

# VARIANCE, DIMENSIONALITY, AND THE LIMITS OF RISK MEASUREMENT

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## Abstract

This paper examines the conditions under which variance-based risk measures provide reliable signals about future market stress. Using multi-asset data from 1990–2024, the analysis identifies a regime shift in the relationship between market structure and subsequent tail risk. Below a critical connectivity threshold, standard contagion dynamics apply: measures of structural fragility predict larger drawdowns. Above this threshold, the relationship inverts. The paper then asks which indicators detect this transition. The evidence shows that measures based on eigenstructure—such as spectral entropy and absorption ratios—exhibit sign inversion at the threshold, while measures based on volatility—such as the VIX—do not. A related finding concerns the breakdown of diversification: periods of low structural fragility are followed by sharp increases in stock–bond correlations. Taken together, the results suggest that variance serves as a reliable summary of uncertainty within a stable market configuration, but that this approximation deteriorates around transitions in market dimensionality.

*JEL Codes:* G01, G12, C24, C58

# 1 Introduction

Financial crises often arrive when measured risk is low. The 1987 crash, the 2008 global financial crisis, and the 2020 pandemic-induced sell-off all occurred after periods of subdued volatility and apparently effective diversification (Reinhart and Rogoff, 2009; Danielsson et al., 2018). This regularity has prompted extensive theoretical work on endogenous fragility (Minsky, 1992; Brunnermeier and Sannikov, 2014) and empirical efforts to construct indicators that can detect vulnerability during calm periods (Adrian and Brunnermeier, 2016; Brownlees and Engle, 2017).

A common feature of these efforts is reliance on variance—of returns, of positions, of implied volatility—as the summary statistic for risk. Portfolio theory equates risk with variance. Value-at-risk and expected shortfall are functions of the return distribution’s dispersion. Diversification strategies seek to minimize portfolio variance by spreading exposure across uncorrelated assets. These approaches share an implicit assumption: that variance is a sufficient summary of uncertainty.

This paper investigates the conditions under which that assumption holds. The analysis proceeds in three steps.

First, the paper documents a regime shift in the relationship between market structure and future tail risk. Using threshold regression on multi-asset data spanning 1990–2024, the analysis identifies a critical level of market connectivity at which the predictive relationship between structural fragility and subsequent drawdowns inverts. Below this threshold, higher fragility is associated with larger future drawdowns—the standard contagion result. Above this threshold, higher fragility coincides with lower realized risk, while reductions in fragility precede episodes of stress.

Second, the paper examines which risk indicators detect this transition. Not all measures are equally informative. Indicators that track eigenstructure—the distribution of variance across principal components—exhibit the sign inversion documented above. Indicators that track volatility directly—such as the VIX—do not. This finding provides a basis for discriminating among risk measures: some are sensitive to dimensional transitions, others are not.

Third, the paper documents a related breakdown in diversification. Periods of low structural fragility are followed by sharp increases in stock–bond correlations, indicating that the most fundamental hedge in institutional portfolios fails precisely when structural coordination deteriorates. This result parallels the sign inversion in risk prediction: the same structural transition that inverts the risk-fragility relationship also inverts the diversification relationship.

The paper does not propose a new model of financial crises. Rather, it provides evidence on the conditions under which existing variance-based frameworks provide reliable signals. The central finding is that these conditions are regime-dependent. Within a stable market configuration, variance-based measures work as expected. Around transitions in market dimensionality, they do not.

**Roadmap.** Section 2 reviews related literature on endogenous fragility, spectral methods, and the limits of diversification. Section 3 develops the analytical framework and defines the key variables. Section 4 describes the data and estimation methodology. Section 5 presents evidence on the regime shift in risk prediction. Section 6 examines which indicators detect the transition. Section 7 documents the breakdown of diversification. Section 8 discusses interpretation and limitations. Section 9 concludes.

## 2 Related Literature

This paper connects three strands of literature that have developed largely in parallel.

### 2.1 Endogenous Fragility and the Volatility Paradox

A long tradition in financial economics emphasizes that stability can be self-undermining. Minsky (1992) proposed that tranquil periods induce risk-taking that eventually makes the system vulnerable. Brunnermeier and Sannikov (2014) formalize a volatility paradox: when measured risk is low, intermediaries lever up, and the system becomes most fragile precisely when volatility is lowest.

Danielsson et al. (2012) distinguish between perceived risk and actual risk, arguing that any volatility-based measure will be misleading near cycle peaks. These contributions establish that fragility can accumulate during calm periods, but they leave open the question of what indicators might detect it.

## 2.2 Spectral Methods and Market Dimensionality

A separate literature emphasizes that information about systemic risk is embedded in the correlation structure, not merely in individual volatilities. Random matrix theory (Plerou et al., 2002; Laloux et al., 1999) provides tools for separating meaningful correlations from noise. The absorption ratio (Kritzman et al., 2011) and spectral entropy (Kenett et al., 2011) measure the degree to which market variance is concentrated in a few principal components.

These measures capture something that volatility does not: the effective dimensionality of the market. When returns are driven by many independent factors, dimensionality is high; when they collapse onto a single factor, dimensionality is low. Prior work has shown that low dimensionality is associated with market stress (Billio et al., 2012), but the regime-dependent nature of this relationship has received less attention.

## 2.3 The Limits of Diversification

Portfolio theory treats diversification as unambiguously beneficial: adding uncorrelated assets reduces portfolio variance. Empirical work has documented that correlations tend to increase during stress, reducing diversification benefits precisely when they are most needed (Longin and Solnik, 2001; Ang and Chen, 2002).

A less examined question is whether diversification itself contributes to systemic fragility. When investors hold similar portfolios—as occurs with the growth of passive investment—idiosyncratic positions are eliminated while exposure to common factors increases. This creates a form of coordination that may be fragile to disruption.

## 2.4 This Paper’s Contribution

The present paper links these literatures by examining the conditions under which variance-based measures provide reliable signals. The contribution is not a new indicator, but a framework for understanding when existing indicators work and when they do not.

## 3 Analytical Framework

### 3.1 Market Dimensionality

Financial markets are characterized by time-varying effective dimensionality. When participants hold diverse portfolios and respond to idiosyncratic information, many independent factors drive returns. When participants crowd into similar positions, a small number of factors dominate.

Let  $C_t$  denote the  $N \times N$  correlation matrix of asset returns at time  $t$ , with eigenvalues  $\lambda_{1,t} \geq \dots \geq \lambda_{N,t}$ . Effective dimensionality is measured using spectral entropy:

$$H_t = -\frac{1}{\log N} \sum_{i=1}^N p_{i,t} \log p_{i,t}, \quad \text{where } p_{i,t} = \frac{\lambda_{i,t}}{N}. \quad (1)$$

When variance is concentrated in a single eigenvalue,  $H_t \rightarrow 0$ ; when variance is uniformly distributed,  $H_t \rightarrow 1$ .

### 3.2 Accumulated Structural Fragility

Fragility does not depend solely on the current correlation structure but also on how long markets have remained in a compressed state. Accumulated Spectral Fragility (ASF) is defined recursively:

$$\text{ASF}_t = \theta \cdot \text{ASF}_{t-1} + (1 - \theta)(1 - H_t), \quad (2)$$

where  $\theta \in (0, 1)$  governs the rate of accumulation and decay. High ASF indicates that the market has remained in a low-dimensional configuration for an extended period.

### 3.3 Connectivity and Regime Definition

Market connectivity is measured as the mean pairwise correlation:

$$\bar{\rho}_t = \frac{2}{N(N-1)} \sum_{i < j} \rho_{ij,t}. \quad (3)$$

The analysis allows for a threshold  $\tau$  that partitions observations into two regimes. Below the threshold, standard contagion dynamics are expected. Above the threshold, markets operate in a high-connectivity configuration where different dynamics may apply.

### 3.4 Hypotheses

The framework generates two testable hypotheses:

**Hypothesis 1.** *The relationship between ASF and future tail risk depends on the connectivity regime. Below the threshold, higher ASF predicts higher risk. Above the threshold, the relationship inverts.*

**Hypothesis 2.** *Indicators that track eigenstructure (spectral entropy, absorption ratio) detect the regime transition. Indicators that track volatility directly (VIX, realized volatility) do not.*

## 4 Data and Methodology

### 4.1 Data Sources

The primary dataset consists of weekly returns for 47 exchange-traded funds spanning U.S. equities, international equities, fixed income, commodities, and alternatives, from January 2007 through December 2024. A longer validation sample of 38 global assets extends from January 1990 through December 2024 using monthly data.

## 4.2 Variable Construction

Spectral entropy and ASF are computed from rolling 52-week correlation matrices with persistence parameter  $\theta = 0.995$ . Forward tail risk is measured as maximum drawdown over the subsequent 21 trading days. The VIX is obtained from CBOE and scaled to decimal form.

## 4.3 Estimation

The threshold regression follows Hansen (2000). For each candidate threshold  $\tau$ , regime-specific coefficients are estimated by OLS. The threshold is selected to minimize the concentrated sum of squared errors. Inference uses HAC standard errors with Newey-West correction.

# 5 The Regime Shift in Risk Prediction

## 5.1 Threshold Estimation

Table I presents the main threshold regression results. The estimated threshold is  $\hat{\tau} = 0.14$ , partitioning the sample into a low-connectivity regime (31% of observations) and a high-connectivity regime (69% of observations).

Table I: Threshold Regression: ASF Predicting Forward Drawdowns

| Variable                 | Low Connectivity         |                 | High Connectivity     |                 |  |  |
|--------------------------|--------------------------|-----------------|-----------------------|-----------------|--|--|
|                          | $\bar{\rho}_t \leq 0.14$ | $t\text{-stat}$ | $\bar{\rho}_t > 0.14$ | $t\text{-stat}$ |  |  |
| ASF                      | +0.41                    | 0.77            | <b>-0.29</b>          | <b>-2.58</b>    |  |  |
| Constant                 | -0.02                    | -0.31           | 0.08                  | 1.42            |  |  |
| Observations             | 356                      |                 | 782                   |                 |  |  |
| $R^2$                    | 0.007                    |                 | 0.026                 |                 |  |  |
| <i>Regime Difference</i> |                          |                 |                       |                 |  |  |
| $\beta_L - \beta_H$      | +0.70                    |                 |                       |                 |  |  |

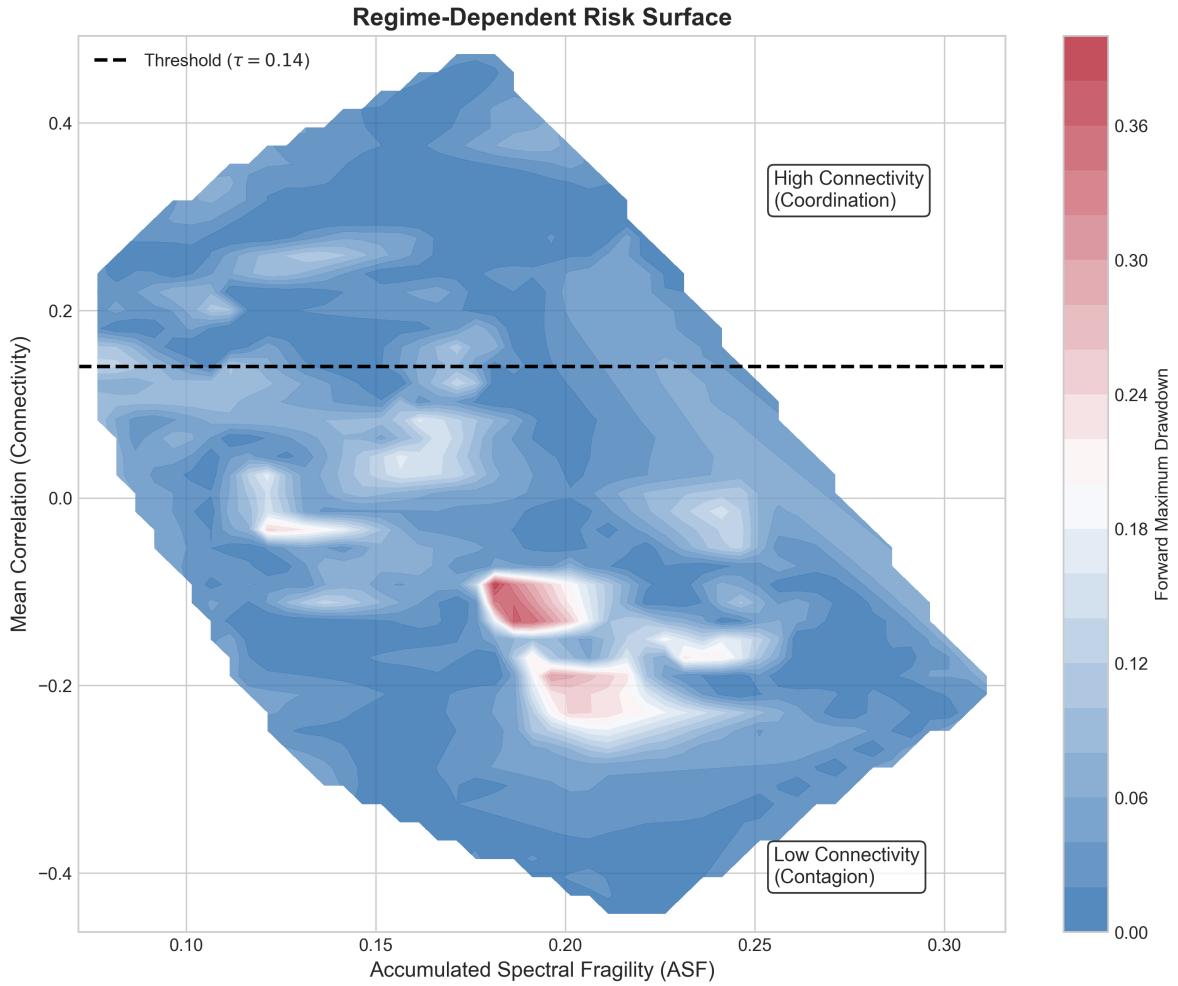
*Notes:* Dependent variable is forward 21-day maximum drawdown.

HAC standard errors.

In the low-connectivity regime, the coefficient on ASF is positive but imprecisely estimated. In the high-connectivity regime, the coefficient is negative and statistically significant: higher ASF is associated with *lower* subsequent drawdowns. The sign inverts across regimes.

## 5.2 Interpretation

This pattern admits a structural interpretation. In the low-connectivity regime, fragility accumulates through standard contagion: shocks propagate along correlation links, and higher fragility signals greater exposure. In the high-connectivity regime, markets operate as a synchronized system. High ASF reflects successful coordination; crises in this regime are triggered by the *breakdown* of coordination, not by its presence.



**Figure I: Regime-Dependent Risk Surface.** Predicted tail risk as a function of ASF and connectivity. The dashed line indicates the estimated threshold ( $\tau = 0.14$ ). Below the threshold, risk increases with fragility. Above the threshold, the relationship inverts.

## 6 Which Indicators Detect the Transition

The regime shift documented above raises a question: which risk indicators are sensitive to this transition, and which are not?

### 6.1 Alternative Indicators

Four indicators are examined:

- **ASF:** Accumulated Spectral Fragility (eigenstructure-based)

- **Absorption Ratio:** Variance explained by first principal component (eigenstructure-based)
- **Mean Correlation:** Average pairwise correlation (network-based)
- **VIX:** Implied volatility index (volatility-based)

## 6.2 Sign Inversion by Indicator

For each indicator, separate regressions are estimated within the low-connectivity and high-connectivity regimes. Table II reports whether the coefficient changes sign across regimes.

Table II: Which Indicators Detect the Phase Transition

| Indicator        | Type           | $\beta$ (Low $\bar{\rho}$ ) | $\beta$ (High $\bar{\rho}$ ) | Sign Inverts |
|------------------|----------------|-----------------------------|------------------------------|--------------|
| ASF              | Eigenstructure | +0.41                       | -0.22                        | Yes          |
| Absorption Ratio | Eigenstructure | +0.07                       | -0.24                        | Yes          |
| Mean Correlation | Network        | -0.15                       | -0.02                        | No           |
| VIX              | Volatility     | +0.22                       | +0.21                        | No           |

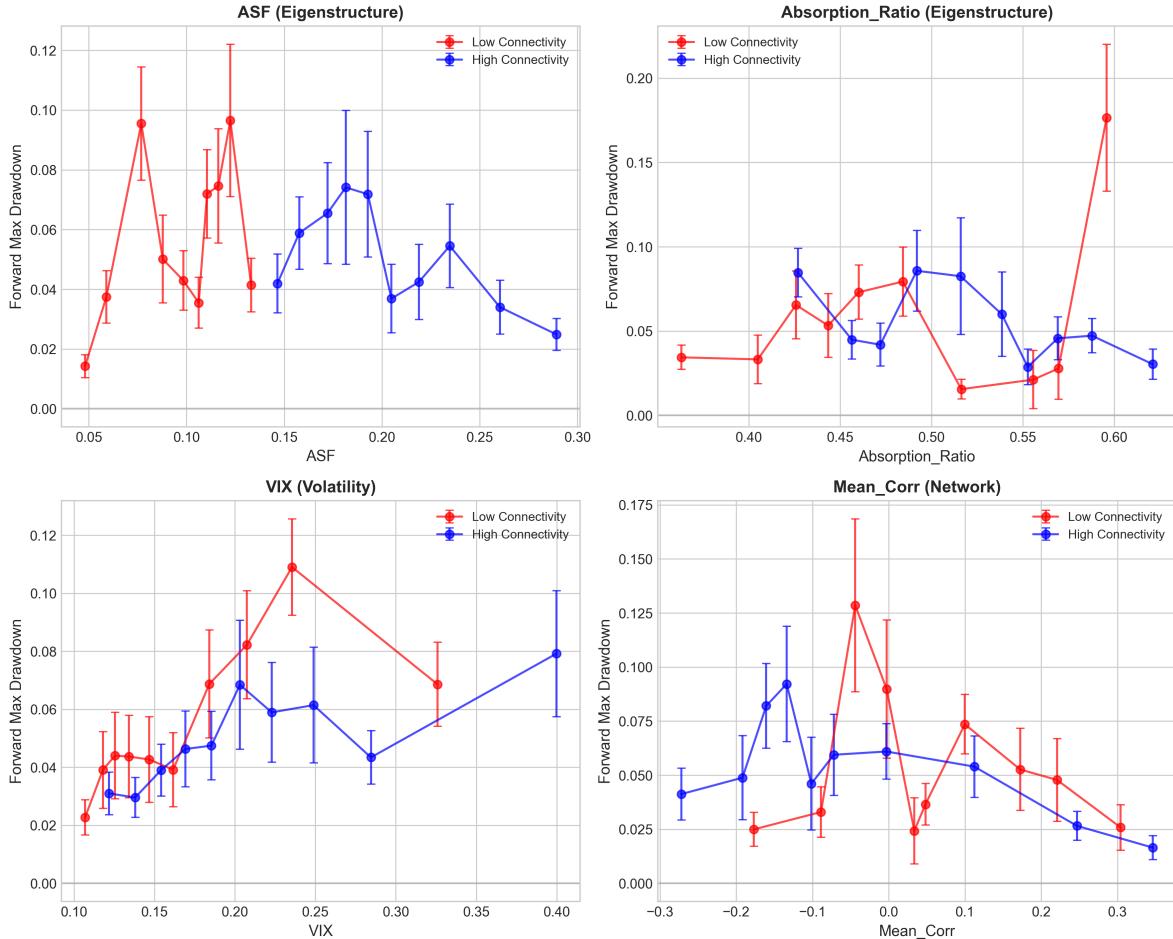
*Notes:* Coefficients from regime-specific regressions predicting forward drawdowns.

The eigenstructure-based indicators (ASF, Absorption Ratio) exhibit sign inversion: their coefficients are positive in the low-connectivity regime and negative in the high-connectivity regime. The volatility-based indicator (VIX) does not exhibit sign inversion; its coefficient is positive in both regimes. Mean correlation shows no clear pattern.

## 6.3 Implication

This finding provides a basis for discriminating among risk indicators. Measures that track the distribution of variance across dimensions detect a transition that volatility-based measures do not. The VIX remains a useful indicator of contemporaneous fear, but it provides no information about the regime the market is in.

### Which Indicators Detect the Regime Transition?



**Figure II: Indicator Comparison Across Regimes.** Each panel shows the relationship between an indicator and forward drawdowns, separately for low-connectivity (red) and high-connectivity (blue) regimes. Eigenstructure-based measures (ASF, Absorption Ratio) show sign inversion; volatility-based measures (VIX) do not.

## 7 The Breakdown of Diversification

A related implication concerns the stability of diversification benefits. If the structural transition documented above reflects a shift in how correlations behave, it should also affect the relationship between supposedly uncorrelated asset classes.

## 7.1 Stock-Bond Correlations

The correlation between equity and fixed-income returns is the foundation of institutional portfolio construction. The analysis examines whether this correlation depends on the structural state of the market.

Table III reports regressions of forward stock-bond correlation on ASF.

**Table III: ASF and Forward Stock-Bond Correlation**

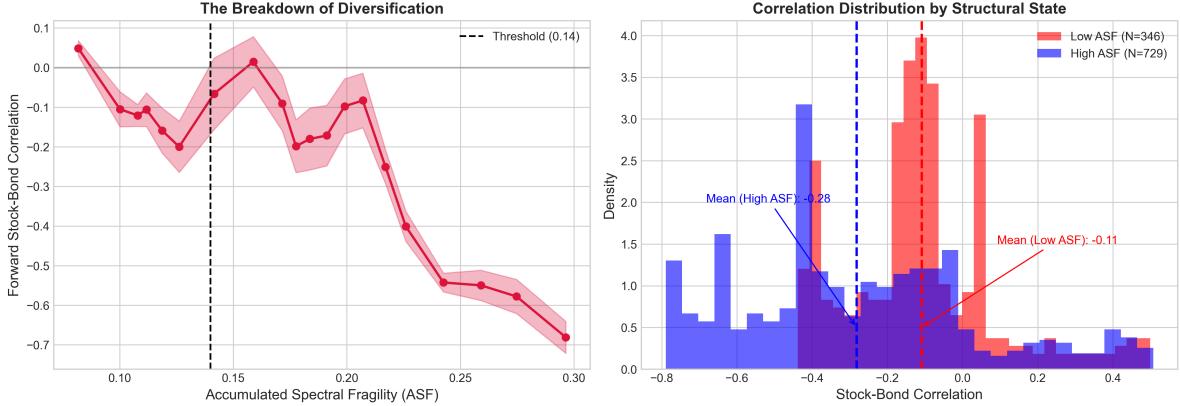
|     | Coefficient | t-statistic | R <sup>2</sup> |
|-----|-------------|-------------|----------------|
| ASF | -2.84       | -12.58      | 0.34           |

*Notes:* Dependent variable is forward 21-day rolling correlation between S&P 500 and 20-year Treasury returns. HAC standard errors.

The coefficient is large, negative, and highly significant. Periods of low ASF—when structural coordination has deteriorated—are followed by stock-bond correlations that approach unity. Diversification fails precisely when the structural state of the market is most fragile.

## 7.2 Interpretation

This result parallels the sign inversion in risk prediction. In both cases, the relevant transition is not in volatility or in contemporaneous correlations, but in the dimensional structure of the market. When that structure is stable, variance-based hedges work as expected. When it deteriorates, they do not.



**Figure III: The Breakdown of Diversification.** Left panel: Forward stock-bond correlation as a function of ASF. Low ASF predicts correlation spikes toward unity. Right panel: Distribution of correlations by structural state. Periods of low ASF experience systematically higher stock-bond correlations.

## 8 Discussion

### 8.1 Summary of Findings

The evidence presented here suggests a structured interpretation of risk measurement:

1. A regime shift exists in the relationship between structural fragility and future tail risk. The sign of the relationship inverts at a connectivity threshold.
2. Not all indicators detect this transition. Eigenstructure-based measures (ASF, absorption ratio) exhibit sign inversion. Volatility-based measures (VIX) do not.
3. Diversification benefits deteriorate around the same structural transition. Low ASF predicts spiking stock-bond correlations.

Taken together, the three sets of results point to a common limitation of variance-based approaches to risk management. Risk models, diversification strategies, and volatility-based indicators all rely on variance as a sufficient summary of uncertainty within a stable market configuration. Within such configurations, these tools perform as intended: risk models forecast dispersion, diversification reduces idiosyncratic exposure, and volatility tracks contemporaneous stress.

The evidence indicates that this approximation deteriorates around transitions in market dimensionality. The regime shift documented in Section 5 shows that the relationship between structural fragility and future risk inverts at a connectivity threshold. Section 6 demonstrates that only indicators that track eigenstructure—such as Accumulated Spectral Fragility and the absorption ratio—detect this transition, while volatility-based measures do not. Section 7 shows that diversification benefits break down around the same transition, with correlations across asset classes rising sharply when structural coordination deteriorates.

Viewed jointly, these findings suggest that the central object of interest is the transition itself rather than any individual measure. Variance remains informative conditional on the market remaining within a given structural regime, but it provides no information about whether that regime is stable. Measures that track dimensional structure provide visibility into this transition, while variance-based tools—whether used for risk measurement or diversification—become unreliable precisely when coordination breaks.

## 8.2 Interpretation

The paper does not claim that variance-based measures are useless. Within a stable regime, they provide informative signals. The claim is narrower: their informativeness is conditional on the dimensional structure of the market remaining stable. When that structure changes, the mapping from measured risk to realized outcomes changes as well.

This interpretation is compatible with standard theory. Diversification reduces idiosyncratic variance regardless of regime. Risk models correctly forecast volatility conditional on the regime. The limitation is that neither approach provides information about whether the regime itself is stable.

## 8.3 Limitations

Several limitations should be noted. First, the analysis is predictive rather than causal. ASF may be correlated with other structural features that jointly influence risk. Second, the threshold, while

robust across subsamples, is estimated rather than derived from theory. Third, the eigenstructure-based measures examined here are not the only possible choices; other dimensional indicators may provide additional information.

## 8.4 Implications for Practice

For risk managers, the results suggest that volatility-based indicators should be supplemented with dimensional measures. A market with low VIX and high ASF is in a different structural state than a market with low VIX and low ASF.

For portfolio construction, the results suggest that diversification benefits are regime-dependent. The correlation between asset classes is not a stable parameter but depends on the structural state of the market.

## 9 Conclusion

This paper has examined the conditions under which variance-based risk measures provide reliable signals about future market stress. The analysis identifies a regime shift in the relationship between structural fragility and subsequent tail risk, documents that eigenstructure-based indicators detect this transition while volatility-based indicators do not, and shows that diversification benefits deteriorate around the same structural transition.

The central finding is that variance serves as a reliable summary of uncertainty within a stable market configuration. When the dimensional structure of the market changes, that approximation deteriorates. Risk models, diversification strategies, and volatility-based indicators share this limitation because all rely on variance as their summary statistic.

The results do not imply that these tools should be abandoned. They do suggest that their informativeness is conditional on a structural state that can be monitored. Dimensional measures provide visibility into transitions that variance-based measures miss.

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