Dynamic Model of Microgrids

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A. Converter Model

The dynamics of the converters involve the low-pass filters, power controller (P-ω and Q-V droop control), voltage controller (PI control), current controller (PI control), output LC filter, and coupling inductance.

Then the converter model can be formulated as follows [1]:

$$\boldsymbol{\omega} = \omega^* - \boldsymbol{K}_{\mathrm{p}}(\boldsymbol{P} - \boldsymbol{P}^*) - \omega_{\mathrm{ref}} \tag{1a}$$

$$\sqrt{(\boldsymbol{v}_{\mathrm{od}}^*)^2 + (\boldsymbol{v}_{\mathrm{oq}}^*)^2} = \boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*) \tag{1b}$$

$$\dot{\boldsymbol{\delta}} = \boldsymbol{\omega} - \omega_{\mathrm{n}} \tag{1c}$$

$$\dot{\boldsymbol{P}} = \omega_{\mathrm{c}}(-\boldsymbol{P} + \boldsymbol{v}_{\mathrm{od}} \odot \boldsymbol{i}_{\mathrm{od}} + \boldsymbol{v}_{\mathrm{oq}} \odot \boldsymbol{i}_{\mathrm{oq}}) \tag{1d}$$

$$\dot{\boldsymbol{Q}} = \omega_{\mathrm{c}}(-\boldsymbol{Q} + \boldsymbol{v}_{\mathrm{oq}} \odot \boldsymbol{i}_{\mathrm{od}} - \boldsymbol{v}_{\mathrm{od}} \odot \boldsymbol{i}_{\mathrm{oq}}) \tag{1e}$$

$$\dot{\boldsymbol{\phi}}_{\mathrm{d}} = \boldsymbol{v}_{\mathrm{od}}^* - \boldsymbol{v}_{\mathrm{od}} \tag{1f}$$

$$\dot{\boldsymbol{\phi}}_{\mathrm{q}} = \boldsymbol{v}_{\mathrm{oq}}^* - \boldsymbol{v}_{\mathrm{oq}} \tag{1g}$$

$$\boldsymbol{i}_{\mathrm{ld}}^* = \boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{od}} - \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{oq}} + \boldsymbol{K}_{\mathrm{pv}} \odot (\boldsymbol{v}_{\mathrm{od}}^* - \boldsymbol{v}_{\mathrm{od}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{d}} \tag{1h}$$

$$\boldsymbol{i}_{\mathrm{lq}}^* = \boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{oq}} + \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{od}} + \boldsymbol{K}_{\mathrm{pv}} \odot (\boldsymbol{v}_{\mathrm{oq}}^* - \boldsymbol{v}_{\mathrm{oq}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{q}} \tag{1i}$$

$$\dot{\boldsymbol{\gamma}}_{\mathrm{d}} = \boldsymbol{i}_{\mathrm{ld}}^* - \boldsymbol{i}_{\mathrm{ld}} \tag{1j}$$

$$\dot{\boldsymbol{\gamma}}_{\mathrm{q}} = \boldsymbol{i}_{\mathrm{lq}}^* - \boldsymbol{i}_{\mathrm{lq}} \tag{1k}$$

$$\boldsymbol{v}_{\mathrm{id}}^* = -\omega_{\mathrm{n}} \boldsymbol{L}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{lq}} + \boldsymbol{K}_{\mathrm{pc}} \odot (\boldsymbol{i}_{\mathrm{ld}}^* - \boldsymbol{i}_{\mathrm{ld}}) + \boldsymbol{K}_{\mathrm{ic}} \odot \boldsymbol{\gamma}_{\mathrm{d}} \tag{1l}$$

$$\boldsymbol{v}_{\mathrm{iq}}^* = \omega_{\mathrm{n}} \boldsymbol{L}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{ld}} + \boldsymbol{K}_{\mathrm{pc}} \odot (\boldsymbol{i}_{\mathrm{lq}}^* - \boldsymbol{i}_{\mathrm{lq}}) + \boldsymbol{K}_{\mathrm{ic}} \odot \boldsymbol{\gamma}_{\mathrm{q}} \tag{1m}$$

$$\dot{\boldsymbol{i}}_{\mathrm{ld}} = -\boldsymbol{r}_{\mathrm{f}} \oslash \boldsymbol{L}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{ld}} + \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{lq}} + \boldsymbol{v}_{\mathrm{id}}^* \oslash \boldsymbol{L}_{\mathrm{f}} - \boldsymbol{v}_{\mathrm{od}} \oslash \boldsymbol{L}_{\mathrm{f}} \tag{1n}$$

$$\dot{\boldsymbol{i}}_{\mathrm{lq}} = -\boldsymbol{r}_{\mathrm{f}} \oslash \boldsymbol{L}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{lq}} - \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{ld}} + \boldsymbol{v}_{\mathrm{iq}}^* \oslash \boldsymbol{L}_{\mathrm{f}} - \boldsymbol{v}_{\mathrm{oq}} \oslash \boldsymbol{L}_{\mathrm{f}} \tag{1o}$$

$$\dot{\boldsymbol{v}}_{\mathrm{od}} = \omega_{\mathrm{n}} \boldsymbol{v}_{\mathrm{oq}} + \boldsymbol{i}_{\mathrm{ld}} \oslash \boldsymbol{C}_{\mathrm{f}} - \boldsymbol{i}_{\mathrm{od}} \oslash \boldsymbol{C}_{\mathrm{f}} \tag{1p}$$

$$\dot{\boldsymbol{v}}_{\mathrm{oq}} = -\omega_{\mathrm{n}} \boldsymbol{v}_{\mathrm{od}} + \boldsymbol{i}_{\mathrm{lq}} \oslash \boldsymbol{C}_{\mathrm{f}} - \boldsymbol{i}_{\mathrm{oq}} \oslash \boldsymbol{C}_{\mathrm{f}} \tag{1q}$$

$$\dot{\boldsymbol{i}}_{\mathrm{od}} = -\boldsymbol{r}_{\mathrm{c}} \oslash \boldsymbol{L}_{\mathrm{c}} \odot \boldsymbol{i}_{\mathrm{od}} + \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{oq}} + \boldsymbol{v}_{\mathrm{od}} \oslash \boldsymbol{L}_{\mathrm{c}} - \boldsymbol{E}_{\mathrm{C}} \boldsymbol{v}_{\mathrm{Bd}} \oslash \boldsymbol{L}_{\mathrm{c}} \tag{1r}$$

$$\dot{\boldsymbol{i}}_{\mathrm{oq}} = -\boldsymbol{r}_{\mathrm{c}} \oslash \boldsymbol{L}_{\mathrm{c}} \odot \boldsymbol{i}_{\mathrm{oq}} - \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{od}} + \boldsymbol{v}_{\mathrm{oq}} \oslash \boldsymbol{L}_{\mathrm{c}} - \boldsymbol{E}_{\mathrm{C}} \boldsymbol{v}_{\mathrm{Bq}} \oslash \boldsymbol{L}_{\mathrm{c}} \tag{1s}$$

Eliminating the associated algebraic variables gives

$$\dot{\boldsymbol{\delta}} = \omega^* - \boldsymbol{K}_{\mathrm{p}}(\boldsymbol{P} - \boldsymbol{P}^*) - \omega_{\mathrm{ref}} \tag{2a}$$

$$\dot{\boldsymbol{P}} = \omega_{\mathrm{c}}(-\boldsymbol{P} + \boldsymbol{v}_{\mathrm{od}} \odot \boldsymbol{i}_{\mathrm{od}} + \boldsymbol{v}_{\mathrm{oq}} \odot \boldsymbol{i}_{\mathrm{oq}}) \tag{2b}$$

$$\dot{\boldsymbol{Q}} = \omega_{\mathrm{c}}(-\boldsymbol{Q} + \boldsymbol{v}_{\mathrm{oq}} \odot \boldsymbol{i}_{\mathrm{od}} - \boldsymbol{v}_{\mathrm{od}} \odot \boldsymbol{i}_{\mathrm{oq}}) \tag{2c}$$

$$\dot{\boldsymbol{\phi}}_{\mathrm{d}} = [\cos(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{od}} \tag{2d}$$

$$\dot{\boldsymbol{\phi}}_{\mathrm{q}} = [\sin(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{oq}} \tag{2e}$$

$$\dot{\boldsymbol{\gamma}}_{\mathrm{d}} = \boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{od}} - \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{oq}} + \boldsymbol{K}_{\mathrm{pv}} \odot ([\cos(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{od}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{d}} - \boldsymbol{i}_{\mathrm{ld}} \tag{2f}$$

$$\dot{\boldsymbol{\gamma}}_{\mathrm{q}} = \boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{oq}} + \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{od}} + \boldsymbol{K}_{\mathrm{pv}} \odot ([\sin(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{oq}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{q}} - \boldsymbol{i}_{\mathrm{lq}} \tag{2g}$$

$$\dot{\boldsymbol{i}}_{\mathrm{ld}} = [-\boldsymbol{r}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{ld}} + \boldsymbol{K}_{\mathrm{pc}} \odot (\boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{od}} - \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{oq}} + \boldsymbol{K}_{\mathrm{pv}} \odot ([\cos(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{od}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{d}} - \boldsymbol{i}_{\mathrm{ld}}) + \boldsymbol{K}_{\mathrm{ic}} \odot \boldsymbol{\gamma}_{\mathrm{d}} - \boldsymbol{v}_{\mathrm{od}}] \oslash \boldsymbol{L}_{\mathrm{f}} \tag{2h}$$

$$\dot{\boldsymbol{i}}_{\mathrm{lq}} = [-\boldsymbol{r}_{\mathrm{f}} \odot \boldsymbol{i}_{\mathrm{lq}} + \boldsymbol{K}_{\mathrm{pc}} \odot (\boldsymbol{F} \odot \boldsymbol{i}_{\mathrm{oq}} + \omega_{\mathrm{n}} \boldsymbol{C}_{\mathrm{f}} \odot \boldsymbol{v}_{\mathrm{od}} + \boldsymbol{K}_{\mathrm{pv}} \odot ([\sin(\boldsymbol{\delta}) \odot (\boldsymbol{U}^* - \boldsymbol{K}_{\mathrm{q}}(\boldsymbol{Q} - \boldsymbol{Q}^*))] - \boldsymbol{v}_{\mathrm{oq}}) + \boldsymbol{K}_{\mathrm{iv}} \odot \boldsymbol{\phi}_{\mathrm{q}} - \boldsymbol{i}_{\mathrm{lq}}) + \boldsymbol{K}_{\mathrm{ic}} \odot \boldsymbol{\gamma}_{\mathrm{q}} - \boldsymbol{v}_{\mathrm{oq}}] \oslash \boldsymbol{L}_{\mathrm{f}} \tag{2i}$$

$$\dot{\boldsymbol{v}}_{\mathrm{od}} = \omega_{\mathrm{n}} \boldsymbol{v}_{\mathrm{oq}} + \boldsymbol{i}_{\mathrm{ld}} \oslash \boldsymbol{C}_{\mathrm{f}} - \boldsymbol{i}_{\mathrm{od}} \oslash \boldsymbol{C}_{\mathrm{f}} \tag{2j}$$

$$\dot{\boldsymbol{v}}_{\mathrm{oq}} = -\omega_{\mathrm{n}} \boldsymbol{v}_{\mathrm{od}} + \boldsymbol{i}_{\mathrm{lq}} \oslash \boldsymbol{C}_{\mathrm{f}} - \boldsymbol{i}_{\mathrm{oq}} \oslash \boldsymbol{C}_{\mathrm{f}} \tag{2k}$$

$$\dot{\boldsymbol{i}}_{\mathrm{od}} = [-\boldsymbol{r}_{\mathrm{c}} \odot \boldsymbol{i}_{\mathrm{od}} + \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{oq}} \odot \boldsymbol{L}_{\mathrm{c}} + \boldsymbol{v}_{\mathrm{od}} - \boldsymbol{E}_{\mathrm{C}} \boldsymbol{v}_{\mathrm{Bd}}] \oslash \boldsymbol{L}_{\mathrm{c}} \tag{2l}$$

$$\dot{\boldsymbol{i}}_{\mathrm{oq}} = [-\boldsymbol{r}_{\mathrm{c}} \odot \boldsymbol{i}_{\mathrm{oq}} - \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{od}} \odot \boldsymbol{L}_{\mathrm{c}} + \boldsymbol{v}_{\mathrm{oq}} - \boldsymbol{E}_{\mathrm{C}} \boldsymbol{v}_{\mathrm{Bq}}] \oslash \boldsymbol{L}_{\mathrm{c}} \tag{2m}$$

B. Load model

Three types of loads are considered: constant impedance load, induction motor load, and variable frequency induction-motor drive load. It is assumed that each load bus connects to a single type of loads.

The constant impedance loads are with the following dynamics:

$$\dot{\boldsymbol{i}}_{\mathrm{Ld}} = -\boldsymbol{r}_{\mathrm{L}}\oslash\boldsymbol{L}_{\mathrm{L}}\odot\boldsymbol{i}_{\mathrm{Ld}} + \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Lq}} + \boldsymbol{E}_{\mathrm{L}}\boldsymbol{v}_{\mathrm{Bd}}\oslash\boldsymbol{L}_{\mathrm{L}} \tag{3a}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Lq}} = -\boldsymbol{r}_{\mathrm{L}}\oslash\boldsymbol{L}_{\mathrm{L}}\odot\boldsymbol{i}_{\mathrm{Lq}} - \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Ld}} + \boldsymbol{E}_{\mathrm{L}}\boldsymbol{v}_{\mathrm{Bq}}\oslash\boldsymbol{L}_{\mathrm{L}} \tag{3b}$$

The induction motor loads are with the following dynamics:

$$\dot{\boldsymbol{v}}_{\mathrm{Md}} = \omega_{\mathrm{n}}\boldsymbol{s}_{\mathrm{M}}\odot\boldsymbol{v}_{\mathrm{Mq}} - [\boldsymbol{v}_{\mathrm{Md}} + \omega_{\mathrm{n}}(\boldsymbol{L}_{\mathrm{Mo}} - \boldsymbol{L}_{\mathrm{Mb}})\odot\boldsymbol{i}_{\mathrm{Mq}}]\oslash\boldsymbol{T}_{\mathrm{Mo}} \tag{4a}$$
$$\dot{\boldsymbol{v}}_{\mathrm{Mq}} = -\omega_{\mathrm{n}}\boldsymbol{s}_{\mathrm{M}}\odot\boldsymbol{v}_{\mathrm{Md}} - [\boldsymbol{v}_{\mathrm{Mq}} - \omega_{\mathrm{n}}(\boldsymbol{L}_{\mathrm{Mo}} - \boldsymbol{L}_{\mathrm{Mb}})\odot\boldsymbol{i}_{\mathrm{Md}}]\oslash\boldsymbol{T}_{\mathrm{Mo}} \tag{4b}$$
$$\dot{\boldsymbol{s}}_{\mathrm{M}} = [(\boldsymbol{a}_{\mathrm{M}} + \boldsymbol{b}_{\mathrm{M}}\odot\boldsymbol{s}_{\mathrm{M}} + \boldsymbol{c}_{\mathrm{M}}\odot\boldsymbol{s}_{\mathrm{M}}^2)\odot\boldsymbol{T}_{\mathrm{M}}^0 - (\boldsymbol{v}_{\mathrm{Md}}\odot\boldsymbol{i}_{\mathrm{Md}} + \boldsymbol{v}_{\mathrm{Mq}}\odot\boldsymbol{i}_{\mathrm{Mq}})\oslash\omega_{\mathrm{n}}]\oslash(2\boldsymbol{H}_{\mathrm{M}}) \tag{4c}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Md}} = -\boldsymbol{r}_{\mathrm{Ms}}\oslash\boldsymbol{L}_{\mathrm{Mb}}\odot\boldsymbol{i}_{\mathrm{Md}} + \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Mq}} + (\boldsymbol{E}_{\mathrm{M}}\boldsymbol{v}_{\mathrm{Bd}} - \boldsymbol{v}_{\mathrm{Md}})\oslash\boldsymbol{L}_{\mathrm{Mb}} \tag{4d}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Mq}} = -\boldsymbol{r}_{\mathrm{Ms}}\oslash\boldsymbol{L}_{\mathrm{Mb}}\odot\boldsymbol{i}_{\mathrm{Mq}} - \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Md}} + (\boldsymbol{E}_{\mathrm{M}}\boldsymbol{v}_{\mathrm{Bq}} - \boldsymbol{v}_{\mathrm{Mq}})\oslash\boldsymbol{L}_{\mathrm{Mb}} \tag{4e}$$

with

$$\boldsymbol{L}_{\mathrm{Mo}} = \boldsymbol{L}_{\mathrm{Ms}} + \boldsymbol{L}_{\mathrm{Mm}} \tag{5a}$$
$$\boldsymbol{L}_{\mathrm{Mb}} = \boldsymbol{L}_{\mathrm{Ms}} + (\boldsymbol{L}_{\mathrm{Mr}}\boldsymbol{L}_{\mathrm{Mm}})\oslash(\boldsymbol{L}_{\mathrm{Mr}} + \boldsymbol{L}_{\mathrm{Mm}}) \tag{5b}$$
$$\boldsymbol{T}_{\mathrm{Mo}} = (\boldsymbol{L}_{\mathrm{Mr}} + \boldsymbol{L}_{\mathrm{Mm}})\oslash\boldsymbol{r}_{\mathrm{Mr}} \tag{5c}$$

where $\boldsymbol{L}_{\mathrm{Ms}}$, $\boldsymbol{L}_{\mathrm{Mm}}$, $\boldsymbol{L}_{\mathrm{Mr}}$, $\boldsymbol{r}_{\mathrm{Mr}}$, $\boldsymbol{r}_{\mathrm{Ms}}$, and $\boldsymbol{H}_{\mathrm{M}}$ are stator inductance, magnetizing inductance, rotor inductance, rotor resistance, stator resistance and inertia of the induction motor load, respectively; $\boldsymbol{a}_{\mathrm{M}}$, $\boldsymbol{b}_{\mathrm{M}}$, and $\boldsymbol{c}_{\mathrm{M}}$ are the torque coefficients of the torque slip characteristics.

The variable frequency induction-motor drive loads are with the following dynamics [2], [3]:

$$\dot{\boldsymbol{i}}_{\mathrm{Ad}} = -\boldsymbol{r}_{\mathrm{A}}\oslash\boldsymbol{L}_{\mathrm{A}}\odot\boldsymbol{i}_{\mathrm{Ad}} + \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Aq}} + (\boldsymbol{E}_{\mathrm{D}}\boldsymbol{v}_{\mathrm{Bd}} - \boldsymbol{v}_{\mathrm{Ad}})\oslash\boldsymbol{L}_{\mathrm{A}} \tag{6a}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Aq}} = -\boldsymbol{r}_{\mathrm{A}}\oslash\boldsymbol{L}_{\mathrm{A}}\odot\boldsymbol{i}_{\mathrm{Aq}} - \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Ad}} + (\boldsymbol{E}_{\mathrm{D}}\boldsymbol{v}_{\mathrm{Bq}} - \boldsymbol{v}_{\mathrm{Aq}})\oslash\boldsymbol{L}_{\mathrm{A}} \tag{6b}$$
$$\dot{\boldsymbol{i}}_{\mathrm{r}} = (\frac{3\sqrt{2}}{\pi}\sqrt{\boldsymbol{v}_{\mathrm{Ad}}^2 + \boldsymbol{v}_{\mathrm{Aq}}^2} - \boldsymbol{r}_{\mathrm{DC}}\odot\boldsymbol{i}_{\mathrm{r}} - \boldsymbol{v}_{\mathrm{DC}})\oslash\boldsymbol{L}_{\mathrm{DC}} \tag{6c}$$
$$\dot{\boldsymbol{v}}_{\mathrm{DC}} = (\boldsymbol{i}_{\mathrm{r}} - \frac{\sqrt{3}}{2\sqrt{2}}\boldsymbol{i}_{\mathrm{Dd}})\oslash\boldsymbol{C}_{\mathrm{DC}} \tag{6d}$$
$$\Delta\dot{\boldsymbol{\omega}}_{\max} = (\frac{\sqrt{3}}{2\sqrt{2}}\boldsymbol{i}_{\mathrm{Dd}} - \boldsymbol{i}_{\max})\odot\boldsymbol{K}_{\max}\oslash\boldsymbol{T}_{\max} \tag{6e}$$
$$\Delta\dot{\boldsymbol{\omega}} = -\boldsymbol{K}_{\mathrm{DP}}\odot(\omega_n - \boldsymbol{\omega}_{\mathrm{m}})\oslash[(\boldsymbol{a}_{\mathrm{D}} + \boldsymbol{b}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}} + \boldsymbol{c}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}}^2)\oslash\boldsymbol{T}_{\mathrm{D}}^0] - \boldsymbol{K}_{\mathrm{DP}}\odot[(\frac{\sqrt{3}}{2\sqrt{2}}\boldsymbol{i}_{\mathrm{Dd}} - \boldsymbol{i}_{\max})\odot\boldsymbol{K}_{\max}\oslash\boldsymbol{T}_{\max}] + \boldsymbol{K}_{\mathrm{DI}}\odot(\omega_{\mathrm{Dref}}\boldsymbol{\alpha} - \boldsymbol{\omega}_{\mathrm{m}} - \Delta\boldsymbol{\omega}_{\max}) \tag{6f}$$
$$\dot{\boldsymbol{\omega}}_{\mathrm{m}} = (\omega_{\mathrm{n}} - \boldsymbol{\omega}_{\mathrm{m}})\oslash[(\boldsymbol{a}_{\mathrm{D}} + \boldsymbol{b}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}} + \boldsymbol{c}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}}^2)\oslash\boldsymbol{T}_{\mathrm{D}}^0] \tag{6g}$$
$$\dot{\boldsymbol{v}}_{\mathrm{Dd}} = \omega_{\mathrm{n}}\boldsymbol{s}_{\mathrm{D}}\odot\boldsymbol{v}_{\mathrm{Dq}} - [\boldsymbol{v}_{\mathrm{Dd}} + \omega_{\mathrm{n}}(\boldsymbol{L}_{\mathrm{Do}} - \boldsymbol{L}_{\mathrm{Db}})\odot\boldsymbol{i}_{\mathrm{Dq}}]\oslash\boldsymbol{T}_{\mathrm{Do}} \tag{6h}$$
$$\dot{\boldsymbol{v}}_{\mathrm{Dq}} = -\omega_{\mathrm{n}}\boldsymbol{s}_{\mathrm{D}}\odot\boldsymbol{v}_{\mathrm{Dd}} - [\boldsymbol{v}_{\mathrm{Dq}} - \omega_{\mathrm{n}}(\boldsymbol{L}_{\mathrm{Do}} - \boldsymbol{L}_{\mathrm{Db}})\odot\boldsymbol{i}_{\mathrm{Dd}}]\oslash\boldsymbol{T}_{\mathrm{Do}} \tag{6i}$$
$$\dot{\boldsymbol{s}}_{\mathrm{D}} = [(\boldsymbol{a}_{\mathrm{D}} + \boldsymbol{b}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}} + \boldsymbol{c}_{\mathrm{D}}\odot\boldsymbol{s}_{\mathrm{D}}^2)\oslash\boldsymbol{T}_{\mathrm{D}}^0 - (\boldsymbol{v}_{\mathrm{Dd}}\odot\boldsymbol{i}_{\mathrm{Dd}} + \boldsymbol{v}_{\mathrm{Dq}}\odot\boldsymbol{i}_{\mathrm{Dq}})\oslash\omega_{\mathrm{n}}]\oslash(2\boldsymbol{H}_{\mathrm{D}}) \tag{6j}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Dd}} = -\boldsymbol{r}_{\mathrm{Ds}}\oslash\boldsymbol{L}_{\mathrm{Db}}\odot\boldsymbol{i}_{\mathrm{Dd}} + \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Dq}} + (\frac{\sqrt{3}}{2\sqrt{2}}\boldsymbol{v}_{\mathrm{DC}} - \boldsymbol{v}_{\mathrm{Dd}})\oslash\boldsymbol{L}_{\mathrm{Db}} \tag{6k}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Dq}} = -\boldsymbol{r}_{\mathrm{Ds}}\oslash\boldsymbol{L}_{\mathrm{Db}}\odot\boldsymbol{i}_{\mathrm{Dq}} - \omega_{\mathrm{n}}\boldsymbol{i}_{\mathrm{Dd}} + (0 - \boldsymbol{v}_{\mathrm{Dq}})\oslash\boldsymbol{L}_{\mathrm{Db}} \tag{6l}$$

with

$$\boldsymbol{L}_{\mathrm{Do}} = \boldsymbol{L}_{\mathrm{Ds}} + \boldsymbol{L}_{\mathrm{Dm}} \tag{7a}$$
$$\boldsymbol{L}_{\mathrm{Db}} = \boldsymbol{L}_{\mathrm{Ds}} + (\boldsymbol{L}_{\mathrm{Dr}}\boldsymbol{L}_{\mathrm{Dm}})\oslash(\boldsymbol{L}_{\mathrm{Dr}} + \boldsymbol{L}_{\mathrm{Dm}}) \tag{7b}$$
$$\boldsymbol{T}_{\mathrm{Do}} = (\boldsymbol{L}_{\mathrm{Dr}} + \boldsymbol{L}_{\mathrm{Dm}})\oslash\boldsymbol{r}_{\mathrm{Dr}} \tag{7c}$$

where $\boldsymbol{L}_{\mathrm{Ds}}$, $\boldsymbol{L}_{\mathrm{Dm}}$, $\boldsymbol{L}_{\mathrm{Dr}}$, $\boldsymbol{r}_{\mathrm{Dr}}$, $\boldsymbol{r}_{\mathrm{Ds}}$, and $\boldsymbol{H}_{\mathrm{D}}$ are stator inductance, magnetizing inductance, rotor inductance, rotor resistance, stator resistance and inertia of the induction motor load, respectively; $\boldsymbol{a}_{\mathrm{D}}$, $\boldsymbol{b}_{\mathrm{D}}$, and $\boldsymbol{c}_{\mathrm{D}}$ are the torque coefficients of the torque slip characteristics.

C. Network model

The dynamics of branches are as follows:

$$\dot{\boldsymbol{i}}_{\mathrm{Bd}} = -\boldsymbol{r}_{\mathrm{B}} \oslash \boldsymbol{L}_{\mathrm{B}} \odot \boldsymbol{i}_{\mathrm{Bd}} + \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{Bq}} + \boldsymbol{E}_{\mathrm{B}} \boldsymbol{v}_{\mathrm{Bd}} \oslash \boldsymbol{L}_{\mathrm{B}} \tag{8a}$$
$$\dot{\boldsymbol{i}}_{\mathrm{Bq}} = -\boldsymbol{r}_{\mathrm{B}} \oslash \boldsymbol{L}_{\mathrm{B}} \odot \boldsymbol{i}_{\mathrm{Bq}} - \omega_{\mathrm{n}} \boldsymbol{i}_{\mathrm{Bd}} + \boldsymbol{E}_{\mathrm{B}} \boldsymbol{v}_{\mathrm{Bq}} \oslash \boldsymbol{L}_{\mathrm{B}} \tag{8b}$$

The Kirchhoff's Current Law of each bus yields

$$\mathbf{0} = \boldsymbol{E}_{\mathrm{C}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{od}} - \boldsymbol{E}_{\mathrm{L}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Ld}} - \boldsymbol{E}_{\mathrm{M}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Md}} - \boldsymbol{E}_{\mathrm{D}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Ad}} - \boldsymbol{E}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Bd}} \tag{9a}$$
$$\mathbf{0} = \boldsymbol{E}_{\mathrm{C}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{oq}} - \boldsymbol{E}_{\mathrm{L}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Lq}} - \boldsymbol{E}_{\mathrm{M}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Mq}} - \boldsymbol{E}_{\mathrm{D}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Aq}} - \boldsymbol{E}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{i}_{\mathrm{Bq}} \tag{9b}$$

D. Full dynamic model in ODE form

Now we obtain the full dynamic model of the MG, in DAE form, as follows:

$$\dot{\tilde{\boldsymbol{x}}} = \tilde{f}(\tilde{\boldsymbol{x}}, \boldsymbol{z}) \tag{10a}$$
$$\mathbf{0} = \tilde{g}(\tilde{\boldsymbol{x}}) \tag{10b}$$

where (10a) collects (2), (3), (4), (6), and (8); (10b) represents (9); and

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{\delta}; \boldsymbol{P}; \boldsymbol{Q}; \boldsymbol{\phi}_{\mathrm{d}}; \boldsymbol{\phi}_{\mathrm{q}}; \boldsymbol{\gamma}_{\mathrm{d}}; \boldsymbol{\gamma}_{\mathrm{q}}; \boldsymbol{i}_{\mathrm{ld}}; \boldsymbol{i}_{\mathrm{lq}}, \boldsymbol{v}_{\mathrm{od}}; \boldsymbol{v}_{\mathrm{oq}}; \boldsymbol{i}_{\mathrm{od}}; \boldsymbol{i}_{\mathrm{oq}}; \\ \boldsymbol{i}_{\mathrm{Ld}}; \boldsymbol{i}_{\mathrm{Lq}}; \\ \boldsymbol{v}_{\mathrm{Md}}; \boldsymbol{v}_{\mathrm{Mq}}; \boldsymbol{s}_{\mathrm{M}}; \boldsymbol{i}_{\mathrm{Md}}; \boldsymbol{i}_{\mathrm{Mq}}; \\ \boldsymbol{i}_{\mathrm{Ad}}; \boldsymbol{i}_{\mathrm{Aq}}; \boldsymbol{i}_{\mathrm{r}}; \boldsymbol{v}_{\mathrm{DC}}; \Delta\boldsymbol{\omega}_{\max}; \Delta\boldsymbol{\omega}; \boldsymbol{\omega}_{\mathrm{m}}; \boldsymbol{v}_{\mathrm{Dd}}; \boldsymbol{v}_{\mathrm{Dq}}; \boldsymbol{s}_{\mathrm{D}}; \boldsymbol{i}_{\mathrm{Dd}}; \boldsymbol{i}_{\mathrm{Dq}} \\ \boldsymbol{i}_{\mathrm{Bd}}; \boldsymbol{i}_{\mathrm{Bq}} \end{bmatrix} \tag{11a}$$
$$\boldsymbol{z} = [\boldsymbol{v}_{\mathrm{Bd}}; \boldsymbol{v}_{\mathrm{Bq}}] \tag{11b}$$

Next, we transform the DAE model to an ODE model. First, taking the derivative of both sides of (11b) and dividing (11a) accordingly gives

$$\dot{\tilde{\boldsymbol{x}}}_1 = f_1(\tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}_2, \boldsymbol{z}) \tag{12a}$$
$$\dot{\tilde{\boldsymbol{x}}}_2 = f_2(\tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}_2, \boldsymbol{z}) \tag{12b}$$
$$\mathbf{0} = \tilde{g}_{\mathrm{der}}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z}) = \boldsymbol{M}_1 \tilde{\boldsymbol{x}}_2 + \boldsymbol{M}_2 \boldsymbol{z} \tag{12c}$$

where (12a) collects (2a)-(2i); (12b) collects (2j)-(2m), (3), (4), (6), and (8); and

$$\tilde{\boldsymbol{x}}_1 = [\boldsymbol{\delta}; \boldsymbol{P}; \boldsymbol{Q}; \boldsymbol{\phi}_{\mathrm{d}}; \boldsymbol{\phi}_{\mathrm{q}}; \boldsymbol{\gamma}_{\mathrm{d}}; \boldsymbol{\gamma}_{\mathrm{q}}; \boldsymbol{i}_{\mathrm{ld}}; \boldsymbol{i}_{\mathrm{lq}}; \boldsymbol{s}_{\mathrm{M}};] \tag{13a}$$
$$\tilde{\boldsymbol{x}}_2 = \begin{bmatrix} \boldsymbol{v}_{\mathrm{od}}; \boldsymbol{v}_{\mathrm{oq}}; \boldsymbol{i}_{\mathrm{od}}; \boldsymbol{i}_{\mathrm{oq}}; \\ \boldsymbol{i}_{\mathrm{Ld}}; \boldsymbol{i}_{\mathrm{Lq}}; \\ \boldsymbol{v}_{\mathrm{Md}}; \boldsymbol{v}_{\mathrm{Mq}}; \boldsymbol{i}_{\mathrm{Md}}; \boldsymbol{i}_{\mathrm{Mq}}; \\ \boldsymbol{i}_{\mathrm{Ad}}; \boldsymbol{i}_{\mathrm{Aq}}; \boldsymbol{i}_{\mathrm{r}}; \boldsymbol{v}_{\mathrm{DC}}; \Delta\boldsymbol{\omega}_{\max}; \Delta\boldsymbol{\omega}; \boldsymbol{\omega}_{\mathrm{m}}; \boldsymbol{v}_{\mathrm{Dd}}; \boldsymbol{v}_{\mathrm{Dq}}; \boldsymbol{s}_{\mathrm{D}}; \boldsymbol{i}_{\mathrm{Dd}}; \boldsymbol{i}_{\mathrm{Dq}} \\ \boldsymbol{i}_{\mathrm{Bd}}; \boldsymbol{i}_{\mathrm{Bq}} \end{bmatrix} \tag{13b}$$

Then, following [4], (12) can be transformed into the following ODE model

$$\dot{\tilde{\boldsymbol{x}}} = \tilde{f}(\tilde{\boldsymbol{x}}, \boldsymbol{z}) \tag{14a}$$
$$\dot{\boldsymbol{z}} = h(\tilde{\boldsymbol{x}}, \boldsymbol{z}) = -\left(\frac{\partial \tilde{g}_{\mathrm{der}}}{\partial \boldsymbol{z}}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z})\right)^{-1} \frac{\partial \tilde{g}_{\mathrm{der}}}{\partial \tilde{\boldsymbol{x}}_2}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z}) f_2(\tilde{\boldsymbol{x}}, \boldsymbol{z}) \tag{14b}$$

Finally, we derive the conditions for the existence of the ODE model and the equivalence of the above transformation. The ODE model exists iff the matrix $\frac{\partial \tilde{g}_{\mathrm{der}}}{\partial \boldsymbol{z}}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z})$ is invertible. We have

$$\frac{\partial \tilde{g}_{\mathrm{der}}}{\partial \boldsymbol{z}}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z}) = -\boldsymbol{E}_{\mathrm{C}}^{\mathrm{T}} \boldsymbol{L}_{\mathrm{C}}' \boldsymbol{E}_{\mathrm{C}} - \boldsymbol{E}_{\mathrm{L}}^{\mathrm{T}} \boldsymbol{L}_{\mathrm{L}}' \boldsymbol{E}_{\mathrm{L}} - \boldsymbol{E}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{L}_{\mathrm{B}}' \boldsymbol{E}_{\mathrm{B}} - \boldsymbol{E}_{\mathrm{M}}^{\mathrm{T}} \boldsymbol{L}_{\mathrm{Mb}}' \boldsymbol{E}_{\mathrm{M}} - \boldsymbol{E}_{\mathrm{D}}^{\mathrm{T}} \boldsymbol{L}_{\mathrm{A}}' \boldsymbol{E}_{\mathrm{D}} \tag{15}$$

with $\boldsymbol{L}_{\mathrm{C}}' = [\mathrm{diag}(\boldsymbol{L}_{\mathrm{C}})]^{-1}$, and $\boldsymbol{L}_{\mathrm{L}}'$, $\boldsymbol{L}_{\mathrm{B}}'$, $\boldsymbol{L}_{\mathrm{Mb}}'$, and $\boldsymbol{L}_{\mathrm{A}}'$ being analogous. We can proved that $\frac{\partial \tilde{g}_{\mathrm{der}}}{\partial \boldsymbol{z}}(\tilde{\boldsymbol{x}}_2, \boldsymbol{z})$ is invertible if the topology of the MG is connected. Further, according to [5], the equivalence between the ODE model and DAE model can be ensured with compatible initial conditions. Let $[\tilde{\boldsymbol{x}}(t_0); \boldsymbol{z}(t_0)]$ denote the initial state at $t = t_0$, then the solutions of the ODE model and DAE model are identical if $g(\tilde{\boldsymbol{x}}(t_0)) = \mathbf{0}$, which allows us to apply the results established for the ODE model to the DAE model and thus the real-life MG.

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