Differential Dynamic Programming Tutorial

Taylor Howell

January 30, 2021

1 Problem Formulation

Differential Dynamic Programming (DDP) solves the following optimization problem:

$$\begin{aligned} \underset{x_{1:T}, u_{1:T-1}}{\text{minimize}} \quad & g_T(x_T) + \sum_{t=1}^{T-1} g_t(x_t, u_t) \\ \text{subject to} \quad & x_{t+1} = f_t(x_t, u_t), \quad t = 1, \dots, T-1, \\ & (x_1 \text{ given}). \end{aligned} \tag{1}$$

For a system with state, $x_t \in \mathbf{R}^n$, control inputs, $u_t \in \mathbf{R}^m$, time index t, and subject to discrete-time dynamics, $f_t : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$, we aim to minimize an objective with stage cost functions, $g_t : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}$, and terminal cost function, $g_T : \mathbf{R}^n \to \mathbf{R}$, over a planning horizon T.