Quiz 4

Name:

For the following problems state the distribution used and reason for doing so.

Suppose a battery manufacturer wants to test out the lifetime of their batteries. They expect their batteries to last an average of 100 hours.
a. What is the probability that a randomly tested battery will last at least 50 hours?

b. If they test 10 batteries, what is the probability that all of the batteries will last longer than 50 hours?

c. Suppose a consumer needs 100 hours of continuous use. What is the probability they will need more than 2 batteries to achieve 100 hours of continuous use? Assume that changing the battery takes no time and one battery is in use at a single time.

Binomial(n,p):
pmf: $f(x)=\binom{n}{x}p^x(1-p)^{n-x}$
expectation: np

Poisson(λ):
pmf: $f(x)=\lambda^x e^{-\lambda}/x!$
expectation: λ

Geometric(p):
pmf: $f(x)=(1-p)^x p$
expectation: $\frac{1}{p}-1$

Negative Binomial(r,p):
pmf: $f(x)=\binom{x+r-1}{x}p^x(1-p)^r$
expectation: $\frac{pr}{1-p}$

Hypergeometric(N_1,N_2,n):
pmf: $f(x)=\frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}}$
expectation: $nN_1/(N_1+N_2)$

Uniform(a,b):
pdf: $f(x)=1/(b-a)$
cdf: $F(x)=\begin{cases}0 & x\le a\\ (x-a)/(b-a) & a<x<b\\ 1 & x>b\end{cases}$
expectation: $(a+b)/2$

Exponential(λ):
pdf: $f(x)=\lambda e^{-\lambda x}$, $x>0$
cdf: $F(x)=1-e^{-\lambda x}$, $x>0$
expectation: λ^{-1}

Gamma(k,θ):
pdf: $f(x)=x^{k-1}e^{-x/\theta}/\big(\Gamma(k)\theta^k\big)$, $x>0$
expectation: $k\theta$

Beta(α,β):
pdf: $f(x)=x^{\alpha-1}(1-x)^{\beta-1}/\mathcal{B}(\alpha,\beta)$, $0<x<1$
expectation: $\alpha/(\alpha+\beta)$