Forecasting Product Returns

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1 Introduction

Reverse logistics activities consist of collecting products from customers and reprocessing them for reuse. Returned products can take the form of end-of-life returns, where the product has been used by the customer, or commercial returns, where the product is returned before use.

Some products are leased to customers (e.g. Xerox copiers to corporate customers) and are collected by the manufacturer at the expiration of the lease. In this case, the timing and quantity of products to be returned are known in advance. The major uncertainty is about the condition of the product. Other products are sold to the customer and are returned when their useful life is over or when the customer wants to trade in the product for an upgrade. In the former category are products such as single-use cameras, toner cartridges and tires. In the latter category are durable products such as personal computers, cars and copiers. Predicting the proportion of such returns is important at a tactical level for procurement decisions, capacity planning and disposal management. At an operational level, detailed predictions of the quantities to be returned in each period, as well as the variability of these quantities, is useful, especially for inventory management and production planning.

Unlike end-of-life returns that have already been sold for profit and now have the potential of generating additional benefits through value recovery, commercial returns represent a lost margin. In catalog sales, an average return rate of 12% is standard, with return rates varying by product category: 5 – 9% in hard goods, 12 – 18% for casual apparel, 15 – 20% for high-tech products, and up to 35% for high fashion apparel [9]. Commercial returns impose high costs on retailers and manufacturers alike. The Gartner group estimates that the cost of processing returns for Web merchandise in 2000 was twice the value of the merchandise itself [22]. Currently, only 44% of returns are sold as new; 2% are trashed, 13% are liquidated, and 41% are sent back to the manufacturer [19].

Retailers and manufacturers strive to design reverse logistics systems that increase the visibility and speed of the return process to maximize asset recovery for commercial returns, especially for seasonal or short life-cycle products. Firms vary in how they address this problem. For example, Ingram Micro Logistics, the distribution arm of Ingram Micro, opened the first automated returns facility in the US in early 2001 [19]. Others increasingly rely on third-party reverse logistics providers such as GENCO Distribution System, UPS, USF Processors, Returns Online [13]. Various software products that are specifically targeted towards returns processing are now available on the market, provided by such companies as Kirus Inc., Retek.com, ReturnCentral and The Return Exchange [13]. Like end-of-life returns, an important lever in managing commercial returns is to accurately predict the return quantities for both tactical and operational level decisions.

Forecasting product returns, narrowly defined, is predicting the timing and quantity of returns within a given system based on past sales and return data. Methods that have been proposed in the literature for either end-of-life or commercial returns are described and compared in §2. The goal of such forecasting schemes is to provide input at an operational level; this section also reviews the literature on integrating forecasts of returns into inventory management decisions.

In this chapter, we take a broader view of forecasting product returns. The proportion of products returned depends to a large extent on a number of factors including the design of the product, the collection system, the customer interface, among others. Significant potential for profit maximization therefore lies in understanding what drives the proportion of returns and designing the system accordingly. In §3, we survey the academic literature, articles from the business press and some case studies to identify factors influencing return rates. In §4, we conclude with directions for future research in exploiting this information for better returns forecasting and management.

2 Forecasting Returns

One method for forecasting return volumes would be to use the time series consisting of past return volumes and apply time-series forecasting methods to it directly, but such a method would ignore the information contained in past sales data. Indeed, the key to forecasting returns is to observe that returns in any one period are generated by sales in the preceding periods. Alternatively, a sale in the current period will generate a return k periods from now with probability ν_k, $k = 1, 2, \ldots$ or not at all. All the methods used in the literature exploit this structure to postulate a return delay distribution and estimate its parameters.

A particular characteristic of the return delay data is that it is right-censored: At a given time, if an item has not been returned, it is not known whether it will be returned or not. For accurate estimation, it is important that the estimation method distinguish between items that are not yet returned and items that will never be returned.

We classify the forecasting methods used in the literature according to the data that they exploit. We say that period-level information is available if only the total sales and return volume in each period are known. For beverage containers, single-use cameras and toner cartridges, this is typically the only data available. We say that item-level information is available if the sale and return dates of each product are known. Electrical motors with electronic data logging technology [17], copiers, and personal computers are typically tracked individually, so this data can easily be obtained for these products. POS (point-of-sale) data technology in retailing also can allow for item-level tracking.

2.1 Period-level Information

A simple estimate of the return probability is to use the proportion of cumulative returns to cumulative sales. This method is known to be used in industry [11, 26]. It is useful only in estimating the return probability; no information about the return delay can be inferred. We refer to this method as "naive estimation."

Let n_t and m_t denote the sales and returns of products in month t, respectively. Goh and Varaprasad [11] propose a transfer function model of the form $m_t = \frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \ldots - \omega^s B^s}{1 - \delta_1 B - \delta_2 B^2 - \ldots - \delta^r B^r} n_{t-b} + \epsilon_t$, where B is the backshift operator, b is the time lag, and ϵ_t is the noise term. The determination of the appropriate transfer function model follows the steps of model identification, parameter estimation and diagnostic checking as described in Box and Jenkins [2].

Note that the transfer function model can be rewritten as $m_t = (\nu_0 + \nu_1 B + \nu_2 B^2 + \ldots) n_t + \epsilon_t$. Once the parameters of the transfer function model have been estimated, the coefficients $\{\nu_k, k \geq 1\}$ are easily calculated. The statistically significant values of these coefficients are used as estimates of the probability of return after k periods, for $k \geq 1$. The probability that a product is eventually returned is given by $\sum_{k=1}^{\infty} \nu_k$.

Goh and Varaprasad use this method to estimate the return quantities of Coca-Cola bottles. Data on sales and returns are available over sixty months from two bottling plants. They find that close to two thirds of the bottles are returned within one month of sales, and almost all containers that will ever be returned will be returned by the third month. The probability that a Coca-Cola bottle will never be returned is found to be less than 5%.

In practice, the data is augmented in each period as new sales and return information becomes available. The incremental nature of the information received makes Bayesian estimation a natural choice. Toktay et al. [26] assume that the return process can be modeled by

$$m_t = pr_D(1)n_{t-1} + pr_D(2)n_{t-2} + \ldots + pr_D(t-1)n_1 + \epsilon_t \quad t = 2, 3, \ldots, \qquad (1)$$

where p is the probability that a product will ever be returned, $r_D(k)$ is the probability that the product will be returned after k periods, conditional on ever being returned, and $\epsilon_t \sim N(0, \sigma^2)$. In this model, if a camera was sold in period t, the probability it comes back in period $t+k$ is $pr_D(k)$. This quantity corresponds to ν_k in [11].

The type of relation in Equation (1) is referred to as a 'distributed lag model' in Bayesian inference [27]. Usually, a specific form of distribution involving one or two parameters is assumed for the lag, which reduces the number of parameters to be estimated. The estimation procedure for a geometrically distributed lag with parameter q (the probability that a sold camera is returned in the next period, given that it will be returned) is illustrated in the appendix. It is also shown how to extend this method to a Pascal distribution, which allows more flexibility in the shape of the delay distribution.

Toktay et al. apply this method to data obtained from Kodak that consists of 22 months of sales and returns of single-use flash cameras. Using a geometric return distribution, they obtain estimates $\hat{p}$ and $\hat{q}$ equal to 0.5 and 0.58, respectively.

Since only two parameters need to be estimated, this method requires less data than the transfer function analysis proposed by Goh and Varaprasad. On the other hand, it lacks the generality of the latter method, since a given distribution is imposed on the data. A partial remedy is to do hypothesis testing with Pascal delay distributions, which allow for a more general delay distribution while remaining relatively parsimonious in the number of parameters to be estimated.

Toktay et al. test the hypotheses of geometric, Pascal lag one and Pascal lag two (as described in the appendix) on the Kodak data. The result supports using a geometric lag model. A geometric return delay makes practical sense for single-use cameras: Since most purchases are impulse decisions [12], prompted by a special occasion, it is likely that the camera will be used and returned quickly after the sale, which is consistent with a geometric distribution.

2.2 Item-Level Information

When items are tracked on an individual basis, it is possible to determine the exact return delay of returned items. For items that have not been returned yet, it is known that the delay is longer than the elapsed time, or possibly infinite (corresponding to a product never being returned). Dempster et al. [7] introduced the Expectation Maximization (EM) algorithm to compute maximum likelihood estimates given incomplete samples. This algorithm can be effectively used to estimate the return delay distribution using censored delay data. The EM algorithm is illustrated in the appendix for a geometric delay distribution.

Hess and Mayhew [14] consider commercial returns and propose a split-adjusted hazard rate model and a regression model with logit split to estimate the return probability and the return delay distribution. In contrast with the papers cited earlier, they augment their models with dependent variables such as the price and fit of the product. The logit model is a discrete choice model, which simultaneously estimates a baseline return rate and the impact of external factors on that rate. By combining the logit model with basic hazard rate or regression models estimating the return delay, the authors avoid the inaccuracy (due to the right-censoring of the data) that would be engendered if only the latter models were used. A description of these models is given in the appendix.

2.3 Comparison of Forecasting Methods

The naive estimate only requires the aggregate sales and return information to date. The data requirements of this method are the lowest. On the other hand, this method ignores the effect of the return delay and consequently generates a biased estimate of the return probability when the time horizon is short (although it is asymptotically unbiased when the return delay is finite). The bias is larger if the return delay is larger. All other models avoid this bias explicitly modeling the return probability and the return delay.

The naive estimate, the distributed lags bayesian inference model and the EM algorithm are particularly suited to updating return flow parameters over time. We illustrate the performance of these methods in Figures 1 – 3, which are generated as follows: The number of sales in each period is a Poisson random variable with parameter 200, 2000, and 20000, respectively, labeled as low, medium and high sales volumes, respectively. The return probability is $p = 0.5$. The return delay is geometric with a mean return delay of eight periods ($q = 0.125$). Parameter estimation starts three periods after returns are first observed; estimates are updated in each period using the most recent sales and return volumes. The evolution of $\hat{p}$ and $\hat{q}$ over the forty periods of data estimation are plotted in Figures 1 and 2 by method and by volume. Figures 3 makes a direct comparison of convergence rates across methods for a fixed sales volume. The estimates are averages over thirty simulation runs.

As expected, the EM algorithm clearly outperforms bayesian inference with a distributed lags model. This is because item-level information is present in the former. Figure 1 shows that the speed of convergence of the EM algorithm depends on the sales volume per period: In this example, two periods, five periods, and twenty periods, respectively, are needed for the confidence interval of the return probability estimate to include the true value of the parameter in the cases of high volume, medium volume and low volume, respectively. While it is to be expected that the accuracy of the estimate in the EM algorithm directly depends on the volume of data, it is particularly striking that the algorithm achieves such accuracy after only two periods in the high-volume scenario.

With period-level data, the convergence of the estimate depends primarily on the number of periods of data available: In Figure 2, the estimate for the return probability reaches the vicinity of 0.5 after eighteen periods of returns for all sales volumes. The demand volume does not impact the point at which the estimate converges, but it is significant in determining the accuracy of the method in the periods up to that point.

Figures 1 and 2 further show that the estimate of the return delay is more robust than the estimate of the return probability; it fluctuates less from period to period under both algorithms.

Figure 3 shows that the methods taking into account the return delay clearly dominate naive estimation, which systematically underestimates the return probability. The bias of this estimate decreases in time, but in this example, it is still 20% less than the true value after forty periods.

Hess and Mayhew apply their methods to simulated data containing 2000 sales whose return delay is exponential with a mean of 3.3 weeks, and show that the hazard model outperforms the regression model. The reason for the superiority of the hazard rate model is that a general hazard rate distribution was assumed, whereas the regression model restricts the analysis to normally distributed errors.

2.4 Inventory Management using Returns Forecasts

Given past sales volumes and estimates of the return probability and the return delay distribution, it is possible to approximate the distribution of returns in future periods. Given n_t, the vector $(m_{t,t+1}, m_{t,t+2}, \ldots)$ that denotes returns from sales in period t has a multinomial distribution with probability vector ν (or pr_D). Based on this fact, Kelle and Silver [15] develop normal approximations for demand over a time horizon of L periods under both period-level and item-level information. For a base-stock level defined by $E[D_L] + k\sigma_{D_L}$, they compare the deviation between the base stock levels obtained under the two information structures. For the range of parameter values that they investigate, the difference ranges between 0.5% to 30%.

These experiments are for a single order only, and assume that the return flow parameters are already known. Kelle and Silver [16] formulate a deterministic dynamic lot sizing problem taking into account future returns in net demand forecasts. The impact of future returns is that net demand may be negative. The authors develop a transformation into the nonnegative demand case. The Wagner-Whitin deterministic lot-sizing procedure can then be applied to determine procurement quantities in each period. In practice, since new sales and returns are recorded in each period, and return flow parameters could be updated periodically, rolling horizon decision making would be more appropriate. In this case, a heuristic that is more robust than the Wagner-Whitin algorithm [1] could be used. It would be interesting to compare the value of the additional information provided by item-level information in this setting.

Toktay et al. develop adaptive procurement policies using dynamically updated return flow parameter estimates in the context of the single-use camera supply chain. They use discrete-event simulation to compare the system (inventory, lost sales and procurement) cost under period-level versus item-level information, and investigate the impact of sales volume and product life-cycle length on the relative benefits of the two informational structures. They conclude that the accurate estimation of the return probability and of the quantity of products that will be returned are the most important levers in achieving low operating cost. In addition, they demonstrate that the relative benefit of using item-level instead of period-level information is highest when the total demand volume for a product over its life-cycle is low. This is consistent with the results discussed in the previous subsection concerning the convergence rate of the two algorithms exploiting different levels of data aggregation. The EM algorithm does significantly better than bayesian inference using the distributed lags model over a given initialization period when the demand volume is low.

3 Factors Influencing Returns

The literature review in §2 shows that papers forecasting end-of-life returns use only sales and return data. Explanatory factors that could increase the accuracy of prediction are not incorporated in the analysis. Hess and Mayhew bring in this dimension in their paper on forecasting commercial returns. They hypothesize that a higher price will increase the probability of return and that items where fit is important are more likely to be returned. On data from a direct marketer of apparel, they find that the return probability is positively correlated with price, but that differences in fit have little impact on the return rate. Hess and Mayhew suggest that for commercial returns estimation can be carried out at a customer level to identify individual return patterns. If this data is not available, they propose that it be carried out on aggregate data to identify patterns at the product or product family level. The goal is to identify higher-profitability customers and products by taking into account not only the sales information but also the return information.

Hess and Mayhew only consider price and fit as dependent variables. In practice, many factors could affect the probability and the delay in returns, for both end-of-life and commercial returns. Incorporating explanatory variables into predicting return flow characteristics could therefore increase the accuracy of the prediction. Equally, if not more important, are the benefits of quantifying the impact of such factors on the volume and timing of returns. Such information would be very valuable in maximizing the profitability of a given product line by optimizing over these factors.

As one possible example of the use of quantifying the effect of explanatory variables on return flows, consider the Kodak single-use camera. Customers take the used cameras to a photofinishing laboratory, where the film is taken out and processed. The laboratories receive a small rebate for each used camera that they subsequently return to Kodak. Due to economies of scale in transportation, small photoprocessors either wait for a long time before sending a batch back to Kodak or do not send in cameras at all, significantly adding to the return delay and influencing the return percentage. The reusable parts (the circuit board, plastic body and lens aperture) of the returned cameras are put back into production after inspection. The circuit board, which can be used several times, is the most costly component in a single-use camera. Therefore, used boards are valuable to Kodak as long as the product design allows them to be reused, although they have minimal salvage value.

The initial design of the product was constrained by the size of the circuit board. Subsequently, Kodak introduced a pocket-size camera that required a smaller circuit board. As a result, a number of larger-size boards would become obsolete by the time they were returned to Kodak. In this setting, an integrated design of the collection policy and the new product introduction decisions would have been valuable to Kodak. To carry out this analysis, the impact of changing incentives provided to consumers and to photoprocessors would need to be assessed. The hypothesis is that the higher the rebates, the more and quicker the returns - the elasticity of return rate and delay to the rebate quantity is a concise measure that captures this interaction.

Once a model of the dependence of returns on such factors as rebate level and ease of return has been developed and its parameters estimated, a cost-benefit analysis can be carried out to investigate the value of investing in collecting used products more rapidly versus delaying the introduction of the new product line. When take back is mandatory, such that the return percentage is close to 100%, the return delay can have a huge impact on the bottom line, especially in the electronics industry where value depreciation is high. In general, the trade-off between investing in collection versus the value that would be generated by this effort needs to be quantified.

One problem that Kodak faced in collecting its products was that opportunistic third parties would load the used camera with new film and sell the camera at a lower price, sometimes claiming it to be a Kodak camera. In addition to potentially reducing the quality-perception of consumers regarding the product, this phenomenon also reduced the return rate. In the tire industry, the technology to remanufacture a tire is relatively cheap, resulting in a proliferation of small third-party remanufacturers. Investing in and promoting a higher-quality proprietary remanufacturing technology allows Michelin to remain one of the main remanufacturers of its own products. As these two examples highlight, the choice of production technology and product design can influence the return rate.

In the tire industry, 5% of car tires, 30% of light truck tires and almost all of the truck tires are retreaded [3]. Tires are bulky items that are costly to transport. Therefore there are important economies of scale in their collection. They are typically collected by garages and dealers, parties that are difficult to reach at low cost by tire manufacturers or retreaders. For this reason, companies exist whose sole business is to purchase tires from garages, and sell them to tire remanufacturers. Clearly, the geographical dispersion in the market, the size of the outlets, and the transportation costs per unit play a role in determining returns. The structure of the collection channel, which is to some extent in the control of manufacturers, also impacts returns. Savaşkan et al. [24] show that if the retailer is in charge of collecting used products (as opposed to the manufacturer or a third-party), then the fraction returned will be higher. Thus, the impact of the collection channel on returns should be incorporated into decisions regarding supply chain design.

The environmental consciousness of consumers would be expected to positively impact the return probability. According to OECD data [20], countries differ in recyling rates. In 1997, glass recycling rates were 26% in the US, 52% in France and 79% in Germany, and paper recycling rates were 40% in the US, 41% in France and 70% in Germany. On the other hand, 65% of Americans and 59% of Germans expressed their willingness to pay a premium on an eco-safe product [10], suggesting that the receptiveness to recycling in Germany is lower than in the US. The significant difference observed in practice could be a function of a variety of other factors such as infrastructure and promotional expenditures. Clarifying the reasons for this difference is relevant to a company who will expand to a new market.

According to CEMA (Consumer Electronics Manufacturing Association) research, a store's return policy is "very important" for 70% of consumers in their decision to shop there [21]. This is also true for e-tailers: In a survey by Jupiter Media Metrix, 42% of online shoppers said they would buy more from the Internet if the returns process was easier [23].

E-tailers vary widely in terms of the returns policies they offer. A search on www.buyersindex.com reveals return policies ranging from conditional returns within several days to unconditional lifetime returns in various product categories. Lenient return policies may increase demand for a retailer's products, but they may also increase the return rate. There are conflicting opinions about this tradeoff. While some managers say that it is a myth that "if a company makes it easy for consumers to return products, they will send back more items" [19], theory claims that this is not a myth, but reality [6].

The reason for this dichotomy may be that the outcome depends on the product, the customer, and the market. A lenient return policy acts as a signal of quality, much like a warranty. Moorthy and Srinivasan [18] show that money-back guarantees are effective signalling devices as customers assume that it is costly for a low-quality retailer to offer this service. This effect would be higher for products for which achieving high quality is costly. Return policies allow the customer to test the good before making the final purchase decision. Che [4] shows that full-refund return policies maximize retailer profits only if customers are sufficiently risk averse or if retail costs are high. The sales medium (on-line versus in-store) would be expected to impact the profitability a given returns policy because it changes the point at which customers are able to test the good. Tailoring the return policy to the target market, the distribution medium and the product remains a significant challenge.

It is claimed that to reduce commercial return rates, retailers can resort to a number of strategies such as clear packaging, follow up calls, toll-free help lines and information sharing about reasons for returns [21]. Determining which of these factors are those that significantly impact commercial returns would be instrumental in allocating resources spent on attempting to reduce return rates.

4 Conclusions

In this chapter, we reviewed the existing literature on forecasting product returns, both for end-of-life and commercial returns. Despite the clear financial impact of product returns on profitability, the literature on this topic is relatively limited. Research has focused on pure forecasting [11, 14, 15] and inventory management incorporating updated forecast information [16, 26].

Integrated returns forecasting and inventory management has been analyzed primarily in the context of end-of-life returns. However, the timing and quantity of commercial returns is a significant determinant of the profitability of a product offering, especially for short life-cycle items. Developing methods to incorporate forecast information about commercial returns in stocking decisions is a potential avenue of research.

Inventory management, an operational-level problem, is not the only facet of supply chain management that is affected by return flow characteristics. We have discussed several system design issues – rebate policy, collection channel design, product design, timing of new product introduction – that would benefit from an integrated approach incorporating the impact of design on return flows.

To address these design problems, two complementary methodologies need to be pursued: empirical and model-based. Return forecasts typically do not take into account explanatory variables that would improve forecast accuracy. Relevant factors are price, rebate level, ease and cost of return, environmental consciousness of consumers, structure of the collection channel, return policy, sales medium, and level of after-sales follow up. Empirical research is necessary to test hypotheses concerning the impact of these explanatory variables on returns behavior. The resulting information can then be used as an input to models of integrated supply chain design or product design and returns management.

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Appendix

Bayesian estimation for the distributed lags model

Recall that p is the probability a sold camera will ever come back and $r_D(d)$ is the distribution governing the return delay. Assume $r_D(d)$ is geometric. Let q be the parameter of the geometric delay distribution, that is, q is the probability that a sold camera is returned in the next period given that it will eventually be returned. Now $r_D(k) = q(1-q)^{k-1}, \;\; k = 1, 2, \ldots$, and $m_t = pqn_{t-1} + pq(1-q)n_{t-2} + pq(1-q)^2 n_{t-3} + \ldots + \epsilon_t, \quad t = 1, 2, \ldots$. We assume that ϵ_t's are iid Gaussian with variance σ^2. Suppose that data is available for the first T periods. Subtracting $(1-q)m_t$ from both sides of the above relation, we obtain $m_t = (1-q)m_{t-1} + pqn_{t-1} + \epsilon_t - (1-q)\epsilon_{t-1}, \quad t = 2, 3, .., T$, which is the form to be used in the analysis. Let $\mathbf{u} = (u_2, u_3, .., u_T)$ where $u_t = \epsilon_t - (1-q)\epsilon_{t-1}$. The covariance matrix for the error term is $E(\mathbf{uu}') = \sigma^2 G$ where

$$G_{(T-1)x(T-1)} = \begin{pmatrix} 1+(1-q)^2 & -(1-q) & 0 & .. & 0 \\ -(1-q) & 1+(1-q)^2 & -(1-q) & .. & 0 \\ : & : & : & : & : \\ 0 & 0 & 0 & -(1-q) & 1+(1-q)^2 \end{pmatrix}$$

The joint pdf for $\mathbf{m} = (m_2, m_3, ..., m_T)$ is

$$f(\mathbf{m} \mid p, q, \sigma, m_1) \propto \frac{\mid G \mid^{-1/2}}{\sigma^T} exp[-\frac{1}{2\sigma^2}(\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})' G^{-1} (\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})]$$

. If we take the prior pdf for the parameters of the model to be $f(p, q, \sigma) \propto \frac{1}{\sigma}$, the posterior pdf becomes

$$f(p, q, \sigma \mid \mathbf{m}, m_1) \propto \frac{\mid G \mid^{-1/2}}{\sigma^{T+1}} exp[-\frac{1}{2\sigma^2}(\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})' G^{-1} (\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})].$$

Integrating with respect to σ, we obtain

$$f(p, q \mid \mathbf{m}, m_1) \propto \frac{\mid G \mid^{-1/2}}{[(\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})' G^{-1} (\mathbf{m} - (1-q)\mathbf{m}_{-1} - pq\mathbf{n})]^{T/2}}.$$

Now the normalizing constant can be calculated, and hence the joint posterior pdf of p and q. It is then straightforward to calculate the marginal densities of p and q and find their expected values to be used as parameter estimates.

To generalize this analysis to Pascal distributions is straightforward. Only the expression relating m_t, m_{t-1} and n_{t-1} changes, and as a consequence, the matrix G. The rest of the analysis is the same. We illustrate this for the case of Pascal of order 2. In this case, $m_t = 2(1-q)m_{t-1} - (1-q)^2 m_{t-2} + pq^2 n_{t-2} + u_t, \quad t = 3, 4, .., T$, where $u_t = \epsilon_t - 2(1-q)\epsilon_{t-1} + (1-q)^2 \epsilon_{t-2}$, and G is a symmetric (T-2)x(T-2) matrix whose nonzero entries are of the form $E(u_k^2) = 1 + 4(1-q)^2 + (1-q)^4$, $E(u_k u_{k+1}) = E(u_{k+1} u_k) = -2(1-q)(1 + (1-q)^2)$, and $E(u_k u_{k+2}) = E(u_{k+2} u_k) = (1-q)^2$.

It is also possible to compare different distributed lag models by assigning prior odds ratios and determining posterior odds ratios, from which posterior probabilities associated with the models can be computed. For example, let us consider three alternative models for the delays: geometric (H_1), Pascal of lag two (H_2) and Pascal of lag three (H_3). Assume prior odds ratios $P(H_i)/P(H_j) = 1 \ \forall i, j$. The posterior odds ratio relating H_i and H_j is given by

$$K_{ij} = \frac{P(H_i) \int\int\int f(\mathbf{m} \mid p, q, \sigma_i, m_1, H_i) f(p, q, \sigma_i | H_i) d\sigma_i dp dq}{P(H_j) \int\int\int f(\mathbf{m} \mid p, q, \sigma_j, m_1, H_j) f(p, q, \sigma_j | H_j) d\sigma_j dp dq}.$$

Now posterior probabilities π_i, $i = 1, 2, 3$ can be calculated using $\pi_i = 1/(1 + \sum_{i \neq j} K_{ji})$.

The Expectation Maximization Algorithm

Let s_i = shipment time of unit i, $i = 1, \ldots n$ and r_i = return time of item i, $i \ldots m$, where $m \leq n$. Set $r_i = \infty$ for items that are not returned, and index the items so that units $i = 1 \ldots m$ have been returned. Let T denote the elapsed time from the sale to the return of a camera, where T_1, $T_2 \ldots T_n$ are independent identically distributed. Let t be the current time. We assume T is geometric with parameter q (probability of return in the next period given the camera will be returned) and let p denote the return probability of a camera. Following the notation of Cox and Oakes (1984), let $x_i = min(r_i - s_i, t - s_i)$ and $v_i = I_{\{r_i \leq t\}}$.

The Expectation Maximization algorithm can be defined as follows:
Denote by $l_o(\phi) = l_o(\phi; T)$ the log likelihood of the data $(T_1, \ldots, T_n)$ that would be observed if there were no censoring and by $l(\phi) = l(\phi; x, v)$, the log likelihood of the data (x, v) that are actually observed. (In our case, $\phi = (p, q)$) Define $Q(\phi\prime, \phi) = E(l_o(\phi\prime; T \mid x, v; \phi))$ to be the conditional expectation of the log likelihood based on T, given the observations (x, v). Then the two steps of the algorithm are:

Expectation step: Given the current estimate $\hat{\phi}_j$ of ϕ, calculate $Q(\phi\prime, \hat{\phi}\prime_j)$ as a function of the dummy argument $\phi\prime$.

Maximization step: Determine a new estimate $\hat{\phi}_{j+1}$ as the value of $\phi\prime$ that maximizes $Q(\phi\prime, \hat{\phi}\prime_j)$.

The likelihood function for the full data set is

$$\begin{aligned} l(p\prime, q\prime; T) &= \prod_{\{i|r_i<\infty\}} p\prime q\prime (1-q\prime)^{r_i - s_i} \prod_{\{i|r_i=\infty\}} (1-p\prime) \\ &= (1-p\prime)^{n-k} p\prime^k q\prime^k (1-q\prime)^{\sum_{\{i|r_i<\infty\}}(r_i - s_i)} \end{aligned}$$

where k equals the number of items that get recycled eventually. The log-likelihood is given by

$$l_o(p\prime, q\prime; T) = k \log p\prime + (n-k)\log(1-p\prime) + k \log q\prime + \sum_{\{i|r_i<\infty\}} (r_i - s_i)\log(1-q\prime).$$

$$\begin{aligned} Q(p\prime, q\prime, p, q) &= E(l_o(p\prime, q\prime; T) \mid x, v; p, q) \\ &= E(k \mid x, v; p, q)\{\log p\prime - \log(1-p\prime) + \log q\prime\} + n\log(1-p\prime) \\ &\quad + E(\sum_{i=1}^{n} I\{r_i < \infty\}(r_i - s_i) \mid x, v; p, q)\log(1-q\prime) \end{aligned}$$

where

$$E(k \mid x, v; p, q) = m + \sum_{i=m+1}^{n} \frac{p(1-q)^{t-s_i+1}}{1-p+p(1-q)^{t-s_i+1}}$$

and

$$E(\sum_{i=1}^{n} I\{r_i < \infty\}(r_i - s_i) \mid x, v; p, q) = \sum_{i=1}^{m}(r_i - s_i) + \sum_{i=m+1}^{n}(t - s_i + \frac{1-q}{q})\frac{p(1-q)^{t-s_i+1}}{1-p+p(1-q)^{t-s_i+1}}.$$

Setting the derivatives of $Q(p\prime, q\prime, p, q)$ with respect to $p\prime$ and $q\prime$ equal to 0 and solving for $p\prime$ and $q\prime$ yields the following recursive relation:

$$\hat{p}_{j+1} = \frac{1}{n}(m + \sum_{i=m+1}^{n} \frac{\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}}{1-\hat{p}_j+\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}})$$

$$\hat{q}_{j+1} = \frac{m + \sum_{i=m+1}^{n} \frac{\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}}{1-\hat{p}_j+\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}}}{m + \sum_{i=1}^{m} r_i - s_i + \sum_{i=m+1}^{n}(t - s_i + 1 + \frac{1-\hat{q}_j}{\hat{q}_j})\frac{\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}}{1-\hat{p}_j+\hat{p}_j(1-\hat{q}_j)^{t-s_i+1}}}$$

The Split Adjusted Hazard Model

Hess and Mayhew [1] use the baseline hazard rate function

$$h_0(t|\boldsymbol{\alpha}, \delta = 1) = \frac{2\alpha_1\alpha_2}{\sqrt{\pi}} exp[-(\alpha_2 t + \alpha_3)^2] + exp(\alpha_4), \tag{2}$$

where the condition $\delta = 1$ indicates that the item will be returned. This functional form allows for flexibility in hazard rate modeling. Let the vector $\boldsymbol{x}$ summarize factors which are candidates for influencing the return flows. The adjusted hazard function incorporating these factors is proposed to be

$$\begin{aligned} h(t|\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{x}, \delta = 1) &= h_0(t|\boldsymbol{\alpha}, \delta = 1)\theta(\boldsymbol{\beta}, \boldsymbol{x}) \\ &= \left(\frac{2\alpha_1\alpha_2}{\sqrt{\pi}} exp[-(\alpha_2 t + \alpha_3)^2] + exp(\alpha_4)\right) exp(\boldsymbol{\beta}'\boldsymbol{x}). \qquad (3) \end{aligned}$$

Adding the information about non-returns results in the split hazard function whose log-likelihood is given by

$$\begin{aligned} logL &= \sum_i log[f(t_i|\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{x}_{it})] \\ &= \sum_i log[f(t_i|\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{x}_{it}, \delta_i = 1)(\delta_i = 1) + 1(\delta_i = 0)] \qquad (4) \\ &\approx \sum_i log[f(t_i|\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{x}_{it}, \delta_i = 1)\pi_i(R_i = 1) + [(1-\pi_i) + S_{it}\pi_i](R_i = 0)]. \quad (5) \end{aligned}$$

The last equation is an approximation: Since it is not known whether an item will be returned or not, a probability π for the return probability is estimated using the logit function $\pi_i = exp(\boldsymbol{x}_{it}\boldsymbol{y}_i)/(1 + exp(\boldsymbol{x}_{it}\boldsymbol{y}_i))$. The variable S_{it} denotes the probability that item i will be returned after time t conditional on being returned. R indicates whether the item was already returned.

The Regression Model with Logit Split

Hess and Mayhew jointly estimate the parameters of the logit model and the regression model in a two-stage process. First, they carry out the maximum-likelihood estimation of the above logit model. Then, for each observed return, they enter a function of the resulting probability of return as a new variable $t_i = \boldsymbol{\beta}'\boldsymbol{x}_i - \rho\sigma\phi[\Phi^{-1}(\hat{\pi}_i)]/\hat{\pi}_i$ in the regression. Here estimates of ρ, the correlation between the time of return and the estimated logit term, and of σ, the standard deviation of the logit error for observation i are used.

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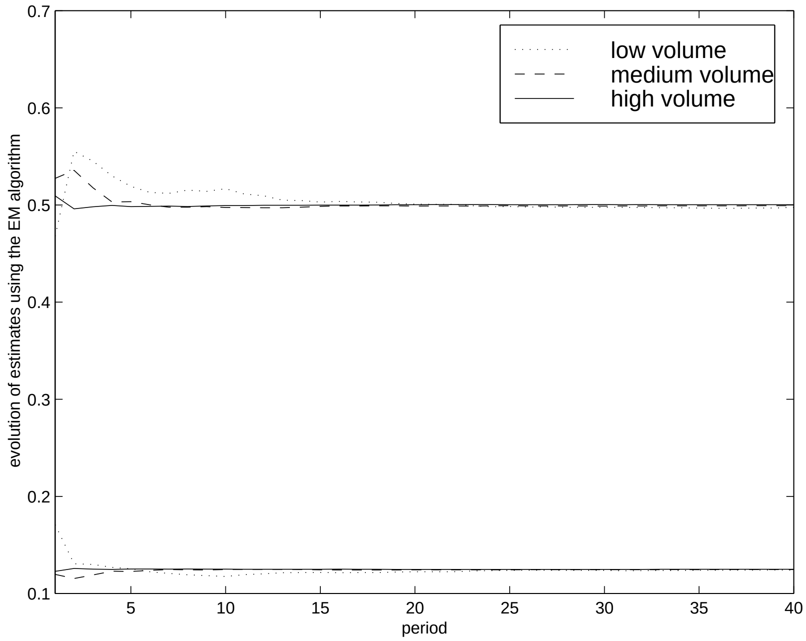

Figure 1: The top (bottom) three lines plot the evolution of the estimate of the return probability (delay) using the Expectation Maximization algorithm. The true values of the return probability and return delay are 0.5 and 0.125, respectively.

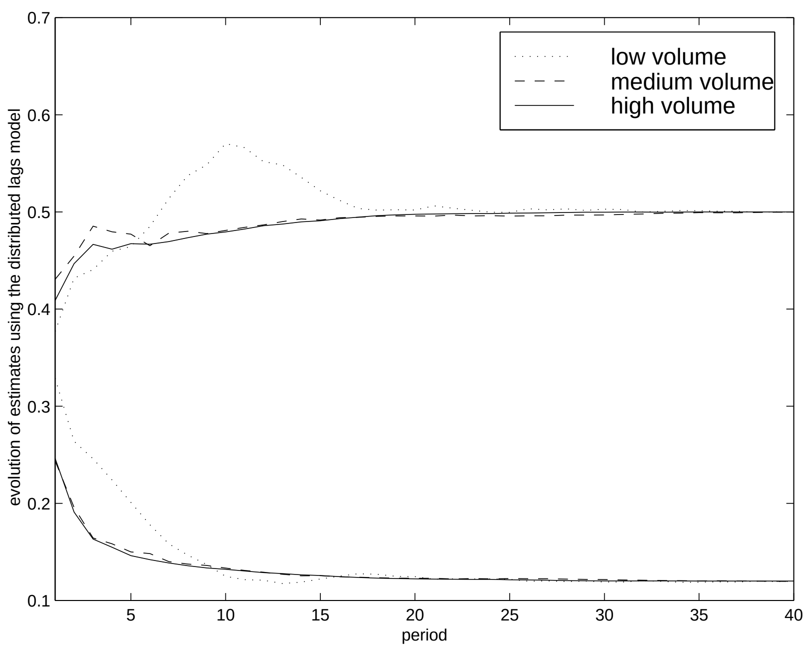

Figure 2: The top (bottom) three lines plot the evolution of the estimate of the return probability (delay) using the distributed lags model. The true values of the return probability and return delay are 0.5 and 0.125, respectively.

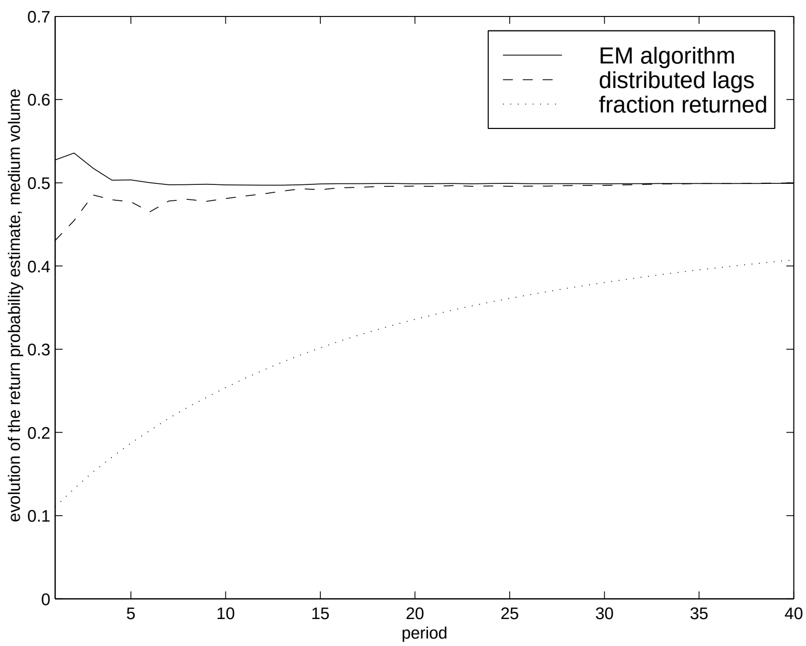

Figure 3: The evolution of the estimate of the return probability under three different estimation methods for a medium sales volume.