Mass and radius constraints for neutron stars from pulse shape modeling

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Aim

- Determination of masses and radii for rapidly rotating neutron stars
- Constrain the number of possible equations of state
- The properties of matter at extremely high densities
- Study the effects of polarization measurements on mass and radius constraints

Contents

Neutron stars

- Most dense objects that can be directly observed
- Typical mass $M = 1.5M_{\odot}$ and typical radius $R = 12$ km
- Supernuclear densities (5 to 10 times the nuclear equilibrium density $n_0 \approx 0.16\ \mathrm{fm}^{-3}$ of neutrons and protons)
- Are created after the gravitational collapse of a core of a massive star ($> 8M_{\odot}$) at the end of its life.

- Composition of the innermost core is unknown.
- Equation of state (EOS) gives the relation between the pressure and density (or mass and radius).
- The EOS of a neutron star is unknown.

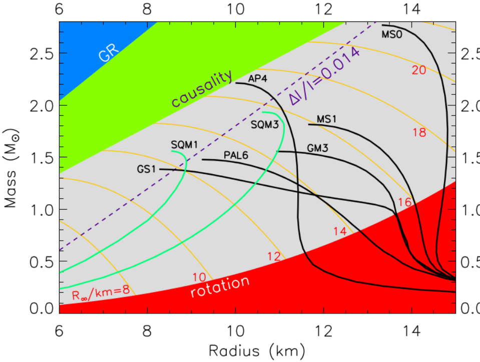


Figure : Figure 2 from Lattimer and Prakash (2004).

Neutron stars

- Observed masses and radii constrain the number of possible EOSs.
- Upper limits for compactness
  - The general relativity (Schwarzschild condition)
  - Causality condition
- Lower limit for compactness
  - Fastest observed rotational frequency

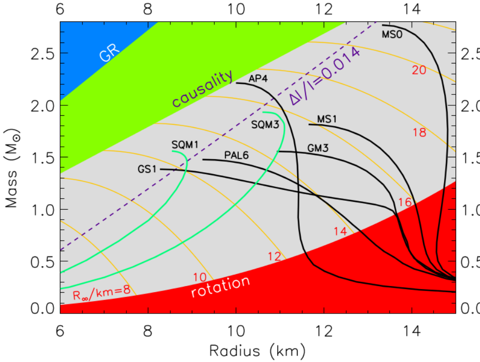


Figure : Figure 2 from Lattimer and Prakash (2004).

Neutron stars

- The highest observed mass rules out many EOSs.
- We aim to find out independent knowledge of both mass and radius.

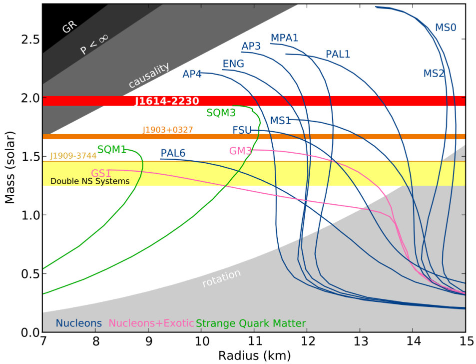


Figure : Figure from Demorest et al. (2010).

Accreting millisecond X-ray pulsars (AMXP)

- A subgroup of low mass X-ray binaries (LMXB)
- Gas from the accretion disk (stripped from the companion) is channeled onto the magnetic poles of a millisecond pulsar.
- A pair of "hot spots"

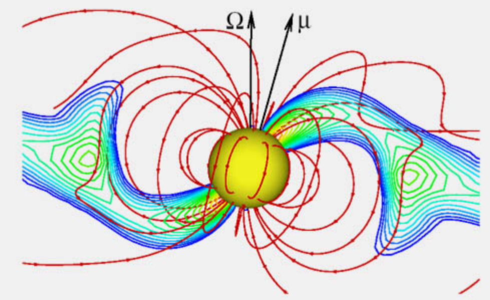


Figure : Figure 4 from Romanova et. al (2004).

Accreting millisecond X-ray pulsars (AMXP)

- Recycling scenario
  - The evolutionary progenitors of recycled radio millisecond pulsars

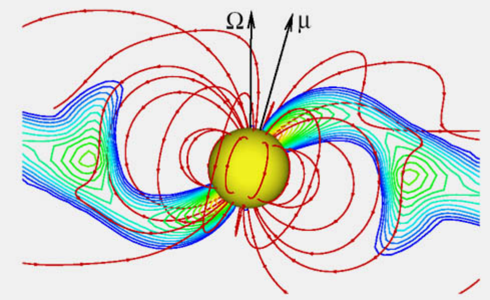


Figure : Figure 4 from Romanova et. al (2004).

Accreting millisecond X-ray pulsars (AMXP)

- Some of AMXPs show outbursts
- SAX J1808.4$-$3658: Nearly coherent oscillations detected during outbursts
- Similar outburst evolution: SAX J1748.9$-$2021 (in the Figure)

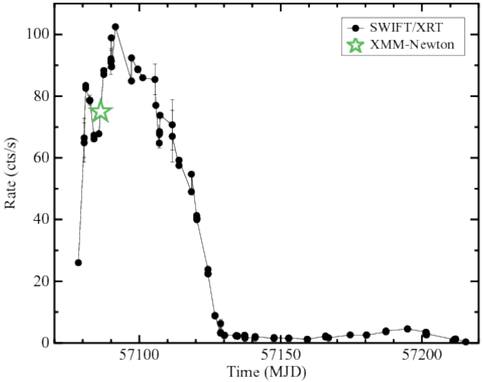
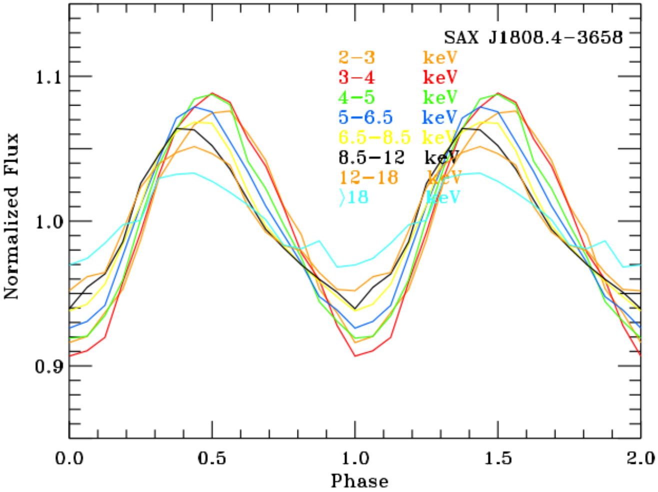


Figure : Figure 1 from Sanna et al. (2016).

SAX J1808.4−3658

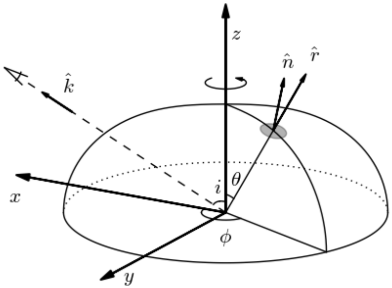


Methods

- Pulse shape modeling
- Bayesian analysis
- Monte Carlo sampling methods

Pulse profile modeling

- Schwarzschild + Doppler approximation (S+D)
- Oblate shape of the star taken into account
- Mass and radius affecting the light bending
- Time delays different from different locations



Pulse profile modeling

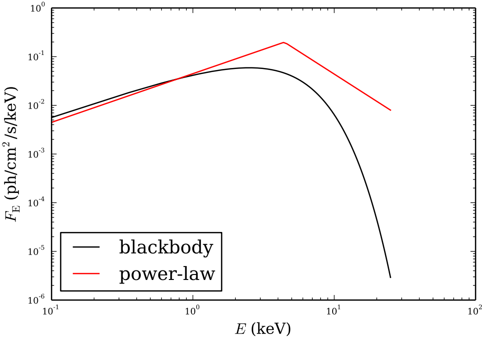
- Observed spectral flux from an infinitesimal spot:

$$\mathrm{d}F_E = (1-u)^{1/2}\delta^4 I'_{E'}(\sigma') \cos\sigma \frac{\mathrm{d}\cos\alpha}{\mathrm{d}\cos\psi} \frac{\mathrm{d}S'}{D^2} \quad (1)$$

- D = distance, ψ = bending angle, δ = Doppler factor, $(1-u)^{1/2}$ = inverse of gravitational redshift, σ = emission angle relative to the spot normal, and α = emission angle relative to the radius vector.
- Integration over the spot surface

Pulse profile modeling

- Energy spectrum (energy dependency of $I'_{E'}$)
- Blackbody + Power-law according to observations
- Heated hot spot + Comptonization from an accretion shock
- In this thesis two-component power-law
- $I'_{E'}(\sigma') \propto \sigma'$ either isotropic or "Hopf" profile



SAX J1808.4−3658

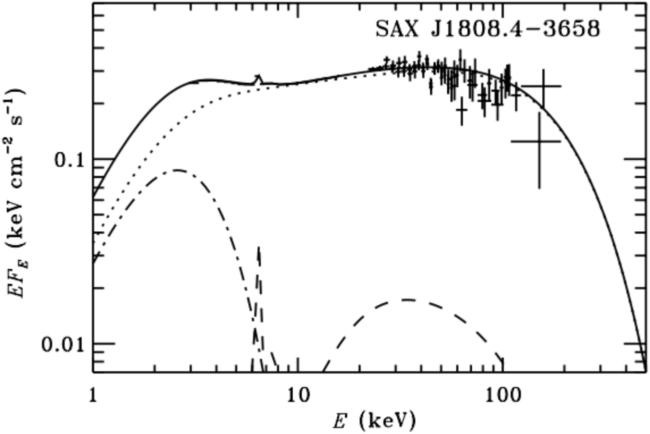


Figure : Figure 3 from Poutanen and Gierlinski (2003).

Bayesian inference

$$p(\mathbf{y}|\mathcal{D}) \propto p(\mathcal{D}|\mathbf{y})p(\mathbf{y}) \qquad (2)$$

- $\mathcal{D}$ = Data
- $\mathbf{y}$ = Parameters of the pulse profile model
- $p(\mathbf{y})$ = Prior probability distributions of the parameters
- $p(\mathcal{D}|\mathbf{y})$ = Probability distribution of the data given the parameters
- $p(\mathbf{y}|\mathcal{D})$ = Probability distribution of the parameters given the data

Ensemble sampler

- Independent walkers moving the parameter space
- Stretch-move algorithm instead of Metropolis-Hastings
- New sample is either accepted or rejected with a certain probability

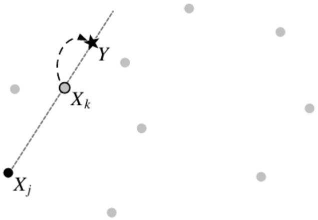


Figure : Figure 2 from Goodman and Weare (2010).

Results

- Synthetic data
- Posterior probability distributions

Synthetic data

- We have generated a synthetic data similar to SAX J1808.4$-$3658 using the pulse profile model.
- Parameters assumed to be physically reasonable
- The variability amplitude A determined mainly by observer inclination i and spot co-latitude θ_s:

$$A = \frac{F_{\mathrm{max}} - F_{\mathrm{min}}}{F_{\mathrm{max}} + F_{\mathrm{min}}} \tag{3}$$

$$A \approx \frac{(1 - r_{\mathrm{S}}/R)\sin i \sin\theta_s}{r_{\mathrm{S}}/R + (1 - r_{\mathrm{S}}/R)\cos i \cos\theta_s}. \tag{4}$$

Synthetic data

$$A \approx \frac{(1 - r_{\rm S}/R)\sin i \sin\theta_s}{r_{\rm S}/R + (1 - r_{\rm S}/R)\cos i \cos\theta_s}. \qquad (5)$$

- Switching i and θ_s has no effect on A.
- However, the variability of polarized flux depends on i and θ_s separately.
- To study this, we have created two datasets which differ only in i and θ_s.
- We assume either uniform or non-uniform prior probability distributions for parameters.
- We obtain posterior probability distributions for the parameters using the ensemble sampler.

Synthetic data

Table : Paramters of the synthetic datasets.

Parameter	Value
Radius R	12.0 km
Mass M	1.5 $\mathrm{M}_{\odot}$
Inclination i	5 $^{\circ}$ or 75 $^{\circ}$
Spot colatitude θ_s	75 $^{\circ}$ or 5 $^{\circ}$
Spot angular size ρ	10.0 $^{\circ}$
Distance D	2.5 kpc
Temperature T_{eff}	2.0 keV

Synthetic data

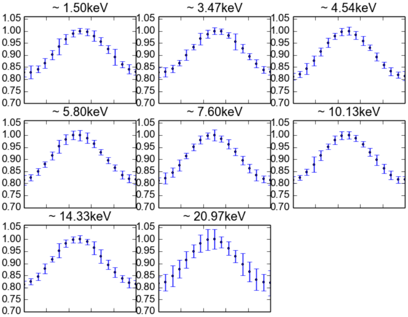


Figure : Equatorial spot

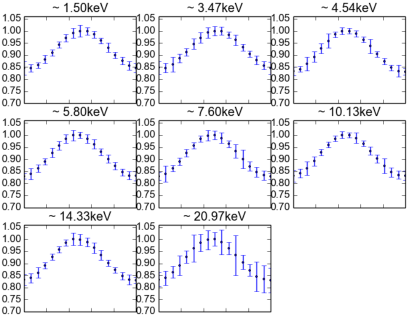


Figure : Polar spot

Posterior probability distributions

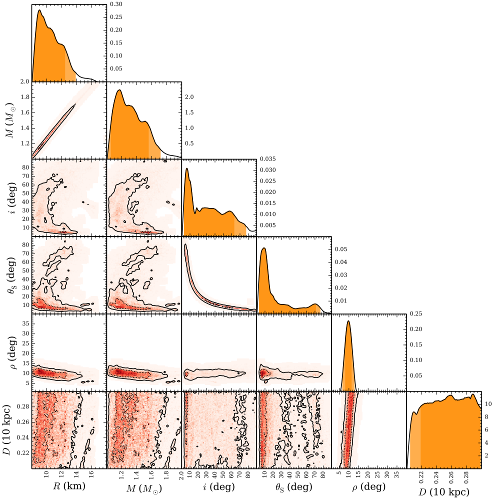


Figure : Polar spot with only uniform priors.

Posterior probability distributions

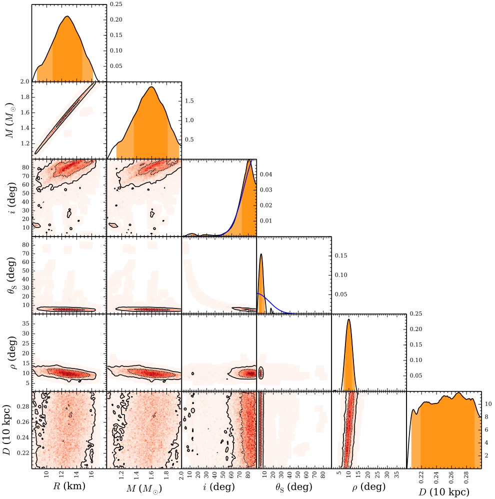


Figure : Polar spot with non-uniform i and θ_s priors (blue lines).

Posterior probability distributions

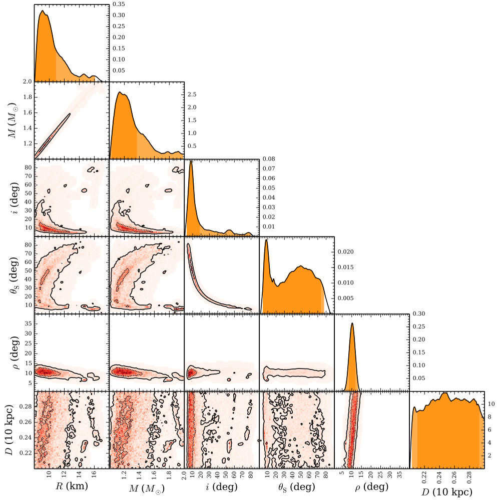


Figure : Equatorial spot with only uniform priors.

Posterior probability distributions

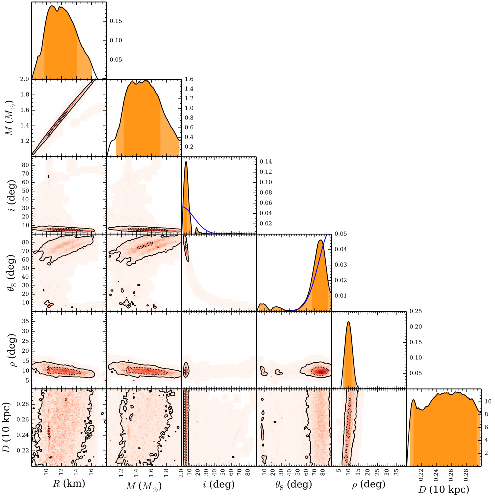


Figure : Equatorial spot with non-uniform i and θ_s priors (blue lines).

Results

- Only upper limits for masses and radii when only non-uniform priors
- Both upper and lower limits when prior information for i and θ_s applied
- Correct $i - \theta_s$ solution not found without prior information.
- The size of the spot is well constrained but the distance is not.

Summary

- AMXPs show coherent oscillations at the spinning frequency of the pulsar.
- These oscillations may be used to constrain masses and radii of pulsars.
- Tighter constraints for mass and radius from polarization
- Future
  - Develope the model
  - From synthetic to real observations

The End