Eyes are smarter than scientists believed

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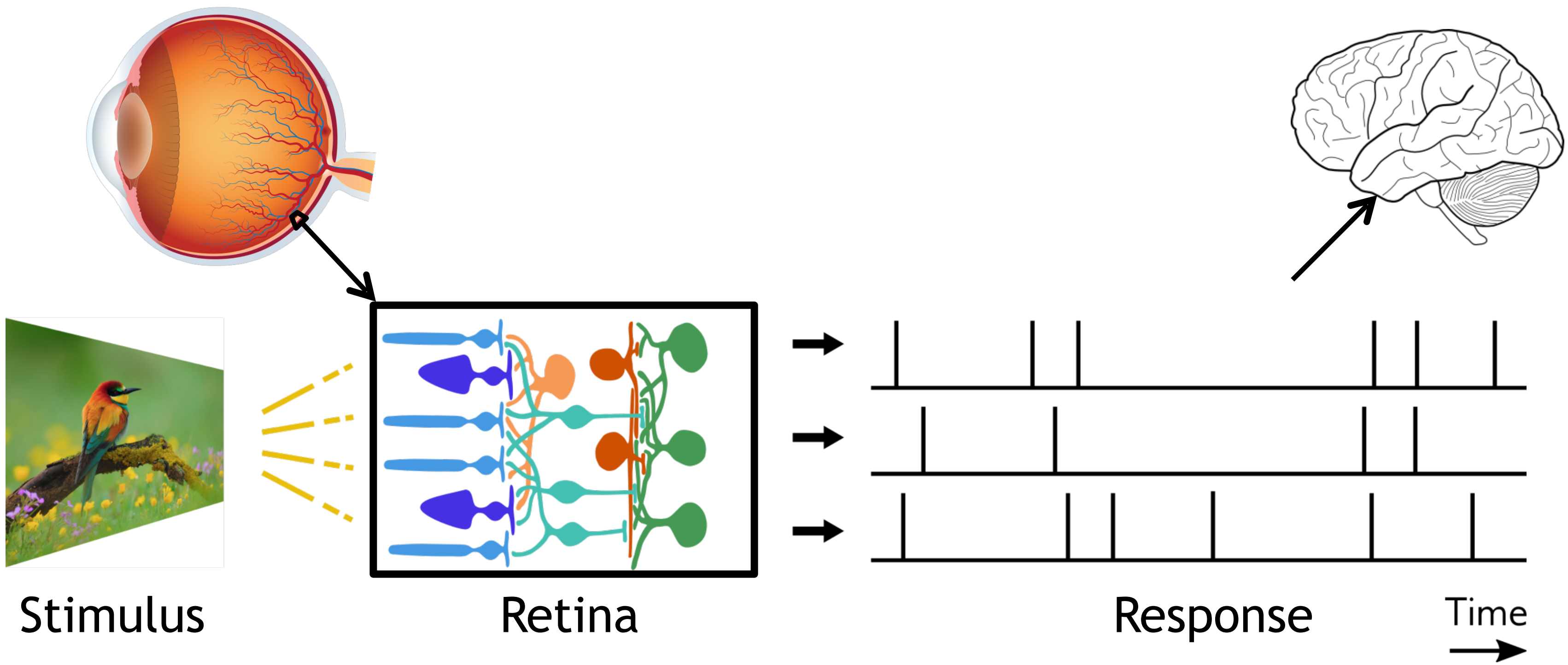
Eyes are smarter than scientists believed

1) The structure of the retina

2) Stimulus processing in the retina

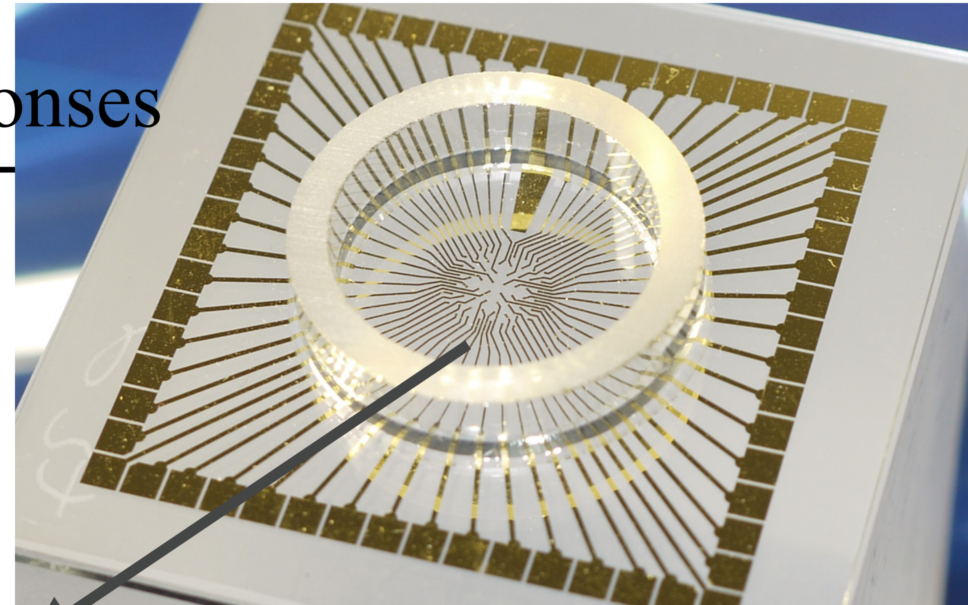
3) Predicting retinal light-response

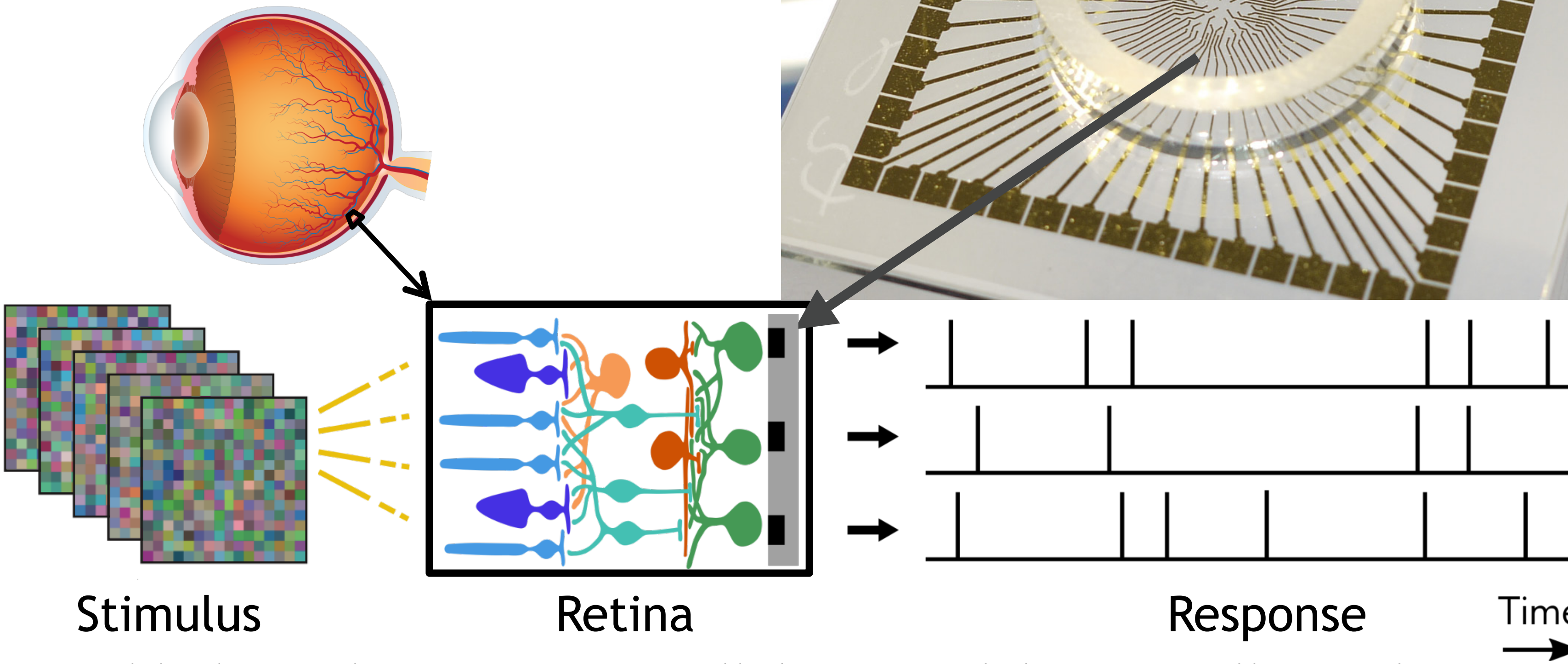
Predicting retinal light responses



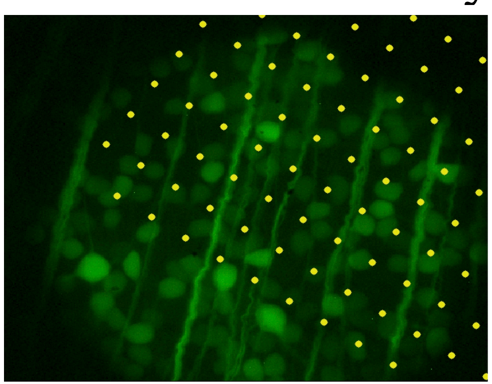
Goal: predict spiking times

Predicting retinal light responses

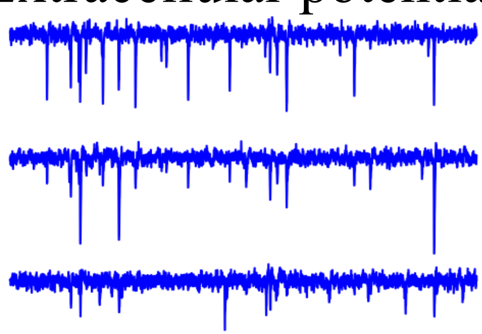




Multi-electrode array

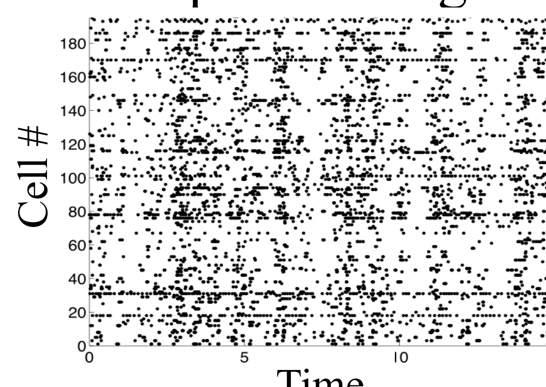


Extracellular potential



Marre '12; Yger '16

Spike sorting



Predicting retinal light responses

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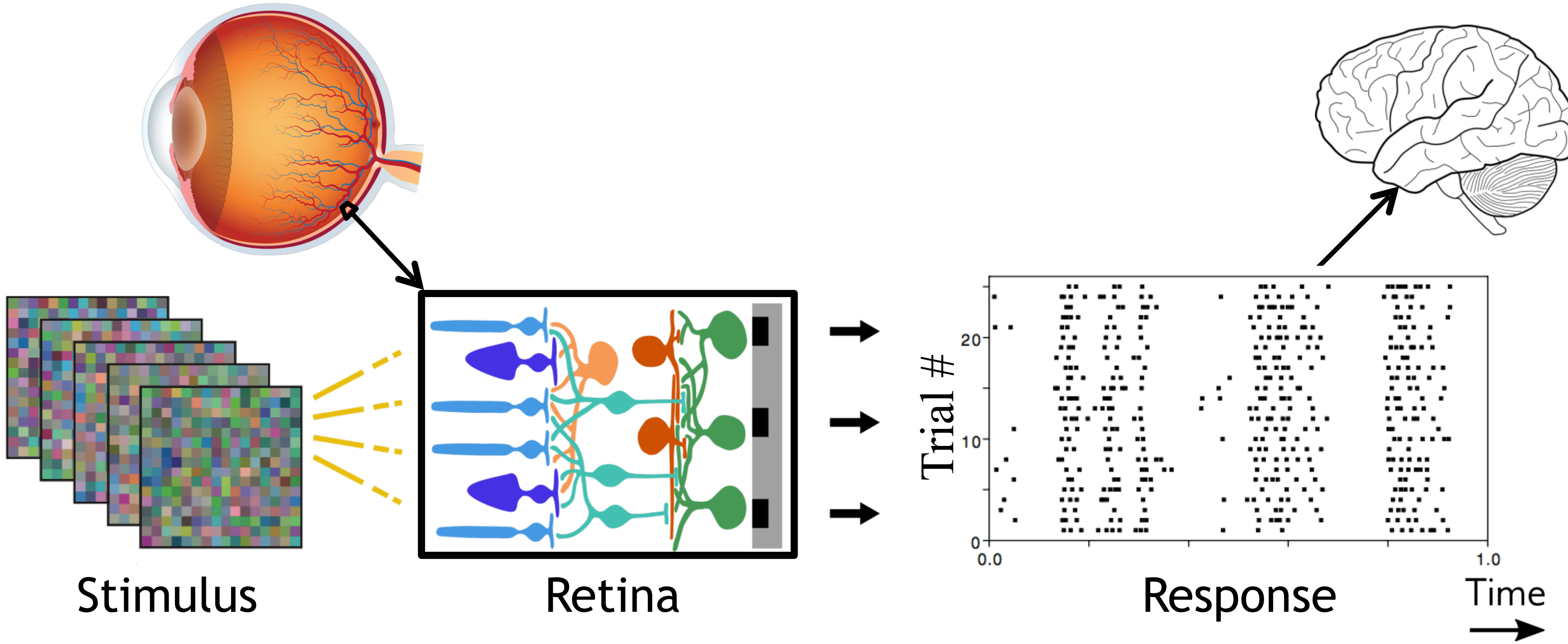
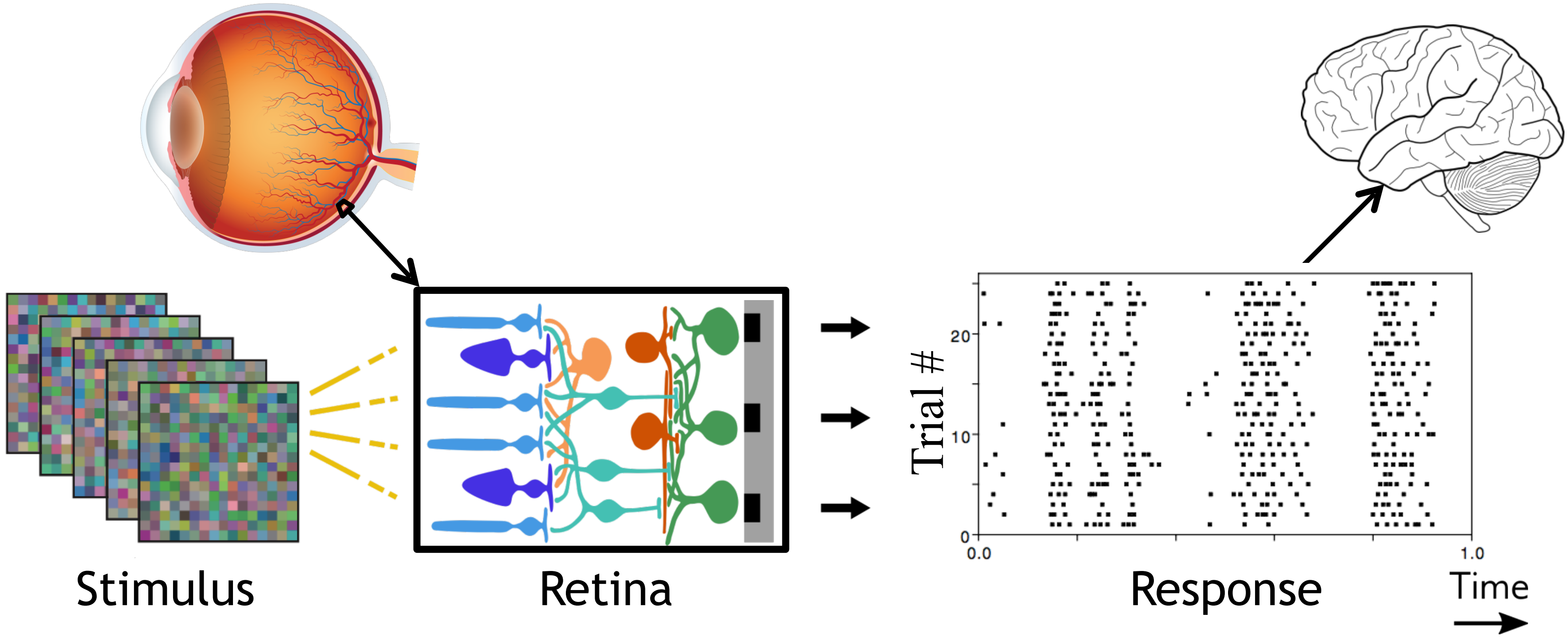


Response is reliable but noisy!

Predicting retinal light responses



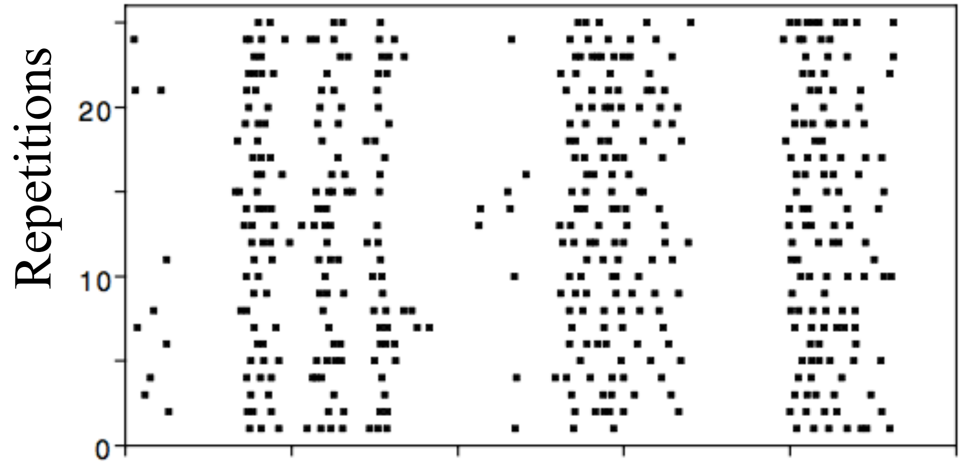
Response is reliable but noisy!

Average over trials

Predicting retinal light responses

Peri-Stimulus Time Histogram (PSTH)

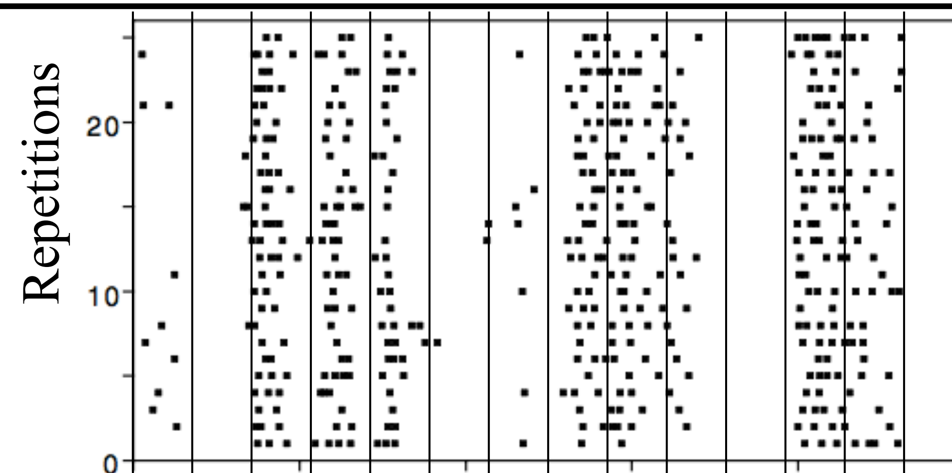
1) repeat video (trials)
2) align responses in time



Predicting retinal light responses

Peri-Stimulus Time Histogram (PSTH)

1) repeat video (trials)
2) align responses in time
3) bin responses at dt=25ms
4) spike counts n_t in each dt



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0	0	7	6	4	1	0	9	7	3	0	6	4	0
0	0	6	2	3	0	0	7	6	3	0	8	6	0
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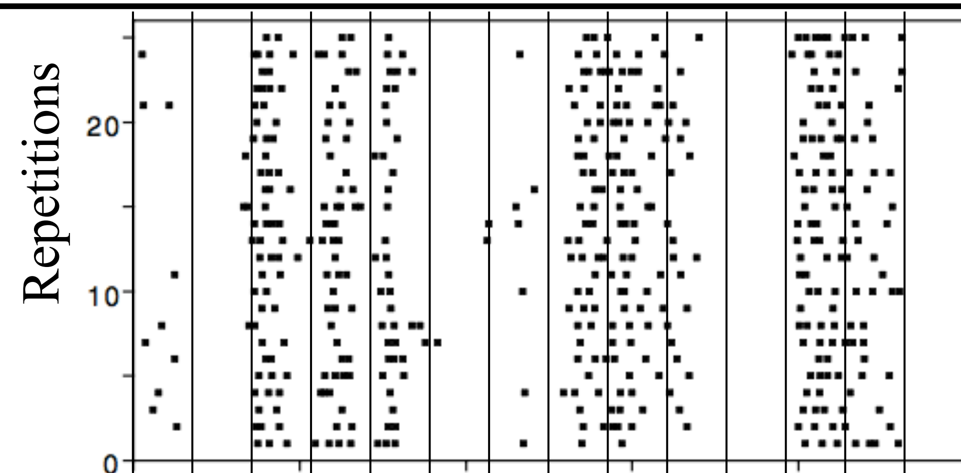
Predicting retinal light responses

Peri-Stimulus Time Histogram (PSTH)

1) repeat video (trials)
2) align responses in time
3) bin responses at dt=25ms
4) spike counts n_t in each dt
5) average over trials
6) estimate spike rate

Can we predict $\langle n_t \rangle$?



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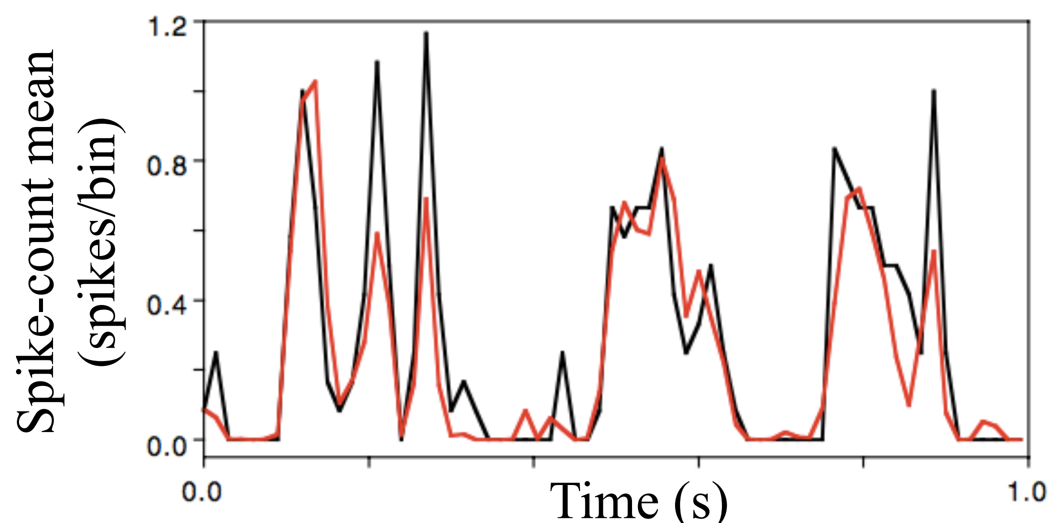

Linear model

$$\tilde{S}_{xyt} = \left(S_{xyt} - \langle S \rangle \right) \Big/ \operatorname{std}(S)$$

$$f(t) = w * \tilde{S}(t) + b = \sum_{x,y,\tau} w_{x,y}(\tau)\ \tilde{S}_{x,y}(t - \tau) + b$$

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Optimise w by minimising: $\quad \mathrm{MSE} = \dfrac{1}{2} \displaystyle\sum_t \big(f(t) - n(t) \big)^2$

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After some algebra: $\quad w_{x,y}(\tau) = \displaystyle\sum_{\tau'} \mathrm{STA}_{x,y}(\tau')\ C^{-1}_{x,y}(\tau', \tau)$

Spike-Triggered Average

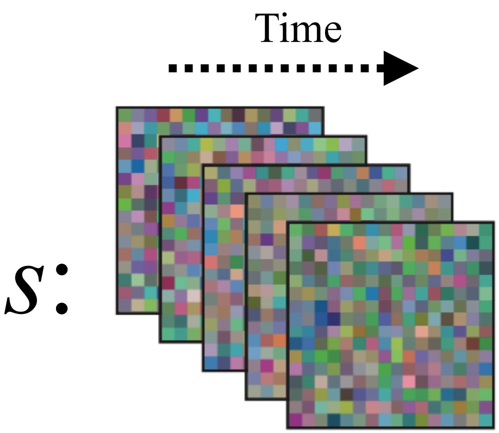
$$\mathrm{STA}_{x,y}(\tau) \equiv \frac{1}{T} \sum_t^T n(t)\ \tilde{S}_{x,y}(t-\tau)$$

Stimulus autocorrelation

$$C_{x,y}(\tau', \tau) \equiv \frac{1}{T} \sum_t^T \tilde{S}_{x,y}(t-\tau')\ \tilde{S}_{x,y}(t-\tau)$$

Linear model

$$C_{x,y}(\tau', \tau) = \delta(\tau - \tau')$$



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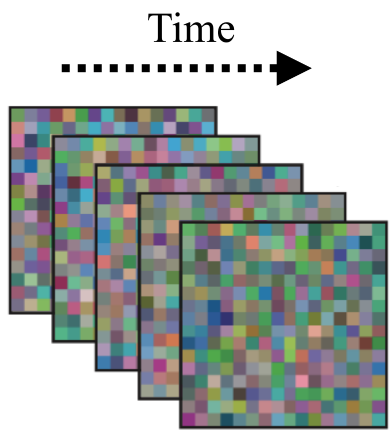

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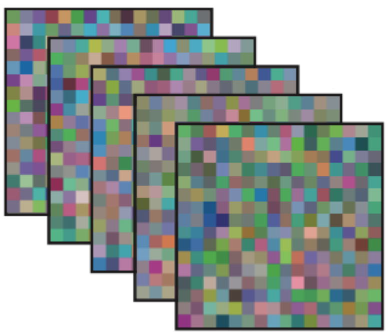
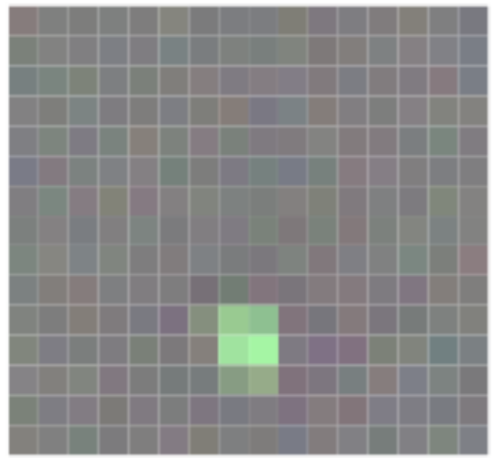
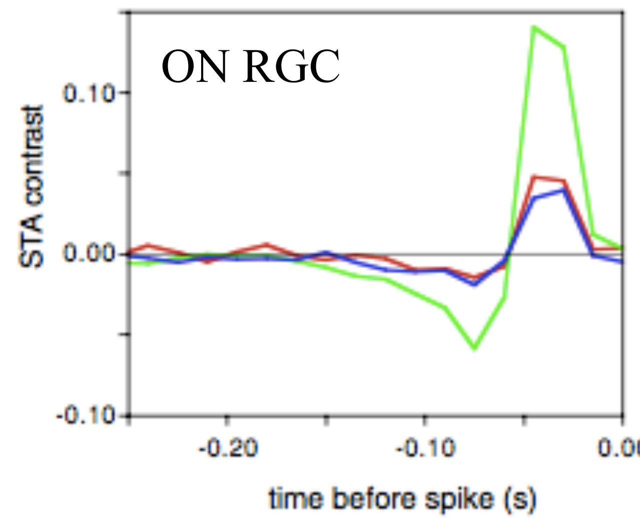
Time

s:







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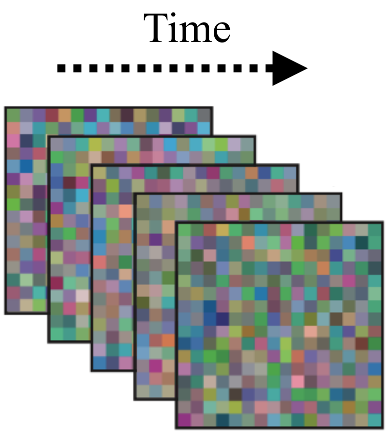
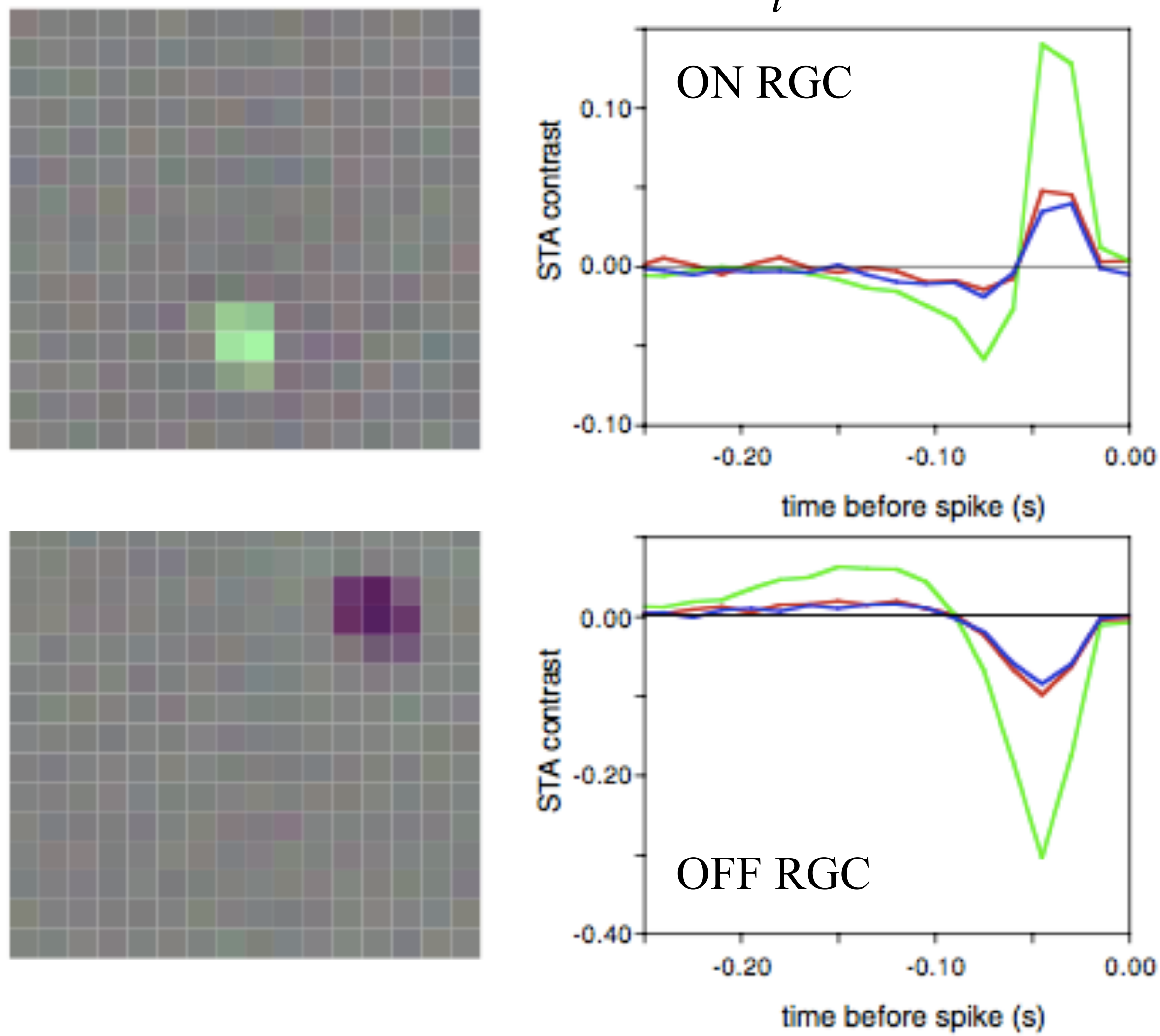
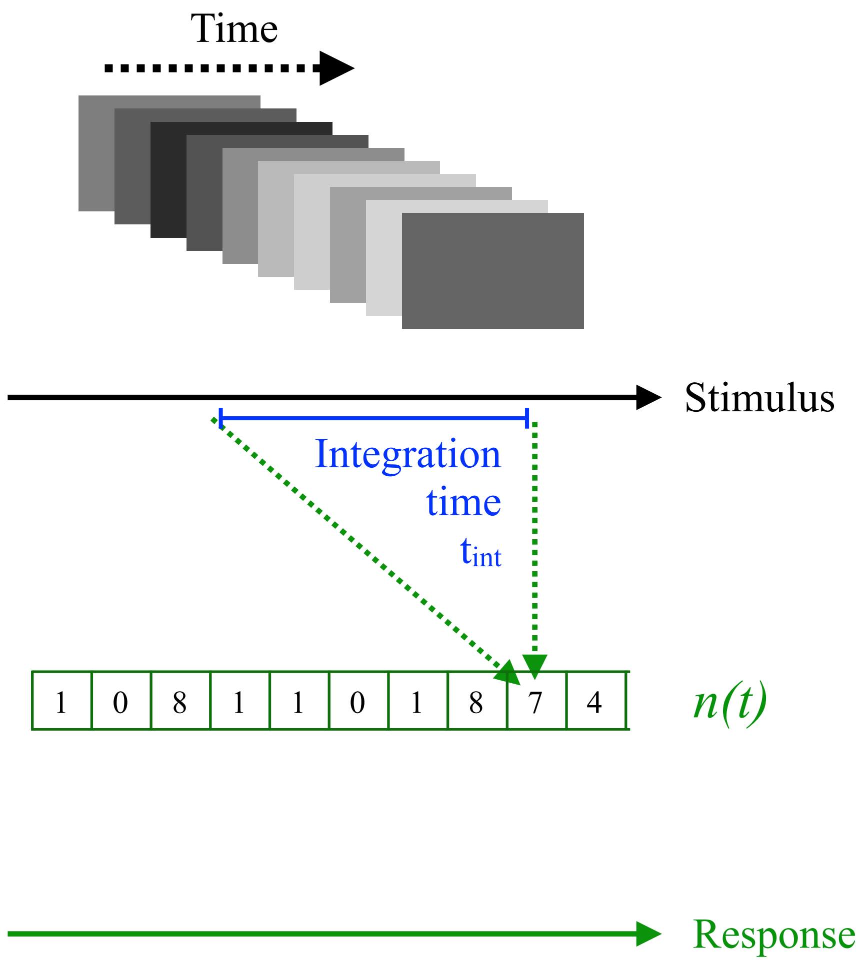




Linear model: full field case

$$f(t) = w * \tilde{S}(t) + b = \sum_{\tau}^{t_{int}} w(\tau)\ \tilde{S}(t-\tau) + b$$

$$\mathrm{MSE} = \frac{1}{2}\sum_{t}\big(f(t) - n(t)\big)^2$$



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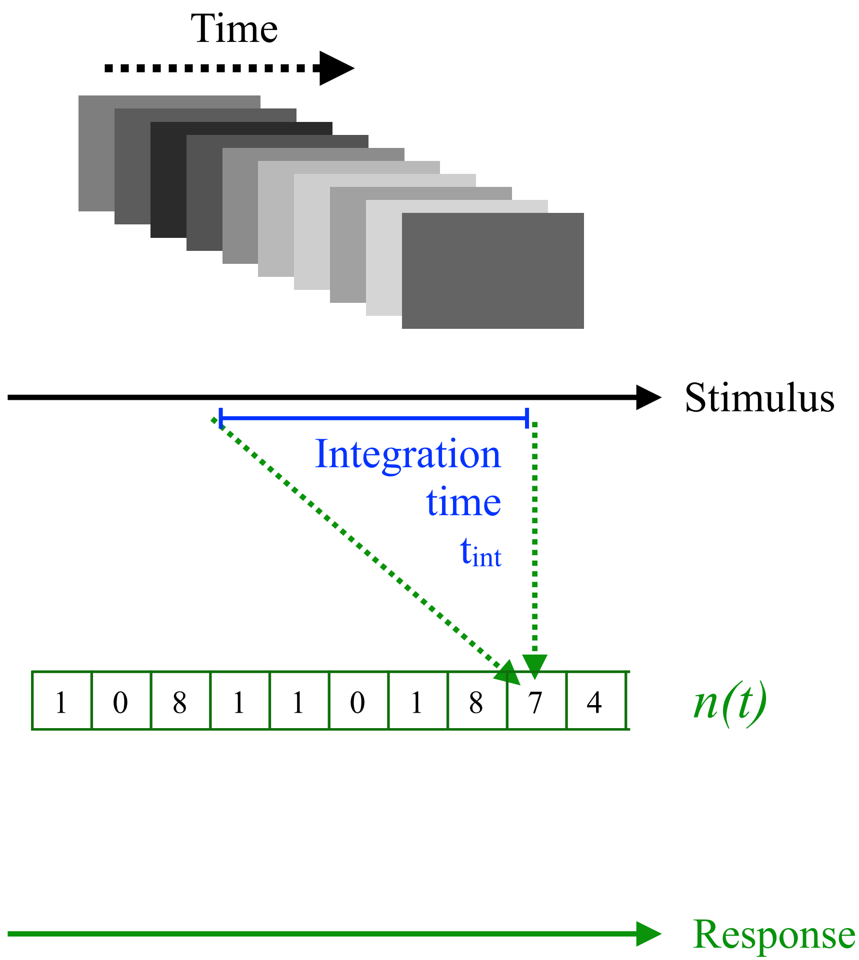

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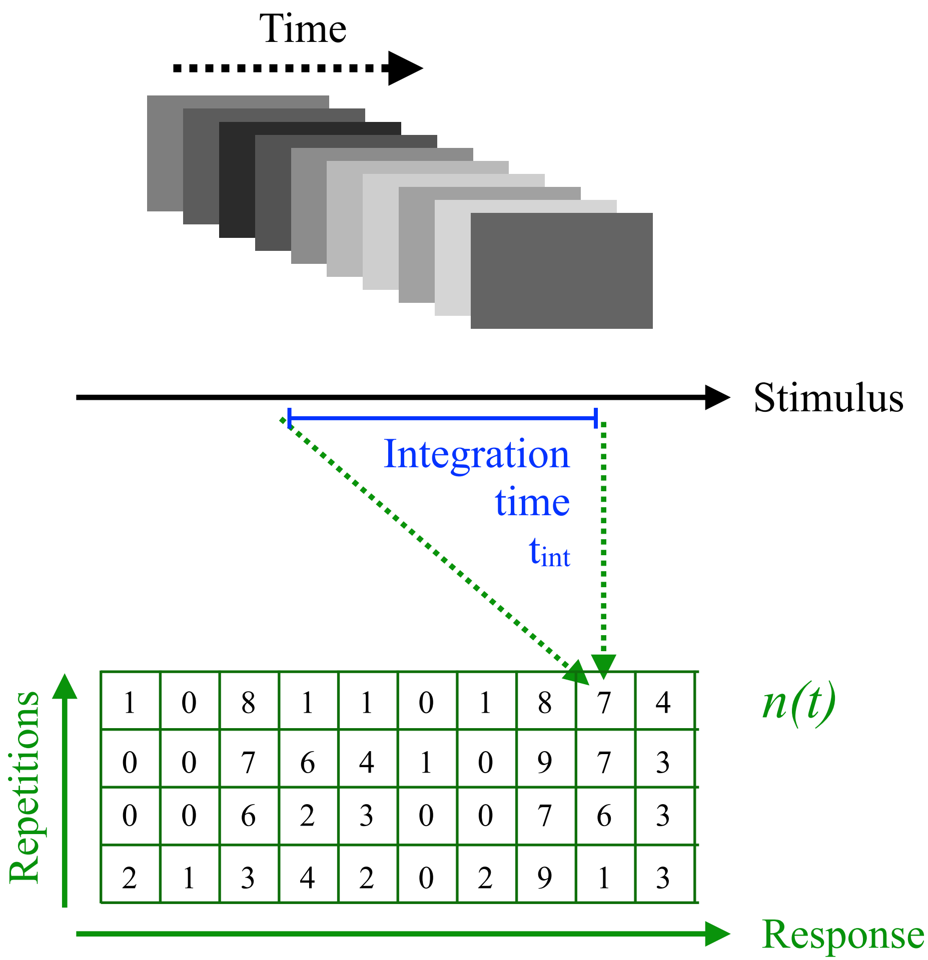

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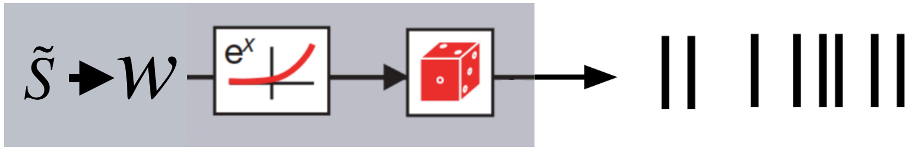
$$= \frac{1}{T} \sum_{t} \left(\frac{1}{R} \sum_{r} n^{r}(t)\right) \tilde{S}(t-\tau)$$

Linear-nonlinear Poisson model (LNP)

$$P_t(n) = \mathrm{Poisson}\left(n \mid f(t) \right) = \frac{f(t)^n}{n!} e^{-f(t)}$$



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Where NL(x) can be exp(x)
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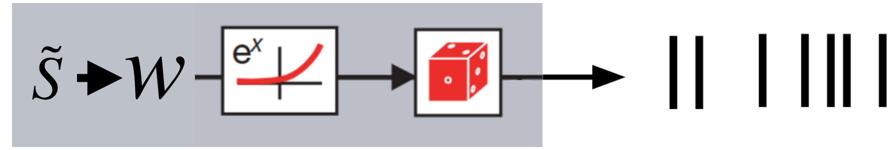

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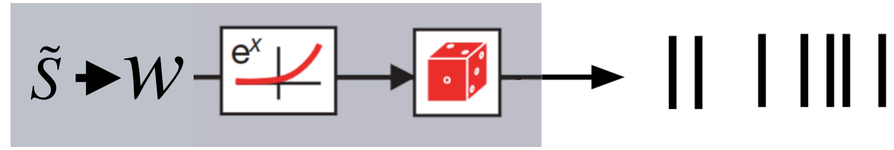

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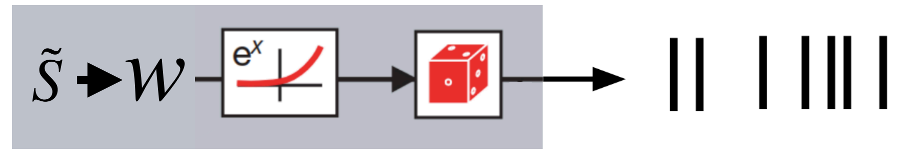

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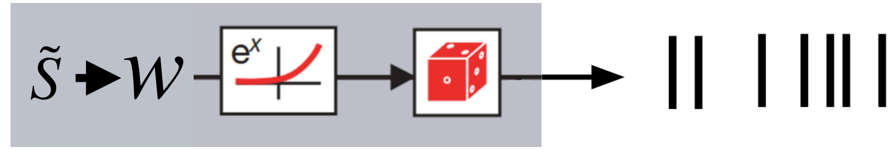

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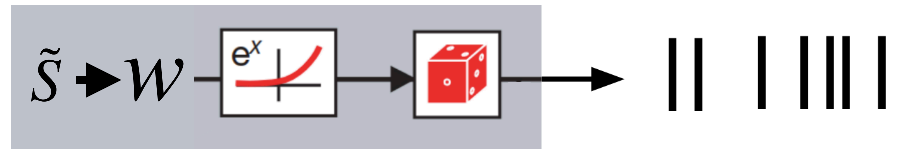

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$$\frac{\partial \ell(w,b)}{\partial b} = \sum_t \left[\big(\, n(t) - f(t) \,\big) \right]$$

Can be solved by steepest gradient

LNP model + L_2-Regularisation

To have a smooth *w*, we should minimise $\left(w(\tau) - w(\tau + 1) \right)^2$

LNP model + L_2-Regularisation

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$$\ell_\lambda(w,b) = \sum_t \left(n(t) \log f(t) - f(t) \right) - \frac{\lambda}{2} \sum_{\tau,\tau'} w(\tau)\, L(\tau,\tau')\, w(\tau')$$

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Laplacian: $$L(\tau,\tau') = \begin{pmatrix} 2 & -1 & & & & & & \\ -1 & 4 & -1 & & & & 0 & \\ & -1 & 4 & -1 & & & & \\ & & -1 & \ddots & & & & \\ & & & & \ddots & -1 & & \\ & & & & -1 & 4 & -1 & \\ & 0 & & & & -1 & 4 & -1 \\ & & & & & & -1 & 2 \end{pmatrix}$$

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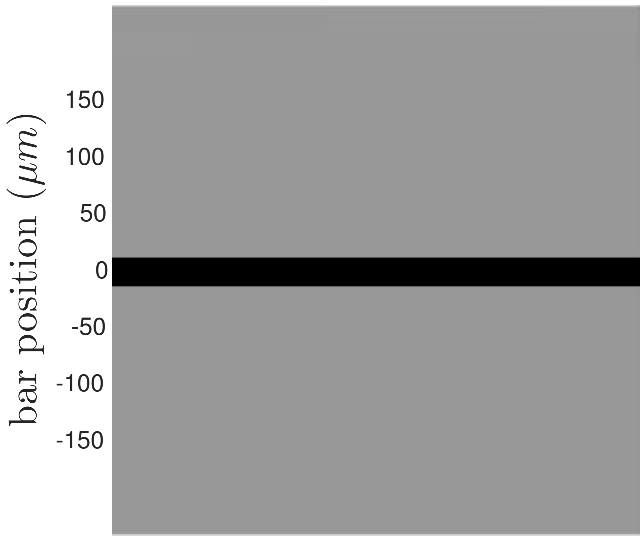
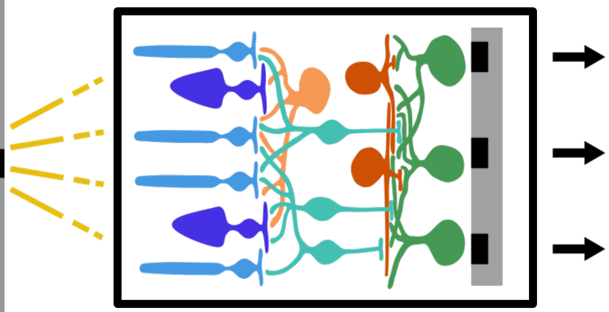
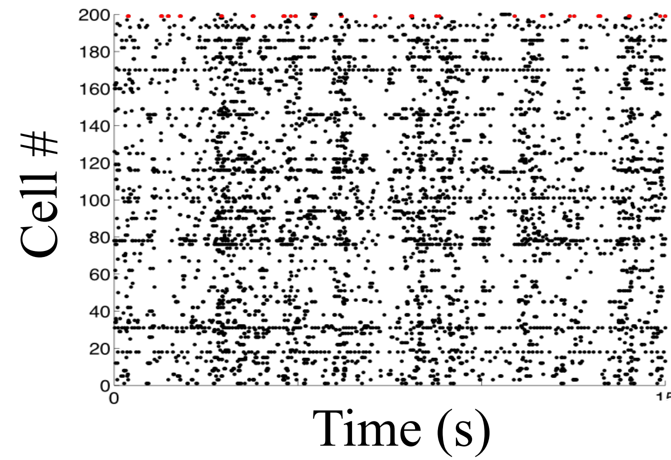
And the strength λ should be optimised over the validation set

Predicting retinal light responses to complex stimuli

What happens if the stimulus is more complex?

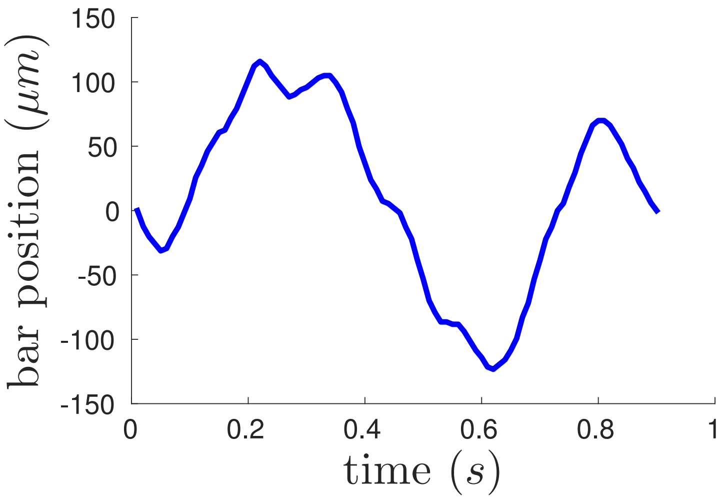






Relevant: test the tracking of moving object

Simple: fully described by one dimensional trajectory



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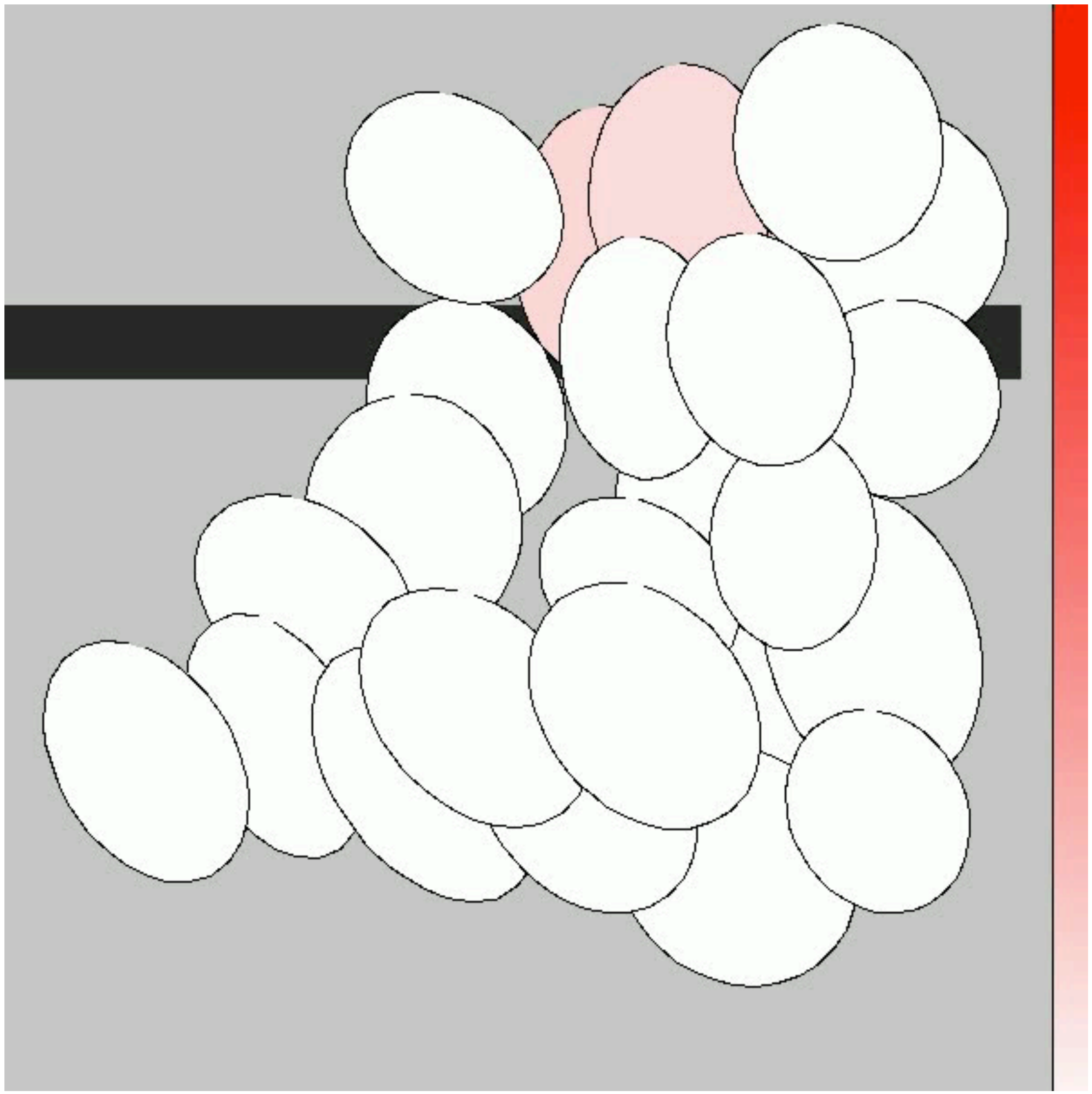

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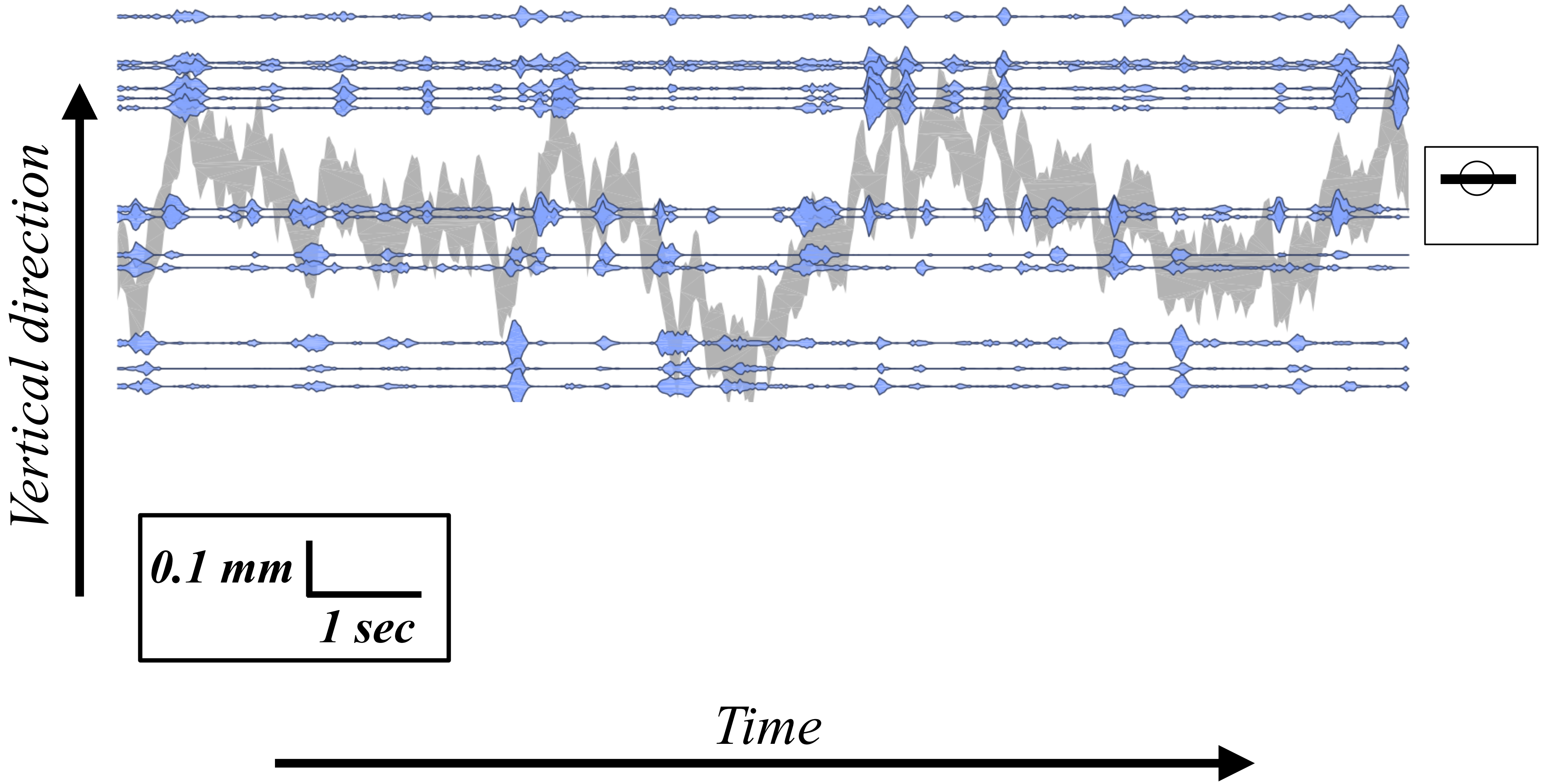
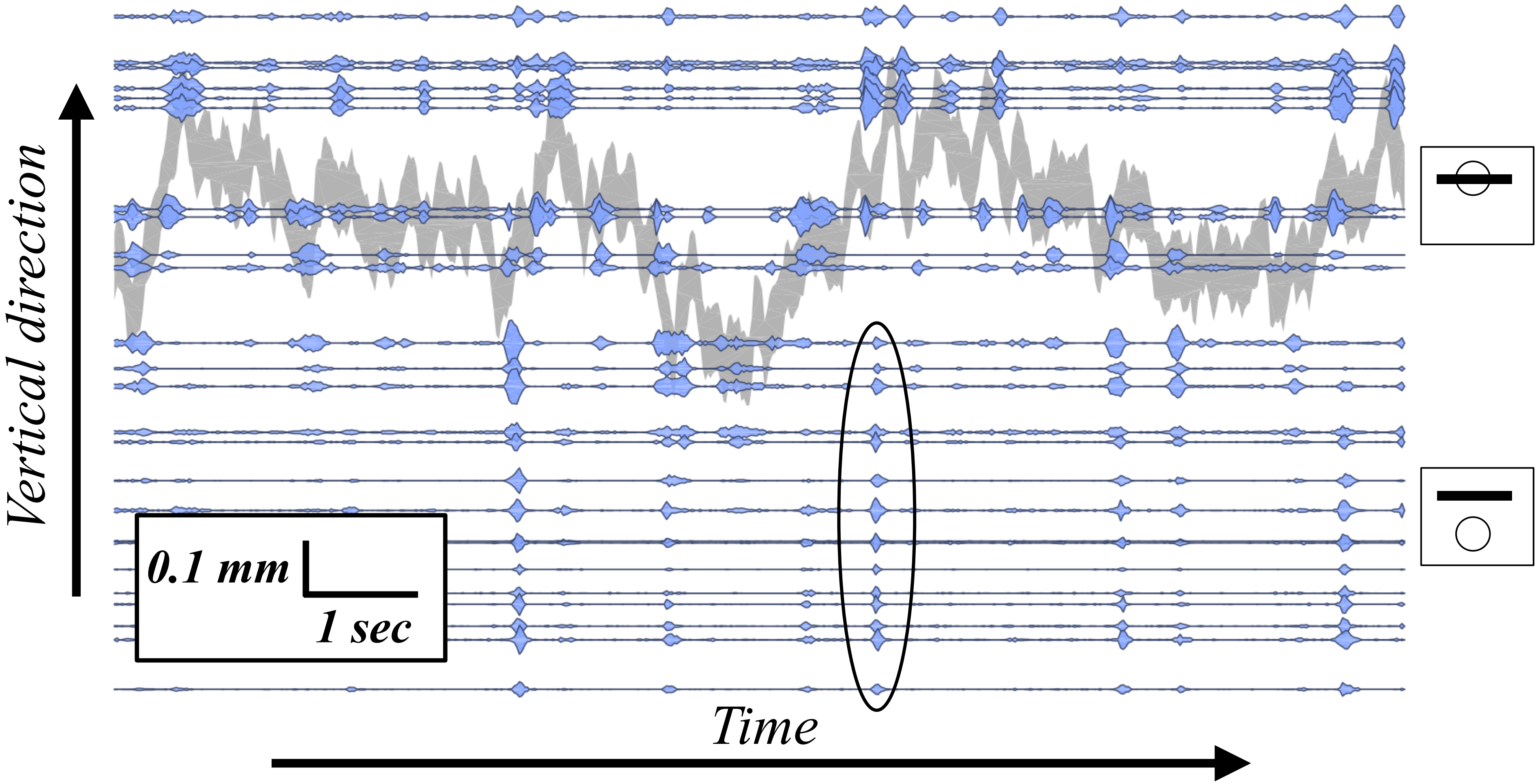
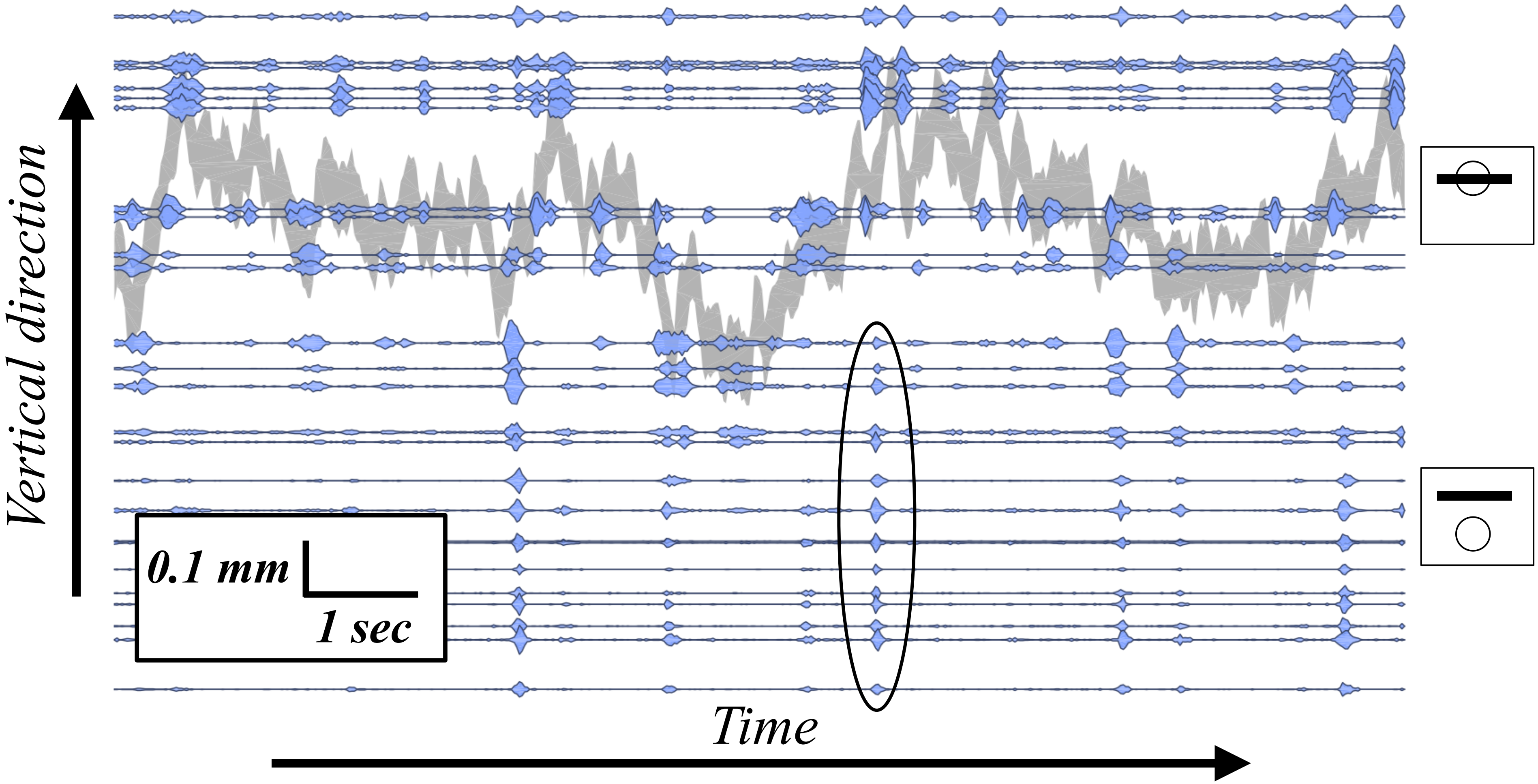


Predicting retinal light responses to complex stimuli



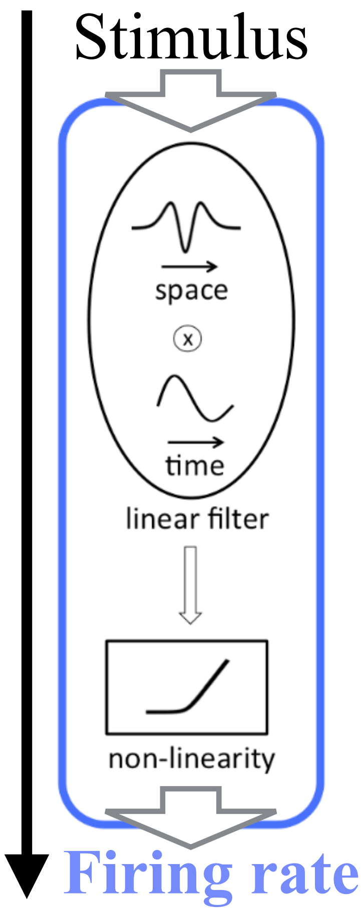
Predicting retinal light responses to complex stimuli

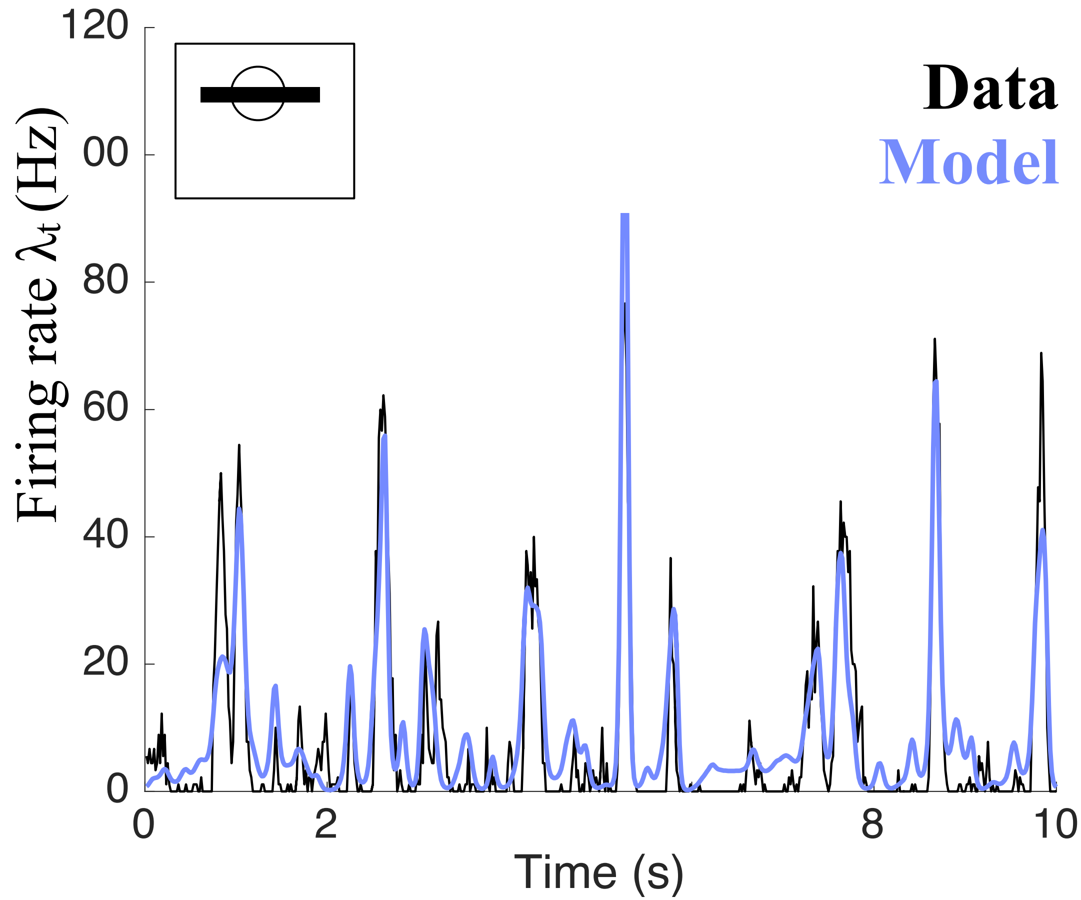


Same type of cell, but different computations!

Predicting retinal light responses to complex stimuli

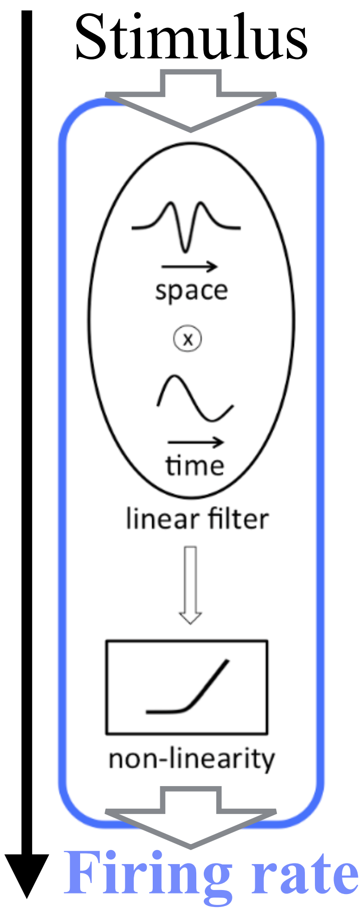
*A simple **LNP** model…*

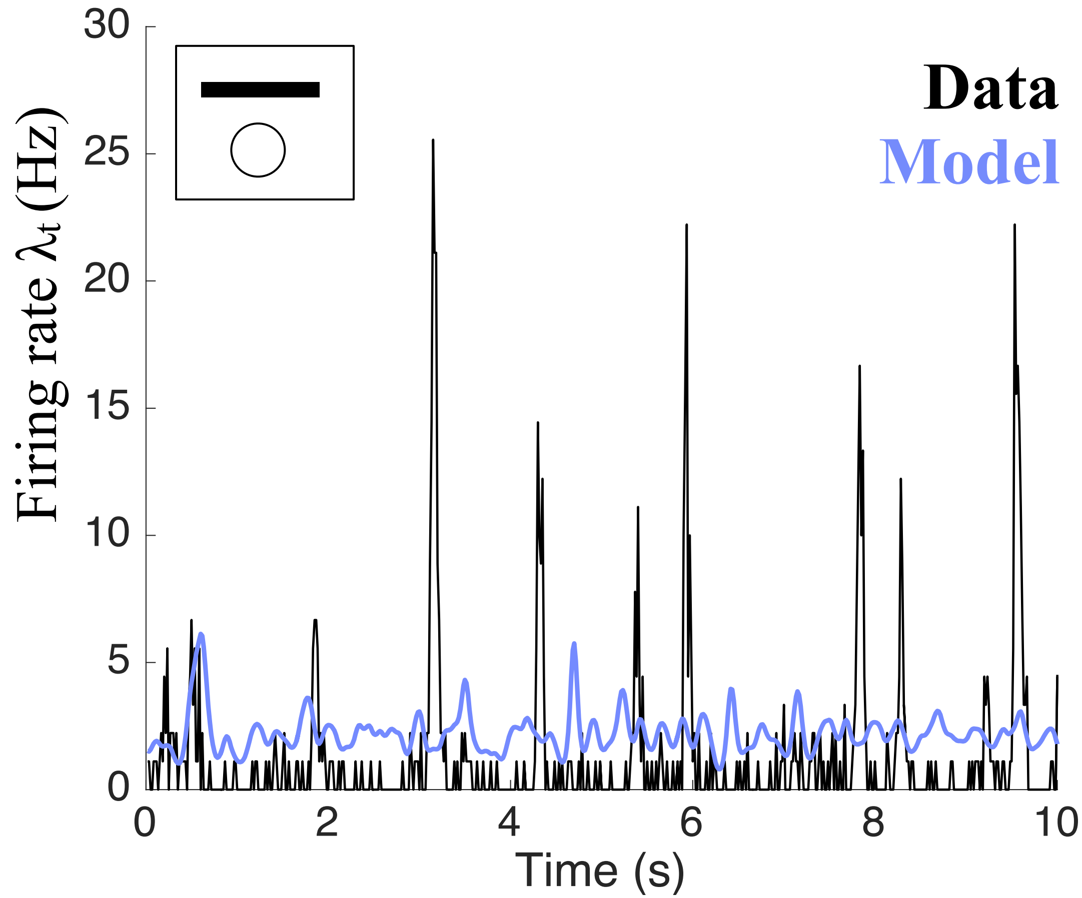




Predicting retinal light responses to complex stimuli

*A simple **LNP** model may not predict the response*





Predicting retinal light responses to complex stimuli

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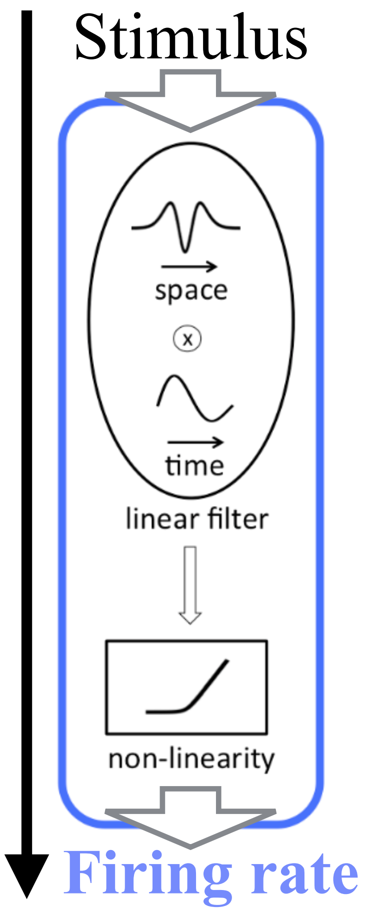
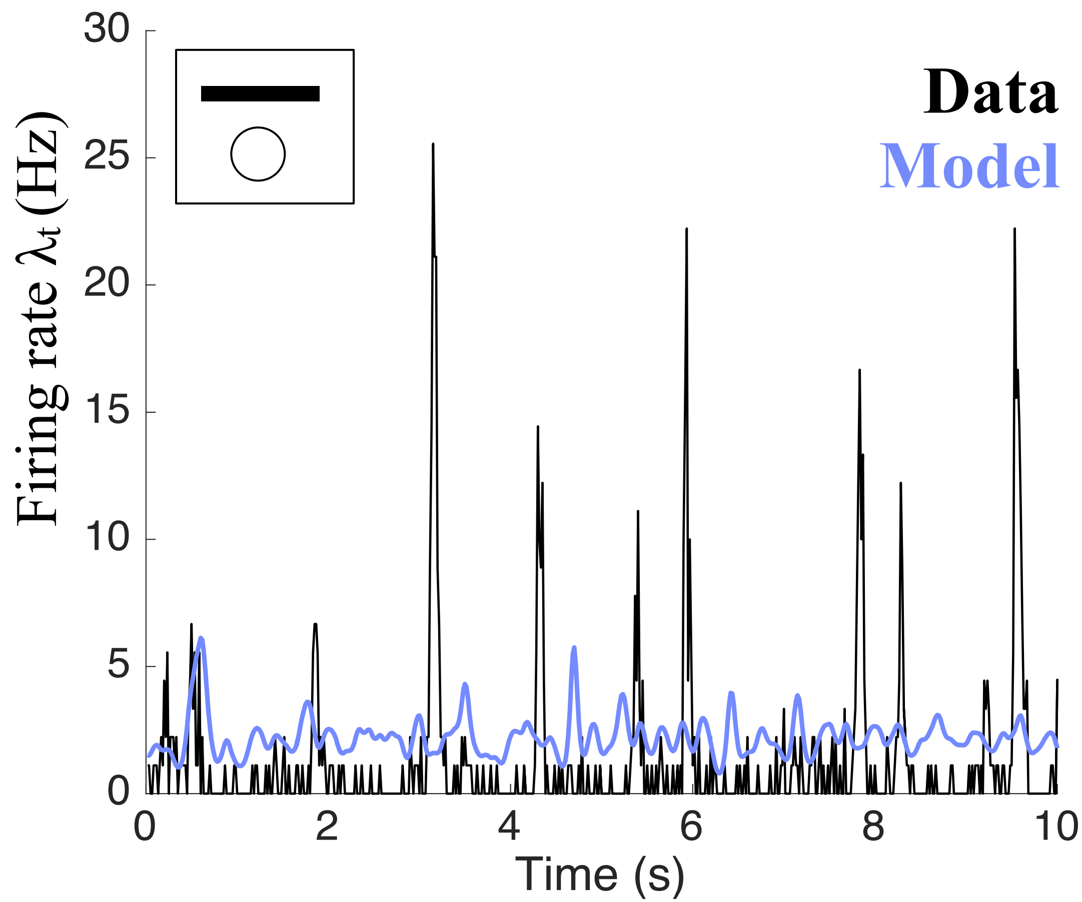
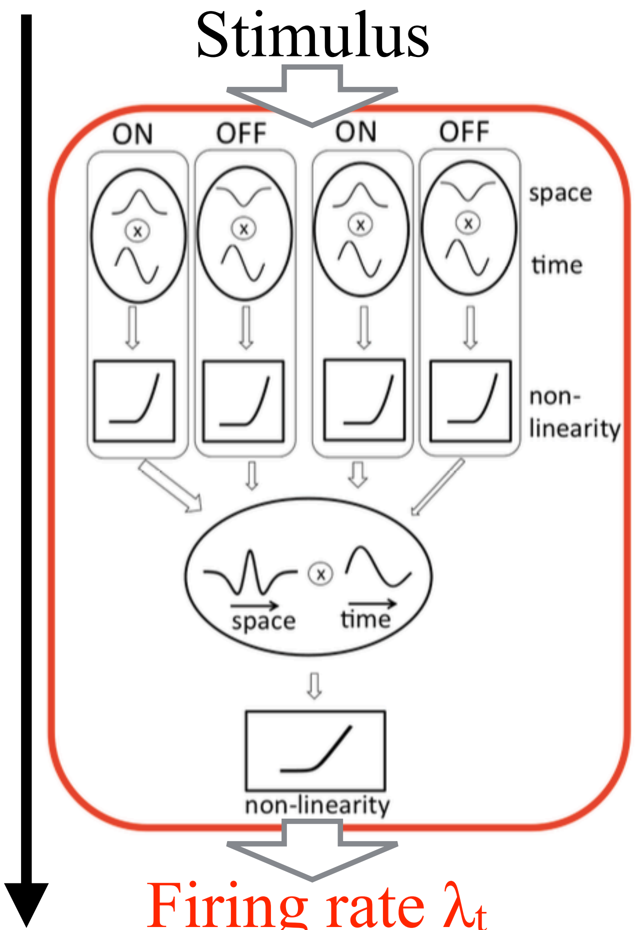




*A **LNP** model is not enough flexible!*

Predicting retinal light responses to complex stimuli

*An **highly non-linear** LN²P model…*



Sub-unit filter

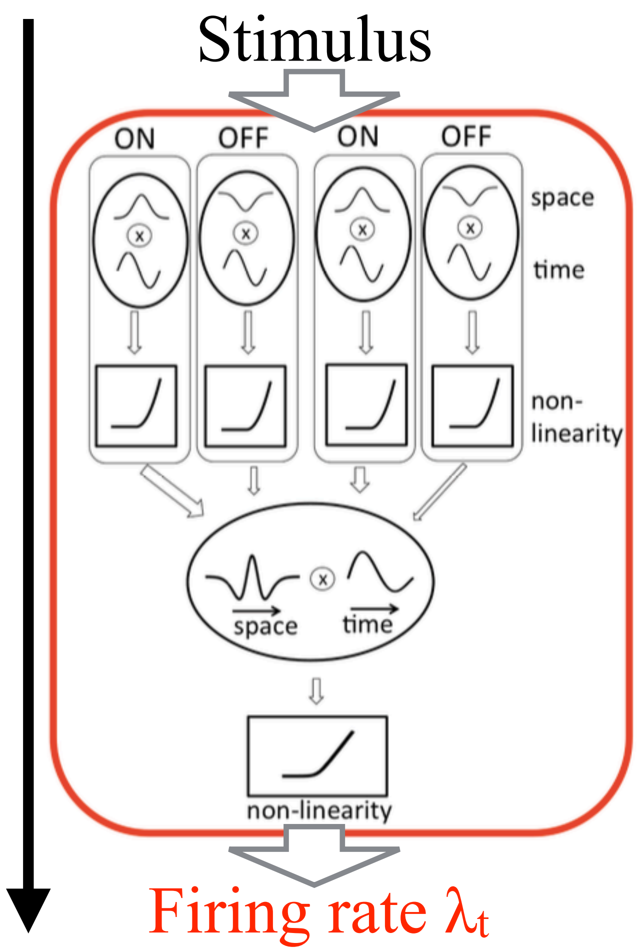
Nonlinear threshold

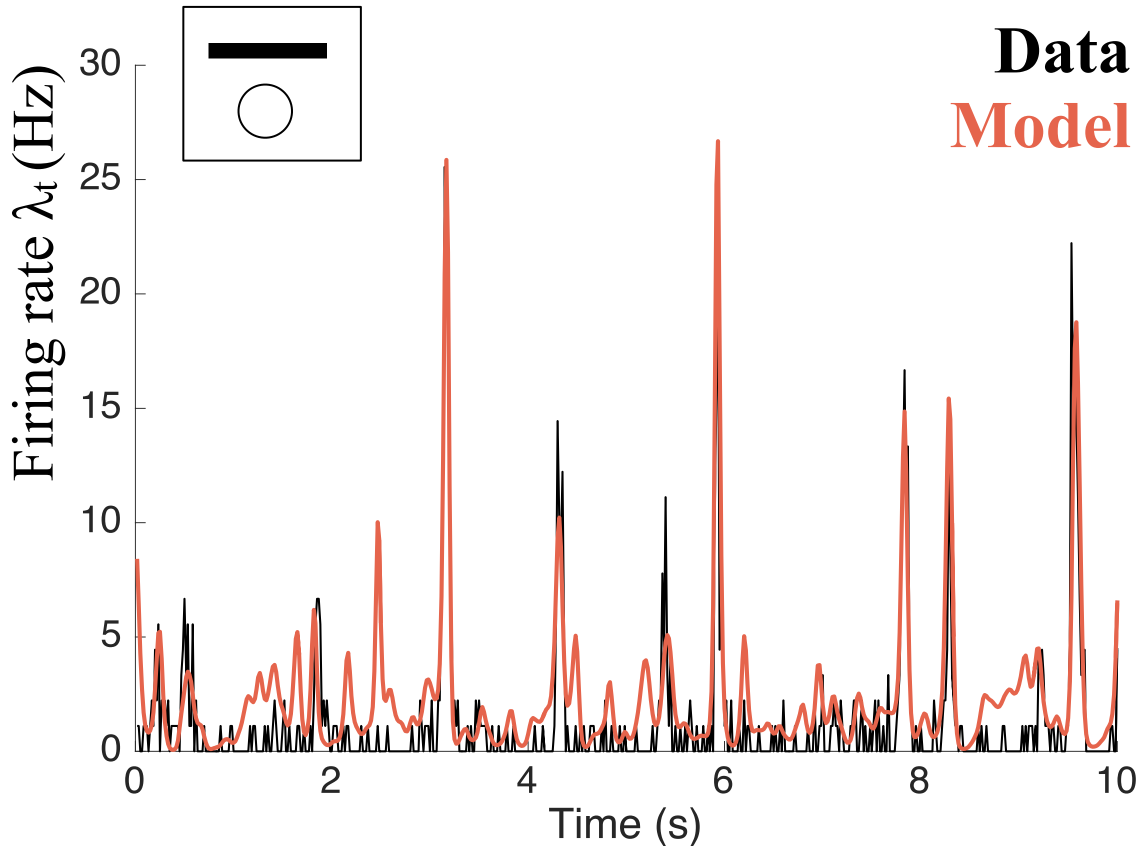
Receptive field filter

Nonlinear threshold

Predicting retinal light responses to complex stimuli

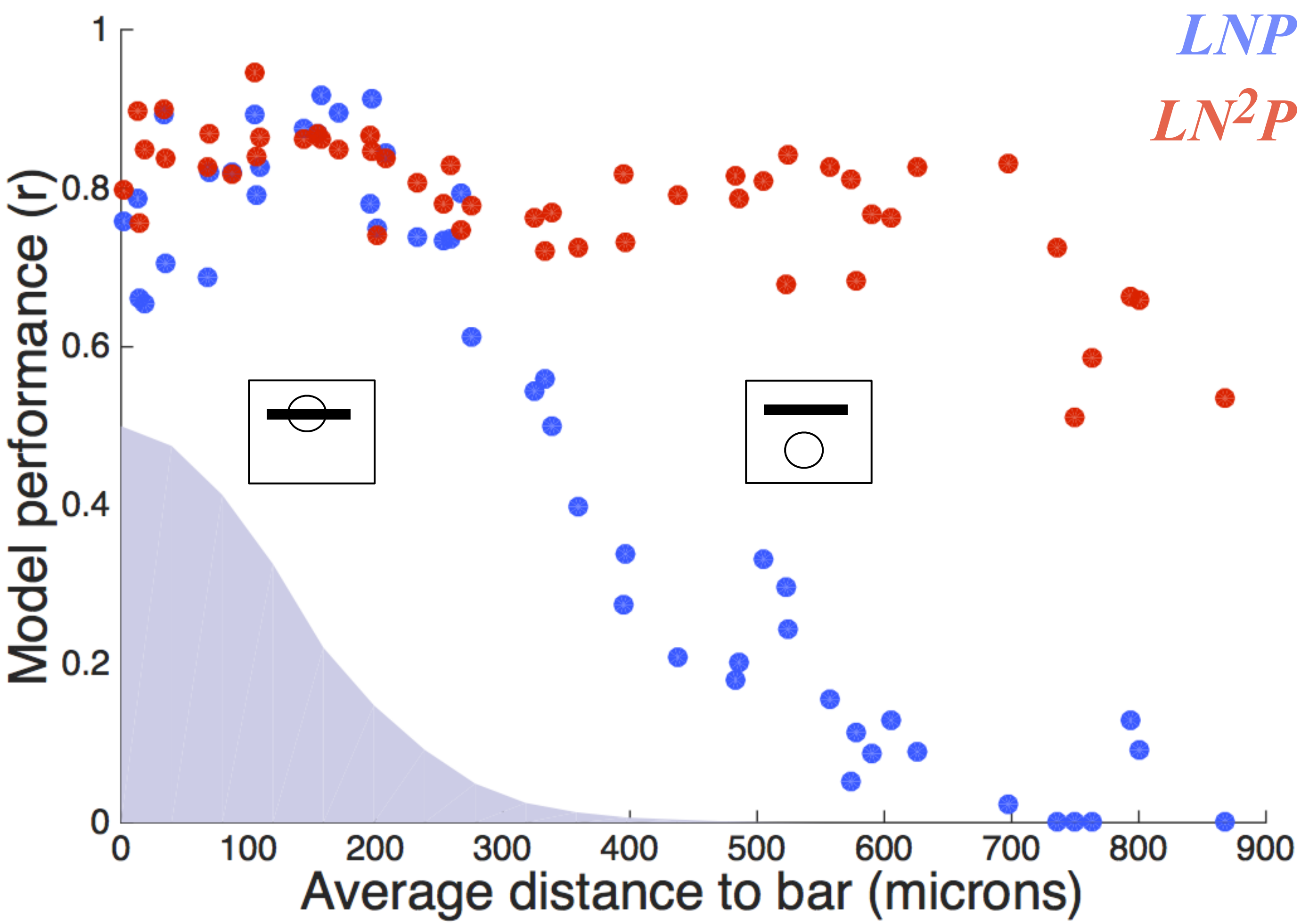
*An **highly non-linear** **LN²P** model predicts the response*





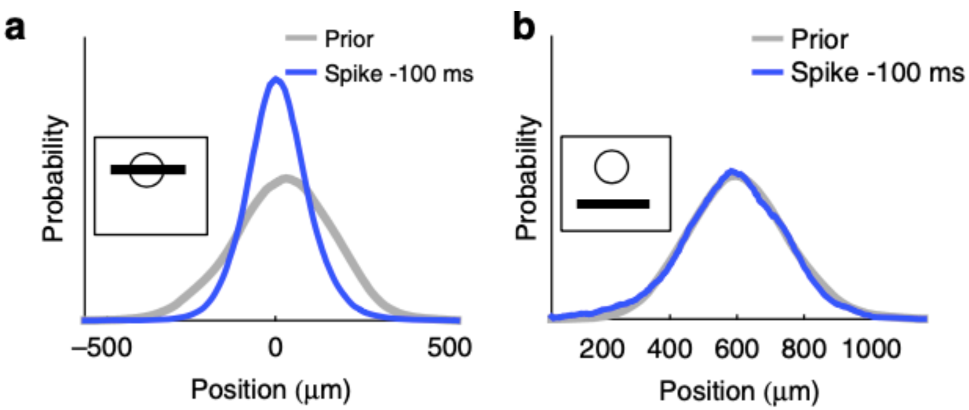
Predicting retinal light responses to complex stimuli

An ***highly non-linear*** *LN²P model predicts the response*



What are the neurons encoding?

How to quantify information carried by the spikes?



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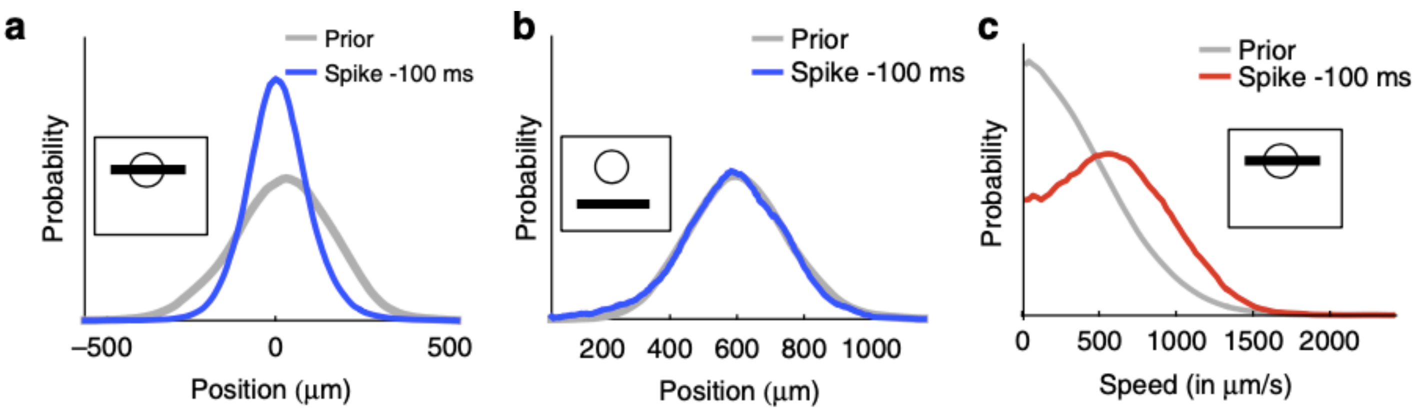

Same can be done for speed…

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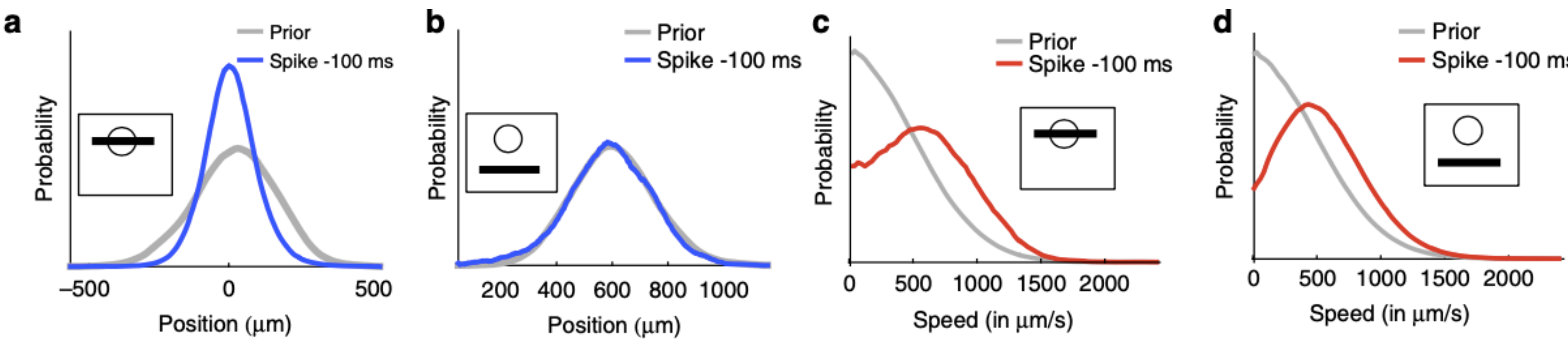
How to quantify information carried by the spikes?



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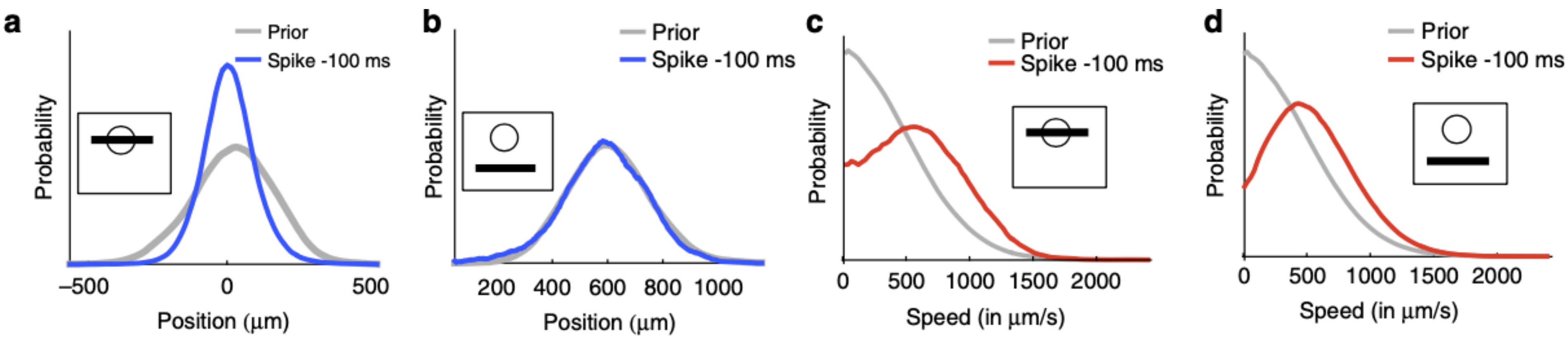

Mutual information between **pos** and **spikes**:

$$\mathcal{H}[\mathrm{pos}] - p(spike)\mathcal{H}[\,\mathrm{pos}\,|\,spike\,] - p(no\text{-}spike)\mathcal{H}[\,\mathrm{pos}\,|\,no\text{-}spike\,]$$

Same can be done for speed…

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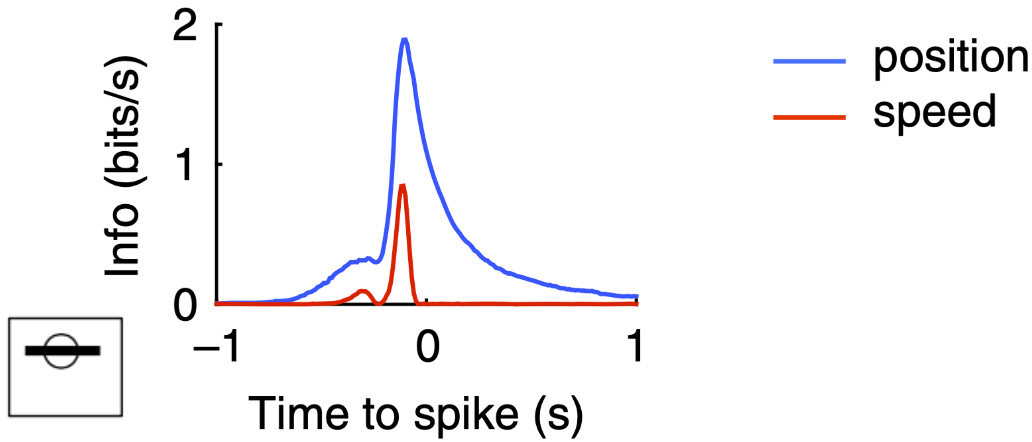

Mutual information between **pos** and **spikes**:

$$\mathcal{H}[\mathrm{pos}] - p(spike)\mathcal{H}[\,\mathrm{pos} \mid spike\,] - p(no\text{-}spike)\mathcal{H}[\,\mathrm{pos} \mid no\text{-}spike\,]$$

Same can be done for speed…

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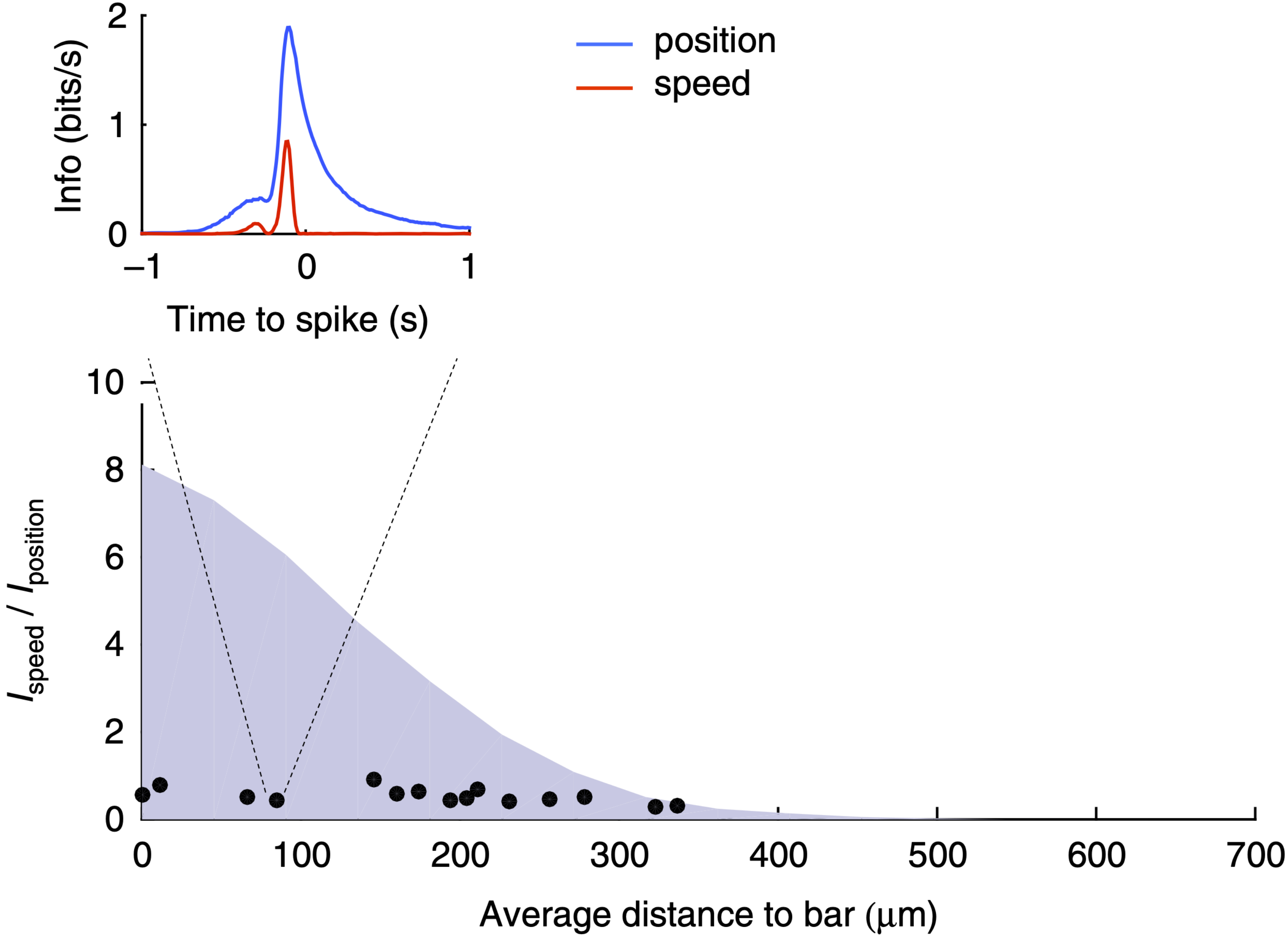

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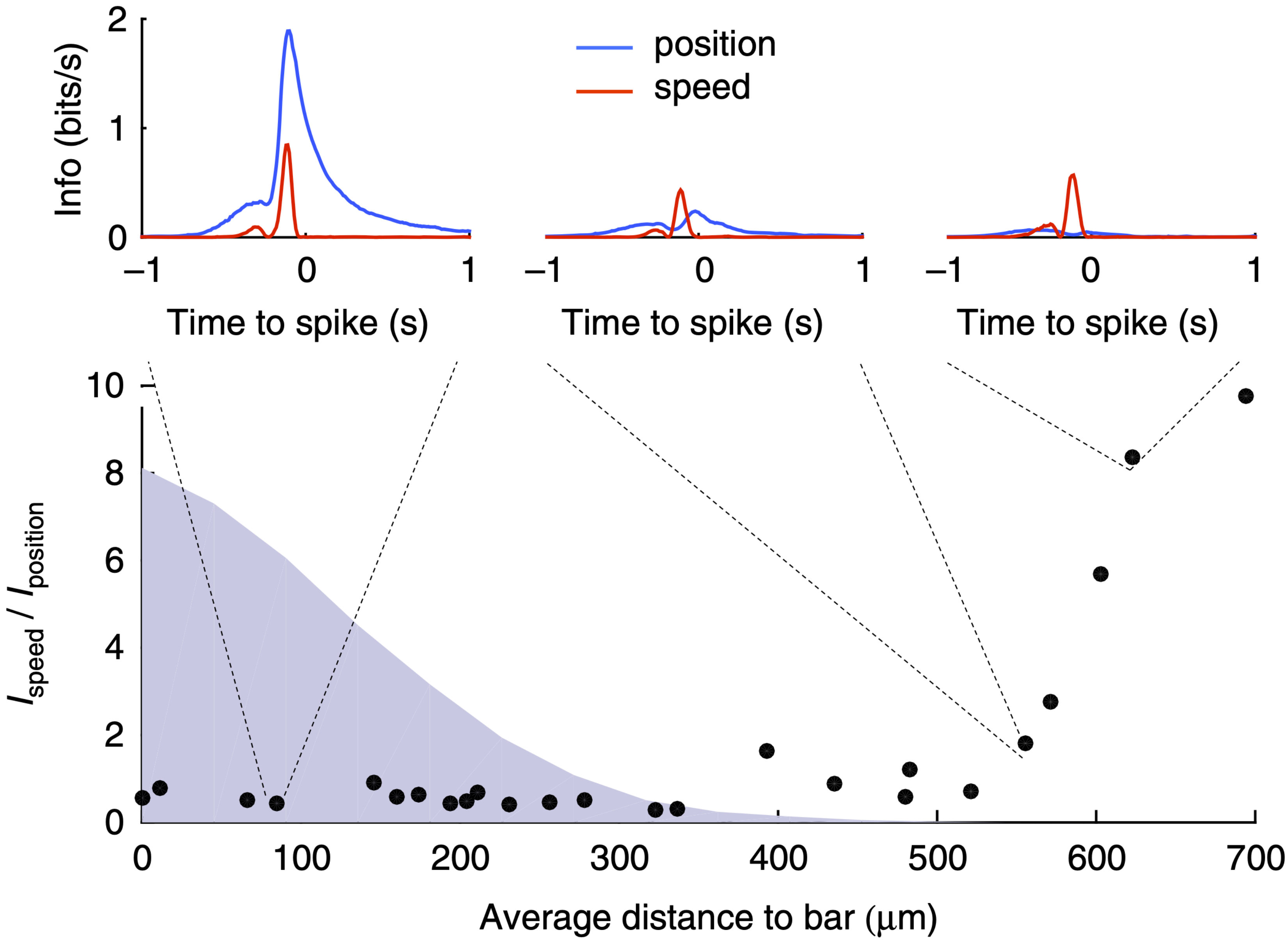
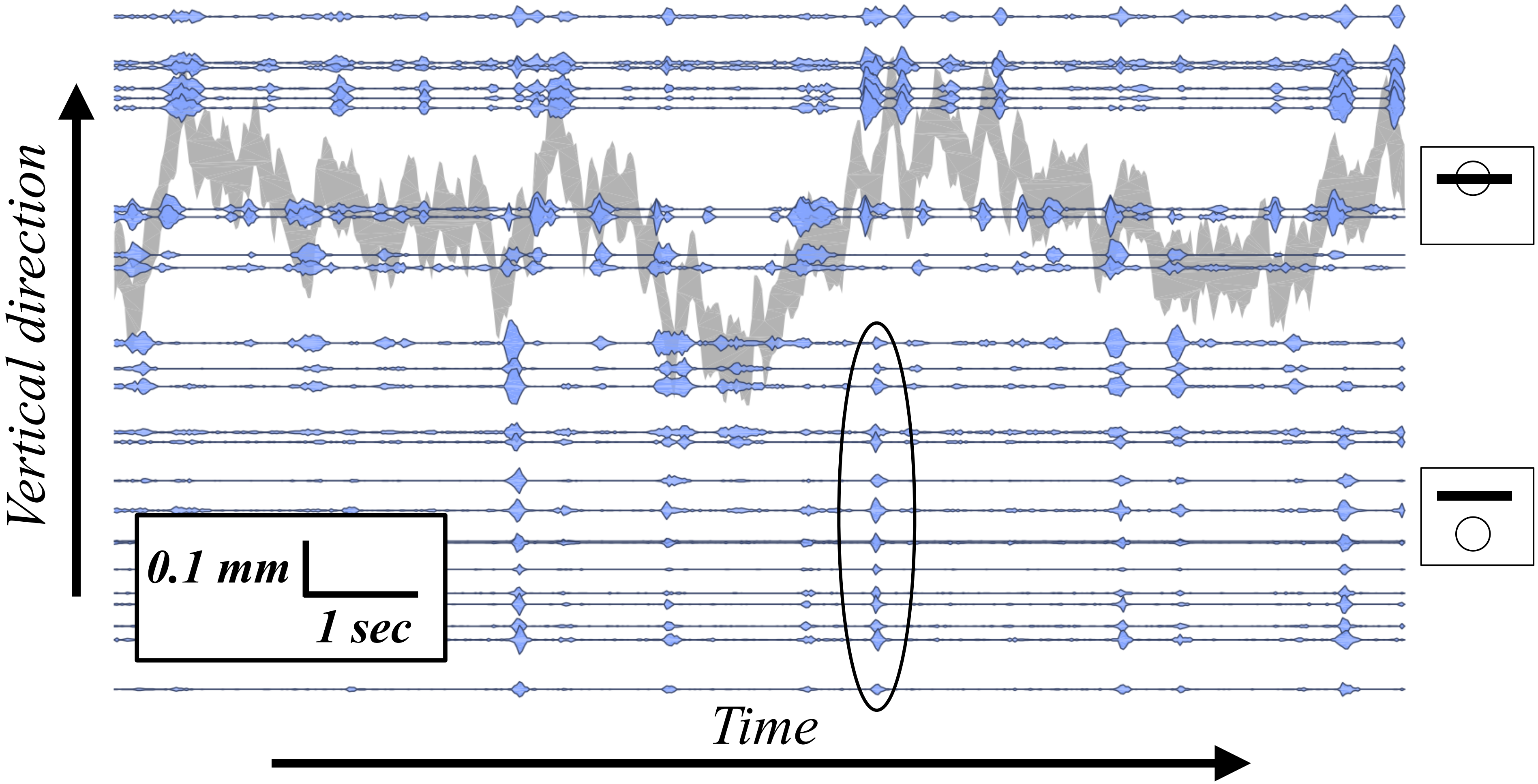


Predicting retinal light responses to complex stimuli



Same type of cell, but different computations!

Eyes are smarter than scientists believed

Ulisse Ferrari

ulisse.ferrari@gmail.com

INSTITUT DE LA VISION PARIS