1 Problem 1

By definition the golden mean $\frac{a}{b}$ satisfies $\frac{a}{b} = \frac{a+b}{a}$. Show that the golden mean has the value $\frac{1+\sqrt{5}}{2}$. Then modify the Fibonacci program to compute and write out the ratio of each term to its predecessor. (Don't forget that you want this ratio to be a double variable.) This $g(n)$ approaches a limit for large n. Show analytically that this limit is the golden mean defined above (hint: start from the definition: $f(n) = f(n-1) + f(n-2)$.)

The golden ratio is defined as $\phi = \frac{a}{b} = \frac{a+b}{a}$. This ratio can be solved for algebraically:

$$\begin{aligned}
a^2 &= ab + b^2 \\
a^2 - ab - b^2 &= 0 \\
a &= \frac{b \pm \sqrt{b^2 + 4b^2}}{2} \\
\phi = \frac{a}{b} &= \frac{1+\sqrt{5}}{2}.
\end{aligned}$$

In the last step, the plus sign was taken in order to keep with the definition that $\phi > 0$.

The Fibonacci sequence is given by

$$f(n) = \begin{cases} 1 & n = 0, 1 \\ f(n-1) + f(n-2) & n > 1 \end{cases}$$

Consider the ratio

$$r(n) = \frac{f(n)}{f(n-1)}.$$

Assume that $\lim_{n\to\infty} r(n) = c$. Claim that $c = \phi$.

Proof. (*Informal*) As n is arbitrarily large, additionally assume

$$\frac{f(n+1)}{f(n)} = \frac{f(n)}{f(n-1)} = c.$$

Then,

$$\begin{aligned}
c &= \frac{f(n+1)}{f(n)} \\
&= \frac{f(n-1) + f(n)}{f(n)} \\
&= \frac{1}{c} + 1 \\
c^2 - c - 1 &= 0 \\
c &= \frac{1+\sqrt{5}}{2} \\
c &= \phi
\end{aligned}$$

□

2 Problem 2

> Define a variable of type double to be the theoretical golden mean from the previous problem (to take the square root, use Math.sqrt) . Modify Fibonacci once again to display the difference between j/i and the golden mean. Let us call this difference for the n'th Fibonacci number q(n). We know from the previous problem that q(n) is supposed to approach zero as n gets large. But how does it approach zero? Plot q(n) vs n for n ranging from 3 to 20. You don't have to use the computer to make the plot. By plotting log q(n) vs n, test the hypothesis that q diminishes exponentially with n i. e. q(n) = (constant) exp(-n/N), where N is some "decay constant". From your graph, obtain an approximate value of N if this makes sense. You can use Java to take natural logs by using the function Math.log(number). You should also save the results to a file. This file should be handed in with the other files of you solution. If you have access to and are familiar with a program like Excel, MatLab, gnuPlot, etc., you can use those to plot your output for you.

This part was different. For this problem, I modified the source code from Problem 1 to calculate $|\phi - r(n)|$ (modified source code included below and in folder). This difference should approach zero as n gets large, as indicated by the previous proof. The results of this calculation are plotted in Figure **??**.

As expected, $r(n)$ converges to ϕ exponentially with a fitted decay constant of $VALUE$.

P2/Fibo.java

```java
// fibo.java
// fibonacci java class 2
// write out fibonacci sequence and ratio
// Athanasios Athanassiadis
// Complexity, Project 1, F2013
// 10/3/2012

import java.applet.Applet;
public class Fibo extends Applet
{

    int i,j,k;
    int n, nsteps, ypos;
    double r, q;
    double g = 1.618;

    public void start()
    {
    nsteps = 20;

    i=1;
    j=1;
        System.out.println(" 0,1,1 ");

    for (n=1; n<=nsteps; n++)
    {
        k = i+j;
        r = (double) k / (double) j;
        q = Math.abs(g - r);
        System.out.println(" " + n + "," + k + "," + q );
        i = j;
        j = k;
    }
    }

}
```