TD 2: Research Design ECO 567A

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$$y = X\theta + \varepsilon$$

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  - Unbiasedness: Expectation of the sampling distribution is the true value
  - Effiency: OLS has the smallest variance among unbiased linear estimators

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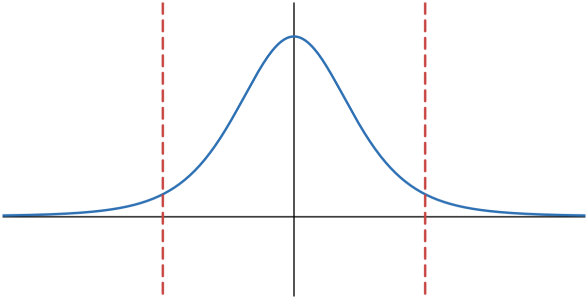

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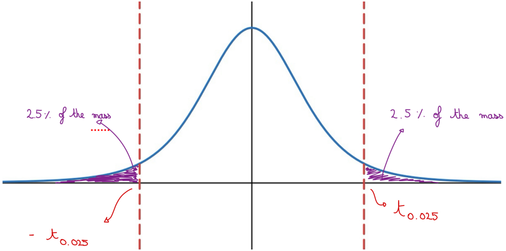
- Choose a confidence level, say 95%, and then:



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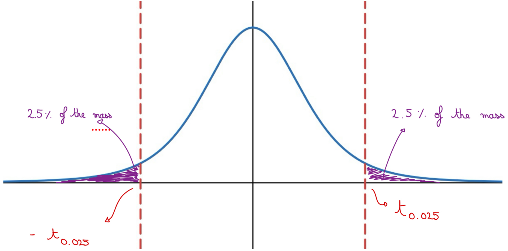

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- 95% confidence interval: $(\hat{\theta}_{OLS} \pm t_{.025} * se(\hat{\theta}_{ols}))$

But...

- Can we assume by default?

$$\varepsilon | X \sim \mathcal{N}\left(0, \sigma^2 I\right)$$

- Or more generally, can we assume?

$$E\left(\varepsilon | X\right) = 0$$

- Today: (i) why we need a **research design** and (ii) how it could look like

Outline

When Do We Have $E[\varepsilon|X] \neq 0$?
- Omitted Variable Bias
- Reverse Causality / Simultaneity
- In the Potential Outcome Framework

What Happens When $E[\varepsilon|X] \neq 0$?

What To Do When $E[\varepsilon|X] \neq 0$?
- Instrumental Variables
- Randomized Control Trials (RCTs)
- Fixed Effects
- Difference-in-Differences

Review Question

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Omitted Variable Bias

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  - Parents' education directly affects income: Better professional network, etc.
  - Parents' education also affects child's education: Role model, information, etc.

Resulting Issue

- When we estimate our wrong model, we have:

$$\mathrm{Cov}(x_i, u_i) = \mathrm{Cov}(\lambda + \mu z_i + \eta_i, \gamma z_i + \epsilon_i)$$

Resulting Issue

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- In the above, we see that there is no issue if:
  - $\mu = 0$: Omitted variable is unrelated to education
  - $\gamma = 0$: Omitted variable does not affect income directly
  - $\mathrm{Var}(z_i, z_i) = 0$: Omitted variable is a constant (not super interesting)

Other Examples

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- So, "income" is an omitted variable related to both pollution and health

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- Governments might target areas that are growing anyways
- And that would have seen poverty decrease even absent the program
- So, "expected growth" could be a problematic omitted variable

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Reverse Causality / Simultaneity

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  - Crowd density has a causal effect on crime: Deterrence?
  - Crime has a causal effect on density: Lower attractivity

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  - $\delta = 0$, which essentially denies reverse causality

When Do We Have $E\left[\varepsilon|X\right] \neq 0$?

In the Potential Outcome Framework

Introducing the Potential Outcome Framework

- Imagine we study a treatment (e.g., ban on polluting cars)

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- We only observe the outcome that is actually realized

$$\text{Observed Outcome } \; Y_i \;\; = \;\; \begin{cases} Y_{1i} & \text{if } X_i = 1 \\ Y_{0i} & \text{if } X_i = 0 \end{cases} = Y_{0i} + \underbrace{(Y_{1i} - Y_{0i})}_{\text{Treatment effect}} * X_i$$

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- Expectation of the sampling distribution: $E(\hat{\beta}) = E\left(Y_i | X_i = 1\right) - E\left(Y_i | X_i = 0\right)$
  - Where: $E[Y_i | X_i = 1] = \alpha + \beta + E[\varepsilon_i | X_i = 1]$
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  - And: $E[Y_i | X_i = 0] = \alpha + E[\varepsilon_i | X_i = 0]$

- We can assess unbiasedness:

$$\begin{aligned} E(\hat{\beta}) &= E[Y_i | X_i = 1] - E[Y_i | X_i = 0] \\ &= \beta + E[\varepsilon_i | X_i = 1] - E[\varepsilon_i | X_i = 0] \\ &= \beta + \underbrace{E[Y_{0i} | X_i = 1] - E[Y_{0i} | X_i = 0]}_{\text{Risk of selection bias}} \end{aligned}$$

What Happens When $E\left[\varepsilon|X\right] \neq 0$?

Monte Carlo Experiment

- Assume a true model

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y_i &= \alpha + \beta * x_i + a_i + \varepsilon_i \\
\alpha &= 10 \\
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- Examine three cases:
  - Case 1: Exogenous a
    - Variable is omitted but it is unrelated to the regressor of interest
  - Case 2: Endogenous a, and we don't know it
    - Variable is omitted, and it is related to the regressor of interest
  - Case 3: Endogenous a, but we know it
    - Variable is related to the regressor of interest, but it is not omitted

Case 1: Exogenous Ability — Theoretically

- True model:

$$\begin{aligned} y_i &= \alpha + \beta x_i + a_i + \varepsilon_i \\ E(a + \varepsilon | x) &= E(a|x) + E(\varepsilon|x) = E(a|x) = 0 \end{aligned}$$

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- Prediction: OLS estimator should be **unbiased**

Case 1: Exogenous ability — Experiment

- Monte Carlo experiment:

$$
\begin{aligned}
x_i &\sim \mathcal{U}(0,20) \\
a_i &\sim \mathcal{N}(0,5) \\
\varepsilon_i &\sim \mathcal{N}(0,5) \\
y_i &= \alpha + \beta x_i + a_i + \varepsilon_i
\end{aligned}
$$

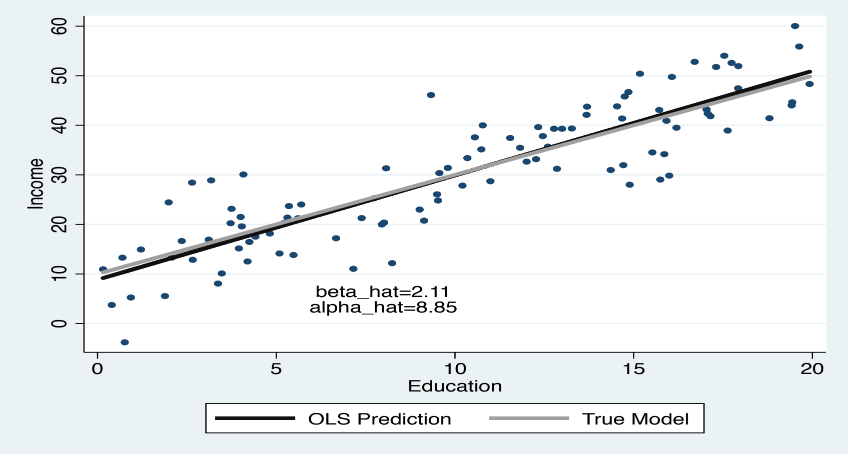
- Data:

Table: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**N**
y	30.268	13.734	-3.795	60.018	100
x	10.169	5.758	0.156	19.923	100
a	0.012	4.899	-13.5	12.748	100
eps	-0.083	5.251	-11.975	11.012	100

Case 1: Exogenous ability — Results

Estimation equation: $y_i = \alpha + \beta x_i + \underbrace{u_i}_{=a_i + \varepsilon_i}$



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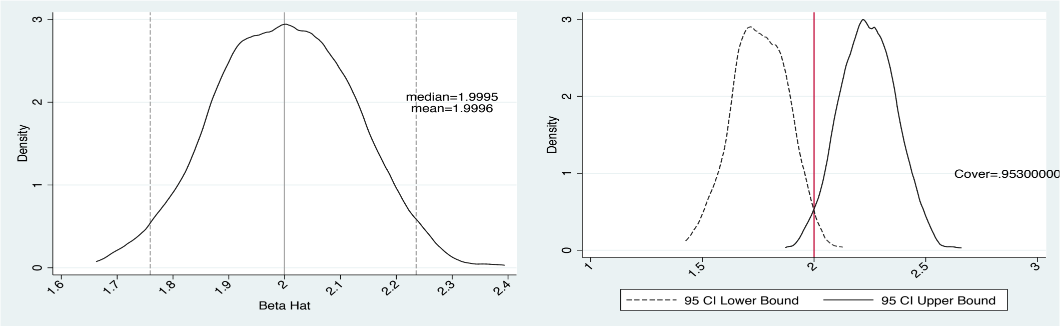

Case 2: Endogenous a, and we don't know it — Theoretically

- True model:

$$\begin{aligned} y_i &= \alpha_1 + \beta x_i + a_i + \varepsilon_i \\ x_i &= \alpha_2 + \delta a_i + \eta_i \\ \delta &= 0.5 \end{aligned}$$

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- Prediction: OLS estimator is **likely biased**

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- Monte Carlo experiment:

$$\begin{aligned} \eta &\sim N(0,1)\ ;\quad a_i \sim N(0,5)\ ;\quad \varepsilon_i \sim N(0,5) \\ x_i &= \alpha_2 + \delta a_i + \eta_i \\ y_i &= \alpha_1 + \beta x_i + a_i + \varepsilon_i \end{aligned}$$

- Data:

Table: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**N**
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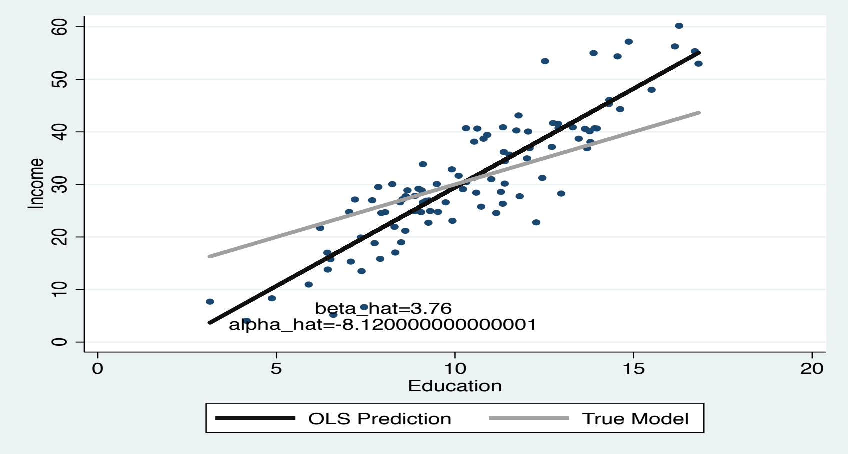

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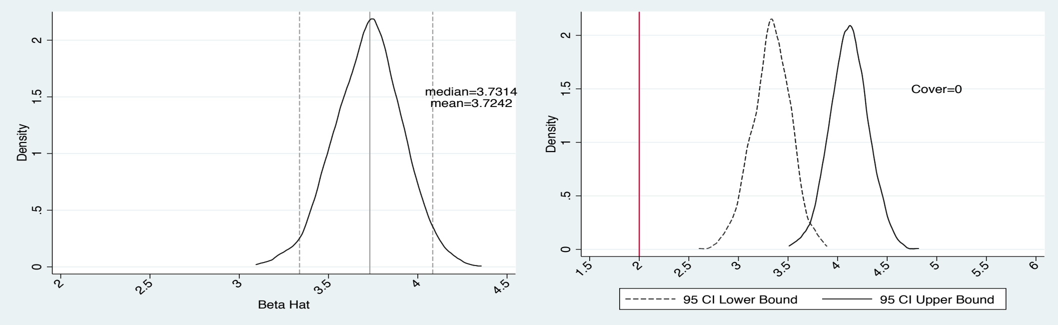

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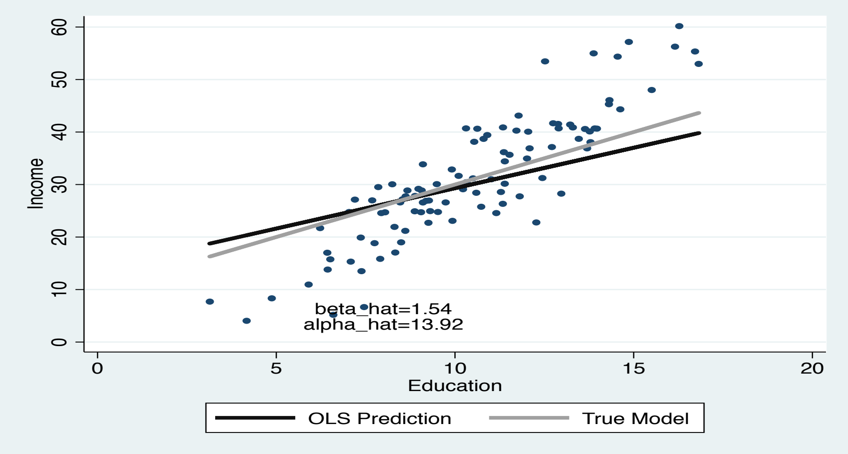
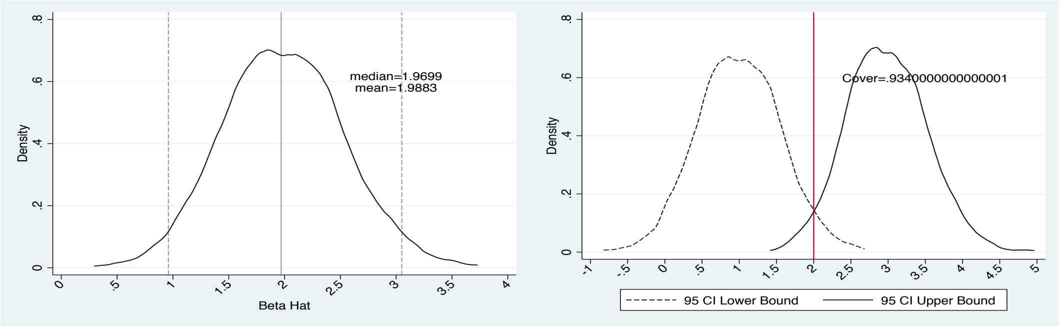


Case 3: Endogenous *a*, but we know it — Results



What To Do When $E\left[\varepsilon | X\right] \neq 0$?

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Instrumental Variables

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- Suppose $E[\varepsilon|X] \neq 0$. E.g., x correlated with a, which also determines y:

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- Instrumental variable approach:
  - Find another z variable which correlates with x...
  - ...but not with a...
  - ...and use it to recover β!

Example

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- Strong first stage can be tested, not the exclusion restriction in general

In Practice: Two-stage Least Squares (2SLS)

1. First, use z to predict x:

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\begin{aligned}
x_i &= \pi_1 + \pi_2 z_i + \underbrace{\gamma_i}_{=\delta a_i + \eta_i} \\
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- Then: $E(u_i|\hat{x}_i) = E(a_i + \varepsilon_i|\hat{\pi}_1 + \hat{\pi}_2 * z_i) = E(a_i|\hat{\pi}_1 + \hat{\pi}_2 * z_i) + E(\varepsilon_i|\hat{\pi}_1 + \hat{\pi}_2 * z_i) = 0$
- $\hat{x}$ uncorrelated with a: Extracted the random component in x, and **OLS is unbiased**

Monte Carlo Experiment — Theoretically

- True model:

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\begin{aligned}
y_i &= \alpha_1 + \beta x_i + a_i + \varepsilon_i \\
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- Prediction: **2SLS estimator should be unbiased**

Monte Carlo Experiment — Specification

$$\begin{aligned} \eta &\sim N(0,1) \\ \varepsilon_i &\sim N(0,5) \\ a_i &\sim N(0,5) \\ z_i &\sim N(0,2) \end{aligned}$$

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Table: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**N**
y	30.581	12.304	-1.62	64.849	100
x	10.502	3.345	1.364	19.494	100
a	0.248	5.039	-13.628	10.604	100
eps	-0.672	4.947	-13.5	10.32	100
eta	0.002	1.02	-2.068	2.252	100
z	0.376	2.167	-4.5	5.626	100

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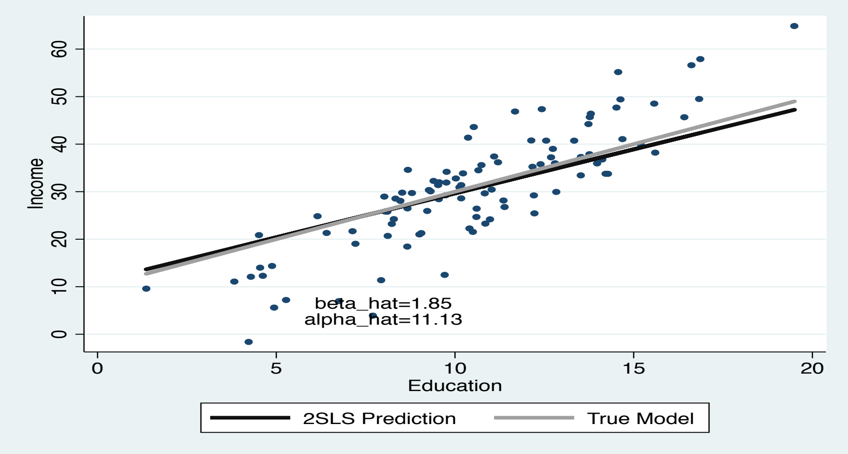

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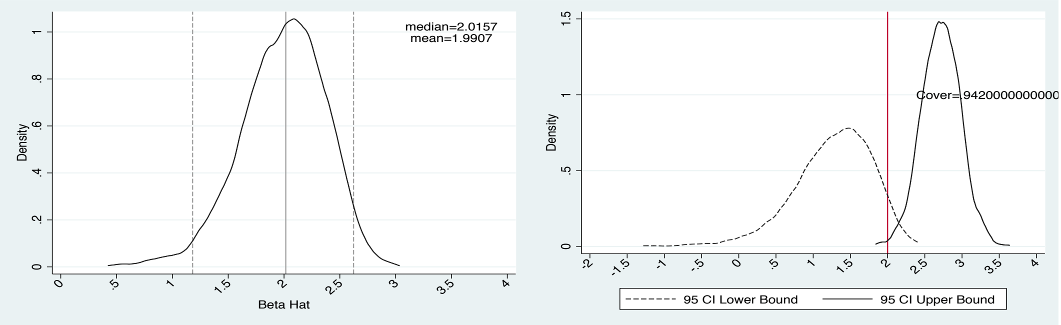

Monte Carlo Experiment — Results

	First Stage	Second Stage		
	x	y	y	y
z	0.88^{***}			
	(0.13)			
x		3.16^{***}	1.97^{***}	
		(0.19)	(0.21)	
a			1.11^{***}	
			(0.14)	
$\hat{x}$				1.85^{***}
				(0.62)
Constant	10.17^{***}	-2.61	9.59^{***}	11.13^{*}
	(0.28)	(2.09)	(2.26)	(6.62)
# Observations	100	100	100	100
R squared	0.329	0.738	0.840	0.083
Mean Dep. Var	10.502	30.581	30.581	30.581

What To Do When $E[\varepsilon|X] \neq 0$?

Randomized Control Trials (RCTs)

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  - Solution? Randomize (part of) disposable income thanks to direct transfers

- Sounds crazy?
  - Boehm, Fize, and Jaravel (2025) spent around €300,000
  - Some customers of a bank receive €300, some don't; allocation is random
  - Use the transfer as an instrument for income and recover the desired parameter

In Practice

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- Potential limitations:
  - Feasibility / Cost
  - Ethical concerns: € transfers are OK, but could we randomize education choices?
  - **External validity**

What To Do When $E[\varepsilon|X] \neq 0$?

Fixed Effects

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  - Instead of solely comparing units i (individuals, countries, cities, etc.) once in time
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Some **unobserved, time-invariant heterogeneity** a_i correlates with x_{it}

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Two Main Approaches

- **Within estimator**:
  - Let: $\bar{y}_{it} = \frac{1}{T}\sum_{t=1}^{T}\left(\alpha + \beta x_{it} + \underbrace{a_i + \varepsilon_{it}}_{u_{it}}\right) = \alpha + \beta \bar{x}_{it} + a_i + \bar{\varepsilon}_{it}$

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  - Consider instead:

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  - Again, exogeneity condition is satisfied: $E\left(\Delta \varepsilon_{it} | \Delta x_{it}\right) = 0$

Fixed Effects

- First-difference estimator

$$\Delta y_{it} \quad = \quad \beta * \Delta x_{it} + \Delta \varepsilon_{it}$$

- Is functionally equivalent to estimating **fixed effect model**:

$$y_{it} \quad = \quad \alpha + \beta x_{it} + \sum_{j=1}^{N} \alpha_j \mathbb{1}\{j = i\} + \varepsilon_{it}$$

  I.e., controlling for a set of dummy variables for all units $\{\mathbb{1}\{j = i\}\}_{j \in [1;N]}$

- **NB**: To lighten notations, we often write that we estimate the model:

$$y_{it} \quad = \quad \alpha + \beta x_{it} + \alpha_i + \varepsilon_{it}$$

What To Do When $E[\varepsilon|X] \neq 0$?

Difference-in-Differences

Context

- Consider again panel data; suppose the true model is:

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- E.g., think about health (y_{it}) and pollution (x_{it})
  - Some regions are most developed industrially, which drives pollution and health (a_i)
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- Again, OLS is biased when estimated on: $y_{it} = \alpha + \beta * x_{it} + u_{it}$

Spirit

- Take the first difference along the time dimension:

$$\Delta y_{it} = \beta \Delta x_{it} + \Delta \gamma_t + \Delta \varepsilon_{it}$$

- Take another difference between units:

$$\Delta y_{it} - \Delta y_{jt} = \beta \left(\Delta x_{it} - \Delta x_{jt} \right) + \left(\Delta \varepsilon_{it} - \Delta \varepsilon_{jt} \right)$$

- Applying OLS to this model yields an unbiased estimator of β if:

$$E \left(\Delta \varepsilon_{it} - \Delta \varepsilon_{jt} \middle| \Delta x_{it} - \Delta x_{jt} \right) = 0$$

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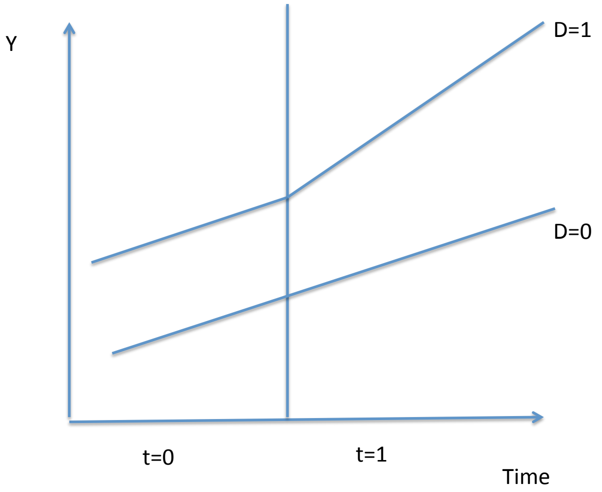
- Method = Compare the T-C difference after the treatment with that before
- **Assumption**: Absent the treatment, both groups evolve similarly in post-period
  - Fundamental **"parallel trends"** assumption that researchers must defend
- Can we test this assumption?
  - We can check that both groups evolve similarly before the treatment
  - But we cannot test this after the treatment is deployed!

Graphical Illustration



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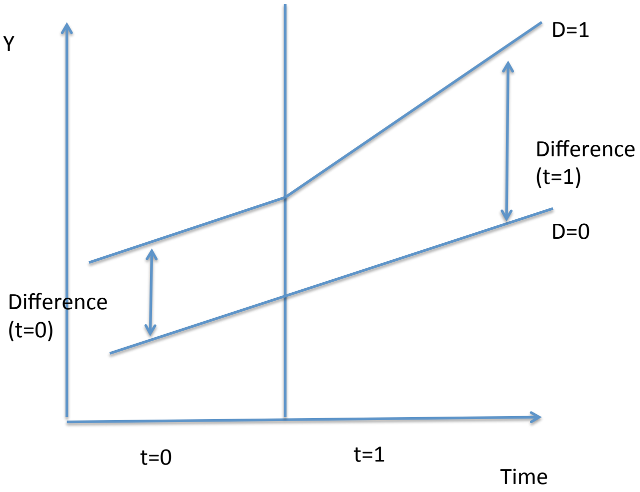

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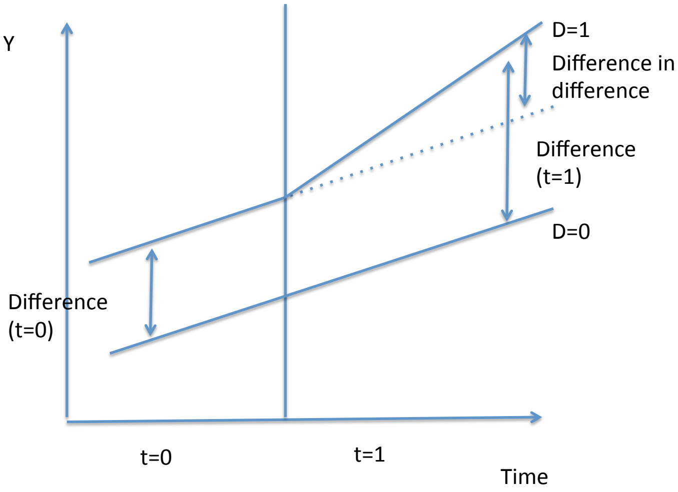

Review Question

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- You want to learn about the relationship between temperature and GDP. You have a cross section for a single year with the GDP and the average annual temperature for many different countries.
  - Under what condition is OLS unbiased?
  - Is this condition likely to be met?
  - What can you do to recover the causal impact of temperature on GDP?

References

- Joshua, D. Angrist. MOSTLY HARMLESS ECONOMETRICS. 2009.