

ECO567A: Solution to 2022 Exam

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This document provides a fully detailed answer to the Winter 2022 exam which you may find on Moodle. We try to have both a mathematical and a graphical approach. You may read this document and make sure you understand each step. It is significantly more detailed than the answers expected in the exam.

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1 Exercise 1

Suppose the market demand for a good is given by $Q = 84 - 15P$, where Q is the quantity demanded and P is the price per unit.

The total cost of producing Q units of the good is $\frac{5}{2}Q^2$, so that the marginal private cost of supplying the good is $5Q$.

Suppose that the total environmental costs of producing Q units are given by Q^2 , so that the marginal environmental cost of supplying the good is $2Q$.

1.1 Question 1: What are the marginal social costs of producing the good?

The marginal *social* cost of producing the good accounts for (i) the marginal private cost of supplying the good and (ii) the marginal environmental cost. It is given by:

$$MSC(Q) = MPC(Q) + MEC(Q) = 5Q + 2Q = 7Q$$

1.2 Question 2: Calculate and illustrate graphically.

(a) **The socially optimal output level Q^* and associated price P^* .** The socially optimal allocation is obtained as the intersection of the marginal social cost curve with the inverse demand function. This corresponds to the optimal allocation when we account for the cost of pollution. We highlight it in Figure 1 below:

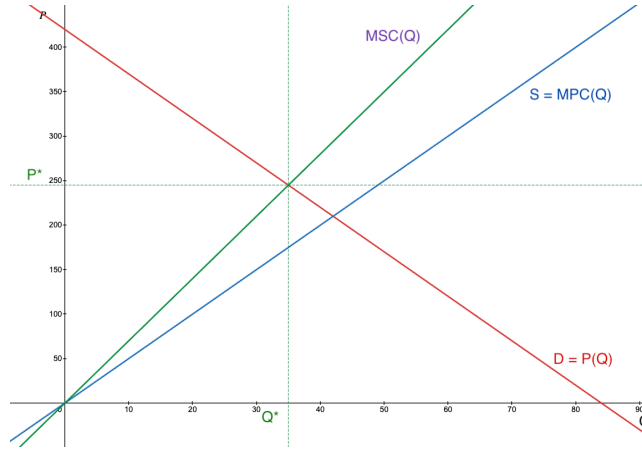


Figure 1: Socially optimal allocation

To determine the socially optimal quantity, we can thus equate the inverse demand function with the marginal social cost function. The inverse demand function $P(\cdot)$ expresses the price as a function of the quantity demanded. We

obtain it by inverting the demand function: $Q = 84 - \frac{1}{5}P(Q) \iff P(Q) = 420 - 5Q$. Then, we solve the equation:

$$P(Q^*) = MSC(Q^*) \iff 420 - 5Q^* = 7Q^* \iff 12Q^* = 420 \iff Q^* = 35$$

We move from quantity to price via the inverse demand function. We plug the socially optimal quantity into the inverse demand function:

$$P^* = P(Q^*) = 420 - 5Q^* = 420 - 5 \times 35 = 245$$

So, the socially optimal allocation is $(Q^*, P^*) = (35, 245)$.

(b) The unregulated competitive equilibrium output level Q_c and associated price P_c . The equilibrium is obtained where the demand and supply curves meet. Graphically, this corresponds to the intersection of the two curves. Figure 2 highlights it.

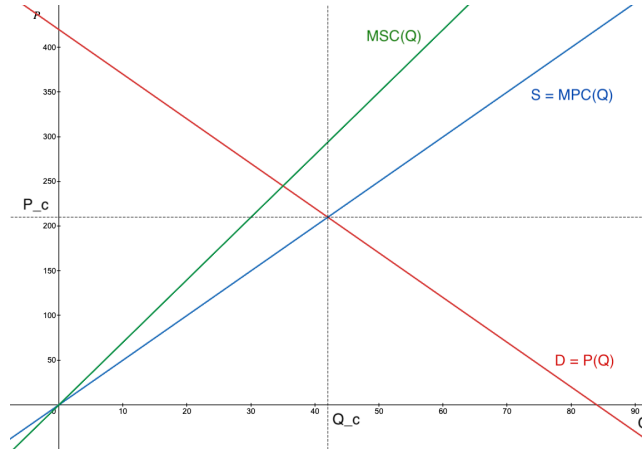


Figure 2: Unregulated equilibrium

Mathematically, we derive the resulting quantity Q^c by solving the equation:

$$\text{Demand curve} = \text{Supply curve} \iff P(Q^c) = MPC(Q^c)$$

Replacing with the expressions defined above:

$$420 - 5Q^c = 5Q^c \iff 420 = 10Q^c \iff Q^c = 42$$

Now that we have the equilibrium quantity, we can deduce the equilibrium price by plugging it into the inverse demand function (or, indifferently, into the marginal private cost function):

$$P^c = P(Q^c) = 420 - 5Q^c = 420 - 5 \times 42 = 210$$

We conclude that the unregulated equilibrium is $(Q^c, P^c) = (42, 210)$.

(c) The resulting inefficiency (i.e., deadweight) loss DWL. What is the deadweight loss? The socially optimal allocation that we have derived in Question 2(a) maximizes total surplus. For any other allocation that we consider (in particular the one obtained with the unregulated equilibrium, etc.), the deadweight loss is the difference between the maximum total surplus obtained with the socially optimal allocation and the total surplus obtained with the allocation considered.

In the case of the unregulated equilibrium, we have:

$$DWL^c = TS^* - TS_c$$

Where TS^* denotes the maximum total surplus obtained with the socially optimal allocation of Question 2(a).

Now, what is total surplus? Consider an allocation with price P and quantity Q . Total surplus corresponds to the sum of the consumer surplus and the producer surplus, to which we subtract the cost of the pollution externality for society. This writes as:

$$TS = CS + PS - EC$$

We said in class that the consumer surplus is defined as the area below the demand curve and above the price line. Mathematically, it corresponds to the integral of the difference between the inverse demand function and the price over the interval of quantities traded and consumed:

$$CS = \int_0^Q [P(q) - P] dq$$

Similarly, we said that the producer surplus (equal to variable profits by the way) is the area above the supply curve and below the price line. Formally, we compute it as the integral of the difference between the price and the marginal private cost function over the interval of quantities produced and sold:

$$PS = \int_0^Q [P - MPC(q)] dq$$

Eventually, the external cost can be written as the integral of the marginal external cost over the interval of quantities traded:

$$EC = \int_0^Q MEC(q) dq$$

So, total surplus is given by:

$$\begin{aligned} TS &= \int_0^Q [P(q) - P] dq + \int_0^Q [P - MPC(q)] dq - \int_0^Q MEC(q) dq \\ &= \int_0^Q [P(q) - MPC(q) - MEC(q)] dq \\ &= \int_0^Q [P(q) - MSC(q)] dq \end{aligned}$$

To get the maximum total surplus, obtained with the socially optimal allocation, we can replace Q by Q^* in the equation above. Similarly, for the unregulated equilibrium, we can replace Q by Q_c . This gives:

$$\begin{cases} TS^* &= \int_0^{Q^*} [P(q) - MSC(q)] dq \\ TS^c &= \int_0^{Q_c} [P(q) - MSC(q)] dq \end{cases}$$

We deduce the deadweight loss in the unregulated equilibrium:

$$\begin{aligned} DWL_c &= TS^* - TS_c \\ &= \int_0^{Q^*} [P(q) - MSC(q)] dq - \int_0^{Q_c} [P(q) - MSC(q)] dq \\ &= \int_{Q_c}^{Q^*} [P(q) - MSC(q)] dq \\ &= \int_{Q^*}^{Q_c} [MSC(q) - P(q)] dq \end{aligned}$$

The deadweight loss is computed as the integral of the difference between the marginal social cost function (how costly producing a given unit is to society) and the inverse demand function (how valuable this same unit is for consumers) over the interval of quantities traded *in excess of the socially optimal quantity*. This difference being positive reflects the idea that the production of each of these excess units costs more to society than the benefit that consumers draw from consuming it. **A valid answer in the exam could directly start from this formula.** We finalize the computations:

$$\begin{aligned} DWL_c &= \int_{Q^*}^{Q_c} [MSC(q) - P(q)] dq \\ &= \int_{Q^*}^{Q_c} [7q - 420 + 5q] dq \\ &= -420(Q_c - Q^*) + 6 \int_{Q^*}^{Q_c} 2q dq \\ &= -420(42 - 35) + 6 [q^2]_{35}^{42} \\ &= -420 \times 7 + 6(42^2 - 35^2) \\ &= 294 \end{aligned}$$

So, the deadweight loss is $DWL_c = 294$ in the unregulated equilibrium. Graphically, the deadweight loss corresponds to the area below the marginal social cost curve and above the demand curve over the quantities traded in excess of the social optimum.

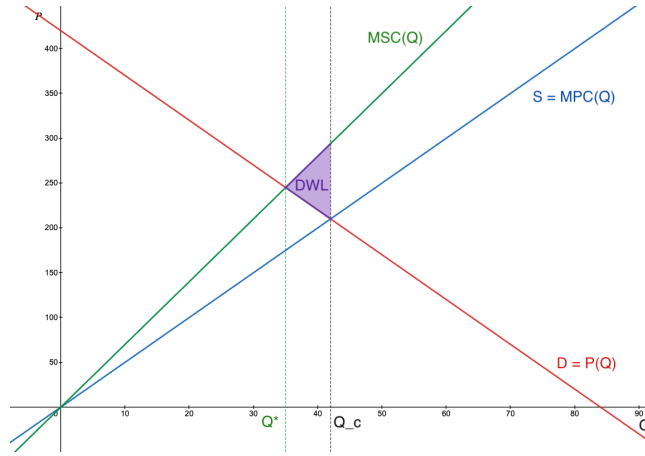


Figure 3: Deadweight loss in the unregulated equilibrium

1.3 Question 3: Determine the optimal production tax per unit of output. What is the amount of tax revenue collected by the government? (5 points)

Denote this tax or subsidy by τ . It is a tax if $\tau > 0$ and a subsidy if $\tau < 0$. For each unit that it produces, the firm pays or receives τ dollars. τ enters the marginal private cost. A tax increases the marginal cost of production, a subsidy reduces it. We have:

$$MPC^{\text{Regulated}}(Q, \tau) = MPC(Q) + \tau$$

In the unregulated equilibrium, in Question 2(b) we used the following condition to derive the equilibrium quantity being traded:

$$P(Q_c) = MPC(Q_c)$$

Given a tax or subsidy τ , this condition becomes:

$$P[Q_c(\tau)] = MPC^{\text{Regulated}}[Q_c(\tau), \tau]$$

So, we can solve for the quantity traded under perfect competition given a tax or subsidy τ , $Q^c(\tau)$:

$$\begin{aligned} P[Q_c(\tau)] &= MPC[Q_c(\tau)] + \tau \\ \iff 420 - 5Q^c(\tau) &= 5Q_c(\tau) + \tau \\ \iff 10Q^c(\tau) &= 420 - \tau \\ \iff Q^c(\tau) &= 42 - \frac{\tau}{10} \end{aligned}$$

So, under perfect competition and given a tax or subsidy τ , a quantity $Q_c(\tau) = 42 - \frac{\tau}{10}$ is exchanged. Knowing this, the regulator chooses the tax or

subsidy to match this quantity with the socially optimal one. We denote it by τ_c below:

$$Q_c(\tau_c) = Q^*$$

We solve for τ_c :

$$Q_c(\tau_c) = Q^* \iff 42 - \frac{\tau_c}{10} = 35 \iff \tau_c = 70$$

Besides, we know that the quantity traded with a per-unit tax $\tau_c = 70$ is the socially optimal one. So, the government draws the following revenues from the tax:

$$G_c = \tau_c \times Q^* = 70 \times 35 = 2450$$

So, the tax or subsidy that the government needs to impose under perfect competition to reach the socially optimal allocation is $\tau_c = 70$. The government collects 2450 of tax revenue.

1.4 Question 4: Could a quota be used to achieve the same result? Which policy instrument would be preferred by producers? (5 points)

Yes, a quota that would limit the quantity exchanged to $Q^* = 35$ would also achieve the socially optimal allocation. This is somewhat mechanical.

However, the redistributive effects of both measures, the Pigouvian tax and the quota, are not the same. Here, we are asked about the instrument that producers would favor. Producers favor the instrument that yields the largest producer surplus. So, we compute the producer surplus in both cases.

First, in the case of the tax, the producer surplus is given by the area between the price *faced by producers* and the supply curve over the interval of quantities being traded. What price do producers face when the tax is implemented? Their price is given by the supply curve, or equivalently the marginal cost curve, evaluated at the quantity reached with the tax:

$$P_{\tau_c, \text{Producers}} = MPC[Q_c(\tau_c)] = MPC(Q^*) = 5Q^* = 175$$

Then, we can compute producer surplus with a similar formula as the one

seen above:

$$\begin{aligned}
PS_{\tau_c} &= \int_0^{Q_c(\tau_c)} [P_{\tau_c, \text{Producers}} - MPC(q)] dq \\
&= \int_0^{Q^*} (175 - 5q) dq \\
&= 175Q^* - 5 \int_0^{Q^*} q dq \\
&= 175Q^* - \frac{5}{2} [q^2]_0^{Q^*} \\
&= 175Q^* - \frac{5}{2} (Q^*)^2 \\
&= 175 \times 35 - \frac{5}{2} 35^2 = \frac{5}{2} 35^2 = 3062.5
\end{aligned}$$

What happens with a quota? The quantity traded is capped at the socially optimal quantity Q^* . Consumers and producers face the same price. The price is such that consumers demand exactly the socially optimal quantity and producers supply the same quantity. It corresponds to the price $P^* = 245$ derived in Question 2(a). Then, producer surplus writes as:

$$\begin{aligned}
PS_{\text{Quota}} &= \int_0^{Q_{\text{Quota}}} [P_{\text{Quota}} - MPC(q)] dq \\
&= \int_0^{Q^*} [P^* - MPC(q)] dq \\
&= \int_0^{Q^*} [245 - MPC(q)] dq \\
&= 245Q^* - 5 \int_0^{Q^*} q dq \\
&= 245Q^* - \frac{5}{2} [q^2]_0^{Q^*} \\
&= 245Q^* - \frac{5}{2} (Q^*)^2 \\
&= 245 \times 35 - \frac{5}{2} 35^2 \\
&= 5512.5 \\
&= \underbrace{2450}_{G_c} + \underbrace{3062.5}_{PS_{\tau_c}}
\end{aligned}$$

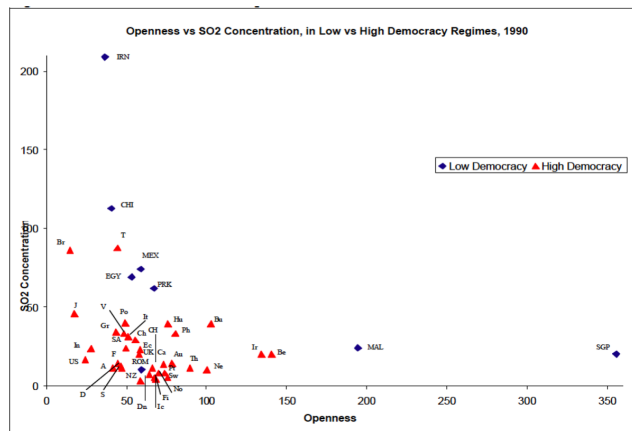
Clearly, producers obtain a larger surplus with the quota than with the Pigouvian tax, and they would thus favor the former. In fact, we notice that the quota allows them to “capture” the amount of taxes collected by the government under the Pigouvian tax.

2 Exercise 2

In a (fairly) recent paper, Frankel and Rose (2005)¹ study the relationship between “trade openness” and environmental quality at the country level. “Trade openness” can roughly be measured as the share of imports and exports in national output (gross domestic product (GDP)). If countries trade a lot, this ratio will be higher. Many people worry that openness to trade endangers the environment, while others argue that openness to trade may encourage stricter environmental regulations, or cleaner production practices. Frankel and Rose (2005) propose to estimate the relationship using cross country data on trade openness and air pollution concentrations.

2.1 Question 1

Frankel and Rose (2005) begin with some simple graphical analysis. Frankel and Rose (2005) collect data on trade openness and various pollutant concentrations at the national level for the year 1990. In Figure ??, Frankel and Rose (2005) plot trade openness on the x-axis and SO_2 concentrations on the y-axis (measured in micro grams per meter cubed). From Figure 1, it appears that SO_2 concentrations fall with trade openness.



(a) Under what condition on trade openness does this graphical analysis identify the causal impact of trade openness on pollution? (5 points) This graphical analysis identifies the causal impact of trade openness on pollution if variation in trade openness across countries is random vis-à-vis variation in SO_2 concentration across countries.

(b) Is this condition likely to hold? Why or why not? (be specific for this context) (5 points) It is highly unlikely that this condition holds.

¹Frankel, Jeffrey A., and Andrew K. Rose. “Is trade good or bad for the environment? Sorting out the causality.” *Review of Economics and Statistics*, 87.1 (2005): 85-91.

First, the relationship probably suffers from an omitted variable bias. For instance, with an argument based on geography, the proximity of countries with the sea may simultaneously affect pollution levels (e.g., winds dissipating pollution) and trade openness (countries with an access to the sea would be more open to trade). Or income per capita may also play a role in explaining both pollution (e.g., better access to abatement technologies) and trade openness (developed economies being possibly more open to trade).

Second, there is a risk of reverse causality, as suggested by the context about the paper. If pollution also affects openness to trade, then this will bias the conclusions drawn from such a naive analysis.

2.2 Question 2

Frankel and Rose (2005) formally estimate the causal relationship with the model:

$$\begin{aligned} EnvDam_i = & \phi_0 + \beta[(X + M)/Y]_i + \phi_1 \ln(Y/pop)_i + \phi_2 \ln(Y/pop)_i^2 \\ & + \phi_3 Polity_i + \phi_4 \ln(LandArea/pop)_i + \epsilon_i \end{aligned} \quad (1)$$

Where:

- $EnvDam_i$ is pollution concentration in country i in 1990,
- $[(X + M)/Y]_i$ is the trade openness of country i in 1990 (Exports (X) plus Imports (M) divided by real output (Y)),
- $\ln(Y/pop)_i$ is the natural log of real per capita output in country i in 1990,
- $Polity_i$ is an index of democratic policies in country i in 1990,
- and $\ln(LandArea/pop)_i$ is the natural log of land area per capita in country i in 1990.

The parameter of interest is β . Frankel and Rose (2005) estimate equation 1 by OLS and report coefficient estimates in Figure 4. The independent variables are reported in the left-most column. Then each successive column represents a different regression. Columns 1-3 estimate equation (1) by OLS taking NO_2 , SO_2 , and PM , respectively as the dependent variable. Point estimates are reported in the various cells with standard errors below in parentheses. (Note that there are no stars reported with these estimates. This does not mean that none of the estimates are statistically significant. Frankel and Rose (2005) just didn't report any stars.)

	OLS (1)	OLS (2)	OLS (3)	IV (4)	IV (5)	IV (6)
	NO ₂	SO ₂	PM	NO ₂	SO ₂	PM
Trade / GDP	-.29 (.17)	-.31 (.08)	-.37 (.34)	-.33 (.19)	-.23 (.10)	-.31 (.41)
Log real GDP per capita	409 (122)	287 (119)	567 (336)	461 (199)	296 (140)	681 (412)
Log real GDP p/c squared	-22.8 (6.9)	-16.6 (6.8)	-35.6 (19.1)	-25.6 (10.9)	-17.1 (7.7)	-42.0 (23.2)
Polity	-3.20 (1.47)	-6.58 (2.05)	-6.70 (3.42)	-3.77 (1.37)	-6.41 (2.27)	-7.78 (4.07)
Log of Area per capita	-5.94 (5.93)	-2.92 (1.39)	-13.0 (6.29)	-6.14 (6.43)	-1.54 (1.96)	-12.6 (6.84)
Observations	36	41	38	35	40	37
R ²	.16	.68	.62	.18	.67	.63
Income Peak	\$7665	\$5770	\$2882	\$8015	\$5637	\$3353

Figure 4: Model estimates

(a) Why do Frankel and Rose (2005) include $\ln(Y/pop)_i$, $Polity_i$, and $\ln(LandArea/pop)_i$ in the regression? (5 points) The authors include these three variables as *control variables* because they are afraid that, without them, their specification would suffer from an omitted variable bias. We already discussed the case of income per capita in the previous question. Regarding the index of democratic polities $Polity_i$, we could imagine that democracies are more open to trade (hence an impact on the regressor of interest...) and give more weight to the citizens' environmental well-being, hence lower pollution levels holding everything else constant (...and a direct effect on the dependent variable). The last control variable, $\ln(LandArea/pop)_i$, may be seen as the mirror image of pollution density. A very dense country may have to rely more on trade (e.g., to secure food resources) and may be more polluted at the same time (e.g., with people concentrating in cities with dense car circulation). These are examples of omitted variable bias concerns that arise without these control variables.

(b) Can we interpret the point estimates in columns 1-3 as causal? (5 points) We could interpret the estimates as causal if variation in trade openness across countries were random vis-à-vis pollution conditionally on control variables. This relaxes a bit the assumption stated in Question 1(a), but this is still unlikely to be true. For instance, the concern that we expressed previously about access to the sea is not taken into account for now.

2.3 Question 3

Question 3 (10 points). To address potential endogeneity, Frankel and Rose (2005) propose to “instrument” trade openness using plausibly exogenous determinants of trade. It is well known in the Trade literature that trade flows decline with distance between two trading partners. Frankel and Rose (2005) use this insight to build a prediction of trade flows between two partners based solely on the distance between them (and a few other variables which you don’t need to worry about). Frankel and Rose (2005) then aggregate up these predicted trade flows across all trading partners and generate predicted trade openness. Does this procedure generate a valid instrument? Why or why not? (10 points)

This procedure yields a valid instrument if we satisfy two conditions: (i) a strong first stage, and (ii) the exclusion restriction assumption.

The strong first stage states that the instrument should predict the regressor of interest to some extent. As stated in the question, distance is a well-known determinant of trade flows and the alternative trade openness “measure” that the authors build is likely to correlate sufficiently strongly with actual trade openness. Besides, the first stage can be tested, so we do not worry too much about it here.

The exclusion restriction assumption states that the instrument built by Frankel and Rose (2005) based on distance should affect pollution *only through trade openness*. This condition is somewhat more problematic since it cannot be tested directly. If we are confident in this assumption, then we can trust the results of the instrumental variable strategy; if not, the strategy may not be valid.

One potential worry would be that pollution may travel across countries so that countries that are relatively close to many other countries would be exposed not only to their own pollution, but also to foreign emissions. Then, distance would have a direct effect on pollution levels, without trade openness as an intermediary factor, and the exclusion restriction assumption would not be satisfied. The authors may need to argue that NO_2 , SO_2 , and PM pollution does not travel across countries to mitigate this concern. They could also exclude clusters of countries very close to one another from the analysis.

2.4 Question 4

Frankel and Rose (2005) re-estimate equation 1 using the instrument for trade openness and present results in columns 4-6 in Figure 2. Again, each column reports point estimates and standard errors from a separate regression, taking a different pollutant as the dependent variable.

(a) Interpret the point estimate on “Trade/GDP” in column 5 (5 points). The point estimate associated with Trade / GDP in Column 5 is -0.23 which implies for instance that a 10 percentage point increase in trade

openness (i.e., exports and imports accounting for 10 more percentage points of real output) raises SO_2 concentration by 2.3 units.

(b) Compute a rough 95% confidence interval for the OLS estimate on “Trade/GDP” in column 5. Can Frankel and Rose (2005) reject the hypothesis that $\beta = 0$ at the 5% level? based on this regression (5 points) We know that the critical value associated with a 95% confidence level is roughly 1.96 and the confidence interval is thus given by:

$$\begin{aligned} & \left[\hat{\beta} - 1.96 \times \widehat{se}(\hat{\beta}); \hat{\beta} + 1.96 \times \widehat{se}(\hat{\beta}) \right] \\ &= [-0.23 - 1.96 \times 0.1; -0.23 + 1.96 \times 0.1] \\ &= [-0.426; -0.034] \end{aligned}$$

Since the null hypothesis value 0 is not part of this interval, the authors can reject the null hypothesis at the 95% confidence level (but it is a close call!).

3 Exercise 3

Why is it difficult to measure the willingness to pay for environmental goods as opposed to traded commodities? (5 points) Measuring consumers’ willingness to pay for environmental goods is particularly difficult because, in most cases, there are no observable transactions or prices remain implicit. For commonly traded commodities, we could collect data on the joint evolution of prices and quantities demanded, and we could then infer consumers’ marginal willingness to pay.

Summarize some evidence we saw in the course with respect to how consumers respond to energy prices? (10 points) Experimental studies have highlighted several deviations from standard economic theory. As an example of bounded rationality, saliency matters for electricity consumption. Sexton (2015) shows that enrollment in automatic billing payment (ABP) programs raised residential and commercial electricity consumption. This result is consistent with the price of electricity becoming less salient, and thus perceived by boundedly rational agents as lower than what they are actually. In the same vein, Ito (2015) exploits variation in the non-linear price schedules of different utilities and shows that consumption responds to the average price of electricity, instead of the marginal one. This indicates that consumers rely on heuristics when choosing how much electricity to consume following price changes.

According to the economic theory we saw in class, is trade good or bad for pollution? According to the empirical evidence we saw, is trade good or bad for pollution? (5 points) On the theoretical side, Copeland and Taylor (1994) highlight three effects of trade on pollution: (i) the *scale* effect, according to which freer trade raises global economic output and

thus pollution; (ii) the *technique* effect, according to which trade-induced income gains spur demand for environmental quality (the latter being considered as a normal good), lead to tougher environmental standards, and ultimately reduce the pollution intensity of economic activity; and (iii) the composition effect, according to which less developed countries adopt lower environmental standards and specialize into dirty activities relatively to more developed countries. The authors argue that the composition effect dominates the two others and should lead to an increase in global pollution.

Antweiler, Copeland, and Taylor (2001) propose to test these conclusions empirically. They find a strong technique effect so that an increase in income from reduced trade barriers lowers SO_2 concentrations overall. Based on empirical evidence, trade would thus be good for pollution as it induces income gains that raise demand for environmental quality, lead to stricter regulation, and lower the pollution intensity of economic activities.

If you wanted to estimate the effect of electricity infrastructure on economic development, would you expect an OLS regression of economic development on electricity infrastructure with time and region fixed effects to deliver an unbiased estimate of the causal effect? If not, in which way would you expect the OLS to be biased? Why? (10 points) The specification with time and region fixed effects would respectively control for any region-invariant and time-invariant determinants of economic development. Therefore, we should only worry about the omitted variables that vary over time and across regions. These would not be accounted for by the fixed effects, and they could bias our OLS estimate if they correlate with electricity infrastructure as well as economic development. We should also consider the risk of reverse causality bias.

Consider first an omitted variable like political institutions. When political institutions are improved (e.g., corruption is reduced), it may jointly affect electricity infrastructure and economic development. On the one hand, we could imagine that electricity infrastructure would expand thanks to a greater access to international capital markets. On the other hand, a rich literature highlights the positive impact of improved political institutions on economic development. Institutions thus create a positive association between electricity infrastructure and economic development. If we do not control for it, our OLS estimate may over-estimate the true effect of electricity infrastructure on economic development.

Consider now reverse causality. Economic development also affects electricity infrastructure through increased demand for energy or larger financing capacities. This creates a positive association between electricity infrastructure and economic development. Again, our OLS estimate may over-estimate the true effect of electricity infrastructure on economic development.