Modeling, Control, and Learning for Quadrupedal and Bipedal Locomotion: Perspective Review

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***Abstract*— Legged robots hold great promise for navigating complex, unstructured environments across a wide range of applications, from industrial, agriculture, and healthcare settings to extreme terrains such as subterranean, underwater, polar, and extraterrestrial environments. While recent advances in both quadrupedal and bipedal locomotion have significantly improved robot agility and robustness, achieving (TBD) remains an open challenge. This perspective paper surveys the state of the art in modeling and control for legged robot locomotion, spanning physics-based methods (e.g., feedback and model predictive control) and learning-based approaches (e.g., reinforcement learning). Beyond reviewing these paradigms in isolation, the core contribution of this work is to highlight their inherent connections, revealing how learned policies and traditional controllers often optimize over similar objectives and rely on shared structure. The paper also offers a forward-looking discussion on how emerging tools, such as generative models and large language models, may enable new modes of generalization, reasoning, and adaptability in legged robot control. By bridging these research threads, this work aims to clarify current opportunities and inspire future directions toward more capable and intelligent legged robots.**

***Index Terms*—Quadrupeds, Bipeds, Locomotion, Modeling, Control, Learning.**

Colors of each author (you may change it): Hao, Ivan, Yan, Guanya, Fan Shi, I-Chia, Jerry: Jerry Link to Google Doc: `https://docs.google.com/document/d/1o0JKuveC7VWEPeBGw9ldP4TVoEWmGLWJzhRos-KOngM/edit?usp=sharing`

I. Introduction (1.5 pages; I-Chia, Ivan)

A. *Why legged robot? (I-Chia)*

Legged robots have huge potential to be applied to industrial, extreme environments, and daily activities to improve the well-being of humans. Nowadays, the development of legged robots is faster than it has ever been.

Legged robots offer unique advantages for mobility in complex, unstructured, and human-centered environments. Unlike wheeled or tracked robots, they are inherently better suited for traversing diverse terrain types, including ladders [1], stairs, sand [2], and even underwater [3] settings (Fig. 2). Compared to aerial systems, legged robots are typically more energy-efficient for locomotion [4] and can support heavier payloads over longer periods [5, 6].

Leg morphology also enables legged robots to navigate spaces designed for humans. Their anthropomorphic or quadrupedal structures allow them to move through buildings, climb stairs, or operate in confined environments inaccessible to other ground robots [7]. Moreover, their familiar forms can improve human-robot interaction by enhancing user comfort and intuitiveness [8].

From a biological and evolutionary perspective, legs have emerged as optimal structures for terrestrial and amphibian locomotion across species [9, 10]. This evolutionary insight informs robot design and motivates the use of legs for adaptive mobility. Beyond their practical capabilities, legged robots also serve as platforms for advancing control theory and understanding the neuromechanics of animal locomotion [11].

Recent years have witnessed a surge in the development of high-performance legged robots, driven by both academic and industrial efforts. As illustrated in Figs. 1 and 2, this progress has expanded the functional capabilities of legged machines and opened new opportunities for real-world deployment. However, the pace of advancement has also introduced a challenge: the field is evolving so rapidly that it can be difficult for newcomers, or even specialists focused on a single subdomain, to maintain a comprehensive view of the landscape.

This paper aims to provide a clear and integrated perspective on the research advancement state of legged robotics, covering key topics such as modeling, control, and learning, along with .... In doing so, we highlight the fundamental interplay between physics-based and data-driven approaches, identify current limitations, and discuss emerging opportunities—including the potential of generative methods and foundation models in shaping the future of legged robotics advancement.

B. *Applications (0.5=>0.2 page; Ivan; Yan edited, April 2)*

Legged robots are increasingly deployed across industrial, domestic, and extreme environments, addressing complex tasks with enhanced mobility and safety. In industry, quadrupeds such as ANYmal endowed with a manipulator arm enable locomotion-manipulation tasks such as valve operation in hazardous areas [12], while bipedal robots such as Agility Robotics' Digit automate warehouse operations such as tote recycling [31] or Honda's E2-DR demonstrated ladder climbing and debris navigation capabilities in inspection tasks [1]. Specialized systems such as MARVEL perform tank inspections through magnetic adhesion [13], and pipe-crawling bipeds reduce human exposure in confined or high-rise spaces [32].

In healthcare, robots such as Aliengo autonomously patrol ICUs to monitor recovery conditions [33, 34]. For assistive mobility, quadrupeds such as Spot guide visually impaired users in urban settings [7]. Bipedal platforms including Neo Gamma

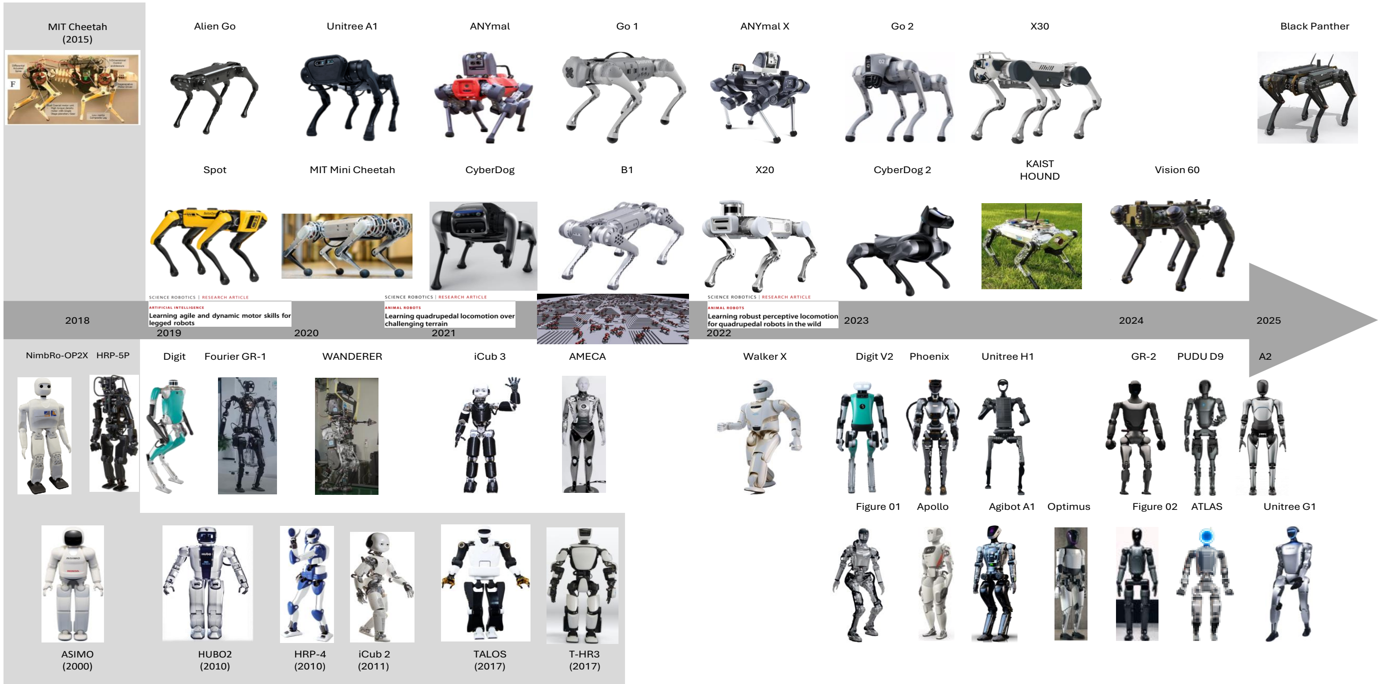

Figure 1: Timeline showing the apparition of remarkable commercial and research oriented quadrupedal and bipedal robots in the last decade. `https://www.linkedin.com/posts/nikolaiensslen_t3v-humanoids-tracker-activity-7297654375185829888-HHml?utm_source=share&utm_medium=member_desktop&rcm=ACoAACPkN0gBp88UkHDcyKzzmAVfHrX9ySOgt_8`

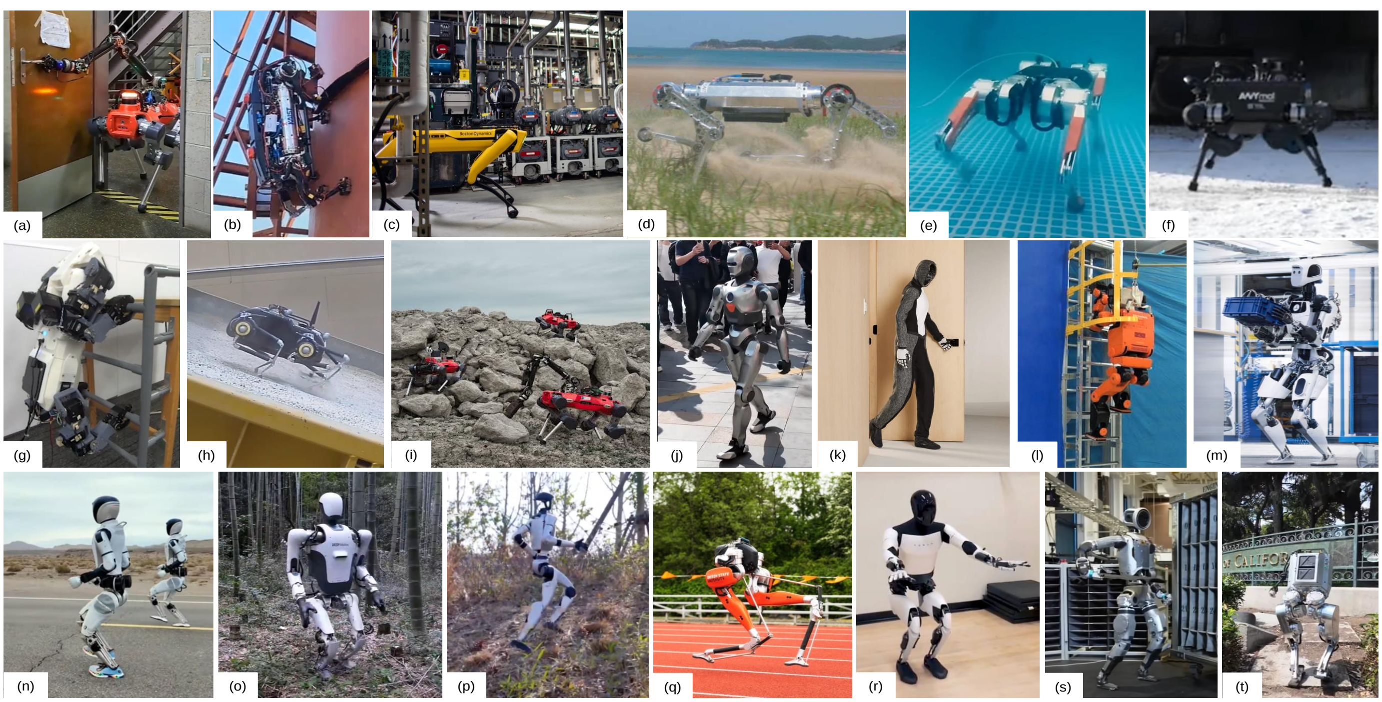

Figure 2: Quadrupedal and bipedal robots performing in real-world environments. To add: quadruped in NMI 2025. (a) ANYbotics' ANYmal opening and passing through a door in an industrial setting [12], (b): KAIST's MARVEL climbing a ferromagnetic surface [13], (c): Boston Dynamics' Spot performing inspection tasks in an industrial facility [14], (d): Quadrupedal robot traversing sandy deformable terrain [2], (e): MAB Robotics' Honey Badger walking underwater [15] (f): ANYbotics' ANYmal traversing an icy slippery terrain [16], (g): Quadruped climbing a vertical ladder [17], (h): ETH Zurich's SpaceBok traversing a steep granular terrain [18], (i): Team of ANYbotics' ANYmals exploring a space analog terrain [19], (j): Engine AI's PM01 Humanoid walking in the street [20], (k): 1X's Neo Gamma home assistance Humanoid [21], (l): E2-DR humanoid climbing a vertical ladder in an industrial setting [1], (m): Apptronik's Apollo moving a plastic crate in an industrial setting [22], (n): A couple of RobotEra's Star 1 running along the Gobi desert [23], (o): Deep Robotics' DR01 traversing a bamboo forest [24], (p): Unitree's G1 running over a steep terrain [25], (q): Oregon State University's Cassie [26, 27], (r): Tesla's Optimus Gen-2 [28], (s): Boston Dynamics' Electric ATLAS [29], (t): Berkeley Humanoid [30].

and Optimus-Gen-2 are pushing the boundaries of household robotics [21, 28].

Quadrupeds are also emerging in precision agriculture, enabling high-resolution crop monitoring with minimal soil impact and terrain adaptability [2, 35]. In vineyards, HyQReal has demonstrated autonomous pruning assistance [36–38].

Legged systems are also suitable for subterranean [39–43], shipboard [44–46], underwater [3, 47–50], and polar environments. ANYmal C100 led DARPA's Subterranean Challenge in mine mapping [42, 43, 51, 52]. Various commercial quadrupedal robots have been assessed in non-inertial shipboard environments where the ground exhibits time-varying and persistent acceleration and rotation [44–46]. Underwater legged robots such as SILVER and RoboIguana enable benthic exploration with minimal ecological disruption [47, 53]. Polar quadrupeds gather climate data on glaciers [16] and ski beneath ice sheets for long-term monitoring [54]. In the world's first Ski Robot Challenge as part of the 2018 Winter Olympics in Pyeongchang, South Korea, humanoids navigated an 80-meter slalom course, showcasing dynamic control in icy environments [55–58].

In space exploration, coordinated teams of ANYmal variants explore Martian terrain with role-specific functionality [19]. Ongoing research explores low-gravity legged jumping for accessing scientifically valuable sites [59–62], and proprioceptive quadrupeds may be used to infer regolith properties for adaptive mission planning [63].

Despite ongoing challenges, legged robots are poised to augment human capability in domains where biological constraints limit access. They also offer insights into wearable robotics and exoskeleton control, with implications for human-machine interaction and rehabilitation.

C. Overview of This Survey

1) Scope/focus This survey targets a broad audience, providing a structured overview of the modeling and control of quadrupedal and bipedal robots, and how learning-based techniques have accelerated progress toward robust, versatile locomotion. As the field advances rapidly, newcomers may find it challenging to navigate the expanding landscape. To this end, we focus on three foundational pillars underpinning recent breakthroughs: modeling, control, and learning. We begin by reviewing physics-based approaches, then explore data-driven methods, and conclude by highlighting their inherent connections. For each category, we discuss representative methodologies, examine their strengths and limitations, and reflect on how these strategies complement one another. For experts, we emphasize the conceptual links between analytical and learning-based control, while outlining emerging challenges and future directions too. Our goal is to offer a clear entry point for newcomers and stimulate further research toward agile, robust, and versatile legged robots. This survey focuses solely on locomotion and excludes manipulation mechanisms, which fall outside the intended scope.

2) Modeling: physics-based and data-driven approaches. Locomotion control relies on models that predict system dynamics. We review recent physics-based dynamics models and simulators for legged locomotion, alongside data-driven approaches that learn dynamics from experience. Finally, we compare these methods and outline future directions for modeling legged locomotion in complex environments.

3) Control: feedback control, optimal control, and reinforcement learning. TBD.

4) Bridging the gap between concepts and implementation. TBD.

5) Recent emerging trends such as foundation models wheeled legged robots. TBD (maybe refer to other surveys).

6) Open problems and future perspectives. TBD.

7) Contrast with recent surveys. This survey provides a unified overview of modeling, control, and learning for bipedal and quadrupedal robots, with emphasis on how artificial intelligence is advancing legged locomotion. Designed for both newcomers and experts, this work complements recent surveys [5, 48, 64–70] by offering an integrated perspective across physics-based and learning-driven approaches. (Todo: Table to list the contribution of each survey and compare with ours, show how this paper is filling the gaps of other surveys)

8) Organization of the paper: TBD.

Jerry:

9) Academia vs industry advancement in legged robot controls Jerry: In recent years, the legged robotics industry has embraced a range of control strategies, from classical methods to learning-based and hybrid approaches. Reference needs to be revised on the control methods used in the industry - also balance between biped robot and quadruped robots - mention unitree, boston dynamics, and anymal robots. Companies like **Boston Dynamics** and **Agility Robotics** pair proven model-based locomotion (e.g. MPC, centroidal QP whole-body control) with task-specific RL modules: Boston Dynamics still runs an MPC stack for routine walking and manipulation [71], but its March 2025 "Walk, Run, Crawl, RL Fun" demo of Atlas uses a motion-imitation policy trained in simulation—likely an actor–critic algorithm similar to PPO—to execute maneuvers such as backflips and crawl transitions [72, 73]. Agility Robotics follows a similar idea: Digit's everyday walking relies on a Hybrid-Zero-Dynamics/CLF-QP controller developed by Jesse Grizzle [74], whereas Cassie's 5 km run and Digit's recent rough-terrain tests employ a PPO-based residual policy that augments the classical controller for speed and disturbance recovery [75, 76]. Others like **Figure AI** and **1X Robotics** pursue end-to-end deep RL pipelines trained entirely in simulation, leveraging domain randomization and imitation learning objectives for zero-shot sim-to-real transfer [77, 78]. Firms like **Tesla**, **Unitree**, and **Disney Robotics** are transitioning from classical gait controllers (e.g., ZMP, inverse kinematics + PID) to hybrid frameworks. For instance, **Tesla**'s Optimus robot utilizes neural networks to control its limbs, enabling it to navigate uneven terrain autonomously without relying on vision, showcasing an integration of learning-based control for improved adaptability [79, 80]. Similarly, **Unitree**'s G1 humanoid robot employs a combination of imitation and reinforcement learning, along with force-position hybrid control, to achieve human-like dexterity and adaptability in various tasks [25, 81]. **Disney Robotics** builds on the DeepMimic framework [82], which employs adversarial imitation learning and phase-conditioned networks to replicate expressive, physically plausible motions from motion capture data [83].

Jerry: Looking into the future, industry–academia alliances should be and are a routine now: ETH Zurich's Robotic Systems Lab co-developed *ANYmal* and continues to test MPC and RL upgrades with spin-off ANYbotics [84, 85]. Agility Robotics, a spin-off of Oregon State's Dynamic Robotics Lab, is still partners with OSU in deep-RL running and perception for Cassie and Digit [86, 87]. Boston Dynamics recently teamed with the Robotics AI Institute to integrate advanced RL into Electric Atlas controls [88, 89]. Apptronik, born from the UT-Austin Human-Centered Robotics Lab, co-publishes whole-body control and supplies Apollo prototypes for Mercedes factory trials [90, 91]. Disney Research collaborates with ETH and CMU on DeepMimic-style motion priors for its biped droids [92, 93]. Unitree's low-cost platforms amplify this trend over the past two years: A1 and Go1 support the university RL studies on high-speed bounding [94], perturbation-adaptive gaits [95], and SLAM-enhanced navigation [96]; its humanoid-hosted CMU-Berkeley H2O zero-shot imitation framework [97]. These cases show a shared pattern: companies provide capital, manufacturing, and test platforms, while academia supplies cutting-edge algorithms—jointly accelerating deployable legged-robot control.

II. Other Topics in Legged Locomotion

Jerry: In robotics, the complete control stack is typically organized as a *sense–plan–act* pipeline. Legged locomotion therefore decomposes into four layers: perception, state estimation, planning, and control. Not surprisingly, this engineering hierarchy mirrors the neural architecture that facilitates vertebrate walking.

Legged robots acquire modalities from cameras, IMUs, and actuator encoders; while vertebrates collect similar visual, vestibular, and proprioceptive signals via the primary visual cortex, vestibular nuclei, and muscle spindles. State Estimator in a robot estimates the body pose, ego-velocities, etc in a robot body, similar to the posterior parietal cortex and cerebellum in the brains that fuse these signals into predicted body states The mission/task planner correspond to dorsolateral prefrontal cortex that selects goals; while grid- or sampling-based global planning over terrain height-maps correspond to the hippocampus–entorhinal map. Local foot-placement planners that optimize walking gait and swing trajectories is mirrored by the supplementary and lateral premotor cortices that controls vertebrates' short-horizon footstep sequencing Finally, the robot's joint-level torque controllers mirror brain-stem locomotor centres, while its embedded hardware constraints and real-time safety interlocks correspond to sub-millisecond spinal reflex arcs. Each neural substrate thus finds a direct analogue in the classical control pipeline for agile legged locomotion; the correspondences are summarized in Figure. 3.

Jerry: Added a draft figure for the comparison figure between human vs humanoid locomotion - Texts to be enlarged - feedback from the meeting: make the brain part larger - maybe include things like which area of the brain that controls the mapping to robotics - also figure out how to make the graph look nicer - use Figma?

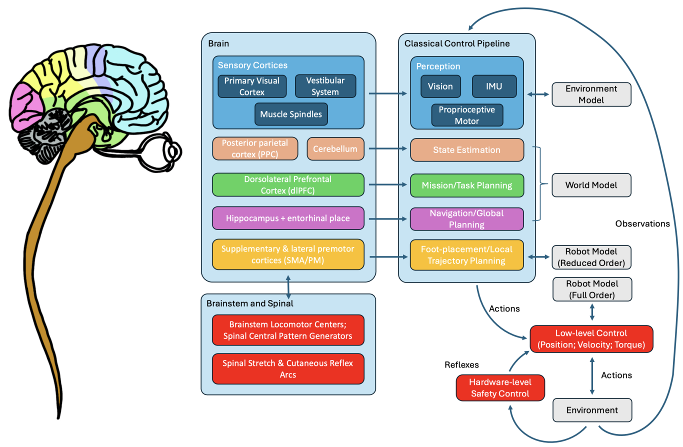

Figure 3: Mapping between vertebrate locomotor circuitry (left) and the classical sense–plan–act hierarchy used in legged-robot control (centre). Colour-matched blocks highlight functional analogues: sensory cortices ↔ perception, posterior parietal cortex / cerebellum ↔ state estimation, dorsolateral prefrontal cortex ↔ mission planning, hippocampus–entorhinal map ↔ global navigation, premotor cortices ↔ local foot-placement, and brain-stem / spinal pathways ↔ joint-level control and safety reflexes. Grey boxes on the right denote the environment, robot, and world models consulted at each layer; arrows indicate the flow of observations, actions, and reflex loops. Figure is inspired by LeCun's cognitive architecture [98] and Ijspeert's neuromechanical control model [99].

The following subsections will briefly survey the state-of-the-art methods at each layer of the legged-robot control stack, interested readers can refer to the references in each of the subsection for further details.

A. Perception

The perception layer ingests heterogeneous exteroceptive (RGB/RGB-D, LiDAR, radar, event) and proprioceptive (IMU, encoder) streams and converts them into metric and semantic world models for downstream modules. Dense local elevation or TSDF grids are reconstructed with stereo/RGB-D matching (SGM [100]) and LiDAR scan registration, then fused over time by SfM/SLAM back-ends such as ORB-SLAM and COLMAP, optionally aided by sparse-to-dense depth completion [101–103]. Semantic cues are extracted by deep CNNs: classification backbones (AlexNet, ResNet), one- and two-stage detectors (YOLO, Faster-R-CNN) for 2-D objects, fully convolutional networks (FCN, DeepLab) for pixel masks, and PointNet++ for 3-D traversability labelling [104–108]. Neural radiance fields further supply photorealistic 3-D reconstructions for simulation and planning [**?**]. The resulting obstacle masks, elevation grids with slope/roughness, and 6-DoF object poses form the observation set fed to the state-estimation layer.

B. State Estimation

Robust locomotion depends on centimeter-level, drift-free estimations, and accurate mappings. The states that legged locomotion are most interested in include the six degree of freedom body pose estimation, velocity estimation, contact or slip state and an accurate local map of the surrounding terrain. Classical probabilistic filters like: Extended/Unscented Kalman Filters, particle filters, and factor-graph optimizers such as sliding-window bundle adjustment and iSAM remain the backbone and are exhaustively discussed in Barfoot's textbook [109]; they underpin visual–inertial EKF-SLAM pipelines on legged robots [110, 111]. These pipelines fuse IMU data, vision, leg-odometry, and foot-contact cues at $\sim$ 1kHz, providing the low-latency state feedback demanded by MPC and whole-body controllers. Emerging learning-based filters augment the stack by regressing contact probabilities or pose priors directly from raw sensor streams, boosting robustness on perceptually ambiguous terrain [112].

C. Mission/Task Planning

D. Navigation/Global Planning

A global planner plans a collision-free body path for legged robots from A to B on a 2.5-D terrain map, reasoning over elevation and slope [113]. Classical navigation has evolved from **graph searches** — BFS/DFS, Dijkstra, A* and their anytime repairs D*-Lite, ARA* — to **sampling planners** such as RRT/RRT*, FMT* and BIT*, which explore high-DOF terrain efficiently [114–123]. Gradient-based optimizers, such as CHOMP, TrajOpt, smooth those trees, and mixed-integer convex programmes convert them into foothold-consistent body paths on rough ground [124–126]. In recent years, **learning-based methods** now supplements optimization methods: offline-RL VAPOR drives a Spot through dense vegetation [127], hierarchical RL enables kilometre-scale urban missions with wheeled-legged robots [128], and traversability-aware policies fuse RGB-D and proprioception to pick safe routes without a global map [129]. Together, these planners deliver collision-free, slope-aware routes that feed the downstream local (foot-placement) layer.

E. Foot-placement/Local Trajectory Planning

This module decides *where, when* and *how* each foot will land over the next few steps. Early robots relied on velocity heuristics such as the Raibert rule, which places the swing leg ahead of the centre of mass for balance [130]. Repeatable patterns soon followed: hybrid-zero-dynamics gait libraries pioneered by Grizzle and collaborators give invariant step templates for bipedal walkers [131], while central-pattern-generator (CPG) networks formalised by Ijspeert generate smooth rhythmic trajectories for many-legged robots [132]. Modern planners cast foothold selection as a constrained optimisation: centroidal-dynamics MPC searches for footholds and swing motions under kinematic, dynamic and energy limits [133, 134], often consulting elevation maps for rough terrain [135]. Reinforcement-learning policies now match this performance, correcting footholds directly from onboard sensors and enabling blind, high-speed runs through natural scenes [136, 137].

III. Modeling of Legged Robots and Contact Interaction with Environment (new) (4 pages, I-Chia)

Before designing a controller for a legged robot, it is essential to understand and describe the underlying system dynamics so the robot can plan and react accordingly. This motivates the development of accurate and efficient models that can serve as a foundation for control design. A 'good' model should consistently (you can trust the model) and efficiently (not using too much computation power and time) offer reliable predictions of the system's evolution over time. This remains a fundamental challenge due to the complex interplay of robot mechanics, robot dynamics, actuation dynamics, and interactions with the environment and external objects. Over the past few decades, the legged robotics community has made significant progress in identifying key physical variables, developing models of different complexities, and adopting new mathematical and computational tools for system representation.

Legged locomotion involves three primary components, i.e., robot, contact, and environment To characterize a 'robot,' a formal representation of how the states of the robot (e.g., the position and orientation of the trunk) evolve in response to the control input(e.g., motor torque command) is needed. This mapping is governed not only by the intrinsic properties of the robot (e.g., mass distribution and geometry) but also by the nature of its contact with the environment and the characteristics of the environment itself. To enable robust motion prediction and control, models must account for the coupled interactions among these three components.

While all control strategies rely on models, the form and role of these models vary. In this section, we review three major modeling approaches: physics-based analytical modeling, physical simulators, and learning-based modeling. These modeling paradigms underpin traditional control methods, model-free reinforcement learning, and model-based reinforcement learning, respectively, which will be discussed in the subsequent sections.

A. Physics-based Analytical Modeling of Legged Locomotion

Physics-based analytical models describe legged robots using analytical mathematical expressions, such as differential equations, grounded in fundamental physical laws such as Newtonian mechanics or Lagrangian dynamics. For decades, these models have served as the primary methodology for repre-

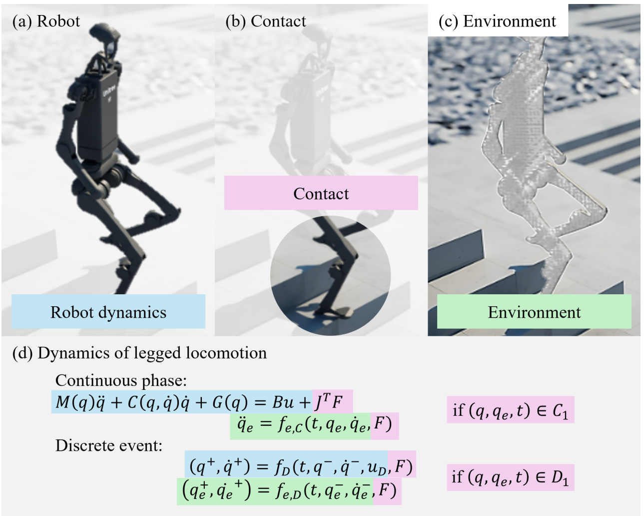

Figure 4: What to model? There are three major components, i.e., (a) robot, (b) contact, and (c) environment, involved in the (d) dynamics of legged locomotion. These three components determine how the robot will behave and are needed to be modeled. (d) The dynamics of legged locomotion can be modeled using a hybrid dynamical system formulation [138] with the robot state q and environment state q_e. The system will evolve with different dynamic equations based on the condition, e.g., $(q, q_e, t) \in C_1$.

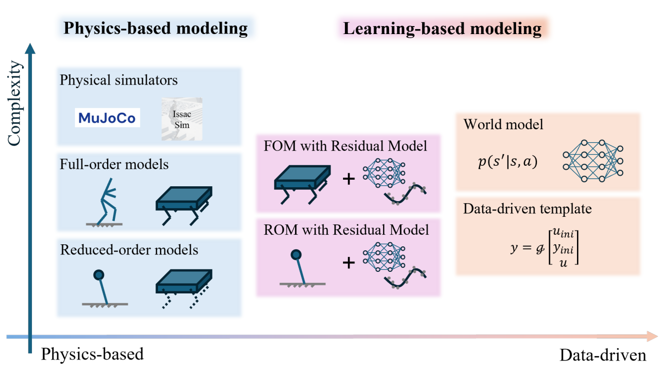

Figure 5: Overview of all different modeling methods spanned by two axes, i.e., the physics-based or data-driven and the complexity.

senting legged systems, offering interpretable formulations and rigorous theoretical guarantees on performance and safety. Their physically grounded nature allows researchers to predict and understand robot behavior with clarity while also providing a foundation for the development of learning-based approaches. In this subsection, we review how physics-based models represent quadrupedal and bipedal systems that involve the robot, contact, and complex environment.

For quadrupedal and bipedal robots composed of rigid links, full-order models can be constructed using rigid-body formulation [139]. These models explicitly capture the dynamics of all degrees of freedom (DOFs), inertia properties, and contact with the environment. When coupled with an explicit model of the environment, as illustrated in Fig 4, full-order models can simulate the coupled behavior of the robot and its surroundings with high fidelity. Such models are used in whole-body control (WBC) [140], offline trajectory planning [139], and physical simulators. However, due to their computational complexity, full-order models are often unsuitable for high-level online planning and reactive control.

To resolve the computational challenges, the hierarchical approach involving reduced-order models (ROMs) is widely used. Reduced-order models (ROMs) simplify the dynamics to a lower-dimensional representation while preserving the key features of the legged locomotion [141]. For instance, [142] describes the bipedal robot walking as a linear inverted pendulum model (LIPM), and [143] captures the dynamics of a quadruped as a single rigid body model (SRBM). These simplified models allow analytical solutions for rigorous performance guarantees, rapid numerical computation, and facilitating real-time motion planning [144, 145]. Then, the hierarchical strategy is to use ROMs to develop efficient controllers that generate an intermediate motion plan, e.g., step location [144], and the whole-body controller (WBC) [140] will determine the motor torque by considering the full-order model to realize the intermediate motion plan.

Hybrid system formulation has been developed to capture one of the defining characteristics of legged locomotion, making and breaking contact. The system's governing dynamics change depending on the robot's contact configuration. For example, the dynamics differ when a leg touches the ground versus when all limbs are in the air. Hybrid system formulation explicitly considers different governing equations with corresponding domains determined by generalized coordinate and time and the discrete event, e.g., the impact triggered when robots make new contact, see Fig 4. This formulation enables the modeling of multi-domain gaits in bipedal walking [146] and offers a compact formulation for step-location control [144].

some paragraphs are too short, merge them?To enable legged locomotion in irregular environments, physics-based analytical modeling has been extended to capture complex terrain conditions. In this paper, we look at three primary aspects of environmental irregularity, i.e., terrain geometry, macro movement, and surface mechanics.

Terrain geometry encompasses a wide range of variations beyond flat and rigid ground. Terrains may be discrete in structure [147, 148], exhibit height variations [149–153], present non-uniform surface normals [154–156], or include confined spaces in horizontal and/or vertical directions [157, 158]. They may also contain complex obstacles, such as vine-like structures [159]. Incorporating terrain geometry into control models typically involves constructing a map that informs planning and reactive behaviors. However, such maps often neglect the dynamic characteristics—namely, the macro movement—of the terrain itself.

Macro movement refers to the motion of the terrain as a whole relative to an inertial frame. When the environment has high inertia, as in large moving platforms, its motion can be modeled as an external, time-varying disturbance [160, 161] or as a variation in system parameters over time [162, 163]. In contrast, locomotion on low-inertia platforms requires reasoning about how the environment responds dynamically to the robot's motion. This necessitates the joint modeling of both the robot and the environment. A representative example is quadruped locomotion on a ball [164], where the robot and the ball are modeled as coupled rigid bodies through contact forces. This domain remains underexplored, particularly in terms of modeling and state estimation of the environment.

Surface mechanics characterize the local mechanical interactions between the robot and the terrain. Irregular terrains may be deformable, granular, partially fluidic, highly slippery [16], or even breakable. Locomotion on such surfaces is especially challenging because ground reaction forces depend on foot geometry [165, 166], terrain material properties, and insertion velocity [167]. Additionally, the foot may sink into the terrain, with friction and resistance forces varying with depth [18, 168], posing modeling challenges distinct from those in rigid-ground locomotion. To capture limb-swing resistance in such media, modified spring-loaded inverted pendulum models have been proposed [168]. Further studies on robotic and animal locomotion over soft, yielding substrates can be found in [169]

In summary, physics-based analytical modeling provides a bottom-up approach to model the legged locomotion system and allows rigorous discussion about the system's behavior. However, when it comes to complex dynamics, analytical modeling often fails to capture the complex behavior while preserving the simple and nice form for controller design.

B. Physical Simulator for Legged Locomotion

Physical simulators such as MuJoCo [170] and Issac Sim simulate the motion of legged robot systems by numerically integrating the full-order dynamics shown in Fig 4. Simulating the motion involves solving the rigid-body dynamics and the contact forces, while the latter is the major challenge. To model contact, simulators typically employ either soft-contact models or hard-contact models [170]. Soft contact models approximate the contact as a spring-damper system, where the contact force is proportional to the penetration depth and its time derivative. While computationally efficient, these models permit surface interpenetration and can exhibit instability or oscillatory behavior if not carefully tuned. In contrast, hard-contact models impose non-penetration constraints and solve for contact forces via constrained optimization, ensuring that both the contact force and inter-body distance remain non-negative. This approach yields more physically accurate simulations of rigid contact but incurs greater computational cost due to the complexity of solving the complementarity problem. The user should select the appropriate method while modeling different contact scenarios.

By incorporating the parallel computation power provided by GPUs, the speed of physical simulation is dramatically increased. Motivated by the reinforcement learning methods, the need for fast simulation is dramatically increased. Physical simulators are essential to RL since they serve as a model of the real world to allow the RL policy to learn the optimal control policy by trying different actions in the simulator. Previously, the simulators were mostly based on CPU and had limited computation speed. To increase the computation speed, several stages, such as contact detection and constraint solving, of the simulation are accelerated by parallel computation. Nowadays, several physical simulators, including MuJoCo, Issac Sim, support GPU acceleration to increase the simulation speed.

C. Learning-based Modeling of Legged Locomotion

Learning-based modeling offers a powerful alternative to traditional analytical methods for capturing the dynamics of legged locomotion using parameterized numerical structures. Unlike the analytical model, which determines the model parameters and structures based on first principle and offline system identification, learning-based models can leverage both offline and/or online data to infer system behavior. This data-driven paradigm allows for better modeling of complex, nonlinear, or poorly understood dynamics that are difficult to capture analytically. Two predominant strategies exist for learning-based modeling: residual modeling and full-model learning.

Residual modeling augments an existing physics-based model by learning the discrepancy between the model and real world. While physics-based models provide interpretability and generalizability, they often suffer from inaccuracies due to simplified assumptions and static parameter estimates. Residual models correct for these deficiencies by incorporating data-driven components. For example, Gaussian process has been employed to enhance locomotion energy efficiency [171] and improve jumping behavior on deformable terrain [172]. An autoregressive moving average vector (AMAV) model has enabled quadruped robots to compensate for unmodeled payloads [173]. In another example, a neural network was used to learn a detailed actuator model, often neglected due to its complexity, which led to improved locomotion performance [174].

In contrast, full-model learning involves constructing a complete model of the robot's dynamics directly from data, without relying on an underlying physics-based structure. These approaches can be more expressive and adaptable, particularly in complex or uncertain environments. Adaptive controllers, for instance, can adjust model parameters online in response to changing conditions [175], enabling bipedal robots to recover from unknown disturbances using linear system formulations and adaptive control strategies [176] Based on the behavioral systems theory, a reduced-order model can be learn from data and be used in a model predictive motion planner [177]. However, the classical method requires the model to be formulated in a specific form, e.g., a linear system, which may not fully capture real-world dynamics. To address this limitation, deep neural networks have emerged as a more flexible representation of system dynamics. Such models can be embedded within control frameworks, including model predictive control [178] and end-to-end learned controllers [179], to improve both adaptability and performance. With a learned model representing the dynamics of legged robots, the sample efficiency can be largely improved by testing the action using the learned model. This methodology enables a quadruped robot to walk in one hour of real-world experiments [180].

Control approach	Used models	Computation
Reinforcement learning $r(s,a)$	Physical simulator, World model, Real world, …	During **offline** training, training a policy requires heavy simulation.
Pros: High performance and adaption to diverse terrain	**Cons:** Lack of performance and safety guarantee. Not explainable to human	
Optimal control $\min_u L(x,u)$ $s.t.\ g(x,u)=0$	ROM (w/ residual model), FOM (w/ residual model), Data-driven template, World model, …	During **run time**, accurately predicting system behavior requires computation power.
Pros: Can respect constraints and ensure safety.	**Cons:** Relies on model accuracy.	
Classical feedback control $u=u(y,y^*)$	ROM, FOM, Data-driven template, …	Relative low computation requirement compared with other two approaches
Pros: Computation efficiency. Rigorous guarantee.	**Cons:** Limited performance and adaptation caused by model accuracy.	

Figure 6: Comparison between the three different control approaches.

D. Limitation, unsolved problem, and challenges

1. How to explicitly model the terrain dynamics accurately and efficiently?
2. Uncertainty during:
    - continuous flow
    - reset map
    - switching condition
    - terrain dynamics
3. High fidelity and efficient simulation environment to capture challenging terrain dynamics.
    - To simulate the sand behavior in IssacGym, instead of really simulating the sand behavior, randomization on friction, terrain roughness, and robot mass are added to simulate the terrain deformation, sliding, or even triggering an avalanche. [181]

Jerry:

E. Limitations, Unsolved Problems, and Challenges of Classical Methods

While physics-based analytical modeling provides rigorous, interpretable, and principled approaches to legged robot locomotion, several fundamental limitations and challenges remain unsolved or inadequately addressed:

Jerry:

F. Imitation learning:

G. Future directions

outlook

talk about relaxing the assumptions of ROMs (height, inertia), stochastic modeling, selection of RoM variables, and more about terrain modeling... A **variable-height** inverted pendulum model is proposed to enable robots to walk over stairs [149]. The **stochastic system** modeling [182] can be applied to explicitly consider the distribution of the uncertainty. Given the stochastic modeling, the robot can maintain its performance even when contact location and timing are uncertain [183]. It has been argued that the **angular momentum** about the contact point could be a better choice [145]. Along with this direction, [184] design a way to automatically synthesize a reduced-order model that is optimal in terms of a user-defined task.

IV. Control Strategies for Robust and Agile Locomotion (5-6 pages)

A. Model-based Feedback Control

(1) Stability criteria for locomotion: Stability is a fundamental performance metric in legged locomotion, broadly referring to a robot's ability to sustain planned movements without falling. To translate this concept into conditions that directly inform controller design, a range of formal stability criteria have been developed.

Ground-referenced criteria are among the most widely used due to their intuitive geometric interpretations. These include the zero moment point (ZMP) [185], divergent component of motion (DCM) [149], virtual repellent point (VRP), and Capture Point. Such criteria are typically applied to simplified analytical models and have proven effective for motion planning and control.

Alternatively, stability can be framed as the formal stability of the full nonlinear dynamics governing locomotion [139]. In this view, stability is achieved when a controller can drive the robot's state toward a desired trajectory—corresponding to sustained walking—while ensuring convergence guarantees grounded in nonlinear control theory.

Based on these criteria, various model-based feedback control methods have been introduced for legged locomotion.

(2) Feedback control based on linearized, continuous-time robot models: These represent some of the earliest and most straightforward control strategies for legged locomotion. The linear inverted pendulum (LIP) model is a linear, time-invariant system described by differential equations. A canonical example is the classical ZMP approach [186], which generates desired center-of-mass (CoM) trajectories by enforcing ground contact constraints (e.g., the ZMP criterion) within the LIP framework, and uses inverse kinematics to execute joint-level tracking. While this approach is computationally efficient and easy to implement, its strong assumptions, such as constant CoM height, limit its applicability in terms of locomotion speed and robustness.

(3) Feedback control based on nonlinear, continuous-time robot model: These methods explicitly capture nonlinear behaviors that are neglected in LIP-based models but naturally arise during agile locomotion in complex environments. Examples include variable CoM height during stair climbing [149], significant centroidal momentum during disturbance recovery, and compliant leg dynamics that improve energy efficiency. By incorporating such nonlinearities, these controllers enable more robust and agile behaviors than their linear counterparts. Notable examples include control strategies based on centroidal dynamics and the spring-loaded inverted pendulum (SLIP) model. In addition, bio-inspired approaches, such as clock-driven hip control, have also been proposed for dynamic locomotion.

(4) Feedback control based on nonlinear, hybrid robot models: Full-order models that capture the hybrid nature of legged locomotion, which comprise continuous dynamics (e.g., swing phases) and discrete events (e.g., foot impacts), offer one of the most accurate analytical representations of robot behavior. A widely studied framework in this category is hybrid zero dynamics (HZD) [139], which applies input-output linearization to transform the nonlinear continuous-time dynamics into a form that can be handled using linear control theory. Unlike LIP-based approaches, HZD does not rely on restrictive assumptions such as constant CoM height, enabling it to support more dynamic behaviors on bipedal robots. However, a key limitation is the reliance on precise modeling required by input-output linearization [45]. A comprehensive review of HZD and related methods can be found in [64].

(5) Feedback control based on linear, hybrid robot models: These approaches leverage simplified linear hybrid dynamics to enable real-time footstep planning for enhanced robustness [144, 145, 162]. Footstep locations are treated as discrete control actions and are optimized based on linear hybrid system theory, allowing systematic synthesis of feedback controllers that reason over both continuous motions and discrete transitions.

B. Model Predictive Control

(1) Convex MPC: Single Rigid Body Dynamics Control Model Predictive Control (MPC) formulates the control problem as a receding-horizon optimization, where actions are optimized over a specific future time window [187]. In legged robots, a key challenge lies in balancing high-frequency onboard computation with the accuracy of the dynamic model. To reduce computational demands, many researchers adopt a simplified Single Rigid Body Model (SRBM), assuming the legs are massless and consolidating the mass in the base link [143, 188]. This simplification works well for robots with light limbs, such as certain quadrupedal robots, where it can even reduce the problem to a linear one. For robots with heavier limbs, like humanoid robots, the SRBM can still be applied under the assumption of quasi-static motion. However, this assumption breaks down for dynamic, agile, whole-body locomotion.

MPC typically optimizes the contact forces during locomotion, while joint torques are handled by the Whole-Body Controller (WBC).

(2) Nonlinear MPC: Full-Model Dynamics Control In such cases, a Multi-Rigid Body Model (MRBM) is required, which introduces significant nonlinearity to the MPC problem, making real-time computation with onboard processors highly challenging [189–191].

Solving non-convex optimal control problems in real time still presents significant challenges, due to the complex dynamics and constraints in the optimization problems. Many different methods have been proposed to exploit the structure within these problems to enhance their solvability. Examples include local convexification methods (e.g., Sequential Quadratic Programming (SQP, [192])), dynamic programming methods (e.g., Differential Dynamic Programming (DDP, [193]) and iterative Linear Quadratic Regulator (iLQR, [194])), and sampling-based methods (e.g., Model Predictive Path Integral (MPPI, [195]) and variants [196]).

(3) Limitations: Two key challenges limit the performance of MPC on actual robots: model accuracy and computational cost. First, while MPC typically uses an averaged dynamic model, its performance is constrained by the accuracy of that model. For legged robots, the simplified Single Rigid Body Model cannot fully capture the system's dynamics, especially when limbs exhibit agile movements that deviate significantly from the nominal conditions. Although the Multi-Rigid Body Model addresses some of these limitations, errors in system identification can still lead to large mismatches between the predicted and actual robot performance. Additionally, environmental factors, such as varying friction coefficients or deformable terrains, introduce further uncertainty during locomotion. Second, legged robots, especially humanoid robots, are highly nonlinear systems with frequent contact switches and multiple constraints. Given limited computational resources and the need for real-time processing, it remains challenging to solve large-scale, complex optimizations on the fly. This computational bottleneck hampers MPC's potential in highly dynamic and agile tasks.

another limitation of MPC: require explicit state estimation.

C. Teaching Legged Robots: Reinforcement Learning for Enhanced Autonomy

A promising methodology to develop motion controllers for legged robots is reinforcement learning. However, for this kind of approach, an accurate model of the robot and its components is needed to obtain a feasible controller. Commonly, only the dynamics and kinematics of the robot are considered and the dynamics of the actuators are neglected. Nevertheless, neglecting such elements can lead to unfeasible performance when the learned controller is deployed in the real robot. As discussed in [197], during highly dynamic maneuvers, effects such as saturation and friction are present. Such effects can lead to important divergences on the commanded torque and the actual position signals. Therefore, accurate modeling the actuator dynamics and interactions is a key point when addressing learning and optimization-based approaches for control. However, the state-of-the-art analytical models of series elastic actuators are not yet accurate enough. To overcome this problem, in [174], a practical learning methodology was developed and experimentally validated. This methodology allows autonomously learning and transferring agile and dynamic motor skills for legged systems. Specifically, this approach overcomes the issue of accurate modeling series elastic actuators by employing a deep neural network to learn the complex actuator/software dynamics of the series elastic actuator on the ANYmal quadrupedal robot, resulting in an accurate mapping from commanded actions to output torques. In addition, the learning framework is complemented with an accurate model of the quadrupedal robot obtained by system identification, thus obtaining a realistic virtual representation of the whole robot (dynamics, kinematics, and actuation). Then, this accurate virtual model is employed to train a reinforcement learning-based controller that is more precise and energy-efficient and allows it to perform highly dynamic and complex motions such as recovering from a fall. Besides, the learned controller was directly deployed on the real robot without the need for tunning or parameter adjustment, which resulted in a successful learning in simulation approach by reducing the sim-to-real gap. Even though this approach showed outstanding results, the authors discussed the following drawbacks: Human intervention is still required: A cost function and an initial state distribution have to be defined and tuned for each task. This is time-consuming even for an individual with expertise on the topic; Handcrafted policies: Developing policies for complex tasks is difficult due to the lack of intuition to embed safety elements such as high impacts, leg swing velocities, and collision with fragile components in a cost function; Lack of transferability: To deploy the controller in a new robot, important modeling and identification efforts must be made. An accurate model of the rigid-body dynamics and the actuators must be obtained and validated, implying a significant amount of time investment; Actuator analytical model complexity: Actuators with coupled dynamics require understanding their behavior for accurate modeling, which can be time-consuming and parameter-intensive. Constructing an analytical model for ANYmal is particularly complex and demands extensive tuning. A single neural network may not generalize across tasks, but a hierarchical policy network could address this limitation.

Despite the advances in control theory and practice, legged locomotion still represents an important challenge nowadays. Such a task involves the integration and coordination of several elements that interact not only among themselves but also with the environment. Therefore, robust and accurate control schemes must be employed to allow legged robots to perform in real environments. Traditionally, the locomotion of legged robots has been addressed using analytical approaches based on trajectory planning. Such an approach requires defining trajectories for all the joints that together allow the displacement of the legged system. These approaches also require a model of the robot dynamics and kinematics together with an accurate knowledge of the system's parameters. However, employing optimization techniques to obtain suitable trajectories has proven to be an affordable option. Such is the case of [198], where trajectory optimization was integrated with reinforcement learning to control a quadrupedal robot (ANYmal).

D. Learning-based Control

This session is a bit unstructured. Maybe consider this outline:

- Sim2real RL from scratch: teacher-student, end2end
  - Introduce the overall framework: 1. Parallel simulation (NVIDIA Isaac, Genesis); 2. RL algorithms; 3. DR, etc (connect it to robust control). 4; Transfer to real world
  - "Classic" before teacher-student
  - Teacher-student and variants. Key ideas: privileged. End2end (Berkeley humanoid transformer, https://arxiv.org/abs/2401.16889)...
- RL with human or animal motion priors: AMP, H2O, etc
  - Learning from scratch problem: reward function, non-natural gait, hard to explore
  - Leveraging priors. Quad:
  - Bipedal humanoids: AMP, H2O, human mimic.
  - Warm-up the RL using model-based methods.
- Structured and hybrid RL methods: Hybrid RL and MPC
  -
- Model-based RL (learning a model first)

(1) Sim-to-Real Reinforcement Learning: Training from Scratch Directly train from scratch: Pioneer [199]. Neural network starts to work on small-size legged robots [200–202]. With the advance of simulation, very efficient CPU-based paralleled simulation RaiSim [203] enable fast and accurate neural-network policy on ANYmal robot [174]. MuJoCo [170] also enables successful sim-to-real deployment on life-size bipedal robot, like Cassie [204, 205]. IsaacGym [206] leaverages GPU-friendly pipeline to further speed up the training and the control policy can be learned within several minutes [207].

Directly learning from scratch becomes difficult to train or converge to an optimal solution given more complex environments due to the limited observation from the robot onboard sensor. Therefore, the new pipeline called Teacher-Student structure is developed to effectively leverage the privileged information during training in the simulation [208]. Teacher can access information such as friction coefficient which is not possible for real robot to observe. The trained policy will distrill to the student policy with limited observation but with advanced network architecture, such as memory modules. This approach could help robot dog successfully cross challenging terrain [208], leveraging perceptive information [209], and even helping win the DARPA Subterranean Challenge [43].

(2) RL with Motion Priors: One common problem of applying reinforcement learning to legged robots is to design reward functions or task specifications for motion naturalness. For example, training a natural walking policy for humanoids requires non-trivial reward design. To effectively extract reference signals from data to facilitate reinforcement learning, people have developed various approaches to learn from human or animal motion priors.

In the field of computer graphics and animation, there are various publicly available human motion datasets collected in motion capture systems, such as AMASS [210]. Given a human (or animal) motion trajectory, different methods have been proposed to translate it to reward functions to facilitate reinforcement learning in simulation. Representative methods include adversarial-learning-based approaches such as Generative Adversarial Imitation Learning (GAIL, [211]) and Adversarial Motion Prior (AMP, [212]), explicit trajectory tracking methods such as Human2Humanoid (H2O, [213]) and HumanPlus [214], and phase-based tracking methods such as DeepMimic [82, 215].

(3) Structured and Hybrid RL and Control: Compared to classic model-based control methods, model-free reinforcement learning approaches, in general, are less data efficient, less interpretable, and lack safety guarantees. To get best of both worlds, people have developed hybrid approaches that synergize model-free RL and model-based control. One common idea is to combine optimal control and RL in a hierarchical way, such as using MPC to generate reference motions for a low-level RL tracking controller [198] or using RL to generate centroidal motion reference for the low-level optimal control layer [216]. Another type of hybrid method is to combine model-based RL with control-theoretic safety filters for safety guarantees [217].

Given that training embodied systems to produce intelligent behavior spanning multiple scales remains difficult. To address the challenges of behavior specification at different scales, credit assignment, and exploration, in [218] a combination of multi-agent RL with pretrained behavior representations at different scales, which they incorporated into the full learning problem in a flexible and adaptive manner. The method makes use of prior knowledge from imitation where available, and the autocurriculum that emerges from self-play in populations of learning agents allows the discovery of complex and robust solutions that would be difficult to specify through reward or learning from imitation. Reward specification, exploration, and credit assignment over long decision horizons and in the presence of multiple agents all pose challenges for end-to-end learning.

(4). Limitations (todo) TRPO, PPO? Adversarial, black box

E. Evaluation Metrics (0.5 page)

Maybe talk about computational costs?

Despite significant advancements in legged robot control and locomotion, a unified, standardized metric for performance assessment remains absent. Key performance indicators, including cost of transport, disturbance robustness, traversability, agility, and versatility, can serve as a performance evaluation criterion.

Energy Efficiency: Based on the analysis of animals' leg locomotion, the Cost of Transport (CoT) emerged as a concept to quantify the energy expenditure of locomotion when traversing from one point to another [219, 220]. Specifically, it quantifies energy consumption relative to distance traveled and body mass, typically expressed as energy per unit distance (joules/meter). The CoT enables meaningful comparisons of locomotion efficiency across different scales and systems. Energy efficiency also depends on hardware design: high-torque-density motors, low-loss transmissions, and lightweight limbs reduce energy waste [4, 221–223]. Computational methods, such as reinforcement learning and motion planners, further optimize energy use by balancing actuator effort, terrain interaction, and stability [224, 225]. However, computational cost is a significant component of energy consumption in legged robots, driven by perception, planning, and control processes. Unlike the Cost of Transport (CoT), which provides a general benchmark by integrating all energy sources, computational cost can be specifically measured by monitoring CPU/GPU power draw during locomotion, separate from actuators using embedded sensors or test benches. The Computing Energy Included Motion Planning (CEIMP) framework, developed for mobile robots, demonstrated that optimizing trajectories by considering both actuation and computational energy can achieve savings of 2.1 to 8.9 times total energy, depending on environmental complexity [226].

Disturbance Robustness: It is a well-known feature in control systems that refers to the capability of a system to maintain desired performance and stability despite the effects of certain external inputs and internal uncertainties. However, the absence of a standardized quantitative measure for robustness in legged robots hinders fair comparison of locomotion algorithms and slows commercialization efforts [227]. Capturability refers to the ability of a robot to regain balance within a finite number of steps, it remains as s common metric for assessing robustness [228–232]. However, its reliance on simplified models and assumptions limits its effectiveness as a robustness standard in real-world scenarios. Recent research has shifted towards more practical evaluation methods, including repeatable disturbance rejection tests using linear impactors [227], reinforcement learning approaches with adversarial attacks to uncover controller weaknesses [233], and algorithms for comparing the adversarial nature of different tests [234].

Traversability: Evaluates a robot's ability to navigate terrain by planning optimal trajectories based on its physical capabilities and environmental challenges [235]. Recent advancements [235–237] integrate multimodal sensing (exteroception, interoception, proprioception) to enhance terrain analysis, improving locomotion success rates in unstructured environments.

Agility: Refers to a robot's ability to adapt to dynamic and unpredictable environments with ease, a key requirement for real-world applications. Recent studies [198, 238, 239] highlight agility's importance for effective locomotion beyond static settings. Notably, research on robotic parkour [240–242] demonstrates advanced agility, balance, and smoothness in challenging environments.

Versatility: Describes a locomotion framework's ability to handle diverse tasks-such as varying speeds, jumping, and navigating stairs or slopes-without specialized algorithms for each. Recent research [205, 243–245] has enhanced versatility in legged robots, enabling quadrupeds and bipeds to adapt to real-world scenarios using various gaits and skills, including skipping, jumping, and even skateboarding [246]. Loco-manipulation comprises the simultaneous coordination of locomotion and manipulation, enabling robots to perform complex tasks such as carrying objects while walking or manipulating tools in confined spaces, all while maintaining balance [12, 247–249]. As a benchmark, it effectively assesses versatility by testing a robot's ability to integrate perception, planning, and control across diverse tasks, objects, and environments.

V. Emerging Advancement about LLM, VLM, etc (1 page)

Recent progress in Foundation Models (FMs) such as Large Language Models (LLMs) and Vision-Language Models (VLMs) has demonstrated groundbreaking capabilities in various domains such as video understanding, image/code generation, and web agent. Integrating FMs with legged robots for locomotion tasks has been trendy. According to the type of the interface between the FM and the robot, there are various paradigms:

- FMs to generate training environments: Language-to-reward [250], Eureka, DrEureka.
- FMs to generate high-level references such as way points:
- FMs for direct control: directly use FMs to generate actions or policies. Example: "Prompt a Robot to Walk with Large Language Models"

Beyond locomotion tasks, FMs have also been widely deployed in loco-manipulation tasks... **My concern is that this section will be very similar to the Section VIII of the humanoid loco-mani survey paper.**

VI. Conclusions (0.25 pages)

VII. References (1.5 pages)

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I. Abstract

Legged robots have the potential to unlock new levels of mobility, stability, and versatility in robotic systems, offering potential applications in industrial facilities, healthcare settings, home assistance, and precision agriculture, as well as in subterranean, underwater, polar, and extraterrestrial environments. Despite the recent advancements in legged robot locomotion, there are still challenges to overcome to achieve full autonomy and adaptive terrain locomotion. This paper provides a comprehensive overview of what has been achieved in advanced modern quadrupedal and bipedal legged robot systems in the fields of modeling and control, including physics-based and data-driven approaches, and examines its future outlook and the role of learning-based methodologies on such fields. The modeling approaches for legged robots are surveyed pointing out their advantages and limitations along with recent techniques considering contact models and environment interaction and their current challenges. The classic and learning-based control techniques for legged locomotion are briefly described, and their relation is discussed to identify how to merge both approaches toward pushing forward the current bounds of agility and versatility of legged robots. This paper provides an overview of achievements to date, identifies existing research gaps, allows further understanding of control and learning-based strategies for legged robot locomotion, outlines potential future opportunities in the field, and promising applications.

Another version: Dynamic legged robots have the potential to unlock new levels of mobility and versatility in diverse environments, offering potential applications in industry, healthcare, home assistance, and extreme environments like polar and space. Despite the recent advancements in legged robot locomotion using either classical or learning-based methods, there are still challenges to overcome to achieve full autonomy and adaptive locomotion. This paper provides a comprehensive and unified overview of what has been achieved in modern quadrupedal and bipedal legged robot systems in terms of modeling and control using either classical and learning-based approaches. The classical and learning-based modeling methods for legged robots on a regular or irregular terrain are surveyed, pointing out their advantages, limitations, and the remained challenge of modeling the interaction with a complex environment. The classical and learning-based locomotion controller designs are reviewed, and their shared and unique features are discussed to identify the way to merge both approaches toward pushing the current boundary of the agility and versatility of legged robots. The application of the emerging large language model (LLM) and visual language model (VLM) on legged locomotion is also discussed. With the unified perspective over classical and learning-based approaches, this paper provides an overview of achievements to date, identifies existing research gaps, allows further understanding of both approaches, and outlines the promising opportunities and applications in the field.

II. WHY LEGGED ROBOTS?

A. Why legged robot? (original version; written by I-Chia)

Legged robots have huge potential to be applied to industrial, extreme environment, and everyday life to improve the well-being of human beings. Nowadays, the development of legged robots is faster than it has ever been. See Figure 1 for fast development of new legged robots. Legged robots have unique advantages compared to wheeled or tracked robots in traversing various types of terrains, including ladders [1], sand [2] , underwater [3] , and even more. See Figure 2 for more insights. Compared to aerial robots, legged robots provide better energy efficiency [4] and load carrying capability [5, 6]. To work close to humans, legged robots have the same morphology as humans or dogs. This allows them to naturally fit the environment designed for humans [7] and also provide familiarisation to human-users [8]. In addition to their empirical usage, the studies of legged robots also push the advancement of control techniques [?] and the understanding of the bio-systems of legged animals [11].

The species' evolution demonstrates that legs represent optimal anatomical structures for terrestrial locomotion across varied landscapes [9]. This adaptation is evidenced by bipedal humans and diverse animal species such as quadrupeds [10], arthropods, and even volant organisms like birds, all of which rely on legs for ground-based mobility. Legged robots offer several advantages over other ground mobile robots, such as wheeled and tracked, especially when traversing rough and unstructured terrains [251]. In addition, they offer a wider variety of locomotion tasks, such as jumping, crawling, and climbing, which allow them to overcome obstacles that the other ground mobile robots cannot.

Therefore, the academic and industrial community are recently making important efforts in the development of such robotic systems. Such efforts have produced outstanding results during the last five years, and the progress is advancing on significant steps, opening the door to new challenges and opportunities to contribute. However, a fast rate of progress could make it difficult for new people who want to contribute to the field to get a concrete and objective idea of the current status of many of the elements comprising the development of functional-legged robots. The same could be true for those who have been focused on only one aspect of development. Precisely, this paper aims to cover the main elements related to the design and operation of legged robots so it could be helpful for experienced and newcomers on the topic by providing a concise description of the aspects of design, actuation, planning, navigation, perception, and control and discussion of their advantages and drawbacks as well as current challenges, achievements and the future direction on these topics.

III. APPLICATIONS (0.5=>0.2 PAGE; WRITTEN BY IVAN)

The advancement of quadrupedal and bipedal robotics is redefining operational paradigms across industries, offering solutions to complex challenges while prioritizing human safety and environmental sustainability. In industrial settings, quadrupedal robots like ANYmal, equipped with an articulated robotic arm, showed coordinated locomotion and manipulation, enabling tasks such as valve operation in hazardous zones [12]. Bipedal robots such as Agility Robotics' Digit has been tested in Amazon warehouses for tote recycling: a highly repetitive process of picking up and moving empty totes once inventory has been completely picked out [31]. Another bipedal robot, the E2-DR demonstrated impressive navigation capabilities by scaling vertical ladders and traversing debris during high-risk industrial inspections [1]. Specialized platforms like the MARVEL quadruped excel in inspecting storage tanks through magnetic adhesion [13], while bipedal crawlers perform multimodal pipe inspections, mitigating risks to human workers [32].

Beyond industrial applications, legged robots are emerging as transformative tools for human assistance. In healthcare, quadrupedal robots like Unitree's Aliengo [33] can autonomously patrol intensive care units, monitoring environmental parameters to improve patients' recovery [34]. For visually impaired individuals, quadrupedal robots like Boston Dynamics' Spot can offer predictable guidance and navigation through urban spaces, providing real-time obstacle alerts that circumvent service animals' training requirements and behavioral variability [7]. Recently, house service and companion bipedal robots like 1X's Neo Gamma [21] or Tesla's Optimus-Gen-2 [28] have been introduced in the market, pushing forward the current boundaries on humanoids.

Modern precision agriculture benefits from quadrupedal robots allowing monitoring of crop health with high accuracy while minimizing soil compaction and allowing traversing deformable surfaces like muddy or grassy fields and gravel paths, a critical advantage for difficult-terrain farming [2, 35]. On behalf of the Vinum project [36] the capabilities of the HyQReal quadrupedal robot have been tested for traversing a vineyard for winter grapevine pruning [37, 38].

Legged robots demonstrated proficiency in navigating subterranean [39–43], marine [3, 47–50], and polar environments. In subterranean operations, quadrupedal robots like the ANYmal C100 dominated DARPA's SubT Challenge, mapping mine networks with superior agility [42, 43, 51]. Underwater, quadrupedal designs such as SILVER or RoboIguana enable benthic exploration through dynamic gaits, avoiding propeller-induced ecosystem disruption [47, 53]. Polar-optimized quadrupeds equipped with slip-detection algorithms enable gathering climate data on glaciers [16], while innovative conceptual designs allow skiing beneath ice sheets for extended polar monitoring [54]. On behalf of the 2018 Winter Olympics in Pyeongchang, South Korea, the world's first Ski Robot Challenge was held [55]. The competition required only humanoid robots with at least 15 DoF and a minimum height of 0.5 meters to complete an 80 meters long slalom course passing through six gates [56–58].

For space exploration, teams of specialized ANYmal robots: scouts, scientists, and hybrids, collaborate to navigate Martian terrain [19], while emerging research focused on low-gravity legged jumping locomotion could redefine celestial bodies' navigation allowing access to regions of remarkable scientific value [59–62]. A novel and under-development space application proposes the use of quadrupedal robots proprioception to measure the geotechnical properties of crusted and icy surface regolith and utilize those measurements to autonomously update the science operations plans [63].

While technical and technological challenges remain, the future of legged robotics is unequivocal. These systems will amplify human exploration capabilities by operating in environments where biological limitations prevail and will play an important role in advancing human assistance and increasing industrial safety. NOTE: Add nature paper and TRO Huang, Yu tmech to mention that legged robot also informed control design in exoskeleton.

IV. Overview of This Survey

1) Scope/focus This manuscript is devoted to a broad audience, aiming introduce the readers into the legged robots field by providing a comprehensive and organized description and discussion on the modeling and control of quadrupedal and bipedal robots and how the learning-based techniques accelerated the development of robust and versatile locomotion controllers. Important advances have been achieved in the field, and its progress is moving at a high rate. Due to this, it could be difficult for those who want to explore by themselves the legged robots field to figure out where to start. Hence, this manuscript addresses the three key factors that have enabled quadrupedal and bipedal robots to attain their current level of performance: modeling, control, and learning. Besides, the advantages and drawbacks of current strategies on each key factor are discussed. In addition, for the most experienced in the field, this survey highlighted the future directions of the aforementioned points regarding their current challenges. Finally, we want to provide the reader with tools such as software, simulation, and training environments developed by different research groups that certainly will help to gain technical experience in controlling such robots. Hopely, this will help increase the number of active research members in this field and attract the attention of some others to address the current challenges, aiming to keep progressing at a high rate on the development of agile, efficient, robust, and affordable legged robot systems. It is worth mentioning that the discussion on the design and control of any mechanism for manipulation purposes that bipedal and quadrupedal robots can admit is out of the scope of this paper since this work is only devoted to those mechanisms allowing these robots to displace over-ground environments.

2) Modeling: physics-based and data-driven modeling. To develop a locomotion controller, a model that predicts how the system will evolve is needed so that the controller can plan and react accordingly. In the modeling section, this paper reviewed the newest physics-based analytical models that capture the physics behind legged locomotion on diverse terrain. Combining physics and modern computation tools, the physical simulators can be treated as a model of the world and accelerate the development of legged locomotion controller. With the emergence and the exciting performance of data-driven approaches, the way of using data to model the system is also presented. Lastly, we discuss the different and common features among these modeling approaches and outline the potential direction in the field of modeling to push the limit of legged locomotion on diverse terrain.

V. Modeling of Legged Robots and Contact Interaction with Environment (old) (4 pages)

Intro to modeling Before designing a controller for a legged robot, we need to understand and describe how the system behaves so we can plan accordingly. This motivates the researchers to find 'good models' of legged robot systems and use them as a foundation to design their controllers. A good model means that the model can consistently (you can trust the model) and efficiently (not using too much computation power and time) provide a sufficiently accurate prediction of how the system will evolve in the future. This is a challenging problem since legged robot systems involve complex dynamics of mechanisms, actuators, and interactions with the external environment and objects. In the past few decades, the legged robot community has been working on identifying the key physical variables to be modeled, developing models with different complexities, and exploring new mathematical and/or computational tools to represent the system.

Different control approaches require different models. In this section, we will start with the physics-based analytical models, which are crucial for the traditional model-based controller design. Physics-based analytical models express the legged robot using analytical mathematical expressions, e.g., differential equations and/or difference equations. This allows rigorous discussion about the behavior of the model, especially with the designed controller, but has limited scalability when dealing with a complex environment. Then, we talked about the learning-based model, which learns to predict the system behavior using collected data either from the real world or a simulator. Learning-based models usually express the system using a numerical expression, for example, a neural network, with numerous adjustable 'knobs' that can be tuned by training data so the network can represent the system well even for a complex scenario. Learning-based model could be used by a traditional model-based controller or a model-based reinforcement learning algorithm. Finally, we talk about the physical simulator, which is the foundation of the model-free reinforcement learning.

A. Physics-based Analytical Modeling of Legged Locomotion

Physics-based analytical models are the models that are constructed using the physics law, e.g., Newton's law. Even though there exists a methodology to build the full-order model of a robot with rigid parts [139], it is hard to use it as a model to develop an online controller and planner for long-term motion planning or reactive behavior since it is not computationally efficient. Therefore, the hierarchical modeling/control strategy is widely used to resolve this issue for both bipeds [145] and quadrupeds [252]. Several models with different levels of complexity could be used to represent a single robot's dynamics in different aspects. Those models with lower dimensions than the full-order model are called the "reduced-order model" (ROM) and can only represent limited (but is means to be crucial) aspects of the system's behavior [141]. Then, the hierarchical strategy is to use ROMs to develop efficient controllers that generate intermediate motion plan, e.g., step length [144], and the whole-body controller (WBC) [140] will determine the motor torque by considering the full-order model to realize the intermediate motion plan.

1) Latest Advancement in legged robot modeling: One of the recent advancements in legged robot modeling lies in the relaxation of assumptions for ROMs. This implies the restriction of ROMs is reduced and the new ROMs can consider more complex movement. Linear inverted pendulum model (LIPM) [142] is a well-known ROM representing the dynamics of a legged robot by assuming the following hypotheses [155]: (H1) All mass is concentrated in the robot's center of mass (COM). (H2) The change rate of angular momentum of the robot is zero. (H3) The contact points lie on the same plane. (H4) The COM of the robot moves along a plane perpendicular to one of the contact points. (H5) The contact points are fixed in the world frame during one step (stationary ground and sufficient high friction). These hypotheses make the LIPM efficient and reasonably accurate to enable robot walking but they also make it restrictive to complex and agile motion. This motivates researchers to develop new ROMs to relax those hypotheses while ensuring efficiency. Regarding relaxation of (H1) and (H2), [143] assumes the robot is a single rigid body (SRB) with fixed rotational inertia and has a small roll and pitch angle. To enable a more agile movement, [253–255] uses the idea of variable inertia modeling to capture the inertia change caused by limb movement. To relax (H3), a point mass model is used to reason about the stability criterion with non-coplanar contacts [256]. Based on LIPM and aiming to relax (H4), a variable-height inverted pendulum model is proposed to enable robots to walk over stairs [149]. In addition to COM, the divergent component of motion (DCM) is used to characterize the stability criterion and enable walking on rough terrains and with COM height variation [150]. When a robot is walking on a moving platform, e.g., a bus, boat, or airplane, hypothesis (H5) breaks. The time-varying versions of LIP-like models are developed to explicitly consider the effect of surface movement [160, 163]. To conclude, in the past decades, the field of legged robot modeling has been intensively developing new ROMs that can handle challenging movements by relaxing assumptions.

Continuous system model to hybrid system model In addition to the hypothesis (H1)-(H5) mentioned above, there are other physical phenomena that the previous models cannot describe. One aspect comes from the mechanism of legged locomotion, i.e., moving forward by making and breaking contacts. Making and breaking contact is a discrete event that only happens at a time instance. However, during this discrete event, there are discrete dynamics that govern the state jump, control inputs the robot can utilize, and also disturbances that can destabilize the robot. For example, the step length can be treated as a control input [144], and the sudden velocity jump of COM caused by impact can be viewed as a disturbance during the discrete event. To explicitly describe these phenomena and consider the dynamics for both continuous phase and discrete phase, the tools from hybrid dynamical systems, switched systems [230], and impulsive systems [138] can be used. By considering the fixed stepping period, the LIPM can be formulated as a hybrid version [162, 163] or switched system version [230] to study the effect of step length control strategy on walking stability. By using a hybrid system formulation, the multi-domain walking of a bipedal robot that involves heel strike and toe strike can be realized [146]. Maybe talk about saltation matrix. See tutorial in [257]

Uncertainty is always a major challenge for legged locomotion. The previously mentioned ROMs mostly assume the system is deterministic. Although the concept of robust control can be applied to a deterministic system by considering the bound of disturbance or model uncertainty, the resultant control performance might be too conservative. Therefore, the stochastic system modeling [182] can be applied to explicitly consider the distribution of the uncertainty. Given the stochastic modeling, the robot can maintain its performance even when contact location and timing are uncertain [183]. To-Do: find more?

Another advancement of legged robot modeling lies in the selection of physical variables of a reduced-order model. Although the linear position, the state of LIPM, is an intuitive and widely-used state to build a reduced-order model, it has been argued that the angular momentum about the contact point could be a better choice [145]. The authors claim that the angular momentum is a better choice since 1) it better represents the state of the whole robot, 2) it has a higher relative degree with respect to motor torque than linear position, and 3) it is impact invariant under some conditions. This choice of state leads to the angular momentum linear inverted pendulum (ALIP) model. Along with this direction, [184] design a way to automatically synthesize a reduced-order model that is optimal in terms of a user-defined task. In addition to using a single reduced-order model to represent the system, [258–260] use models with different fidelity in the same time to find an optimal balance between computation cost and prediction accuracy.

- LIP
  - Height-variation: 3D-DCM [151] (TBD)
- SRB

  We focus on the SRB-related model (With a focus on how to cooperate with more complex dynamics, kinematics consideration)
  - hierarchical-MPC-based scheme [258] How to best use different models in a MPC horizon [259] Cafe-MPC [260] (quadruped whole-body dynamics + SRBM later in MPC), SRB + foot step [261]
- Centroidal dynamics

consider the rotational effect of upper body: Angular DCM [153] (TBD**).

Task-optimal reduced-order model [184]

1) Robot model to Robot-Environment Contact? model:

Introduction to Robot-Environment model In the previous section, most of the works focus on the model which only captures the legged robots' characteristics. This is because most of the legged robots are operated in a regular and static environment and the dynamic behavior of the environment can be neglected. However, legged robots are designed to be operated in and to interact with complex and dynamic environments, e.g., walking on moving boats [160], opening doors [238], assisting human subjects [262], etc. Since the dynamic effect and constraint caused by moving boats, doors, and human subjects are not negligible, all these skills need knowledge about how the robot and the environment behave to have high performance while ensuring safety. Therefore, standing on the solid foundation of legged robot modeling, how to model the environment together with legged robots becomes a valuable topic and encourages numerous researchers into this field.

General formulation of legged robot on different terrains One of the robotics applications that motivates the study of robot-environment modeling is the locomotion on irregular terrain. To model a legged robot on a regular terrain, one can use the full-order rigid-body model with a simple holonomic constraint. That is, express the robot's dynamics using

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) =& Bu_c - J_t(q)^T F_{t,c}, \\ & (q, q_t, t) \in C, \\ (q^+, \dot{q}^+) =& f_d(t, q^-, \dot{q}^-, u_d, F_{t,d}), \\ & (q, q_t, t) \in D, \end{aligned} \tag{1}$$

where q and $\dot{q}$ are the robot's generalized coordinate and its time derivative. $M(q), C(q,\dot{q})\dot{q}$, and $G(q)$ are the inertia matrix, the sum of the Coriolis and centrifugal terms, and the gravitational term, respectively. u_c and B are the control input vector during the continuous phase and the input matrix. $J_t(q)$ is the contact Jacobian between the robot and the terrain while $F_{t,c}$ is the interaction force during the continuous phase. The robot system follows the continuous dynamics when the time and the state of the robot and terrain are in the set C. The superscript $^+$ and $^-$ indicates the state right after and before the time instance. The function $f_d(\bullet)$ is the discrete state transition function of time t^-, pre-impact state $q^-, \dot{q}^-$, discrete control input u_d, and terrain interaction force $F_{t,d}$ to model the sudden jump of the state while the time and the state of the robot and terrain are in the set D. Based on the assumptions of regular terrain, i.e., horizontally flat, rigid (no deformation), static, sufficient friction, and respecting the bilateral contact constraint, one can apply a holonomic constraint

$$J_t(q)\ddot{q} + \dot{J}_t(q,\dot{q})\dot{q} = 0 \tag{2}$$

to solve for the contact force $F_{t,c}$ and $F_{t,d}$. This formulation (regular terrain formulation on full-order model) is used for a passivity-based whole-body controller [145] and an inverse dynamics whole-body controller [263]. The regular terrain assumptions could also be found in the reduced-order model [142–144].

we want to show math, but the key is to let broader readers understand the state of the art, what is solved, what is not solved yet. In contrast to the locomotion on regular terrain, the modeling of legged robots on irregular terrain becomes more complicated since it involves the dynamics of the terrain(environment). this sounds a research paper, not perspective survey paper Without loss of generality, we assume the dynamics of the terrain can be modeled using a hybrid dynamical system formulation and the state of the terrain(environment) can be approximated by a finite-dimensional generalized coordinate q_t. The continuous part is assumed to be modeled using a second-order differential equation $f_{t,c}$ while the discrete part is modeled using a nonlinear function $f_{t,d}$. The terrain(environment) dynamics can be expressed as

$$\begin{aligned} \ddot{q}_t &= f_{t,c}(t, q_t, \dot{q}_t, F_{t,c}), \qquad (q, q_t, t) \in C \\ (q_t^+, \dot{q}_t^+) &= f_{t,d}(t, q_t^-, \dot{q}_t^-, F_{t,d}), \quad (q, q_t, t) \in D \end{aligned} \tag{3}$$

Comparing equation (3) with equation (2), it is obvious that it is harder to solve for the interaction force $F_{t,c}$ and $F_{t,d}$ with equation (3). This imposes additional difficulty and complexity on modeling the force applied on legged robots and results in a more challenging locomotion control problem. For example, one small modification on equation (2) by assuming non-zero contact acceleration, i.e., $J_t(q)\ddot{q} + \dot{J}_t(q, \dot{q})\dot{q} = a(t)$ where $a(t)$ is the time-varying contact surface's acceleration, turns the time-invariant system into a more complicated time-varying system [160]. While the dynamics of general terrain remains an open question, the interaction between legged robots and rigid objects, like doors or tables, can be modeled using a multi-robot formulation [140, 164].

In addition to the full-order modeling of the robot-environment system, there is an emerging trend on the development of reduced-order models for legged robots on irregular terrains. Despite the numerous ways to characterize and categorize the irregular terrain, in the following, we focus on the three major aspects, i.e., geometry, macro movement, and terrain mechanics, and review the development of reduced-order models on these types of terrains.

Irregular terrain: Geometry The geometry of a terrain could have diverse variations other than just horizontally flat, which is the assumption of a regular terrain. For example, the geometry of terrain could be discrete, having height variation, having a varying surface normal, having confirmed space in horizontal and/or vertical direction, and even have vine-like obstacle [159]. To handle the variation of the terrain geometry, different reduced-order models or constraints for the locomotion controllers are designed.

Discrete terrain means the available contact surfaces are not continuously connected together, such as stepping stones. This terrain geometry imposes constraints when searching for the foot placement, and this is challenging since the locomotion becomes aperiodic and highly dynamic [147, 148]. Climbing ladders and stairs are the two obvious examples for discrete terrains, while they also need to handle the height variation. To allow the robot to climb ladders or stairs, the CoM height needs to vary, which breaks the assumption of LIPM [142]. This leads to the variable height inverted pendulum [149], 3D-DCM model [150, 151], angular momentum linear inverted pendulum model [152], and their successors [153]. Imagine walking on bumpy roads, the surface normal of the terrain is no longer horizontally flat and could vary place-by-place. This imposes additional complexity on the direction of friction cone and the constraint of ground reaction force [154]. A centroidal model is used to model the robot's motion under multi-contact with different surface normal [155, 156]. To traverse within a confined space, the robot needs to respond safely to the height [157] or lateral constraints [158]. A vertically actuated spring-loaded inverted pendulum (vSLIP) model was proposed to model the underactuated robot to walk in an unknown height-constrained environment [264]. Notably, a shared fracture of the locomotion control within confined space is the presence of a point cloud map or height map of the environments. This map is independent of the robot and represents a static characteristic of the environment and is essential to specify the constraints of robot height [264] or the foot placement [148, 158]. However, there is no a map that considers the macro movement (dynamic characteristic) of the terrain yet.

Irregular terrain: Macro movement The macro movement of a terrain describes the motion of the terrain as a whole with respect to the inertia frame. In this paper, we categorize the terrains' macro movement by two questions: 1) Can it accelerate relative to the inertia frame? 2) Can the motion of the terrain be affected by the motion of the robot? A regular terrain is static, which means it does not accelerate relative to the inertia frame and this static character will not be affected by the interaction between the robot and the environment. Large-inertia environments, like the vessel and the trains, could be modeled as an environment that can have acceleration, but the terrain's motion will not be affected by the robot since the terrain has a significant amount of mass. Small-inertia environments, like small boats and a flexible cantilever beam, can be modeled as an environment that can accelerate and its motion can be affected by the robot. Locomotion on a terrain with non-trivial macro movement brings challenging since the dynamic motion of the terrain has to be considered.

To enable legged robot walking on a moving platform with large inertia, the (reduced-order) model of the robot needs to consider the dynamic effect caused by the moving platform. By assuming the platform has sufficient large inertia and using a pendulum-like model, the surface motion will cause a time-dependent non-homogeneous disturbance [160, 161] and/or a time-dependent system parameter variation [162, 163] according to the direction of the platform's motion. Based on the accessibility to the surface's motion information, the disturbance and the parameter variation can be treated as known [160], unknown [265], or partially known [162, 163]. Although the assumption of large-inertia moving platform already turns a time-invariant LIP model to a time-varying version, the case with a small-inertia moving environment could be more challenging. For the locomotion on a small-inertia moving platform, the model needs to reason about how the platform will move according to the interaction between the robot and the environment, which depends on the state of both the robot and the environment. One preliminary work in this direction is the quadruped locomotion on a ball [164], where the robot and the ball are both modeled using rigid-body dynamics and are coupled together through the contact force. Another relevant work also models the robot and the objects to be interacted as rigid bodies and formulates a dynamics equation whose state includes the state of both the robot and the objects [140]. This field remains challenging since the modeling and the state estimation of the environment are not extensively explored.

Irregular terrain: Terrain mechanics Terrain mechanics describes the local mechanical properties of the terrain surface. For regular terrain, it is assumed that the terrain surface is rigid and has sufficient high friction. However, the terrain mechanics of an irregular terrain can be deformable, granular, a mixture of water and sand, highly slippery, and even breakable. To handle these variations, a model for the terrain needs to be designed and integrated with the robot's model.

For the slippery ground, the assumption that most reduced-order model has, i.e., non-slip ground contact, could fail easily. [16] develops the slip detection and modifies the friction constraint in the dynamic model to make a quadruped walk stably on a slippery ground. Locomotion on deformable and granular terrain is challenging since the ground reaction force depends on the shape of the feet [165, 166], the terrain material, and insertion speed [167]. The foot will also sink into the granular terrain and the friction could change based on the insertion depth [18, 168], which is different from the case on a regular terrain. To model the resistant force when swinging the limbs in a resistive medium, like sand or water, a modified spring loaded inverted pendulum model [168] is proposed. More research on the locomotion of both robots and animals on a soft-yielding ground can be found in [169].

General formulation for locomotion on different terrains A general problem formulation for locomotion on general terrains is set up for the reader to understand a few key aspects of this problem. Let's first set up the model between a legged robot and the terrain. This model is crucial for further motion planning, low-level controller design, and the estimation of the state for either robot and terrain. For the model of a legged robot, we assume 1) he robot can be modeled using a hybrid dynamical system formulation with continuous phase dynamics and a switching event, 2) the continuous phase dynamics can be modeled using rigid body dynamics. Therefore, the robot dynamics can be expressed as

$$
\begin{aligned}
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) =& Bu_c - J_t(q)^T F_{t,c}, \\
& (q, q_t, t) \in C, \\
(q^+, \dot{q}^+) =& f_d(t, q^-, \dot{q}^-, u_d, F_{t,d}), \\
& (q, q_t, t) \in D,
\end{aligned} \tag{4}
$$

where q and $\dot{q}$ are the robot's generalized coordinate and its time derivative. $M(q), C(q,\dot{q})\dot{q}$, and $G(q)$ are the inertia matrix, the sum of the Coriolis and centrifugal terms, and the gravitational term, respectively. u_c and B are the control input vector during the continuous phase and the input matrix. $J_t(q)$ is the contact Jacobian between the robot and the terrain while $F_{t,c}$ is the interaction force during the continuous phase. The robot system follows the continuous dynamics when the time and the state of the robot and terrain are in the set C. The superscript $^+$ and $^-$ indicates the state right after and before the time instance. The function $f_d(\bullet)$ is the discrete state transition function of time, state, discrete control input u_d, and terrain interaction force $F_{t,d}$ to model the sudden jump of the state while the time and the state of the robot and terrain are in the set D.

Next, let's look at the modeling of the terrain. Since terrain is a high-dimension system and subject to complex physical law, the modeling of terrain remains an open question. Here, we try to model the terrain in a way that we can still have some insights. We assume the dynamics of the terrain can be modeled using a hybrid dynamical system formulation. The continuous part is assumed to be modeled using a second-order differential equation $f_{t,c}$ while the discrete part is modeled using a nonlinear function $f_{t,d}$. The terrain dynamics can be expressed as

$$
\begin{aligned}
\ddot{q}_t =& f_{t,c}(t, q_t, \dot{q}_t, F_{t,c}), \quad (q, q_t, t) \in C \\
(q_t^+, \dot{q}_t^+) =& f_{t,d}(t, q_t^-, \dot{q}_t^-, F_{t,d}), \quad (q, q_t, t) \in D
\end{aligned} \tag{5}
$$

where q_t and $\dot{q}_t$ are the terrain's generalized coordinate and its time derivative.

Based on the two equations (1), (3), we would like to highlight a few aspects:

- *Hybrid nature of the system.* The unique feature of a legged robot is the fact that it moves by breaking and making contact with terrain. This breaking and making contact could cause the robot's state and its velocity to jump. The contact event happens when the configuration of the robot and the terrain is matched in a given time. These concepts can be modeled using the hybrid impulsive dynamical system formulation with time and state-dependent switching conditions [266].

- *Terrain dynamics.* In this formation, we not only model the robot as a dynamical system but also emphasize that the behavior of the terrain can also be modeled as a dynamical system. This directly points us to more questions: 1) Do we need to model the terrain as a dynamical system? 2) What are the feasible state representation for a terrain? 3) How does the terrain state change based on time, current state, and external forces? These problems could be partially answered later by the existing literature but they remain open equations in general.
- *Interaction force between the robot and the terrain.* The two dynamical systems are connected through the interaction force $F_{t,c}$ and $F_{t,d}$ during continuous phase and discrete event, respectively. Based on the perspective of robot control, the robot hopes to drain sufficient and proper force from the terrain to execute its own task. However, the robot also need to consider the dynamic of the terrain since it poses limitation about what kind of force is obtainable. If there is no other way for the robot to estimate the state of terrain, the interaction force $F_{t,c}$ and $F_{t,d}$ are the only communication channel between the two players.
- *Applicability to the reduced-order model.* The equations (1), (3) can not only serve as the modeling framework for the full-order model. It can also provide insight when setting up reduced-order model when computation efficiency is an essential consideration.

Regular terrain In this subsection, we introduce the assumptions of a regular terrain which is widely used in the legged robot community. Then, we show how these assumptions simplify and constrain the problem formulation we discussed in the previous subsection, including modeling, motion planning, low-level control, and state estimation.

Assumptions of regular terrain. Intuitively, a regular terrain is flat, rigid, and static. To be precise, we list the assumptions and elaborate the details:

- *Horizontally flat.* The surface is flat while its surface normal is aligned with the gravity.
- *Rigid.* The surface is not deformable, not penetrable, and can provide unlimited vertical supporting force.
- *Static.* The whole terrain and the contact surface are not moving with respect to the inertia frame.
- *Sufficient friction.* The surface of terrain is not very slippery.

(The full-order model for a regular terrain) Under the assumptions of a regular terrain, the modeling of the robot and the terrain can be simplified. First, since the geometry of the terrain is not changing, the switching conditions in (1, 3) are no longer a function of the terrain state q_t. Furthermore, since the terrain is assumed to be rigid and static, the terrain dynamics is ignored, and therefore the equation (3) no longer exists. To determine the interaction force between the terrain and the robot, a holonomic constraint is used to enforce the non-accelerating contact surface condition

$$J_t(q)\ddot{q} + \dot{J}_t(q)\dot{q} = 0 \tag{6}$$

and to develop the constrained robot dynamics during the continuous phase. As for the discrete jumping event, the rigid body contact model can be used to derive the reset map for the robot's discrete dynamics [267].

(The Reduced-order model for a regular terrain) Several reduced-order models have been proposed to capture the dynamics of a legged robot on a regular terrain. For example, the inverted pendulum model [144, 268], spring-loaded inverted pendulum model [269, 270], angular momentum inverted pendulum model [145], to name a few. TBD

Irregular terrain: Macro movement After introducing the general problem formulation and the special cases for a regular terrain, the properties of irregular terrain and their effect on the modeling, motion planning, control, and estimation are investigated. The physical properties of the terrain can be categorized using three aspects, i.e., macro movement, terrain mechanics, and geometry. The macro movement describes the characteristics of movement of the terrain as a whole. The terrain mechanics describes the material properties of the terrain surfaces. The geometry describes the geometrical properties of the terrain.

(Definition of macro movement) The macro movement of the terrain describes the movement of the terrain as a whole relative to the inertia frame. For any terrain, the macro movement can be further categorized using the two dimensions listed below:

- (MM1)*Does it accelerate?*
- (MM2)*Can the movement of terrain be affected by external forces?*

Based on the two dimensions, the terrains can be categorized into three group since one of the combination does not sense.

The first group of terrain is that with non-accelerating macro movement and the macro movement of the terrain will no be affected by external forces. The regular terrain belongs to this categorize. This group of terrain also include those with constant linear velocity but with zero angular velocity since rotation induces centrifugal acceleration.

The second group of terrain is that with accelerating macro movement and their macro movement is not affected by the external force. This means that the robot's movement and induced ground reaction force will not affect the motion of the terrain. The terrain with sufficient large inertia or self-actuation force belong to this group, e.g.; boat, vessel, airplane, trains. In this situation, the terrain dynamics can be simplified by only treating the kinematic motion profile of the terrain instead of its dynamic equation.

(Full-order modeling) Full-order model becomes time-varying system [46](?)

(Reduced-order modeling) LIP with time-varying disturbance and system parameter uncertainty [162, 163, 271]

The third group the terrain is that with accelerating macro movement and their macro movement could be affected by the external force, including the robot induced interaction force. Unlike the previous two groups of terrains, the dynamic of the system cannot be ignored.

(Full-order modeling) Quadureped walking on a ball [164]

(Reduced-order modeling)

Irregular terrain: Terrain mechanics (Definition of Terrain mechanics)

- Deformable
- Penetrable (provide more friction!?)
- Friction
- Granular, Fluid, half-half

Irregular terrain: Geometry (Definition of geometry)

- Surface normal
- Discrete surfaces (stairs)
- limited surface area (3D, narrow bridge)
- entanglement

VI. Real-world Applications (1.5 pages)

0) Industrial Operations: Quadrupedal and bipedal robots have the potential to enhance workplace safety and efficiency by performing hazardous tasks in industrial settings.

Recently, quadrupedal robots have demonstrated advanced mobility and environmental interaction capabilities. The integration of a robotic arm with the ANYmal platform enabled coordinated locomotion and manipulation, advancing beyond simple manipulator mounted configurations [12]. While quadrupeds have limitations in industrial settings like vertical ladders and tight spaces, humanoid robots excel in these human-designed environments. The E2-DR humanoid was developed for industrial inspections and disaster response, featuring vertical climbing abilities and effective navigation through confined areas and debris [1]. For industrial inspections of pipes and tall structures, specialized quadrupedal and bipedal robots have been developed. MARVEL, a quadrupedal climbing robot, can navigate ferromagnetic surfaces including painted, rusty, or dusty storage tanks [13]. Additionally, a bipedal crawling robot has been designed for pipe inspection, capable of multimodal locomotion to navigate pipe segments and overcome obstacles like flanges and angled sections [32].

The remaining challenges for autonomous quadrupedal and especially bipedal robots for industrial use are safe environment interaction, object manipulation, and vertical mobility in confined spaces. All of these while ensuring safe human-robot interaction during the performed tasks.

0) Human Assistance: Robots outperform humans in repetitive tasks and execute complex operations accurately and efficiently. Quadrupedal and bipedal systems show promise in assisting humans across residential, office, healthcare, and service environments, potentially enhancing quality of life. A key challenge lies in ensuring safe human-robot interaction within dynamic, unstructured settings like hospitals and public spaces, where dense pedestrian traffic creates operational complexities.

Quadrupedal robots demonstrate autonomous navigation capabilities, but their effectiveness as non-visual guides for people with blindness or low vision remains uncertain compared to trained service dogs. Recent research [7] defined technical requirements and evaluated their feasibility for this application. While lacking the social rapport and protective instincts of guide dogs, quadruped robots offer advantages including predictable behavior in stressful scenarios and allow voice-interface communication about environmental features (stairs, ramps, obstacles) and hazards. These systems require no specialized training and avoid behavioral risks associated with live animals. However, no quadrupedal robots are currently designed exclusively for this role.

In healthcare settings, sleep quality significantly impacts patient recovery, yet hospital environments often disrupt rest. Poor sleep can prolong hospital stays and impede healing [272]. Key disruptors include noise, light exposure, and temperature fluctuations [273]. Recent innovations employ quadrupedal robots in intensive care units to monitor vital signs and environmental conditions (CO levels, temperature, air quality), ensuring optimal sleep conditions [34]. This approach enhances patient recovery and reduces nurses' workload through automated surveillance, allowing more time for direct care. Despite advancements, deploying quadruped and biped robots for human assistance faces challenges in human-robot interaction and ensuring safety.

0) Agriculture: Modern agriculture faces increasing demands and environmental challenges, driving the adoption of advanced robotic systems to enhance productivity and sustainability [35]. Quadrupedal robots have emerged as a promising development in agricultural automation, leveraging AI-powered control systems to navigate complex terrains like uneven soil, gravel, and crop fields. Recent innovations enable these robots to traverse deformable surfaces (soft sand, inflatables) and rigid substrates (asphalt), expanding their operational versatility across diverse farming conditions [2]. Unlike traditional wheeled robots, quadrupedal systems excel in precision agriculture through adaptive mobility, supporting critical tasks like crop monitoring, data collection, and targeted resource management while minimizing soil compaction.

A study compared legged and wheeled robots for agricultural terrain navigation, analyzing energy efficiency and localization performance [274]. Legged robots demonstrated consistent power consumption across varied terrains, while wheeled counterparts exhibited lower overall energy use and superior localization accuracy. While legged robots excel on challenging terrain, enhancing their energy efficiency remains crucial for widespread agricultural adoption.

0) Terrestrial Operations:

Subterranean: Navigating subterranean environments presents challenges for robots due to variable geometry and unstable terrain, where uneven or slippery surfaces test even advanced systems [275]. While aerial vehicles were initially considered for such applications, their limited autonomy and payload capacity shifted focus to ground or hybrid solutions. Collaborative aerial-ground systems were explored, but breakthroughs emerged with DARPA's Subterranean (SubT) Challenge, where quadrupedal robots demonstrated superior mobility in these environments [42, 43, 51]. The CERBERUS team showed advanced capabilities using an upgraded ANYmal C100 quadruped robot at the DARPA SubT Challenge, while the CSIRO Data61 team employed a Boston Dynamics' Spot. Notably, six of eight finalist teams utilized legged platforms, highlighting their growing feasibility for subterranean operations [51] and research in general. Recent research prioritizes multi-agent systems for underground navigation, including heterogeneous configurations (solely quadrupedal robots) [276] and hybrid teams combining aerial and quadrupedal robots [39–41].

Legged robots demonstrated superior performance in subterranean environments, outperforming conventional mobility systems through their adaptability to irregular terrain. This capability makes them optimal for underground applications including mine inspection and cave-based search and rescue missions.

Marine: Legged underwater robots offer significant advantages over traditional propeller-based vehicles for marine exploration. These robots provide enhanced stability, precise movement, and minimal environmental impact while navigating complex underwater terrains [3]. Unlike propeller systems, which can disturb sediment and harm marine life, legged robots enable careful interaction with the seabed while maintaining maneuverability in confined spaces [48]. Considering such benefits, SILVER, the first underwater quadrupedal robot with dynamic hopping and static gaits, pioneered benthic exploration was developed [53]. While soft-legged quadrupeds have demonstrated locomotion and grasping capabilities [49], recent research has shifted toward hexapod designs for seabed exploration [3, 50, 277]. A new marine iguana-inspired quadruped combines undulatory swimming with walking, enabling efficient travel and precise positioning in benthic environments [47].

Legged robots' adaptability to difficult terrain and stable power usage make them ideal for underwater tasks like scientific seabed exploration, where conventional propulsion systems face limitations [48].

Polar: The Arctic's critical role in understanding climate change and global environmental patterns necessitates thorough exploration, yet extreme conditions make human-driven research hazardous. Recently, legged robots have demonstrated exceptional capability in navigating challenging terrains, making them a feasible option for safe Arctic research and data collection [2, 16, 208, 278]. A quadrupedal robot, ANYmal, was equipped with a slip-detection algorithm enabling it to navigate frozen terrain and recover from slippage [16]. Additionally, an arctic under-ice quadrupedal robot concept was developed, featuring morphing capabilities that enable skiing beneath ice while using legs for stability during thruster-based movement, designed for extended polar environmental monitoring missions [54].

Legged robots equipped with specialized sensors enable safe, continuous data collection in challenging arctic terrains, advancing our understanding of polar ecosystems and their climate impacts while reducing human exposure to harsh conditions. However, this application is still underdeveloped in contrast to others.

0) Space Exploration: Legged robots are emerging as superior alternatives to traditional wheeled and tracked vehicles for space exploration, offering enhanced mobility across challenging terrains and hazardous environments while enabling access to previously unreachable areas of scientific significance [19].

A team of three specialized ANYmal quadrupedal robots was proposed for planetary exploration, each with distinct roles (scout, hybrid, and scientist) to optimize data collection and overcome wheeled robots mobility limitations [19]. This collaborative approach enhances exploration efficiency in challenging environments through distributed task allocation.

While commercial quadrupedal robots excel at walking and obstacle navigation, jumping locomotion remains underexplored. The challenges include managing impact forces, developing robust leg structures, and implementing sophisticated control algorithms for take-off, mid-air stability, and landing. Recent research has focused on jumping quadrupeds for low-gravity space exploration [59–62], indicating the feasibility of autonomous jumping navigation, though further development is needed in control systems and path planning.

VII. Bipeds vs. Quadrupeds (1 page)

While all previous sections discuss the modeling, planning, and control for both bipeds and quadrupeds, in this section, the different and shared features of bipeds and quadrupeds are discussed. This section aims to provide insights regarding how to generalize the success of quadrupeds to bipeds or the other way around. To do so, this section highlights the intrinsic properties of bipeds and quadrupeds and lists out existing works that transfer methodology from quadrupeds to bipeds.

A. Difference between Bipeds and Quadrupeds

Stability. Stability is one of the key measures when discussing locomotion and is also one of the biggest differences between bipeds and quadrupeds. In general, quadrupeds are more stable than bipeds. This is caused by several factors.

1. Numbers of legs: A Quadruped has four legs while a biped only has two legs. When the robot is standing still, the CoM projection must stay in the support polygon, i.e., the convex hull of all contact points with the ground []. A quadruped has four legs, causing a larger support polygon, and is easier to maintain static balance. For a biped, a small perturbation can push CoM projection away from the support polygon and it requires a well-designed controller to maintain static balance and reject disturbance []. When the robot is dynamically moving, having more legs means having more flexible choices of contact sequences and larger action space. For bipeds, each decision of making and breaking contact is crucial since there is no redundancy.

2. Distance between legs: Self-collision between legs is a constraint that is needed to be considered when designing a legged locomotion controller, especially for aggressive motion. In general, avoiding self-collision is a computationally heavy constraint []. For quadrupeds, the four legs are located at the corner of the trunk and are far away from each other. This naturally reduces the chance of self-collision between legs. In contrast, a common biped has two legs compactly located side-by-side. This makes self-collision a non-neglectable issue when reasoning about the stepping position and imposes additional difficulty to maintain balance [265].

3. Limb mass: Swinging the leg is inevitable when repositioning the leg to the next contact configuration. The ratio of limb mass to the total mass of a biped is higher than that of a quadruped. This means the inertia effect of swinging limbs, which is ignored and treated as disturbance by most of the ROMs for legged robots, is more serious on a biped compared to a quadruped. This makes the bipeds more unstable than quadrupeds, especially when aggressive swing leg motion is required [255, 279].

Load-carrying

Energy efficiency

Cost and accessibility

B. Shared feature between bipeds and quadrupeds

Goal:

- How can we generalize the success of controlling quadrupeds to bipeds?
- What should we be careful about when generalizing the methods to bipeds?

C. Difference between Bipeds and Quadrupeds

- Stability.
  - Quadrupeds are more stable since
    - More legs: increase flexibility to make contact. A lot of choices of gaits and contact sequences.
    - Larger distance between legs: less possible to have self-collision.
    - Lower center of mass: combined with a larger distance between legs, the CoM is easier to stay inside the support polygon.
    - Negligable limb mass: the swing leg motion cause less effect on the overall dynamics
  - It is more challenging to maintain the stability of a biped.
    - Fewer legs: each decision of contact timing and position matters
    - Shorter distance between legs: self-collision of leg impose additional constraint for step planning [265]
    - Higher center of mass: since the legs are close to each other, the CoM is easier to be outside of the support polygon.
    - Non-negligible limb mass: the swing leg motion could cause a significant effect while performing the agile motion [255, 279].
- Performance (agility, load carrying, ...)
  - Quadrupeds

        * Require larger region of terrain to place the four legs [280]
        * Better load-carrying capability
        * Better energy efficiency
    - Bipeds
        * Only request a smaller region of terrain [280]
        * So far, the load-carrying capability is less than quadrupeds. [?]
        * More agile
        * High energy consumption.
- Cost and accessibility
    - Quadrupeds are generally cheaper and more accessible since
        * Inherent stability reduces the need for powerful and precise motors. (although the number of motors are similar)
        * Lower risk of breaking the robot when it fails down.
        * Low mechanical complexity.
    - Human-like bipeds are more expensive since
        * Bipeds need better motors in terms of power and precision. A single leg needs to support the weight of the whole robot. The control of leg motion determines the stability of the robot.
        * Higher risk of breaking the robot when it fails down.
        * High mechanical complexity. Hard to fix.

D. *Common things*

- The modeling/control approaches that can be applied on both quadrupeds and bipeds:
    - LIP model can be used to guide a humanoid robot to perform both quadrupedal locomotion and bipedal locomotion [280].
    - SRB model can be applied on quadrupedal robot and also humanoid robot. But since the humanoid robot ... needs modifications [255, 279]
    - Learning-based control method:
        * [281], [282] use RL to make a quadruped performing bipedal locomotion.
        * [283] use RL to make a quadruped perform quadrupedal locomotion and transit to bipedal locomotion.

HARNESSING VIRTUAL WORLDS: SIMULATION ENVIRONMENTS FOR LEGGED ROBOT CONTROL DEVELOPMENT (THESE TEXT NEEDS TO BE REPOSITIONED)

E. State-of-the-art:

5) Single-environment simulation: Single environment simulation trains a reinforcement learning policy one model at a time, this method often results in training times in the tens of hours.

F. Future:

6) Massive Parallel Simulation for Ultra-Fast Legged Robot Training: Massive parallel simulation harnesses the power of parallel processing found in modern graphic processing units to train thousands of robots at once, greatly reducing required training time.

Fig. 8 Single Environment Simulation vs Massively Parallel Simulation

G. Frameworks for learning in simulation

Fan and Guanya will check this for potential incorporation into the LC sec. A prevalent problem in many existing learning-based frameworks is their inability to adapt online to changing conditions. Such online adaptation becomes particularly crucial when the robot operates in an environment with dynamics that differ from those considered or neglected in its model or prior offline training. To address this problem a learning-based framework for a legged robot that provides robustness under unexpected disturbances was proposed in [284] and tested on the MIT Mini Cheetah. The proposed approach simplifies the learning problem by separating it into two main tasks: learning a model to estimate the robot's steady-state response and learning a dynamic model for the system near its steady-state behavior. The general idea of this approach is to obtain a model of the robot and its environmental interaction via learning in simulation, then transfer the learned model to the real robot, and finally, fine-tune the model via online learning. In summary, this approach allows equipping robots with the ability to adapt their skills to new environments with unpredictable dynamics and serves as an example of enhancing an existing model-based controller through the integration of learning-based techniques. Therefore, this online learning feature point out to be the next step towards increased robustness and autonomy for legged robots.

7) IsaacGym and IsaacSim:

7) HumaniodBench:

7) MuJoCo: Fan and Guanya will either incorporate this and the simulation based modeling into the learning control sec. or separately write in two sections - sim. based modeling (detailed description) and learning control (a table). Proposing reward functions when developing reinforcement learning-based controllers for the dynamic motion of legged robots is a challenging task, especially when the objective is to deploy the learned controller in the real robot without needing adjustments or fine-tuning. In order to provide robustness and allow operation for different tasks in unstructured environments, a sparse reward function is commonly employed. However, sparse reward functions lead to unrealistic behaviors or suboptimal performance. To overcome this issue, in [285], a Model Predictive Control (MPC) scheme was introduced to assess the performance of a humanoid robot while performing different tasks (standing, walking, and pushing a box) employing different state of the art Reinforcement Learning (RL) with common reward functions against the introduced one based on MPC with custom reward function shaping terms. The novel benchmark for whole-body humanoid control HumanoidBench [286] was the employed framework to train the controllers and the MuJoCo MPC [287] was used to develop the MPC scheme. The results obtained with the MPC scheme were substantially better and more consistent than those obtained with the state-of-the-art approaches in terms of the HumanoidBench Score. The approach is based on formulating the tasks in terms of reward functions rather than a cost function by performing a transformation before feeding the data into the MPC. It was observed that feeding the reward without the transformation leads to physically undesirable behaviors resulting in lower rewards. The results showed that even though some of those reward residuals are already encoded in the HumanoidBench default reward, providing such information individually to the optimizer improved the overall performance. In addition, it was observed that the episode length heavily impacts the generated behaviors. Hence, the recommendation is to perform longer episode lengths and repeated tasks with changing goals.