Lecture 4: Amenities and Quantitative Spatial Economic Models ECO 567A

Geoffrey Barrows[1]

[1]CREST, CNRS, Ecole polytechnique

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Syllabus

- Part I: Demand for Local Environmental Quality
  - Intro (Jan 10)
  - Demand I - Estimation (Jan 17)
  - Demand II - Sorting and Environmental Justice (Jan 24)
  - Amenities and Quant. Spatial Economic Models (Jan 31)
- Part II: Supply of Local Environmental Quality - Energy
  - Energy Production (Feb 7)
  - Energy Demand (Feb 14)
  - Energy Efficiency Innovation (Feb 21)
  - Trade and Pollution (March 7)
- Part III: Global Externalities
  - Climate Change (March 14)

Last Week

- Roback (JPE, 82) lays out simple spatial equilibrium model with
  - Utility equalized across space
  - Identical Agents
  - No trade costs

  $\Longrightarrow$ Distribution of labor and production across space is indeterminate

- Can we augment the model so that model predicts population, production, consumption?
  - More realistic
  - Take to the data

Endogenous Amenity Supply

Diamond, Rebecca. "The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000." American Economic Review 106.3 (2016): 479-524.

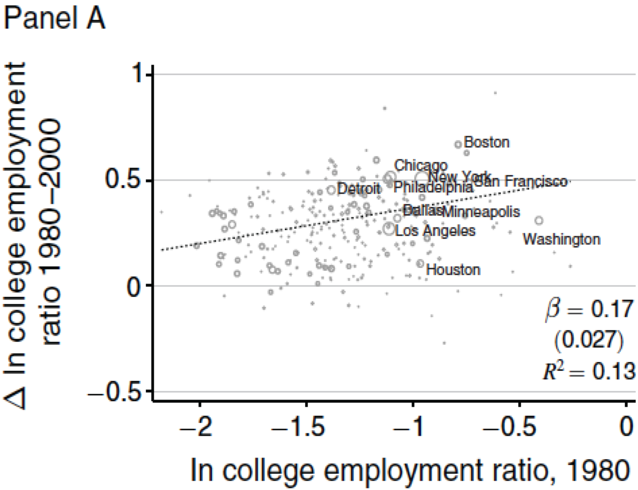
Skill Wage Gap has gone up

TABLE 2—OBSERVED CHANGES IN WAGES AND LOCAL REAL WAGES, 1980–2000

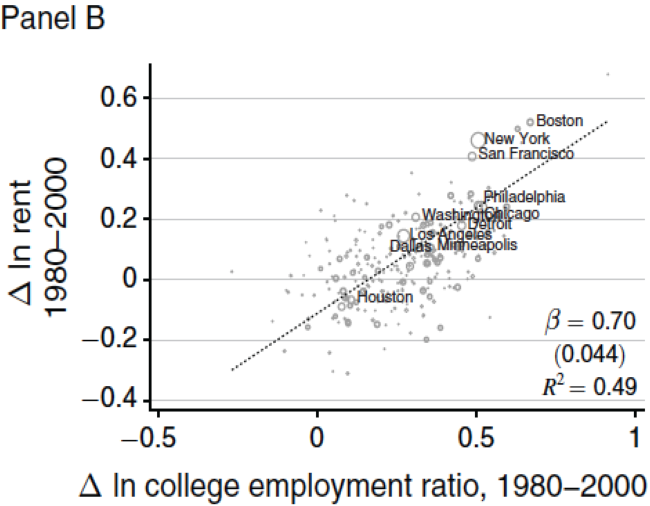
Year	College/high school grad wage gap (1)	College/high school grad rent gap (2)	Local real wage gap (3)
1980	0.383	0.048	0.353
	[0.0014]	[0.0004]	[0.0014]
1990	0.544	0.145	0.454
	[0.0010]	[0.0007]	[0.0009]
2000	0.573	0.119	0.499
	[0.0009]	[0.0004]	[0.0009]
Change, 1980–2000	0.190	0.072	0.146

Notes: Wage gap measures the log wage difference between college and high school graduates. Rent gap measures the log rent difference between college and high school graduates. Note that rent is measured as the city-level rent index and does not reflect differences in housing size choices. Local real wage gap measures the wages net of local rents gap.

High Skill Cities Became More High Skilled



High Skill Cities Became More Expensive



Skill Wage and Rent Gap have Both Gone Up

TABLE 2—OBSERVED CHANGES IN WAGES AND LOCAL REAL WAGES, 1980–2000

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Diamond

- With increased spatial concentration of High Skill labor, differential wage growth may be offset by differential rent growth
- But if amenities of “High Skill” cities also improve, then High Skill workers also benefit in terms of utility
- Additionally, there could be productivity feedbacks of amenities

Diamond

- Build a model with endogenous amenities
- Estimate structural parameters using Census data from the US
- Study the determinants of increased skill concentration (wages vs rents vs amenities) and the welfare consequences

Model

- Labor Demand
- Labor Supply – Household location choice
- Housing Supply
- Amenity Supply

Labor Demand

- Traded output of Firm d in city j in time t (Y_{djt}) combines Labor N_{djt} and capital K_{djt}

$$Y_{djt} = N_{djt}^{\alpha} K_{djt}^{1-\alpha}$$

- Labor Combines High Skill (H) And Low Skill (L)

$$N_{djt} = \left[\theta_{jt}^{L} L_{djt}^{\rho} + \theta_{jt}^{H} H_{djt}^{\rho}\right]^{\frac{1}{\rho}}$$

- Productivities

$$\theta_{jt}^{L} = f_L\left(H_{jt}, L_{jt}\right) e^{\epsilon_{jt}^{L}}$$
$$\theta_{jt}^{H} = f_H\left(H_{jt}, L_{jt}\right) e^{\epsilon_{jt}^{H}}$$

Labor Demand

- Firms choose inputs to minimize costs ($W^L_{jt}, W^H_{jt}, \kappa_t$ input prices)

$$\begin{aligned} \underset{L_{djt},H_{djt},K_{djt}}{\text{Min}} \quad & W^L_{jt} * L_{djt} + W^H_{jt} * H_{djt} + \kappa_t * K_{djt} \\ \text{subject to} \quad & Y_{djt} = N^{\alpha}_{djt} K^{1-\alpha}_{djt} \end{aligned}$$

- First Order Conditions

$$\begin{aligned} W^H_{jt} &= \alpha N^{\alpha-\rho}_{djt} K^{1-\alpha}_{djt} H^{\rho-1}_{djt} f_H (H_{jt}, L_{jt}) e^{\epsilon^H_{jt}} \\ W^L_{jt} &= \alpha N^{\alpha-\rho}_{djt} K^{1-\alpha}_{djt} L^{\rho-1}_{djt} f_L (H_{jt}, L_{jt}) e^{\epsilon^L_{jt}} \\ \kappa_t &= N^{\alpha}_{djt} K^{-\alpha}_{djt} (1-\alpha) \end{aligned}$$

- Log-Linearized Labor Demand at City Level

$$\begin{aligned} w^H_{jt} &= \gamma_{HH} \ln H_{jt} + \gamma_{HL} \ln L_{jt} + \epsilon^H_{jt} \\ w^L_{jt} &= \gamma_{LH} \ln H_{jt} + \gamma_{LL} \ln L_{jt} + \epsilon^L_{jt} \end{aligned}$$

Households

- Households (i) choose local good M and traded good O to maximize utility with local price R_{jt} and traded prices P_t

$$\begin{aligned} \underset{M_{it},O_{it}}{\text{Max}} \quad & \ln\left(M_{it}^{\zeta}\right) + \ln\left(O_{it}^{1-\zeta}\right) + s_i\left(A_{jt}\right) \\ \text{subject to} \quad & P_t O_{it} + R_{jt} M_{it} \leq W_{jt}^{edu} \end{aligned}$$

- Marshallian Demands for M and O

$$\begin{aligned} M_{it} &= \frac{\zeta W_{jt}^{edu}}{R_{jt}} \\ O_{it} &= \frac{(1-\zeta) W_{jt}^{edu}}{P_t} \end{aligned}$$

- Indirect Utility for household i living in city j at time t

$$\begin{aligned} V_{ijt} &= \ln\left(\frac{W_{jt}^{edu}}{P_t}\right) - \zeta \ln\left(\frac{R_{jt}}{P_t}\right) + s_i\left(A_{jt}\right) \\ &= w_{jt}^{edu} - \zeta r_{jt} + s_i\left(A_{jt}\right) \end{aligned}$$

Demand for Amenities

- Demand for Amenities

$$
\begin{aligned}
s_i(A_{jt}) &= a_{jt}\beta_i^a + \boldsymbol{x_{jt}^A}\boldsymbol{\beta_i^x} + \boldsymbol{x_j^{st}}\boldsymbol{\beta_i^{st}} + \boldsymbol{x_j^{div}}\boldsymbol{\beta_i^{div}} + \sigma_i\epsilon_{ijt}
\end{aligned}
$$

$$
\begin{aligned}
\beta_i^a &= \boldsymbol{\beta^a}\boldsymbol{z_i} \\
\boldsymbol{\beta_i^x} &= \boldsymbol{\beta^x}\boldsymbol{z_i} \\
\beta_i^{st} &= \boldsymbol{st_i}\boldsymbol{\beta^{st}}\boldsymbol{z_i} \\
\beta_i^{div} &= \boldsymbol{div_i}\boldsymbol{\beta^{div}}\boldsymbol{z_i} \\
\sigma_i &= \beta^\sigma\boldsymbol{z_i} \\
\epsilon_{ijt} &\sim \text{Type I Extreme Value}
\end{aligned}
$$

- z_i is a 3X1 vector of dummy variables for white, black, and immigrant

a_{jt} is endogenous amenities, X_{jt}^A is exogenous amenities, *st* indicates state, *div* indicates census division, σ_i is a normalization so that standard deviation of $\epsilon_{ijt} = 1$

Demand for Location j

- Indirect Utility for household i living in city j at time t

$$V_{ijt} = \delta^z_{jt} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \boldsymbol{\beta^{st}} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \boldsymbol{\beta^{div}} \boldsymbol{z_i} + \epsilon_{ijt}$$

- With common utility

$$\delta^z_{jt} = \left(w^{edu}_{jt} - \zeta r_{jt} \right) \boldsymbol{\beta^w} \boldsymbol{z} + a_{jt} \boldsymbol{\beta^a} \boldsymbol{z} + \boldsymbol{x^A_{jt}} \boldsymbol{\beta^x} \boldsymbol{z}$$

Demand for Location j

- Probability that a given household i chooses to live in j

$$
\begin{aligned}
Pr(V_{ijt} > V_{ij't}) &= Pr(\delta_{jt}^{z_i} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i} + \epsilon_{ijt} \\
&> \delta_{j't}^{z_i} + \boldsymbol{x_{j'}^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_{j'}^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i} + \epsilon_{ij't})
\end{aligned}
$$

- Yields standard conditional logit formula

$$
\begin{aligned}
H_{jt} &= \sum_{i \in H_t} \frac{\exp\left(\delta_{jt}^{z_i} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i}\right)}{\sum_k^J \exp\left(\delta_{kt}^{z_i} + \boldsymbol{x_k^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_k^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i}\right)} \\
L_{jt} &= \sum_{i \in L_t} \frac{\exp\left(\delta_{jt}^{z_i} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i}\right)}{\sum_k^J \exp\left(\delta_{kt}^{z_i} + \boldsymbol{x_k^{st}} \boldsymbol{st_i} \beta^{st} \boldsymbol{z_i} + \boldsymbol{x_k^{div}} \boldsymbol{div_i} \beta^{div} \boldsymbol{z_i}\right)}
\end{aligned}
$$

Supply of Housing

- Price of housing equals marginal cost of housing construction (function of construction cost CC and Land cost LC)

$$P_{jt}^{house} = MC\left(CC_{jt}, LC_{jt}\right)$$

- Housing rental price (ι_t is interest rate)

$$R_{jt} = \iota_t * MC\left(CC_{jt}, LC_{jt}\right)$$

- Log linearization

$$\begin{aligned} r_{jt} &= \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(LC_{jt}\left(HD_{jt}\right)) \\ &= \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(HD_{jt}) \end{aligned}$$

with

$$\gamma_j = \gamma + \gamma^{geo} * e^{x_j^{geo}} + \gamma^{reg} * e^{x_j^{reg}} \quad , \quad HD_{jt} = L_{jt} \frac{\zeta W_{jt}^L}{R_{jt}} + H_{jt} \frac{\zeta W_{jt}^H}{R_{jt}}$$

- x_j^{geo} is share of land not suitable for development, x_j^{reg} is index of local growth control policies, HD_{jt} is housing demand (from M demanded by agents)

Supply of Amenities

- Endogenous Amenity a_{jt} is an index of
  - School Quality, retail environment, crime, environment, transportation infrastructure, quality of the job market (beyond wages)
- Assume Endogenous Amenity a_{jt} responds to skill mix

$$a_{jt} = \gamma^a \ln\left(\frac{H_{jt}}{L_{jt}}\right) + \epsilon^a_{jt}$$

Skill Mix and Amenities

Panel A. Retail amenities

	Apparel stores per 1,000 residents	Eating and drinking places per 1,000 residents	Movie theaters per 1,000 residents
Δ College emp. ratio	0.477***	0.182***	0.230
	[0.0928]	[0.0539]	[0.166]

Panel B. Transportation amenities

	Bus routes per capita	Public transit index	Avg. daily traffic: interstates	Avg. daily traffic: major roads
Δ College emp. ratio	1.045***	0.0161	−0.169*	−0.0513
	[0.376]	[0.338]	[0.0979]	[0.0704]

	Property crimes per 1,000 residents	Violent crimes per 1,000 residents	Gov. spending on parks per capita	EPA air quality index
	Panel C. Crime amenities		*Panel D. Environment amenities*	
Δ College emp. ratio	−0.231*	0.115	0.263	−0.539***
	[0.122]	[0.155]	[0.172]	[0.171]
	Gov. K–12 spending per student	**Student–teacher ratio**	**Patents per capita**	**Employment rate**
	Panel E. School amenities		*Panel F. Job amenities*	
Δ College emp. ratio	0.129**	0.00423	0.104	0.0105
	[0.0639]	[0.0631]	[0.234]	[0.00787]

Equilibrium

Equilibrium is menu of wages, rents, amenities $(w_{jt}^{L*}, w_{jt}^{H*}, r_{jt}^{*}, \frac{H_{jt}^{*}}{L_{jt}^{*}})$ with population levels H_{jt}^{*}, L_{jt}^{*} such that 4 market clear:

1. High Skill Labor

$$
\begin{aligned}
H_{jt}^{*} &= \sum_{i \in H_t} \frac{\exp\left(\delta_{jt}^{z_i} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \beta^{\boldsymbol{st}} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \beta^{\boldsymbol{div}} \boldsymbol{z_i}\right)}{\sum_{k}^{J} \exp\left(\delta_{kt}^{z_i} + \boldsymbol{x_k^{st}} \boldsymbol{st_i} \beta^{\boldsymbol{st}} \boldsymbol{z_i} + \boldsymbol{x_k^{div}} \boldsymbol{div_i} \beta^{\boldsymbol{div}} \boldsymbol{z_i}\right)} \\
w_{jt}^{H*} &= \gamma_{HH} \ln H_{jt}^{*} + \gamma_{HL} \ln L_{jt}^{*} + \epsilon_{jt}^{H}
\end{aligned}
$$

2. Low Skill Labor

$$
\begin{aligned}
L_{jt}^{*} &= \sum_{i \in L_t} \frac{\exp\left(\delta_{jt}^{z_i} + \boldsymbol{x_j^{st}} \boldsymbol{st_i} \beta^{\boldsymbol{st}} \boldsymbol{z_i} + \boldsymbol{x_j^{div}} \boldsymbol{div_i} \beta^{\boldsymbol{div}} \boldsymbol{z_i}\right)}{\sum_{k}^{J} \exp\left(\delta_{kt}^{z_i} + \boldsymbol{x_k^{st}} \boldsymbol{st_i} \beta^{\boldsymbol{st}} \boldsymbol{z_i} + \boldsymbol{x_k^{div}} \boldsymbol{div_i} \beta^{\boldsymbol{div}} \boldsymbol{z_i}\right)} \\
w_{jt}^{L*} &= \gamma_{LH} \ln H_{jt}^{*} + \gamma_{LL} \ln L_{jt}^{*} + \epsilon_{jt}^{L}
\end{aligned}
$$

Equilibrium

Equilibrium is menu of wages, rents, amenities $(w_{jt}^{L*}, w_{jt}^{H*}, r_{jt}^{*}, \frac{H_{jt}^{*}}{L_{jt}^{*}})$ with population levels H_{jt}^{*}, L_{jt}^{*} such that 4 market clear:

3. Housing

$$
\begin{aligned}
r_{jt} &= \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(HD_{jt}) \\
HD_{jt} &= L_{jt}\frac{\zeta W_{jt}^{L}}{R_{jt}} + H_{jt}\frac{\zeta W_{jt}^{H}}{R_{jt}}
\end{aligned}
$$

4. Amenities

$$
\begin{aligned}
a_{jt} &= \gamma^{a} \ln\left(\frac{H_{jt}}{L_{jt}}\right) + \epsilon_{jt}^{a} \\
\delta_{jt}^{z} &= \left(w_{jt}^{edu} - \zeta r_{jt}\right)\beta^{w} z + a_{jt}\beta^{a} z + x_{jt}^{A}\beta^{x} z
\end{aligned}
$$

Estimation Strategy

1. Estimate $\delta_{jt}^{educ,z}$ by conditional Logit

2. Estimate all structural parameters simultaneously by GMM

$$
\begin{aligned}
\delta_{jt}^{H,z} &= \left(w_{jt}^{H} - \zeta r_{jt}\right) \boldsymbol{\beta}^{\boldsymbol{w},\boldsymbol{H}}\boldsymbol{z} + a_{jt}\boldsymbol{\beta}^{\boldsymbol{a},\boldsymbol{H}}\boldsymbol{z} + x_{jt}^{A}\boldsymbol{\beta}^{\boldsymbol{x},\boldsymbol{H}}\boldsymbol{z} \\
\delta_{jt}^{L,z} &= \left(w_{jt}^{L} - \zeta r_{jt}\right) \boldsymbol{\beta}^{\boldsymbol{w},\boldsymbol{L}}\boldsymbol{z} + a_{jt}\boldsymbol{\beta}^{\boldsymbol{a},\boldsymbol{L}}\boldsymbol{z} + x_{jt}^{A}\boldsymbol{\beta}^{\boldsymbol{x},\boldsymbol{L}}\boldsymbol{z} \\
w_{jt}^{H*} &= \gamma_{HH} \ln H_{jt}^{*} + \gamma_{HL} \ln L_{jt}^{*} + \epsilon_{jt}^{H} \\
w_{jt}^{L*} &= \gamma_{LH} \ln H_{jt}^{*} + \gamma_{LL} \ln L_{jt}^{*} + \epsilon_{jt}^{L} \\
r_{jt} &= \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(L_{jt}\frac{\zeta W_{jt}^{L}}{R_{jt}} + H_{jt}\frac{\zeta W_{jt}^{H}}{R_{jt}}) \\
a_{jt} &= \gamma^{a} \ln\left(\frac{H_{jt}}{L_{jt}}\right) + \epsilon_{jt}^{a}
\end{aligned}
$$

Structural parameters to estimate

1. Labor Demand

$$\gamma_{HH}, \gamma_{HL}, \gamma_{LH}, \gamma_{LL}$$

2. Labor Supply

$$\beta^{st}, \beta^{div}, \beta^{w,educ}, \zeta^{educ}, \beta^{a,educ}$$

3. Housing Supply

$$\gamma, \gamma^{geo}, \gamma^{reg}$$

4. Amenity Supply

$$\gamma^{a}$$

Structural parameters to estimate

1. Labor Demand

$$\gamma_{HH}, \gamma_{HL}, \gamma_{LH}, \gamma_{LL}$$

2. Labor Supply

$$\beta^{st}, \beta^{div}, \beta^{w,educ}, \zeta^{educ}, \beta^{a,educ}$$

3. Housing Supply

$$\gamma, \gamma^{geo}, \gamma^{reg}$$

4. Amenity Supply

$$\gamma^{a}$$

Estimation

- Let Δ indicate difference between 1980 and 1990 or 2000
- We want to estimate

$$\begin{aligned}\Delta\delta_{jt}^{z} &= \boldsymbol{\beta}^{\boldsymbol{w}}\boldsymbol{z}\left(\Delta w_{jt}^{edu} - \zeta\Delta r_{jt}\right) + \boldsymbol{\beta}^{\boldsymbol{a}}\boldsymbol{z}\Delta a_{jt} + \boldsymbol{\beta}^{\boldsymbol{x}}\boldsymbol{z}\Delta x_{jt}^{A} \\ &= \boldsymbol{\beta}^{\boldsymbol{w}}\boldsymbol{z}\left(\Delta w_{jt}^{edu} - \zeta\Delta r_{jt}\right) + \boldsymbol{\beta}^{\boldsymbol{a}}\boldsymbol{z}\Delta a_{jt} + \Delta\xi_{jt}^{z}\end{aligned}$$

- $\Delta\xi_{jt}^{z}$ indicates unobserved components of amenities

Estimation

- Let Δ indicate difference between 1980 and 1990 or 2000
- We want to estimate

$$\begin{aligned}
\Delta\delta^{z}_{jt} &= \boldsymbol{\beta^{w} z}\left(\Delta w^{edu}_{jt} - \zeta\Delta r_{jt}\right) + \boldsymbol{\beta^{a} z}\Delta a_{jt} + \boldsymbol{\beta^{x} z}\Delta x^{A}_{jt} \\
&= \boldsymbol{\beta^{w} z}\left(\Delta w^{edu}_{jt} - \zeta\Delta r_{jt}\right) + \boldsymbol{\beta^{a} z}\Delta a_{jt} + \Delta\xi^{z}_{jt}
\end{aligned}$$

- $\Delta\xi^{z}_{jt}$ indicates unobserved components of amenities
- Endogeneity problem : $\Delta w^{edu}_{jt}, \Delta r_{jt}, \Delta a_{jt}$ are correlated with $\Delta\xi^{z}_{jt}$
- $\Longrightarrow$ Need 3 instruments ΔZ_{jt} that are correlated with $\Delta w^{edu}_{jt}, \Delta r_{jt}, \Delta a_{jt}$, but uncorrelated with $\Delta\xi^{z}_{jt}$

Tutorial on Method of Moments (MM)

- For a single equilibrium condition

$$E\left[\Delta Z'_{jt} * \Delta \xi^{z}_{jt}\right] = 0$$

$$E\left[\Delta Z'_{jt} * \left[\Delta \delta^{z}_{jt} - \left(\boldsymbol{\beta^{w} z}\left(\Delta w^{edu}_{jt} - \zeta \Delta r_{jt}\right) + \boldsymbol{\beta^{a} z} \Delta a_{jt}\right)\right]\right] = 0$$

- Suppose we have 4 cities and 3 instruments, and suppress Δ
- In matrix notation

$$\begin{pmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} \\ Z_{2,1} & Z_{2,2} & Z_{2,3} \\ Z_{3,1} & Z_{3,2} & Z_{3,3} \\ Z_{4,1} & Z_{4,2} & Z_{4,3} \end{pmatrix}' \begin{pmatrix} \xi_1\left(\beta^W, \zeta, \beta^a\right) \\ \xi_2\left(\beta^W, \zeta, \beta^a\right) \\ \xi_3\left(\beta^W, \zeta, \beta^a\right) \\ \xi_4\left(\beta^W, \zeta, \beta^a\right) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

- or multiplying out

$$\xi_1(\cdot) * Z_{1,1} + \xi_2(\cdot) * Z_{2,1} + \xi_3(\cdot) * Z_{3,1} + \xi_4(\cdot) * Z_{4,1} = 0$$

$$\xi_1(\cdot) * Z_{1,2} + \xi_2(\cdot) * Z_{2,2} + \xi_3(\cdot) * Z_{3,2} + \xi_4(\cdot) * Z_{4,2} = 0$$

$$\xi_1(\cdot) * Z_{1,3} + \xi_2(\cdot) * Z_{2,3} + \xi_3(\cdot) * Z_{3,3} + \xi_4(\cdot) * Z_{4,3} = 0$$

- $\Longrightarrow$ 3 equations in 3 unknowns $(\beta^W,\ \zeta,\ \beta^a)$

Instrumental Variables : Bartick Shocks

- National trends in wages reflect trends in productivity
- Increased productivity tends to raise labor demand, and hence the wage
- Strategy : use "exogenous" trends in wages at the industry level for individuals outside of the city multiplied by initial industry concentration in the city

$$\Delta B^H_{jt} = \sum_{ind} \left(w^H_{ind,-j,t} - w^H_{ind,-j,1980} \right) \frac{H_{ind,j,1980}}{H_{J,1980}}$$

$$\Delta B^L_{jt} = \sum_{ind} \left(w^L_{ind,-j,t} - w^L_{ind,-j,1980} \right) \frac{L_{ind,j,1980}}{L_{J,1980}}$$

- "Bartick Shocks" ΔB^H_{jt} ΔB^L_{jt} should correlate with Δw^H_{jt} and Δw^L_{jt} , though not $\Delta \xi^z_{jt}$

Instrumental Variables via GMM

- GMM stands for Generalized Method of Moments
- In this case, the assumption is

$$
\begin{aligned}
&E\left[\Delta Z'_{jt} * \Delta\xi^{z}_{jt}\right] = 0 \\
&E\left[\Delta Z'_{jt} * \left[\Delta\delta^{z}_{jt} - \left(\boldsymbol{\beta}^{\boldsymbol{w}}\boldsymbol{z}\left(\Delta w^{edu}_{jt} - \zeta\Delta r_{jt}\right) + \boldsymbol{\beta}^{\boldsymbol{a}}\boldsymbol{z}\Delta a_{jt}\right)\right]\right] = 0
\end{aligned}
$$

  with

$$
\Delta Z_{jt} \in \{\Delta B^{H}_{jt}, \Delta B^{L}_{jt}, \Delta B^{H}_{jt} x^{reg}_{j}, \Delta B^{L}_{jt} x^{reg}_{j}, \Delta B^{H}_{jt} x^{geo}_{j}, \Delta B^{L}_{jt} x^{geo}_{j}\}
$$

- Via GMM, we choose $\beta^{w}, \zeta, \beta^{a}$ for both high skill and low skill that make the expectation hold

Data

- US Census 1980, 1990, 2000, 5% public use sample
- 300 metro areas (commuting zones)
- Household wage education (college vs non college), rents, industry, demographic characteristics
- City-level amenities – crime, enviro, etc
- Housing supply characteristics

Worker Preference for Cities

TABLE 5—GMM ESTIMATES OF MODEL PARAMETERS

	Non-college	College	Non-college	College	Non-college	College	Non-college	College
	(1)		(2)		(3)		(4)	
Panel A. Worker preferences for cities								
Wage	4.155***	5.523***	3.757***	−1.783***	4.026***	2.116***	3.261***	4.976***
	[0.603]	[1.797]	[0.561]	[0.682]	[0.727]	[1.146]	[1.064]	[1.671]
Rent	−2.418***	−1.404	−2.329***	1.105***	−2.496***	−1.312***	−2.944***	−2.159***
	[0.349]	[0.833]	[0.348]	[0.423]	[0.451]	[0.711]	[0.551]	[0.821]
Implied local expenditure share	0.582***	0.254**	0.62	0.62	0.62	0.62	0.903***	0.434***
	[0.0678]	[0.078]	—	—	—	—	[0.261]	[0.0810]
Amenity index	—	—	—	—	0.274*	1.012***	0.771***	0.638***
					[0.147]	[0.115]	[0.307]	[0.185]

Estimates of Productivity

Largest Increases in College Productivity	
msa	Δ Productivity
San Jose, CA	0.237
Milwaukee, WI	0.236
Tulsa, OK	0.213
San Francisco-Oakland-Vallejo, CA	0.202
New York-Northeastern NJ	0.170
Hartford-Bristol-Middleton- New Britain, CT	0.168
Oklahoma City, OK	0.163
Philadelphia, PA/NJ	0.160
Chicago, IL	0.153
Birmingham, AL	0.131

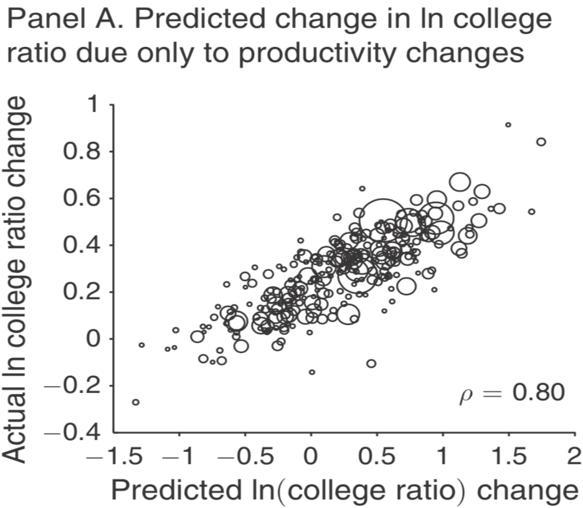
Estimates of Amenities

Best Amenities for College Workers, 1980	
msa	Amenity
Los Angeles-Long Beach, CA	2.071
San Francisco-Oakland-Vallejo, CA	1.853
Washington, DC/MD/VA	1.761
Denver-Boulder, CO	1.666
Seattle-Everett, WA	1.569
New York-Northeastern NJ	1.529
Chicago, IL	1.500
Dallas-Fort Worth, TX	1.500
Phoenix, AZ	1.465
Minneapolis-St. Paul, MN	1.456

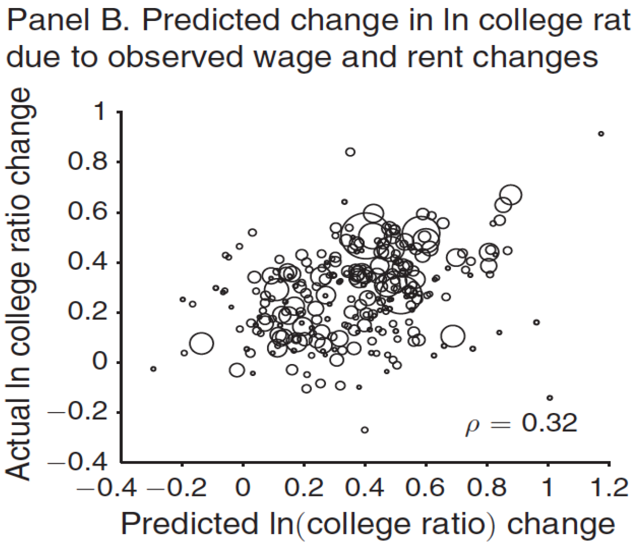
Determinants of City Choice

- Hold certain channels fixed at 1980's levels, for example hold amenities and rents fixed, but let wages evolve
- Compute counterfactual utility derived from living in each city for each household
- Assign the household to live in the counterfactual 1st-choice city
- Compute counterfactual city-level college ratio changes
- Compare to observed changes

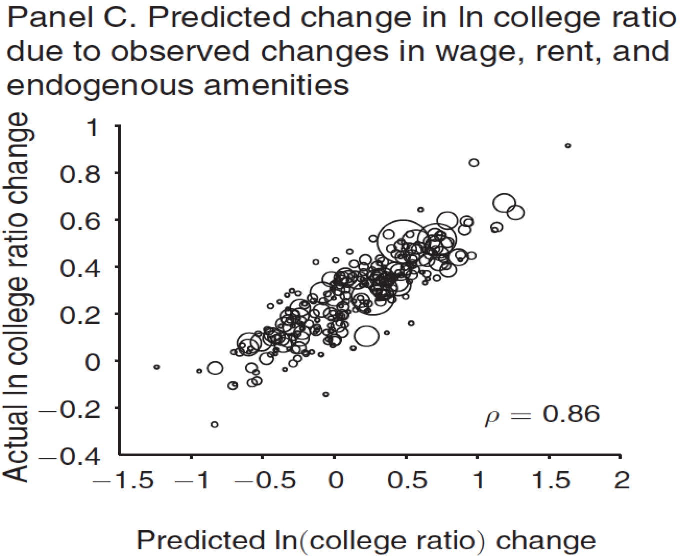
Determinants of City Choice



Determinants of City Choice



Determinants of City Choice



Decomposition of Well-Being Inequality

- Hold certain channels fixed at 1980's levels, for example hold amenities and rents fixed, but let wages evolve
- Compute counterfactual utility derived from living in each city for each household
- Compare observed utility level in 1980 to counterfactual utility level from counterfactual 1st-choice city in future decade
- take average over college and noncollege workers

Decomposition of Well-Being Inequality

Year	(1)	(2)	(3)	(4)
1980	0.383	0.383	0.383	0.383
	—	—	—	—
1990	0.540	0.519	0.570	0.730
	[0.0022]	[0.0024]	[0.0316]	[0.1344]
2000	0.601	0.577	0.639	0.956
	[0.0033]	[0.0012]	[0.0364]	[0.2398]
Change: 1980–2000	0.218	0.194	0.256	0.573
	[0.0033]	[0.0012]	[0.0364]	[0.2398]
Wages	—	—	—	—
Rents		—	—	—
Endog. amenities from resorting of workers			—	—
Endog. amenities from national supply of college graduates				—

Notes: Well-being gap is measured by the difference in a college and high school graduate's willingness to pay to live in his first-choice city from the choices available in 2000 versus his first choice in 1980. For example, the well-being gap due to wage changes only accounts for the welfare impact of wage changes from 1980 to 2000, while the well-being due to wages and rents accounts for both the impacts of wages and rents. The well-being gap is normalized to the college wage gap in 1980. Standard errors for welfare estimates use the delta method.

Conclusions from Diamond

- Reduced form evidence that amenity quality responds to local skill mix
- Changes in productivity is important driver of observed changes in collage vs noncollege skill mix in cities
- Changes in endogenous amenity quality is important driver of observed changes in collage vs noncollege skill mix in cities
- Taking into account endogenous amenity changes, the well-being gap between college-educated and noncollege-educated workers increased by 25 percentage points (from .38 to .63), which is 30% higher than the observed wage gap.

Tutorial on Conditional Logit

- Utility for agent i for choosing option j is

$$V_{ij} = \delta_j + \epsilon_{ij}$$

- Random component of utility is distributed EV Type I

$$(PDF)\ \ f(\epsilon_{ij}) = e^{-\epsilon_{ij}} e^{-e^{-\epsilon_{ij}}}\ \ ,\ \ (CDF)\ \ F(\epsilon_{ij}) = e^{-e^{-\epsilon_{ij}}}$$

- Then the difference between two draws $\epsilon_{ijk} = \epsilon_{ij} - \epsilon_{ik}$ is distributed Logistic

$$(CDF)\ \ G(\epsilon_{ijk}) = \frac{e^{\epsilon_{ijk}}}{1 + e^{\epsilon_{ijk}}}$$

- Implies that the probability that i chooses option j

$$\begin{aligned}
\text{Prob}(i \text{ chooses } j) &= Pr(V_{ij} > V_{ij'}) = Pr(\delta_j + \epsilon_{ij} > \delta_{j'} + \epsilon_{ij'}) \\
&= Pr(\delta_j - \delta_{j'} > \epsilon_{ij'} - \epsilon_{ij}) = \frac{e^{\delta_j}}{\sum_k e^{\delta_k}}
\end{aligned}$$