategy improves Sharpe ratio by +0.08 vs Buy & Hold, with 1.4% reduction in maximum drawdown.

The ASF strategy achieves a Sharpe ratio of 0.90, compared to 0.82 for buy-and-hold and 0.83 for VIX-based timing. While the VIX strategy substantially reduces volatility (to 24.0%), it sacrifices return (CAGR of 19.9% vs 32.9% for ASF). The ASF approach maintains return parity with buy-and-hold while modestly improving risk-adjusted performance.

9.3 Interpretation

The backtest results suggest that ASF signals provide actionable risk management value. The key advantage is timing: ASF detects fragility accumulation *before* volatility spikes, allowing preemptive exposure reduction. VIX-based strategies react *after* stress arrives, often exiting at

the worst moment. The economic value of a 0.08 Sharpe improvement on a $1 billion portfolio translates to approximately $8 million in annual risk-adjusted returns.

10   Discussion

10.1   Summary of Observations

The analysis documents three related patterns:

1. The relationship between structural fragility and future tail risk changes sign at a connectivity threshold.

2. Indicators based on eigenstructure detect this transition. Indicators based on volatility do not.

3. Diversification benefits weaken around the same transition.

10.2   Variance and Diversification

Consider what risk models and diversification have in common. Risk models work by measuring variance. Diversification works by reducing variance. Both assume that variance is a sufficient summary of risk. And both, as this evidence suggests, become unreliable around structural transitions.

Within a stable market configuration, variance tells you something meaningful. High variance signals turbulence. Low variance signals calm. Diversification across uncorrelated assets reduces portfolio variance, which reduces risk. The framework is internally consistent and empirically useful.

The problem arises at transitions. When markets shift between structural regimes—from coordination to fragmentation, or vice versa—the mapping from variance to risk changes. Low variance no longer means low risk. Diversification no longer means protection. The tools continue to function, but their outputs become misleading.

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Risk models and diversification are not separate phenomena. They are two faces of the same dependence on variance. Both hedge against the risks that variance captures. Neither protects against the risks that variance misses—specifically, the risk of coordination failure.

10.3   Minsky and the Volatility Paradox

The patterns documented here are consistent with ideas that have circulated in financial economics for decades, though the empirical formalization has remained elusive.

Hyman Minsky argued that stability is destabilizing. During tranquil periods, economic agents take on more risk: they extend credit, increase leverage, and reduce buffers. Brunnermeier and Sannikov formalized a related idea as the "volatility paradox": when measured volatility is low, intermediaries lever up, and the financial system becomes most vulnerable. Low volatility is not a signal of safety—it is a symptom of fragility accumulation.

The evidence presented in this paper is consistent with both frameworks. Fragility, as measured by eigenstructure compression, accumulates during periods of low volatility. The market becomes increasingly coordinated. Risk indicators remain quiet. And then, when coordination breaks, the system moves together in the wrong direction.

What this paper adds is a potential framework to measure and observe fragility accumulation in real time. The dimensional structure of the market—specifically, how long it has remained in a compressed, low-entropy state—may provide that visibility.

10.4   Converging Views

The analysis of this work does not undermine the utility of the varience based-tools. They have proven valuable across many decades and many market environments. They measure real phenomena— dispersion, fear, uncertainty—and they do so reliably.

The implication is that relying on them alone may not be enough.

History offers a pattern. Before 1987, volatility was low and diversification appeared to work. Before 2008, credit spreads were tight along and risk models saw little to worry about. Before

2020, implied volatility was subdued and portfolios were positioned for stability. In each case, the instruments functioned correctly. In each case, they did not signal what was coming.

If we continue to rely exclusively on variance-based measures, there is little reason to expect a different outcome. The tools will work as designed. They will track variance, not a regimen shift.

Variance-based measures tell you about conditions within a regime. Dimensional measures tell you about the stability of the regime itself. Neither is sufficient alone. Together, they offer a clearer understanding of the financial markets.

10.5   Limitations

Several limitations should be acknowledged. The analysis is observational, not causal. The threshold is estimated from historical data and may not be stable over time. The eigenstructure measures examined here are not necessarily the best choices—other approaches to tracking market dimensionality may prove more effective. The patterns documented could have alternative explanations that this analysis does not rule out.

10.6   For Practitioners

For those managing risk: if the patterns documented here hold, it may be worth considering dimensional measures alongside traditional volatility-based indicators. A market with low VIX and stable eigenstructure is in a different state than a market with low VIX and compressed eigenstructure.

For portfolio construction: the evidence suggests that correlations between asset classes may not be stable across regimes. If so, hedges that work in one market state may behave differently in another.

10.7 Policy Considerations

The observations documented in this paper may have implications for financial regulators and policymakers, though any conclusions should be drawn cautiously given the empirical nature of the analysis.

Stress Testing and Systemic Risk Monitoring. Current stress testing frameworks rely heavily on volatility-based scenarios. The evidence suggests that periods of low volatility may coincide with accumulating structural fragility. If so, stress tests conducted during calm periods might underestimate systemic vulnerability. Regulators might consider supplementing volatility-based stress scenarios with measures that track market dimensionality—not to replace existing approaches, but to provide a complementary view of structural conditions.

Early Warning Systems. Central banks and financial stability boards have developed various early warning indicators for systemic risk. Many of these emphasize credit spreads, leverage ratios, and volatility measures. The patterns documented here suggest that eigenstructure-based indicators could complement these existing tools. In particular, tracking how long markets have remained in a compressed, low-dimensional state may provide information about vulnerability that volatility alone does not capture.

The Growth of Passive Investment. A substantial share of equity market capitalization is now held in passive vehicles that track common benchmarks. This concentration may contribute to the coordination dynamics described in this paper: as more investors hold similar portfolios, idiosyncratic positions are eliminated while exposure to common factors increases. The policy question is not whether passive investment is good or bad—it has clear benefits for individual investors—but whether its aggregate effects on market structure warrant monitoring. The framework developed here provides one way to track such effects over time.

Procyclicality of Risk Management. Many risk management systems adjust exposures based on measured volatility. This creates potential for procyclicality: low volatility leads to increased leverage, which may contribute to fragility accumulation. The evidence is consistent with this dynamic, though causation is difficult to establish. Policymakers concerned about procyclicality might consider whether regulatory frameworks could incorporate measures that are less sensitive to short-term volatility and more sensitive to structural conditions.

A Note of Caution. These policy considerations are offered tentatively. The threshold identified in this paper is estimated from historical data and may not be stable over time. The patterns could have alternative explanations. Implementing policy changes based on a single empirical regularity would be premature. The appropriate response may simply be to monitor dimensional measures alongside existing indicators, and to observe whether the patterns documented here persist in future market conditions.

11   Conclusion

Variance-based risk measures have proven valuable across many market environments. This paper does not question that. What it suggests is that there may be a specific set of conditions—around transitions in market dimensionality—where these measures become less informative than usual.

The evidence points to a threshold in market connectivity. Below it, the standard relationships hold. Above it, the patterns appear to invert. Some indicators, particularly those tracking eigenstructure, seem to detect this shift. Others, including VIX, do not seem to.

If these observations hold, the implication is that existing tools may benefit from being supplemented with measures that track dimensional structure. This would not replace variance-based thinking. It would add a layer of visibility into conditions where variance alone may not tell the full story.

Whether this proves useful in practice is an open question. What seems worth noting is the possibility itself: that there may be states of the market where our usual instruments, while functioning

correctly, are measuring something that matters less than we assume.

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