Practice Final Exam

1. In RNA sequencing experiments, the counts of expressed genes can be modeled as a Poisson distribution with pmf $f(x) = \Pr(X = x) = \lambda^x e^{-\lambda}/x!$. On the other hand, non-expressed genes have zero counts. This creates an issue in modeling the data, as genes with zero counts could be expressed and the zero is due to random sample or they could just not be expressed. To accommodate this, we can model all of the counts as a zero inflated Poisson distribution. To do this, we assume that for each gene there is some probability p of being expressed. Conditional upon being expressed, the counts follow a Poisson distribution with parameter λ. If the gene is not expressed, with probability $1 - p$, then the count is zero. Suppose we have 100 we investigate with counts $x_1, \ldots, x_{100}$.

   a. If $\lambda = 2$ and $p = 0.25$, calculate the probability that $X = 0$.

   b. Write out the log-likelihood function.

   c. Set up the maximum likelihood equations. Do not solve.

   d. Calculate the expectation and variance of a zero-inflated Poisson distribution with parameters p and λ.

   e. Set up the Method of Moments equations. Do not solve.

2. A survey was given to a random sample of college students to assess the support of gay marriage. 242 females and 207 males were surveyed. 176 females expressed support while 134 males expressed. a. Construct 95% confidence intervals for the percentage of support for gay marriage among college females and college males, seperately.

b. Construct a 95% confidence interval for the difference between the two proportions. Use the fact that the pooled standard error is $\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ with $\hat{p}$ equal to the pooled proportion.

c. If we were to do a hypothesis test to test for a difference between the two proportions, what would be the null and alternative hypotheses?

d. Is there a statistically significant difference between the two proportions at the 95% confidence level?

3. Suppose that Y, X_1, X_2 and ϵ are all Gaussian (aka Normal) random variables and that $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$.

   a. If X_1, X_2, and ϵ are all independent standard normal random variables (mean 0 and variance 1) and $\beta_0 = 10$, $\beta_1 = 2$, $\beta_2 = 5$, and $\beta_3 = 1$, what is the variance of Y?

   b. If X_1, X_2, and ϵ are all independent standard normal random variables (mean 0 and variance 1) and $\beta_0 = 10$, $\beta_1 = 2$, and $\beta_2 = 5$, what is the correlation of X_1 and Y?

   c. Suppose we also have information on X_3, another standard normal random variable. We have 20 total observations of Y, X_1, X_2, and X_3. We fit linear models using X_1 alone, X_1 and X_2, and all three X's. State which of the three models will be chosen according to each of the below criteria.

criterion	X_1	X_1, X_2	X_1, X_2, X_3
BIC	125.5	128.2	128.1
AIC	122.5	124.2	123.1
adj R^2	-0.003	-0.045	0.048
5-fold CV	135.9	162.3	104.2

4. In a manufactured product you want to be certain that the delivered product is the weight advertised. The goal is 100 grams.

   a. To test the process you weigh 5 products and get weights of 106, 104, 99, 95, and 97. Calculate the sample mean and standard deviation.

   b. Obviously 5 samples is not sufficient so you take 20 more samples (for 25 total) and get a sample mean of 101.84 and sample standard deviation of 3.3. State the null and alternative hypotheses and then compute a p-value under the null distribution.

   c. If the average weight is in fact 101 with variance equal to 4, what is the power with 25 observations compared to null distribution of $N(100, 4^2)$?

5. Suppose that we want to estimate the size of the wolf population in Yellowstone. One method to do this is to go out, capture wolves, tag them, and release them. You then go out later and capture more wolves, noting how many are recaptured (this is known as mark-recapture). Suppose that there are in fact 400 wolves in Yellowstone. In the first round you capture and tag 50 wolves.

   a. Suppose you capture another 50 wolves in the second round. What is the expected number of wolves you will capture?

   b. Suppose you capture another 50 wolves in the second round. What is the probability you capture 10 tagged wolves in the second round of captures?

   c. The Lincoln-Peterson estimator for the total number of captured individuals is $\frac{Kn}{k}$ where K is the total number of capture in the second round, n is the number of tagged wolves in the first round, and k is the number of tagged wolves seen in the second round. Write out the formula to calculate the expectation of the Lincoln-Peterson estimator when $N = 400$, $n = 50$, and $K = 50$ (the only random part is k).

6. In basketball, baseball, and hockey playoffs, teams play each other for the best of 7 or 5 series.

   a. If team A and team B play each other and team A has a 60% chance of winning each game (whether home or away), what is the probability that team A will win the best of 7 series?

   b. Suppose that team A has a 60% chance of winning home games and a 50% chance of winning away games and that games alternate home and away (starting with a home game for A). What is the probability that team A wins a best of 7 series?

   c. Does the order that the teams play matter (as long as team A plays 4 home games and 3 away games)?

7. Your friend wants to sell his car, but they tell you they won't accept less than \$10,000. You think the car is worth more like \$8,000. Suppose offers will follow an exponential distribution with mean \$8,000.

   a. What is the probability the first offer will be greater than \$10,000?

   b. What is the probability your friend will have to take more than 5 offers before getting one that is \$10,000 or larger?

   c. What is the expected amount of time they would have to wait for an offer larger than \$10,000?

   d. Suppose they change their strategy to taking the largest of the first 5 offers. What would the expected sale price be?

8. Suppose you're on the game show *Let's Make a Deal*. The host reveals 3 doors. One contains a car and the other two contain goats, but you want the car. You pick one of the 3 doors. The host, Monty Hall, says, "Let's make a deal." He goes to the nearest (lowest in numerical order) door that contains a goat (for example, if you selected Door 1 and the car was indeed behind Door 1, then the host would always open Door 2, never Door 3). What are the probabilities that you will win the car if you stay with your original choice vs if you switch?

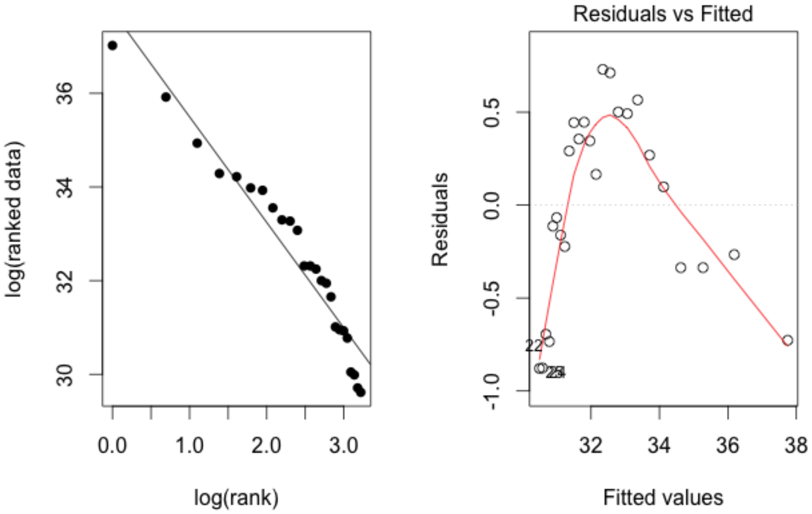
9. A popular model for long tailed distributions (meaning they are likely to take on large values) is power law type distributions. This has been used to model word frequency and city population. If X is power law distributions the X has cdf that is proportional to $x^{-\alpha}$ for $x > 0$ and some $\alpha > 0$.

   a. If $\alpha = 3$ then $f(x) = cx^{-3}$. Calculate the normalizing constant c.

   b. If $\alpha = 3$, calculate the expectation of X.

   c. Set up the maximum likelihood equation.

   d. A common method to fitting power law data is to plot the log of the ranks versus the log of the ranked data. Below is a figure of a power law linear fit. Does this fit look homoscedastic?



10. A random sample of 100 voters in a community produced 59 voters in favor of measure A. You would like to test to see if measure A passes.

    a. State the null and alternative hypotheses.

    b. Construct a 95% confidence interval for the support of measure A.

    c. Compute the p-value. Will measure A win with 90% confidence?

    d. Suppose the true support for measure A is 58%. What is the minimum sample size needed for a power of 90% at a 95% confidence level?