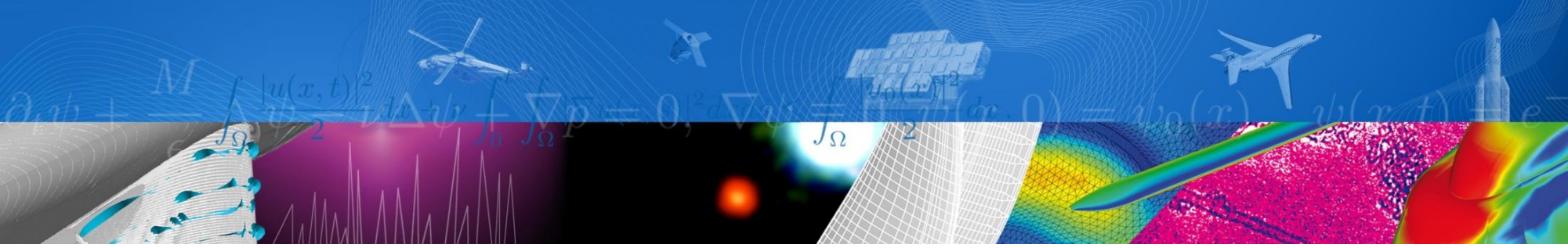

Filtrage Particulaire appliqué à la navigation et à l'estimation d'état

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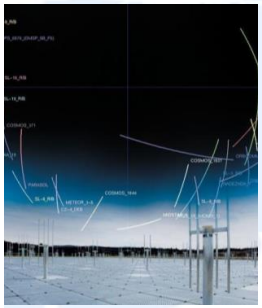
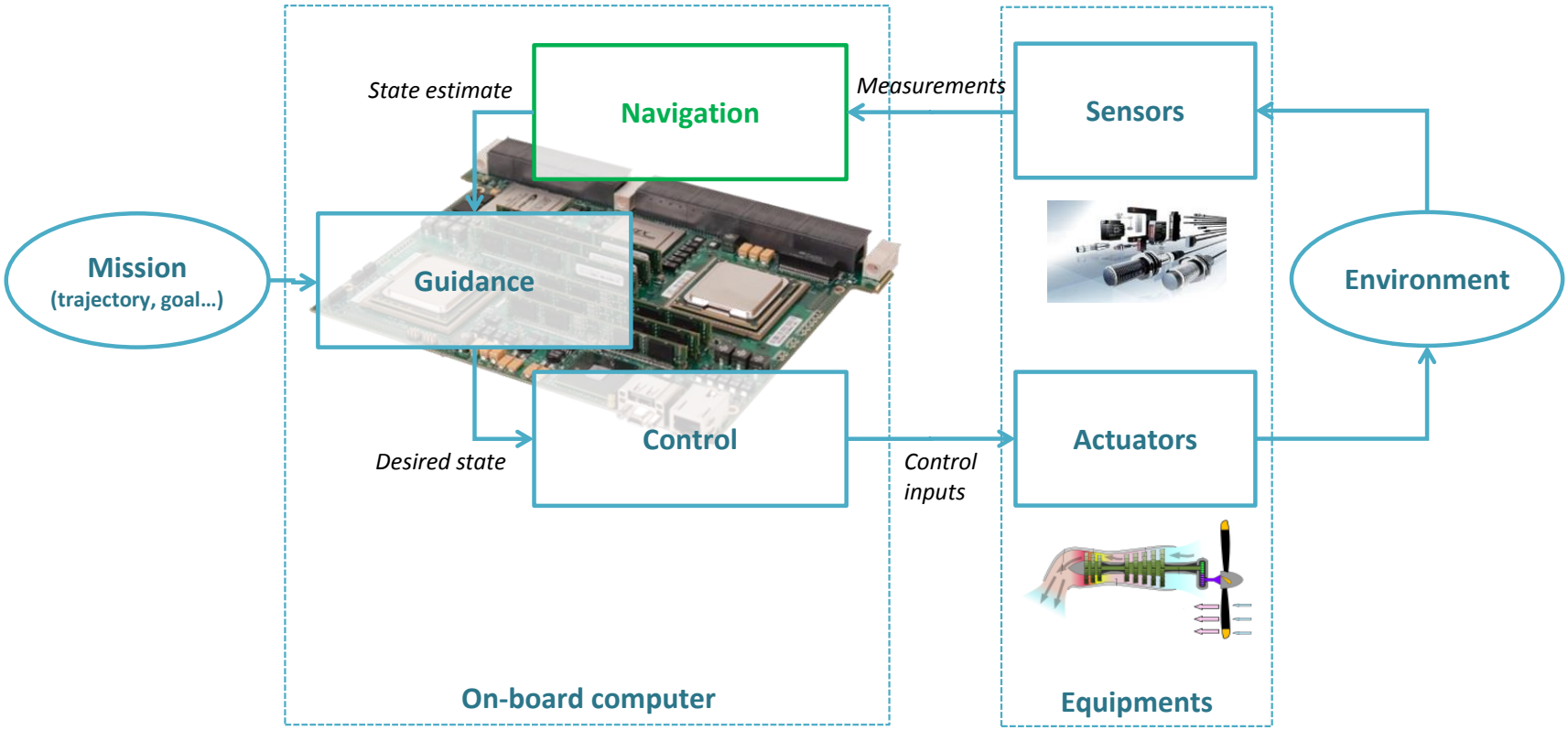


Motivation

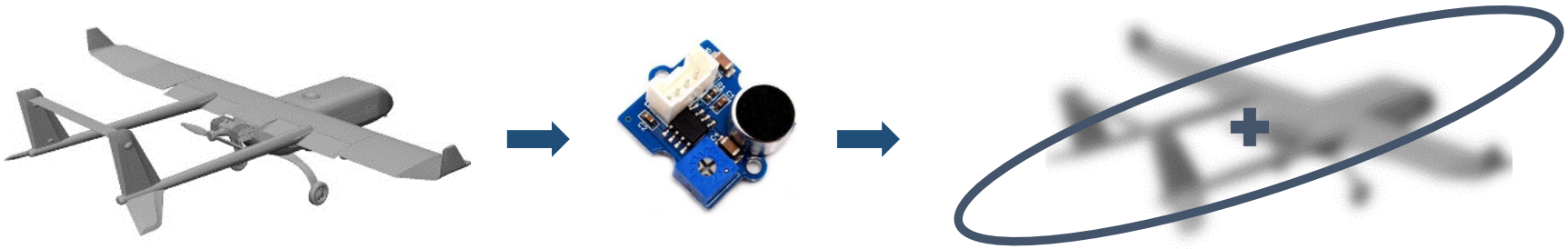
Autonomous systems need to perform accurate state estimation (e.g., self localization, objective assessment...)

... with limited embedded calculation capabilities.

Guidance, Navigation and Control (GNC)



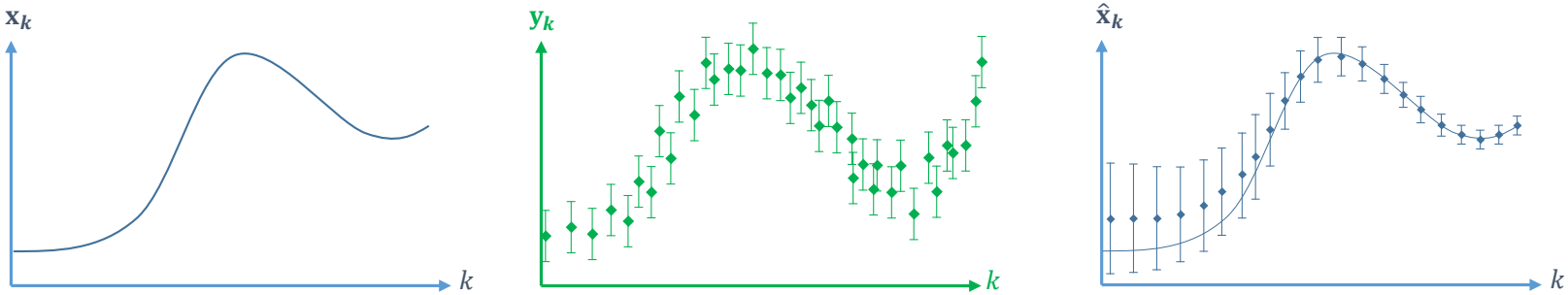
State estimation for navigation



$\mathbf{x}_k$ = position, velocity...

$\mathbf{y}_k$ = measurements

$\hat{\mathbf{x}}_k$ = estimated position, velocity...



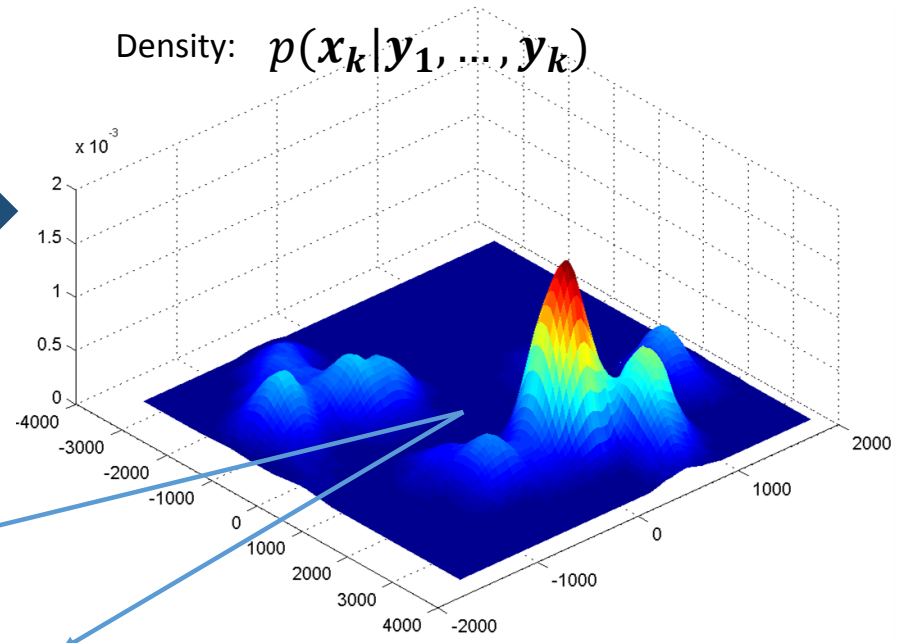
State estimation consists in retrieving the vehicle's state from noisy and incomplete measurements and uncertain evolution model.

Different ways to model the state density

$\boldsymbol{x_k}$ = position, velocity...

$\boldsymbol{y_k}$ = measurement

Density: $p(\boldsymbol{x_k}|\boldsymbol{y_1}, \dots, \boldsymbol{y_k})$



Kalman Filters

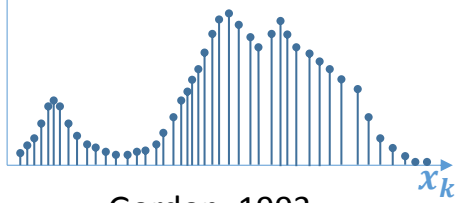
Particle Filters

Gaussian approximation

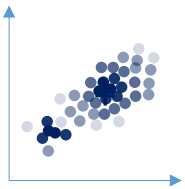
Kalman, 1960

Diracs mixture

$\hat{p}(\boldsymbol{x_k}|\boldsymbol{Y_k}) = \sum_i w_k^i \delta_{x_k^i}$



Gordon, 1993



What we know

Theoretical state evolution (dynamical model):

$$\begin{aligned} \dot{\mathbf{x}} &= F(\mathbf{x}, \mathbf{u}) \\ \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) \end{aligned} \tag{1}$$

Theoretical observation equation (sensor model):

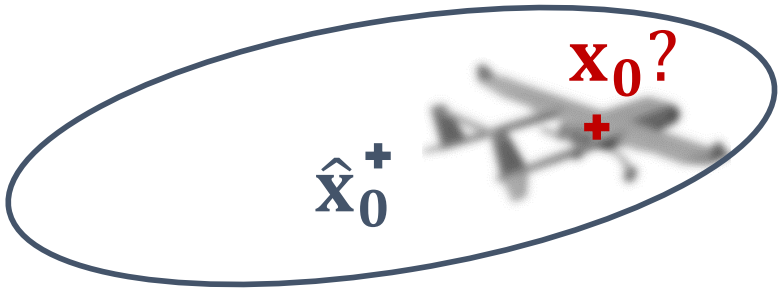
$$\mathbf{y}_k = h(\mathbf{x}_k) \tag{2}$$

However, these equations are not totally representative of the actual system, e.g.:

- unexpected wind, friction, unmodeled dynamics...
- sensor noise, unmodeled disturbances...

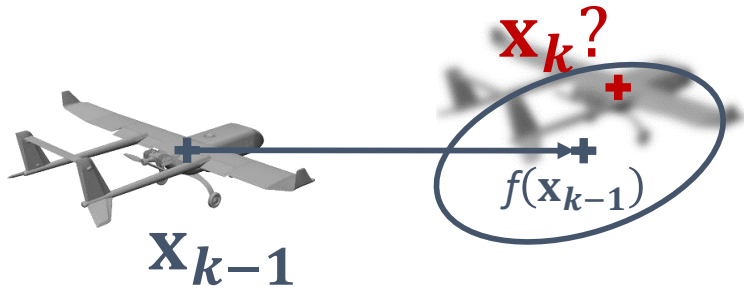
What we **don't** know (uncertainties)

- Initial state uncertainty



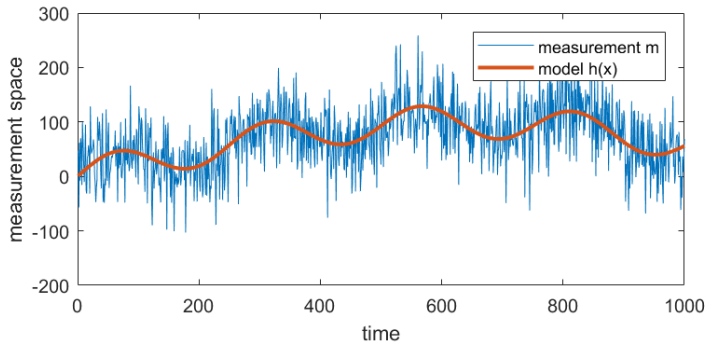
- Process noise (dynamics)

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{v}_k)$$



- Measurement noise (and potentially some bias)

$$\mathbf{y}_k = h(\mathbf{x}_k, \mathbf{v}_k^m)$$



A way to model uncertainties: probability distribution functions

- State distribution:

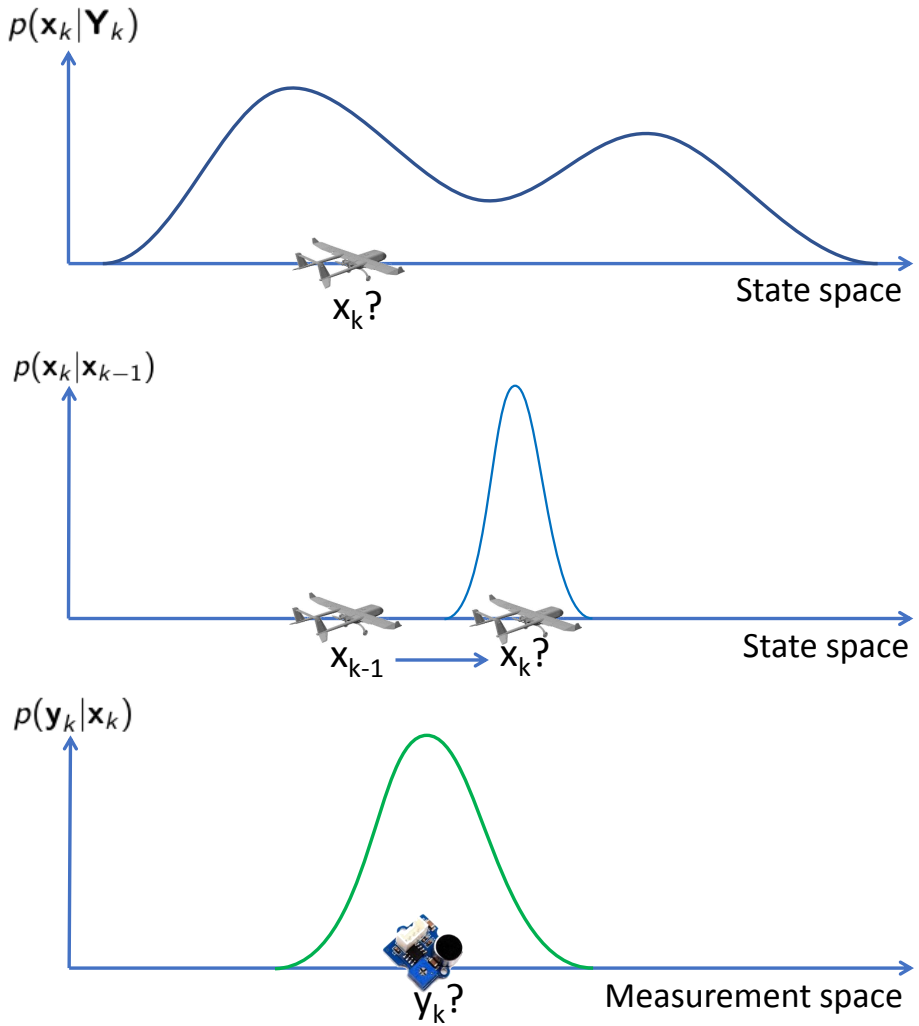
$$p(\mathbf{x}_k|\mathbf{y}_1,\ldots,\mathbf{y}_k) \triangleq p(\mathbf{x}_k|\mathbf{Y}_k)$$

- Process noise distribution

$$p(\mathbf{x}_k|\mathbf{x}_{k-1})$$

- Measurement distribution

$$p(\mathbf{y}_k|\mathbf{x}_k)$$



Optimal filter equations (Bayesian filtering)

State density propagation (dynamics, Chapman-Kolmogorov equation):

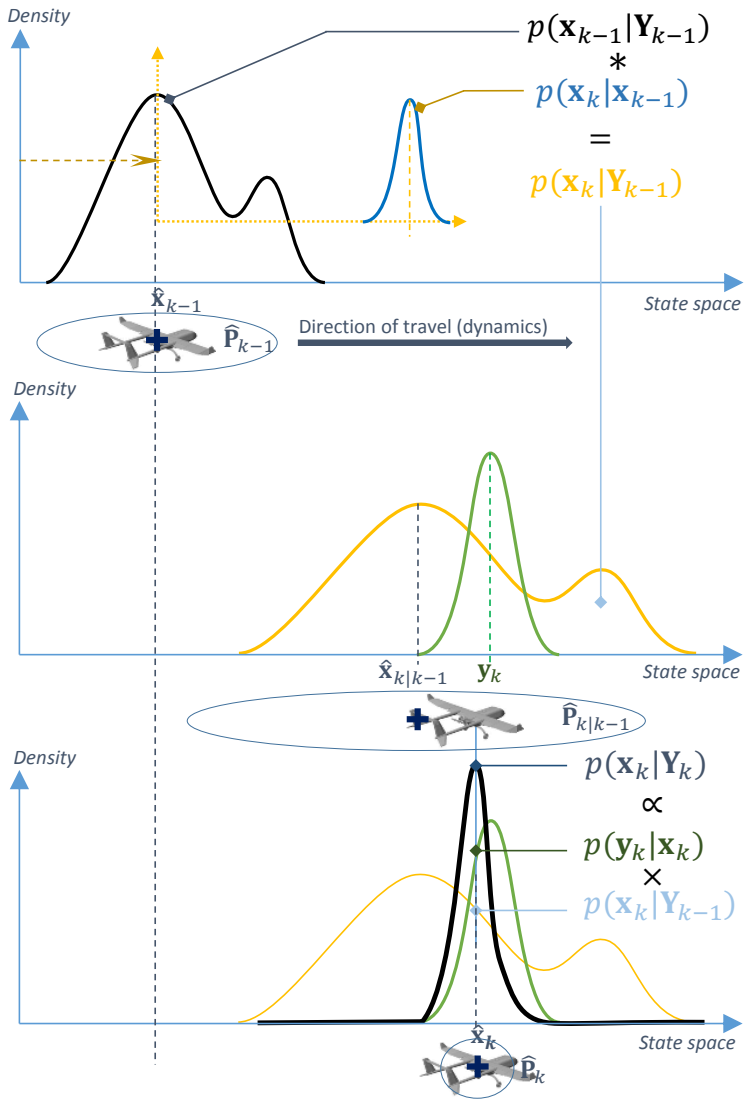
$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \tag{3}$$

State density correction/update (measurements, Bayes rule):

$$p(\mathbf{x}_k|\mathbf{Y}_k) = \frac{p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1})}{p(\mathbf{y}_k|\mathbf{Y}_{k-1})} \tag{4}$$

(Measurements $\mathbf{y}_i$ and $\mathbf{y}_j$ ($\forall\ i \neq j$) are assumed to be statistically independent)

Optimal filter equations (Bayesian filtering)



(a) Prediction

Convolution $(*)$ of the **prior conditional density** with the state transition density. The transition density accounts for the deterministic dynamics f_k and its uncertainty (process noise $\mathbf{w}_k$).

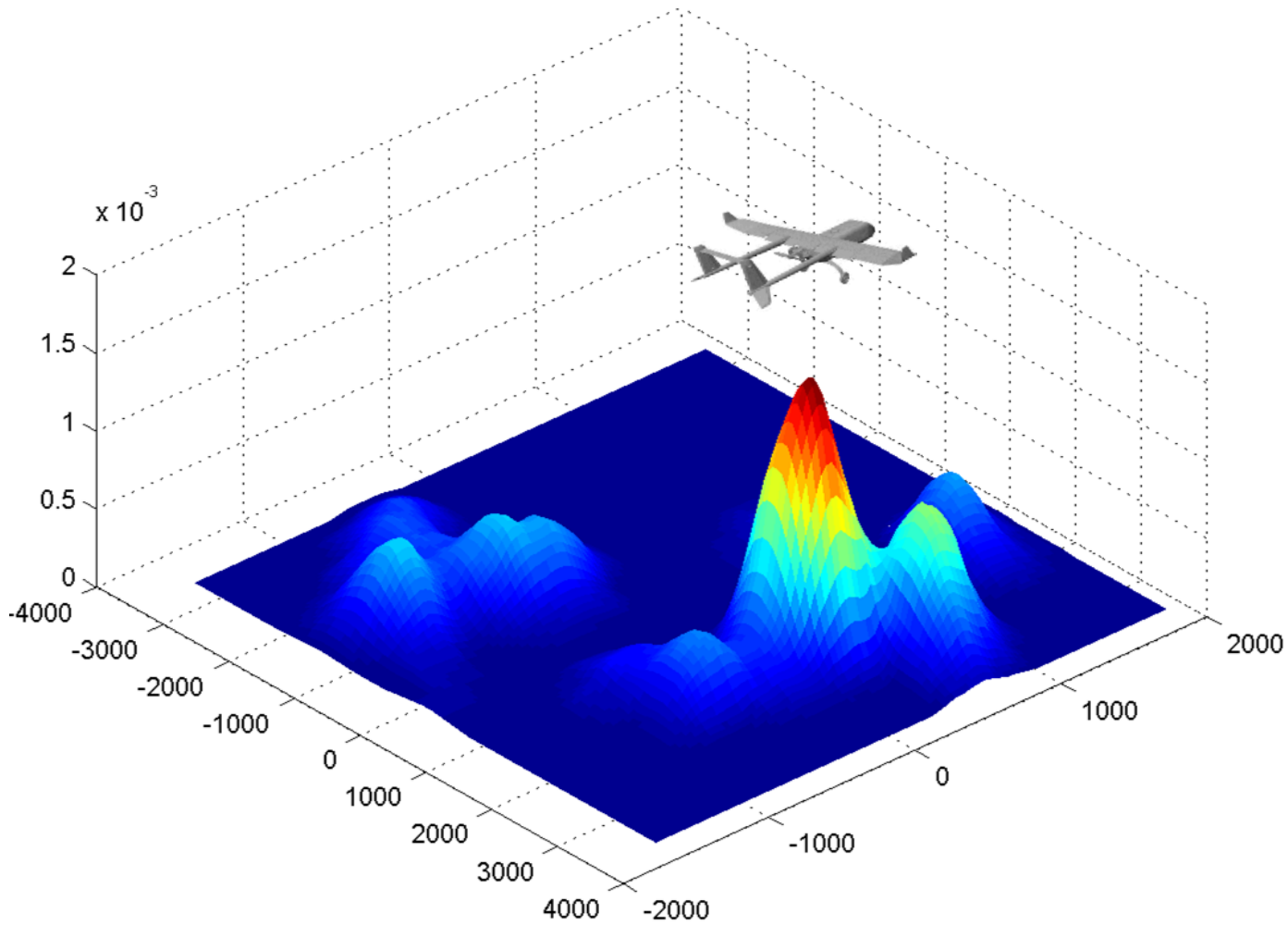
(b) Predicted density, new measurement

Step (a) results in the **predicted conditional density**, whose support is usually larger than the prior density.
A measurement $\mathbf{y}_k$ is now available. It will introduce information in the estimation.

(c) Correction

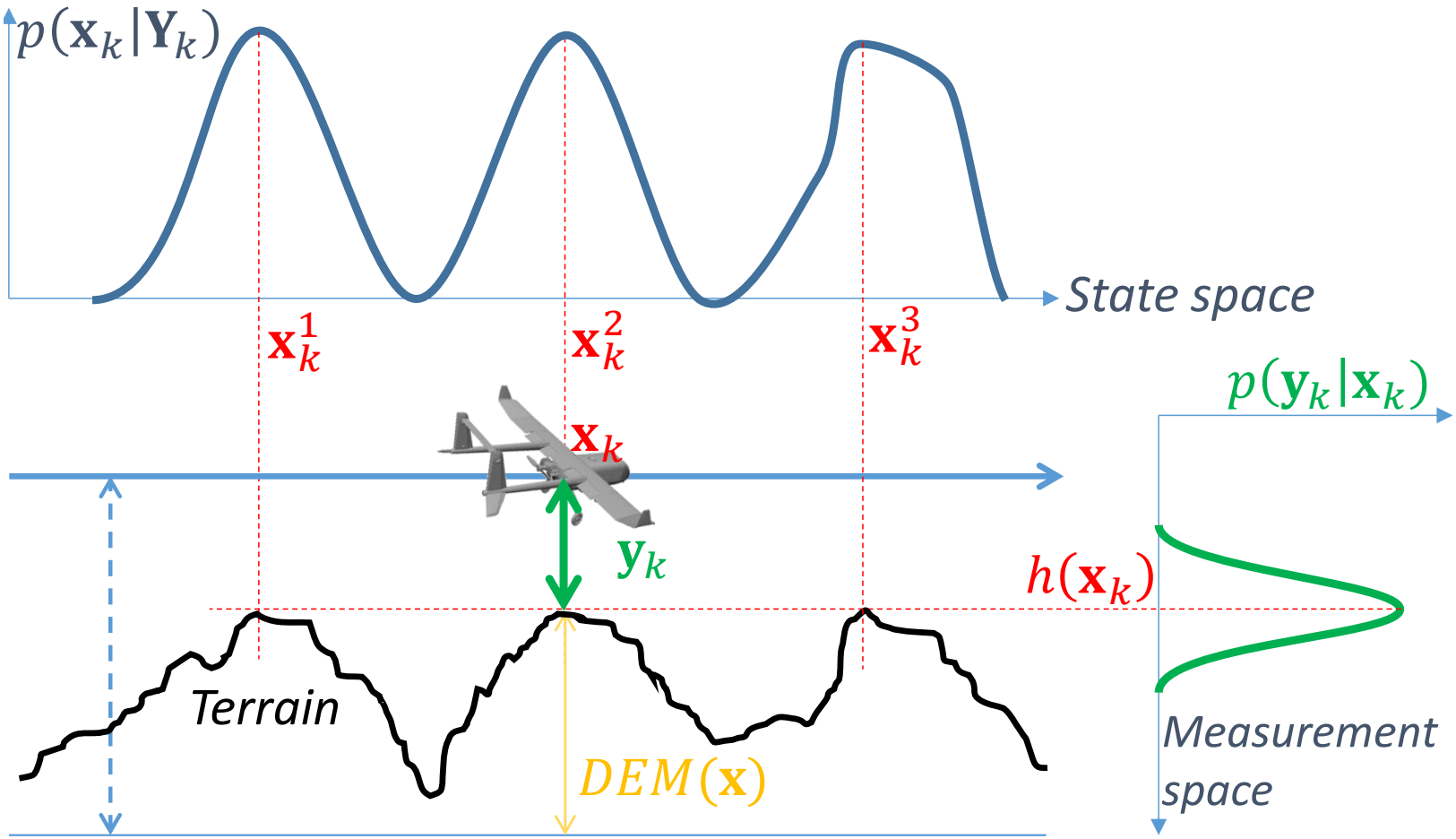
The predicted density is multiplied with the measurement density, leading to the **posterior conditional density**.
Its support is usually smaller than the predicted density. It yields a refined estimate $\hat{\mathbf{x}}_k$ and covariance $\hat{\mathbf{P}}_k$.
Then, one can iterate back to step (a) for further time-steps.

Multimodal state density (example: Terrain Aided Navigation)



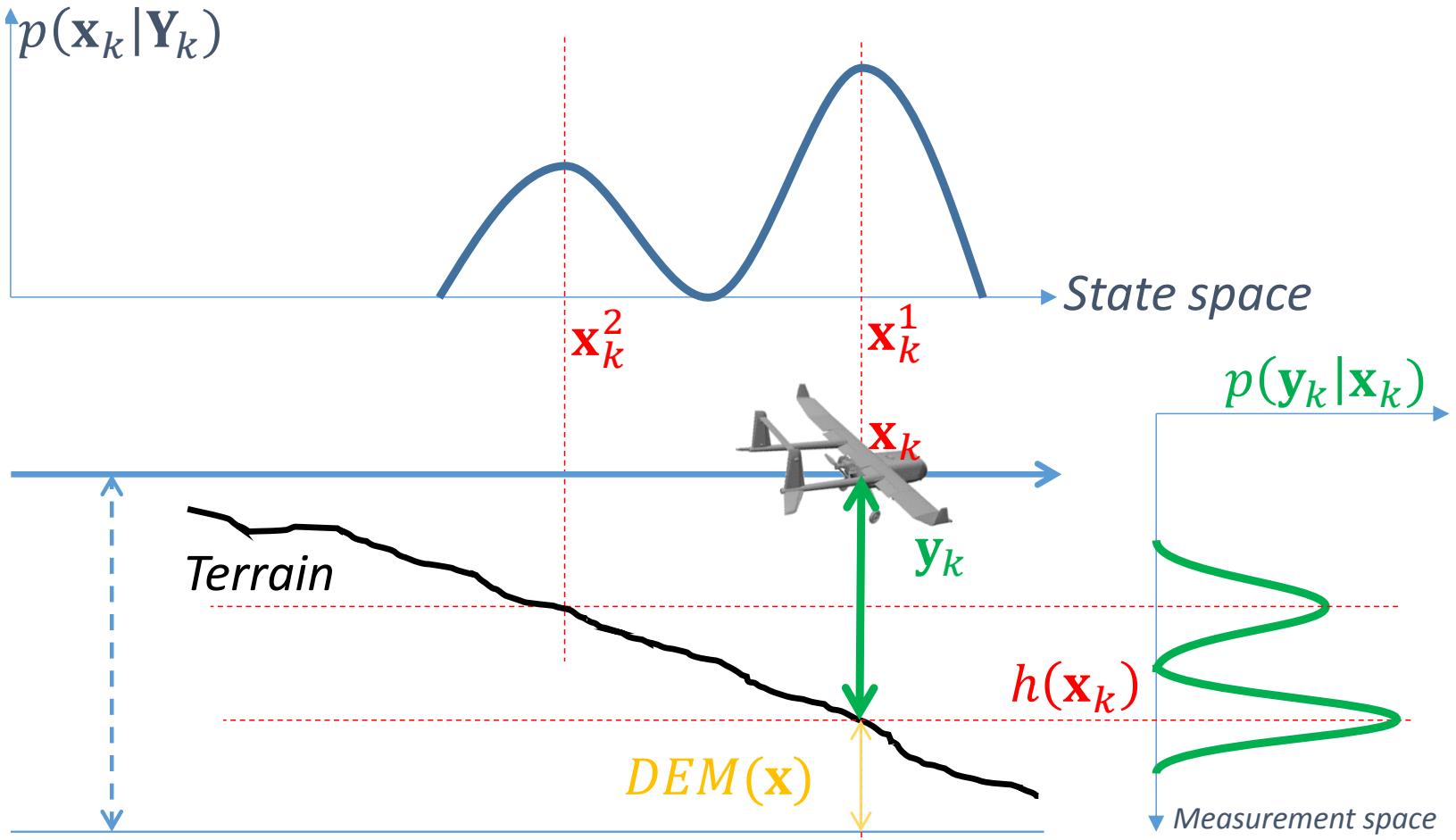
State density $\boldsymbol{p}(\mathbf{x}_k|\mathbf{Y}_k)$

Multimodal state density (example: Terrain Aided Navigation)



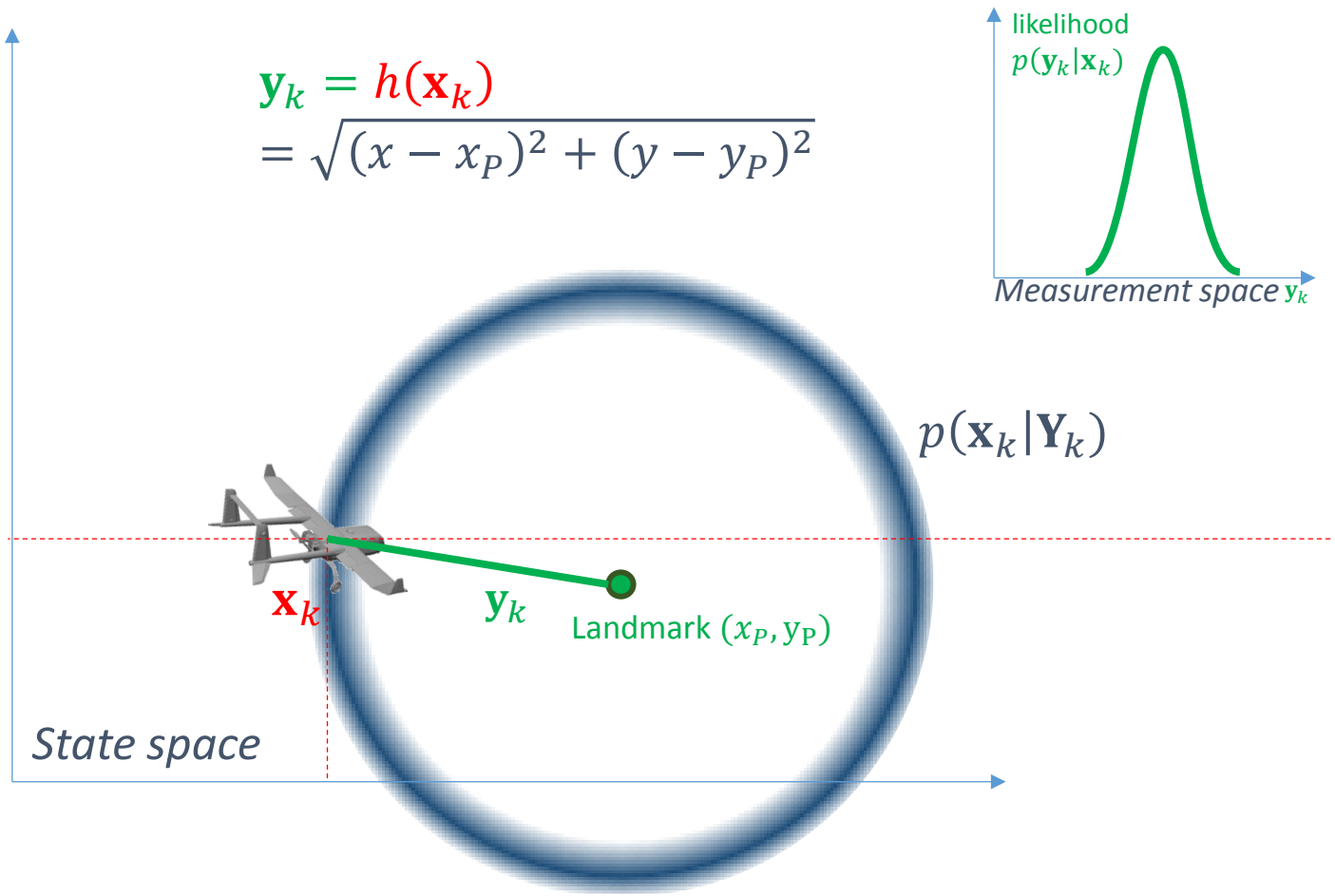
Non injective observation equation (e.g. terrain elevation) may yield several admissible states (peaks in posterior state density).

Multimodal likelihood



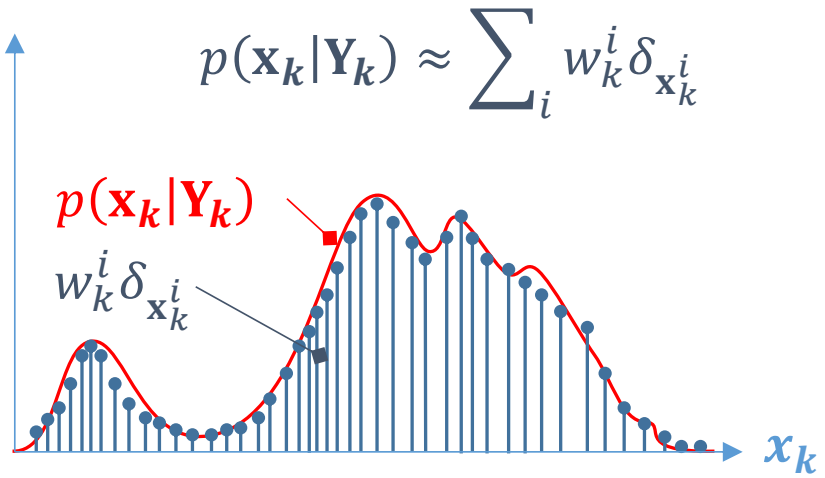
Multimodal measurement noise (likelihood density) yields several admissible states.

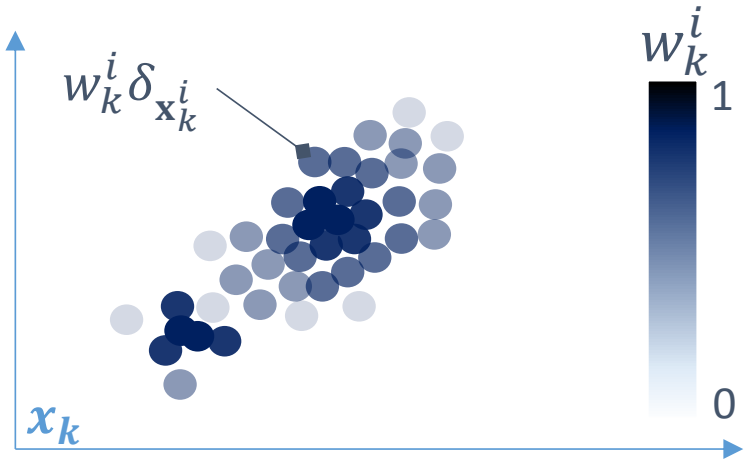
Incomplete measurements



Example: range only measurement towards a target landmark lead to an infinity of admissible states distributed along a circle around the robot.

State density approximation





$$p(\mathbf{x}_k|\mathbf{Y}_k) \approx \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \quad (5)$$

$\mathbf{x}_k$: actual state
$\mathbf{x}_k^i$: particle i state
w_k^i : particle i weight $(\sum_i w_k^i = 1)$

Particle Filter (theory): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_{k-1}) & = \int & p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \\ \sum_{i=1}^{N} w_{k-1}^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^{N} w_{k-1}^i \delta(\mathbf{x}_k - (f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{v}_k^i)) \end{array} \tag{6}$$

State density correction/update (measurements, Bayes rule):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_k) & \propto & p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1}) \\ \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^{N} w_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_k^i)\delta(\mathbf{x}_k - \mathbf{x}_k^i) \end{array} \tag{7}$$

Particle Filter (in practice): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k, \mathbf{v}_k^i) \tag{8}$$

$\mathbf{v}_k^i \ \ \forall i \in [1, N]$: random sample of process noise

State density correction/update (measurements, Bayes rule):

$$w_k^i \propto w_{k-1}^i p(\mathbf{y}_k | \mathbf{x}_k^i) \tag{9}$$

Particle Filter: state estimation

Maximum *a posteriori* (hard to compute in the general case):

$$\widehat{\mathbf{x}}_k = \underset{\mathbf{x}_k}{\operatorname{argmax}}\, p(\mathbf{x}_k|\mathbf{Y}_k) \tag{10}$$

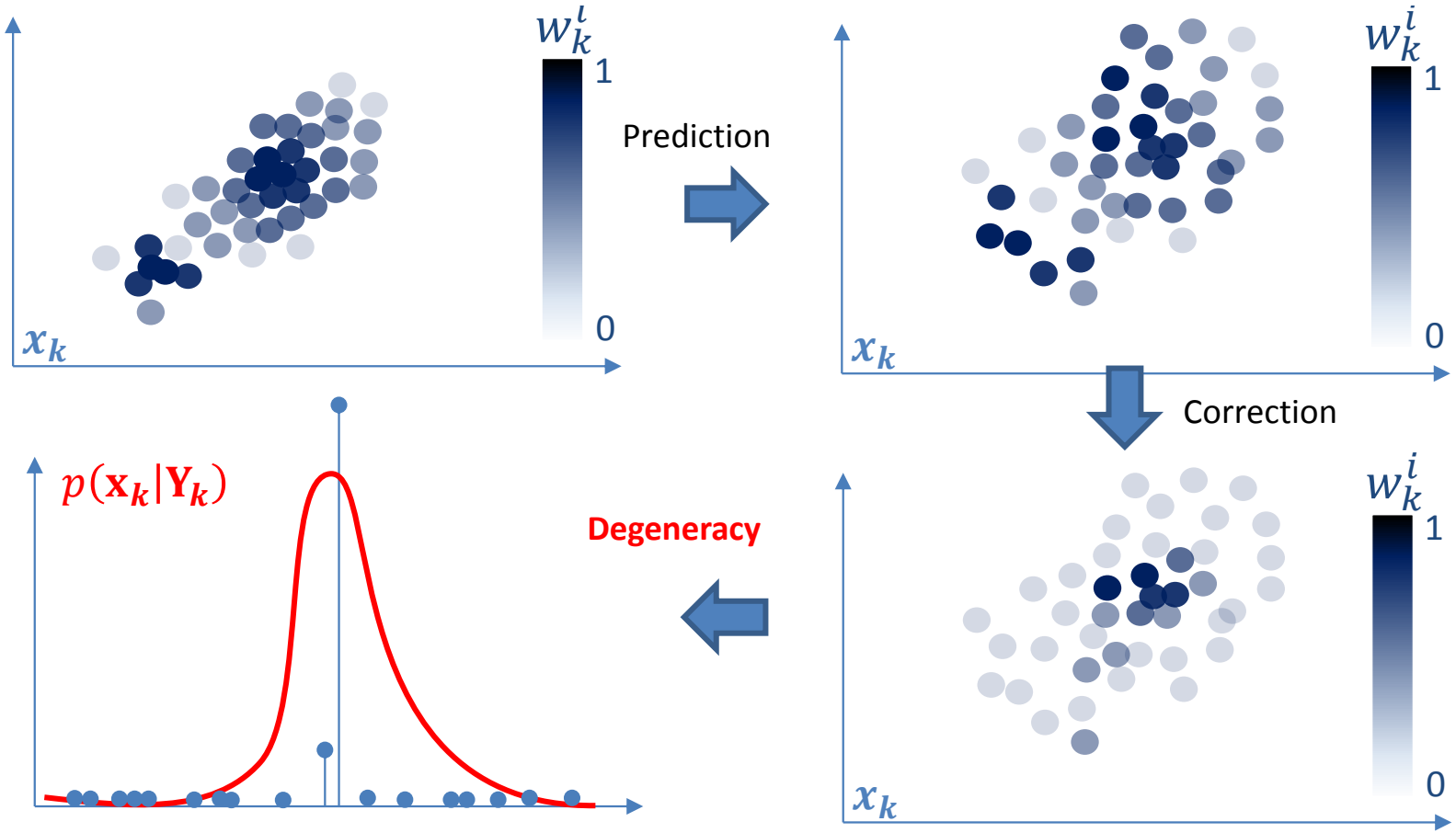
Least square approximation:

$$\widehat{\mathbf{x}}_k \approx \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \tag{11}$$

Empirical covariance:

$$\widehat{\mathbf{P}}_k = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)(\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)^T \tag{12}$$

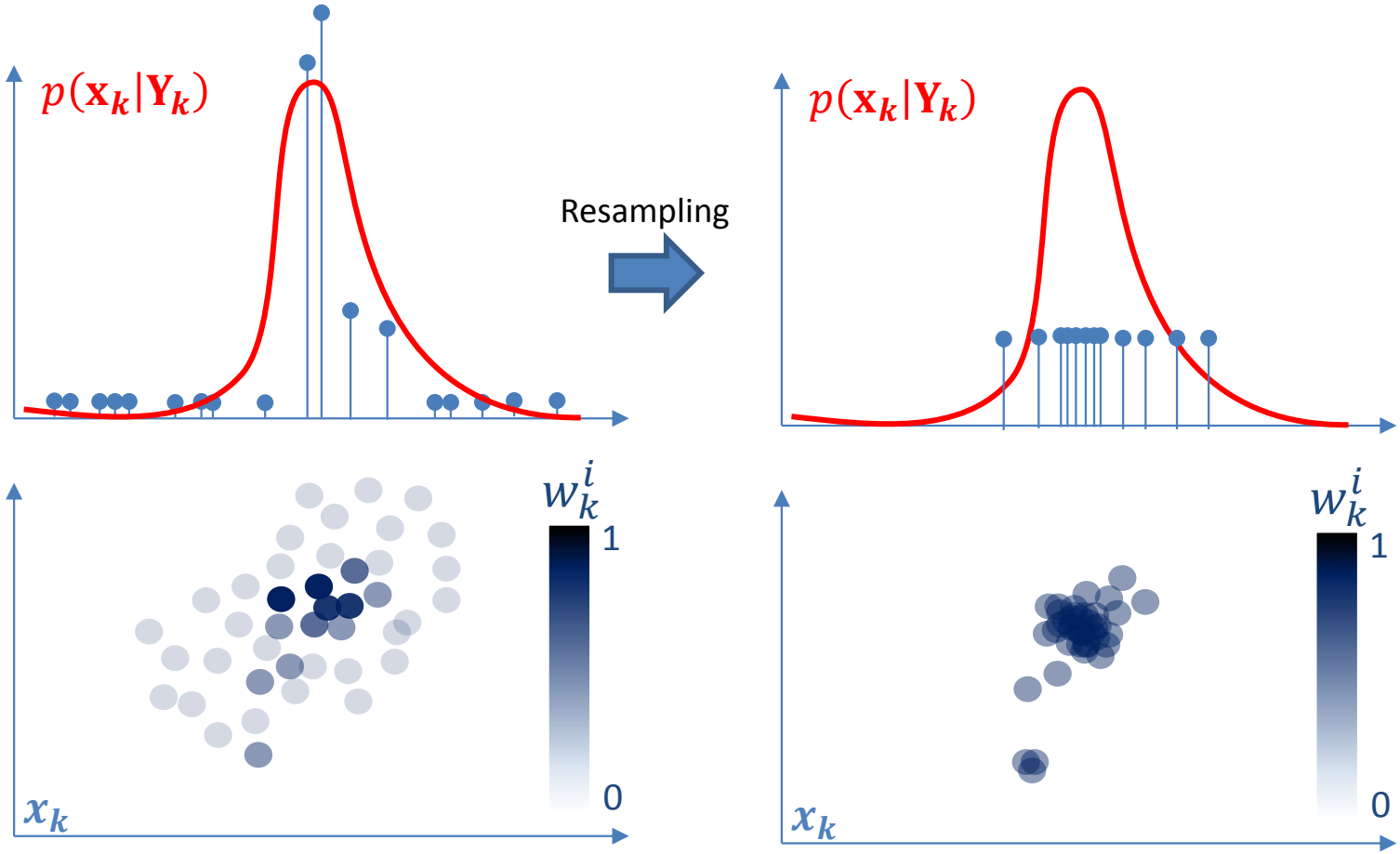
Prediction, correction, and degeneracy phenomenon



After a number of [prediction-correction] cycles, a majority of particle weights will tend to 0 while a small number (or only one) will tend to 1:

That is the weights degeneracy phenomenon.

Resampling



The resampling step aims to duplicate strong-weighted particles to keep an appropriate description of the state density **when degeneracy is about to occur**.

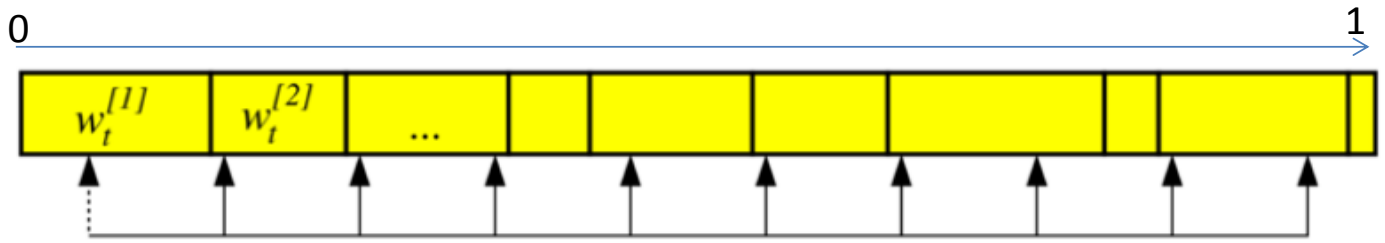
Low-weighted particles are destroyed to keep N unchanged.

Multinomial Resampling

The most commonly used resampling technique is Multinomial Resampling:

Input: particle weights $\{w^i\}_{i\in[1,N]}$
Output: number of new instances per particles $\{n^i\}_{i\in[1,N]}$

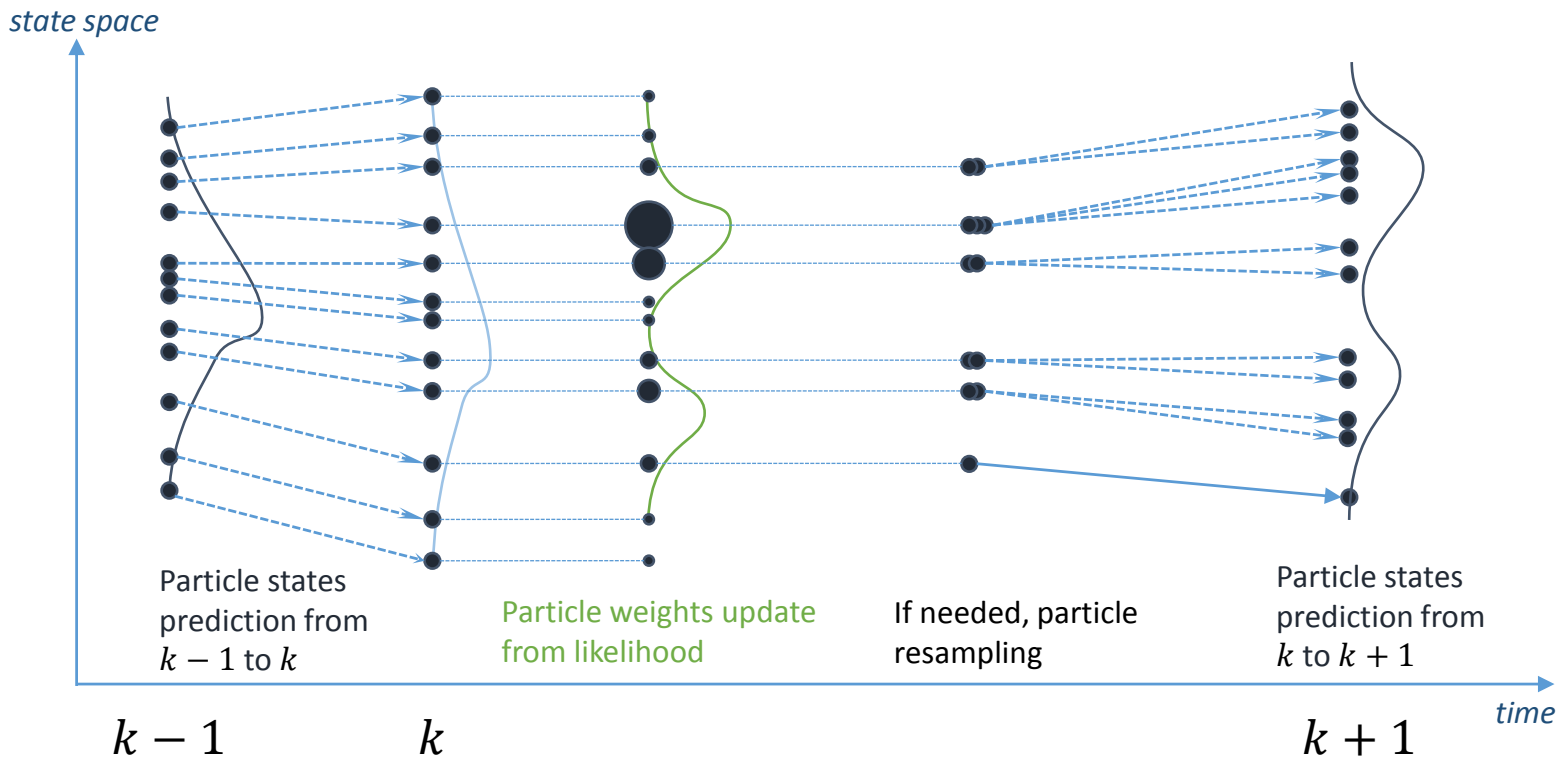
1: Initialise the duplication counters to $n^i = 0\ \forall i \in [1, N]$
2: **for** $i = 1$ to N **do**
3: Draw $u^i \sim \mathcal{U}_{[0,1]}$
4: Find $j \in [1, N]$ such that $u^i \in \left]\sum_{l=1}^{j-1} w^l, \sum_{l=1}^{j} w^l\right]$
5: Count $n^j = n^j + 1$
6: **end for**
7: Return $n^i\ \forall i \in [1, N]$



for $i = 1$ to N **do**
Draw $u^i \sim \mathcal{U}_{[0,1]}$

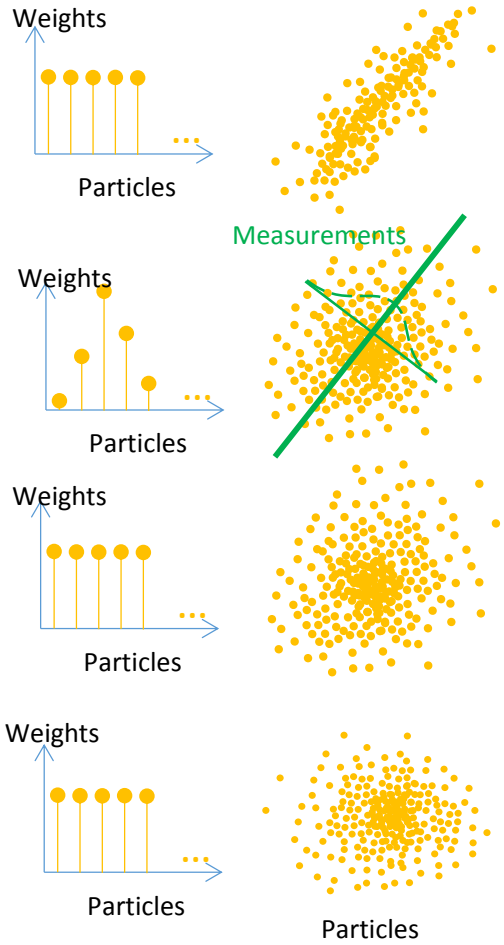
New particle distribution

Prediction, correction, and resampling



SIR-PF scheme

Particle Filter



Low-weighted particles are replaced with copies of high-weighted particles

$$p(\mathbf{x}_k|\mathbf{Y}_k) = \sum_{i=1}^{N} \omega_k^i \delta_{\mathbf{x}_k^i}$$

$$\omega_k^i \propto p\left(\mathbf{y}_k|\mathbf{x}_k^i\right)\omega_{k-1}^i$$

$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \sum_{i=1}^{N} \omega_{k-1}^i \delta_{\mathbf{x}_k^i}$$

$$\mathbf{x}_k^i = f\left(\mathbf{x}_{k-1}^i, \mathbf{u}_k\right) + \mathbf{v}_k^i$$

$$p(\mathbf{x}_0) = \sum_{i=1}^{N} \omega_0^i \delta_{\mathbf{x}_0^i}$$

$$\left\{\mathbf{x}_0^i\right\}_{i=\{1,\dots N\}} \sim p(\mathbf{x}_0)$$

Steps

Resampling

Correction

Prediction

Initialization

PF formulation example with Gaussian uncertainties

Initialization: draw:

$$\left\{ \mathbf{x}_0^i \sim \mathcal{N}(\widehat{\mathbf{x}_0}, \widehat{\mathbf{P}_0}) \right\}_{i \in [1,N]} \tag{13}$$

Prediction step:

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k, \mathbf{v}_k^i) \tag{14}$$

where $\mathbf{v}_k^i \sim \mathcal{N}(0, \mathbf{Q}_k) \quad \forall i \in [1, N]$

Correction step:

Weights update (where $(\mathbf{y}_k - h(\mathbf{x}_k^i)$ is the *innovation* term):

$$\tilde{w}_k^i = w_{k-1}^i \exp\left(-\frac{1}{2} (\mathbf{y}_k - h(\mathbf{x}_k^i))^T \mathbf{R}_k^{-1} ((\mathbf{y}_k - h(\mathbf{x}_k^i)) \right) \tag{15}$$

Weights normalization: $w_k^i = \frac{\tilde{w}_k^i}{\sum_i \tilde{w}_k^i}$

Estimation:

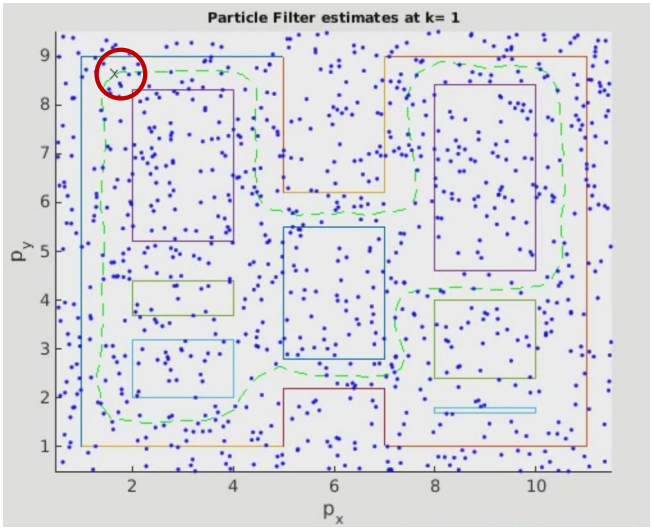
$$\widehat{\mathbf{x}_k} = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \qquad \widehat{\mathbf{P}_k} = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}_k})(\mathbf{x}_k^i - \widehat{\mathbf{x}_k})^T \tag{16}$$

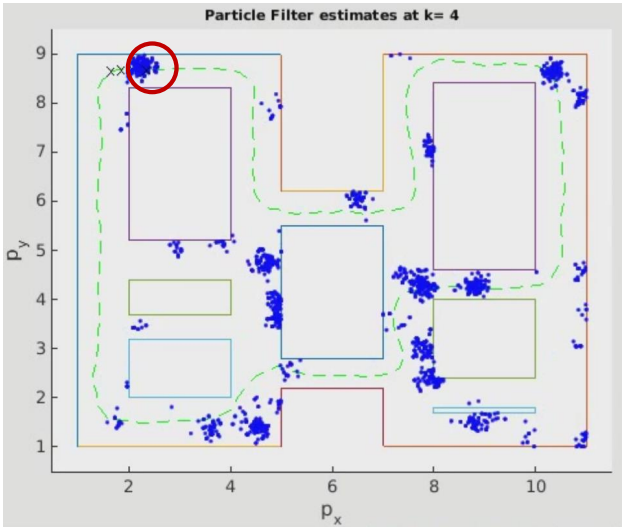
Resampling (cf slide 22) if:

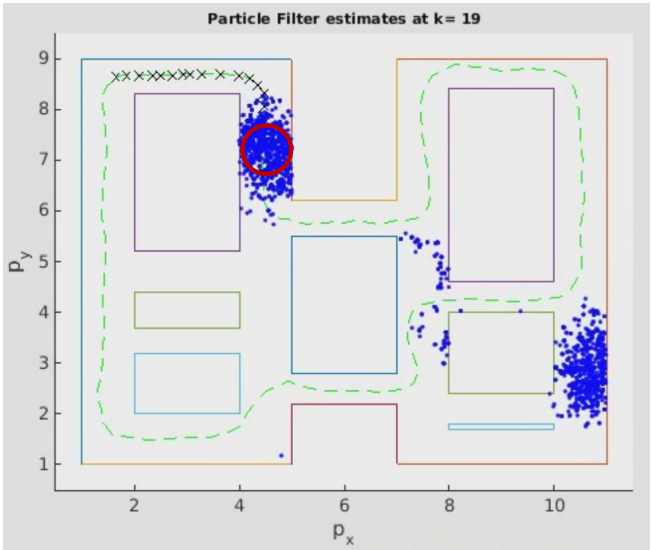
$$N_{eff} = \frac{1}{\sum_i (w_k^i)^2} < \theta_{eff} N \tag{17}$$

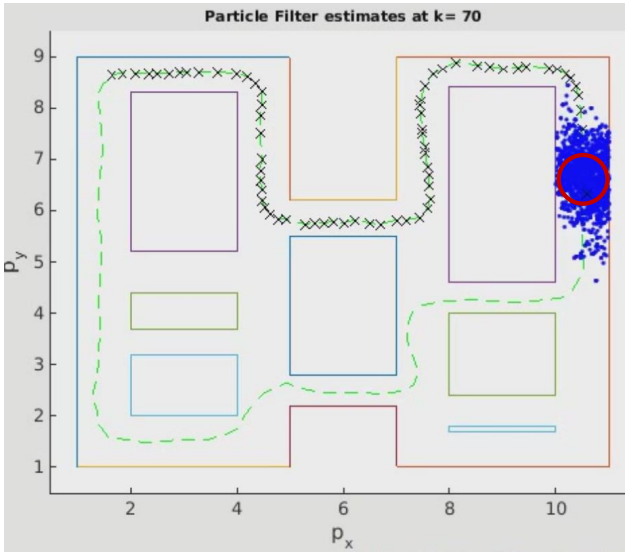
where $\theta_{eff} \in [0, 1]$, (usually 0.5)

Ex1: Indoor map navigation (odometry only)



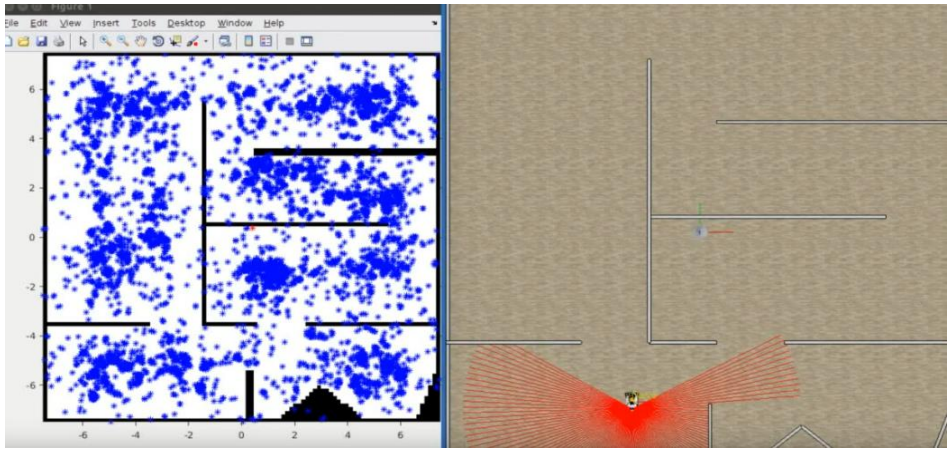


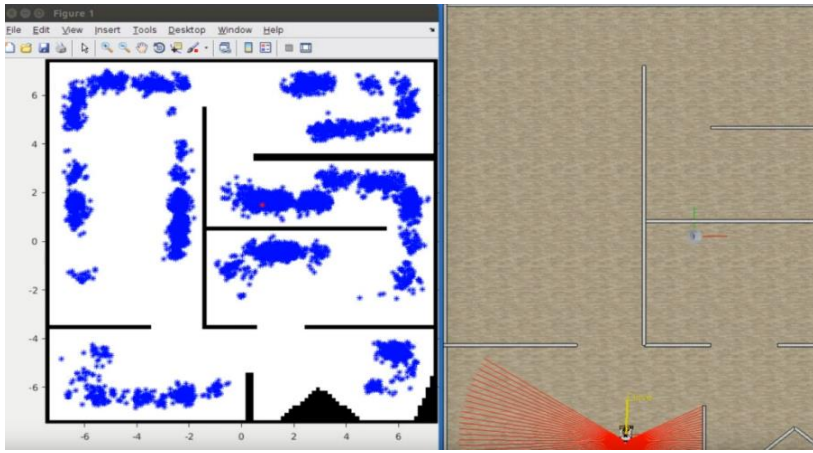


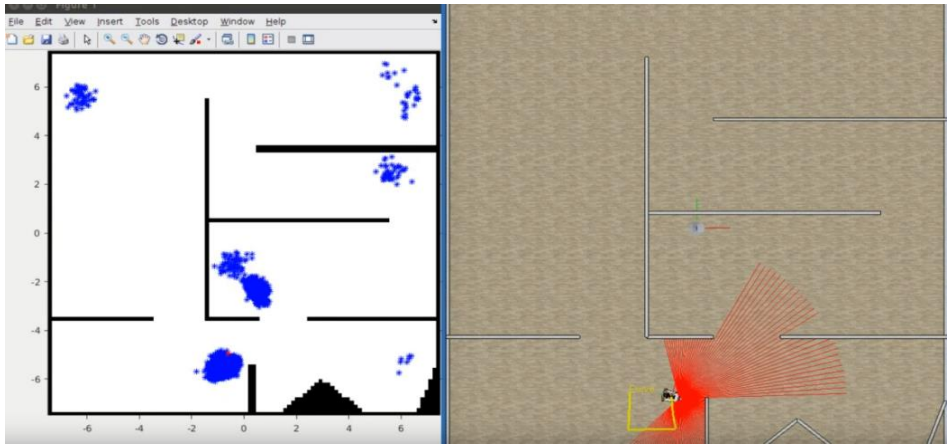


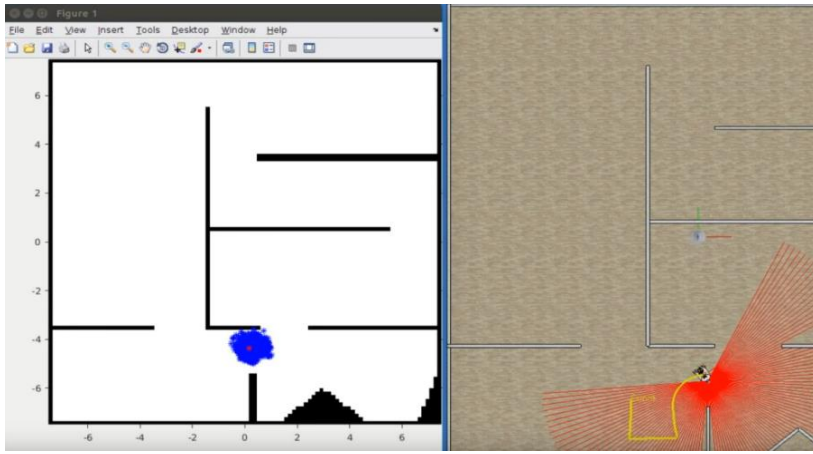
Position estimation using Particle Filter
https://www.youtube.com/watch?v=qsNGoHi7o2U

Ex2: Indoor map navigation with LIDAR data



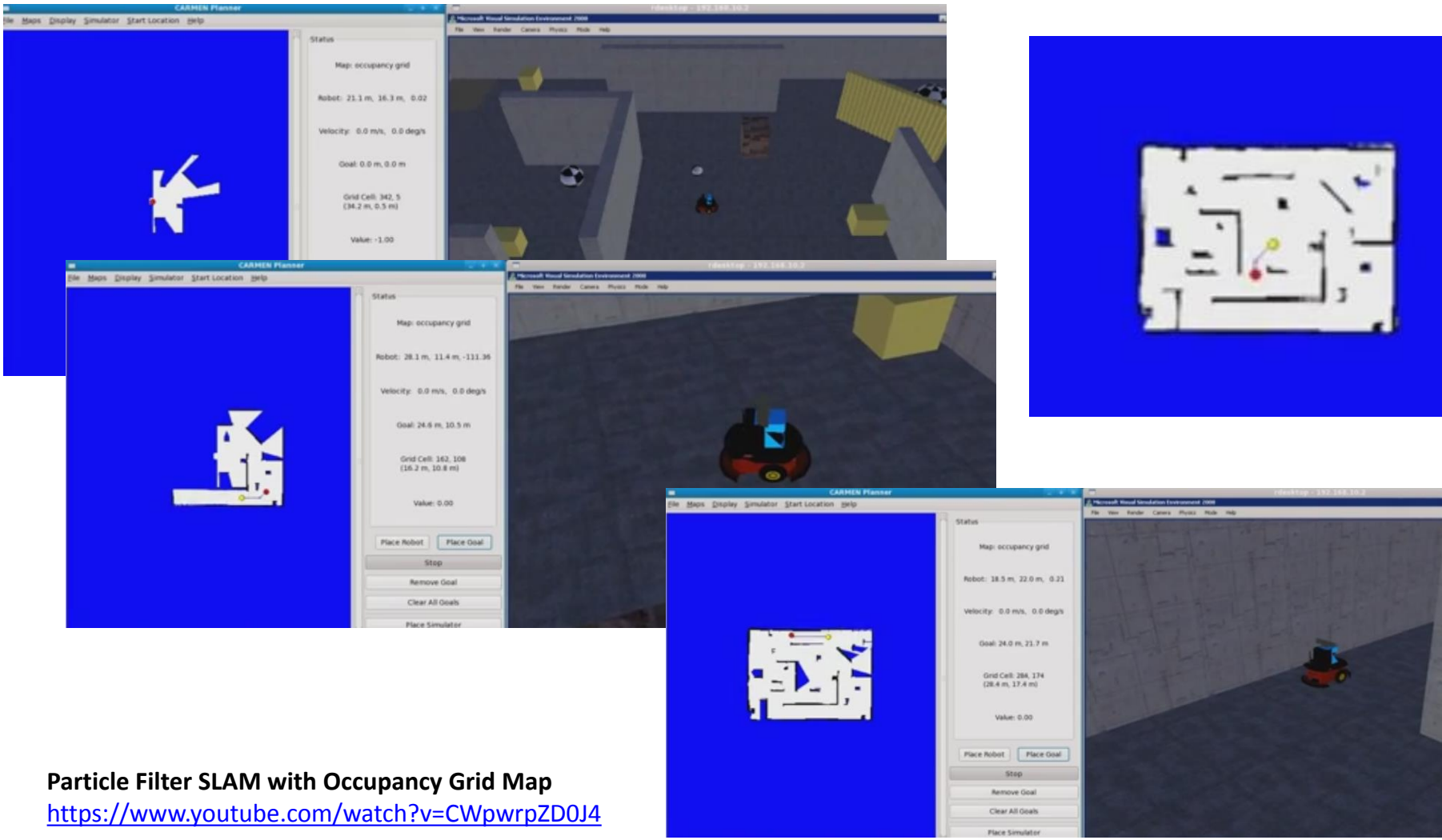






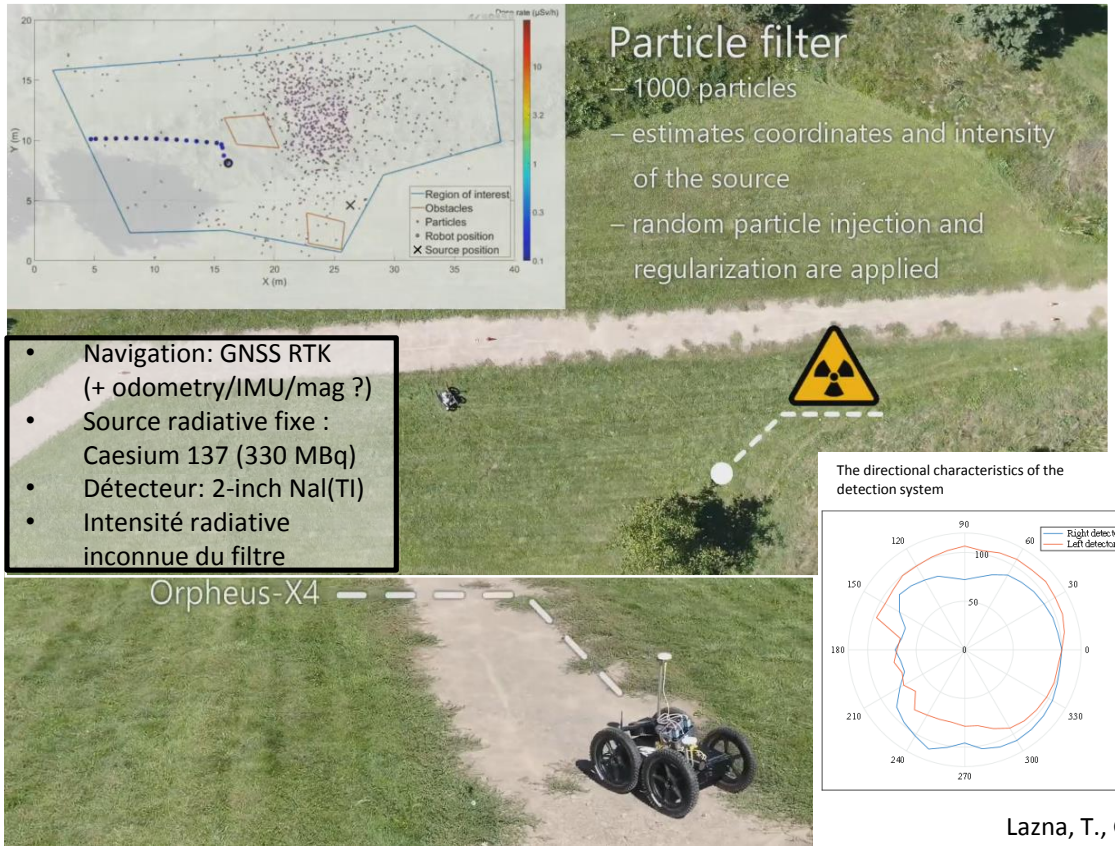
EMOR - Project 3: particle filter (with LIDAR data)
https://www.youtube.com/watch?v=Xlt1aoz5fUw

Ex3: Particle Filter SLAM

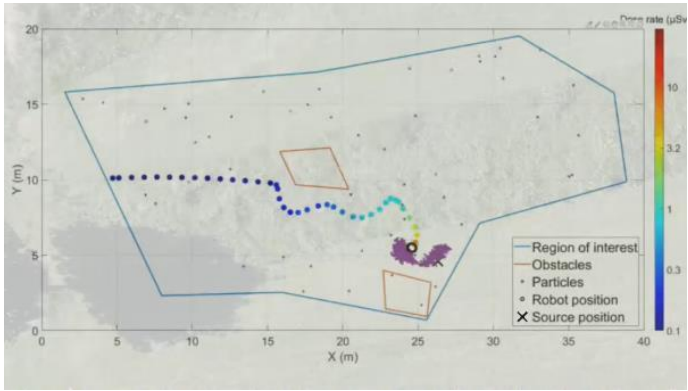


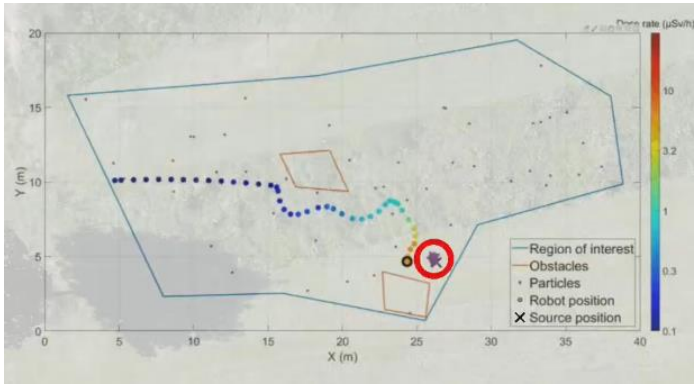
Particle Filter SLAM with Occupancy Grid Map
https://www.youtube.com/watch?v=CWpwrpZD0J4

Ex4: Autonomous Radiation Source Localization Using a Particle Filter



The directional characteristics of the detection system

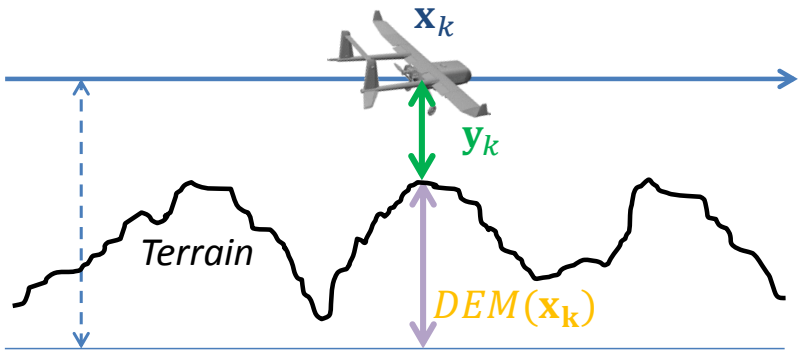


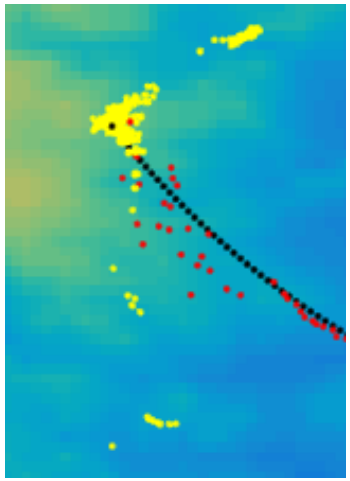


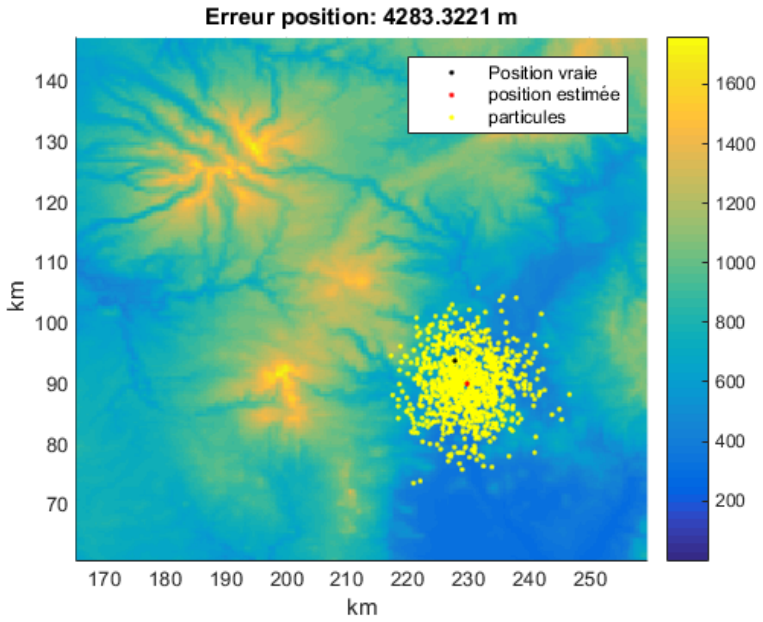
Autonomous Radiation Source Localization Using a Particle Filter
https://www.youtube.com/watch?v=Ox4J_Jov2XE

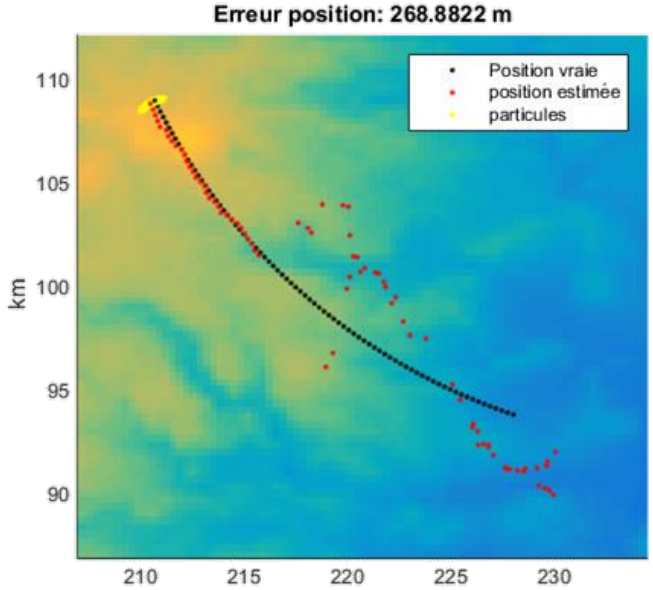
Lazna, T., Gabrlik, P., Jilek, T., & Zalud, L. (2018). Cooperation between an unmanned aerial vehicle and an unmanned ground vehicle in highly accurate localization of gamma radiation hotspots. *International Journal of Advanced Robotic Systems*, *15*(1), 1729881417750787. https://journals.sagepub.com/doi/pdf/10.1177/1729881417750787

Ex5: Terrain aided navigation









SIR-PF advantages and limitations

Advantages

- Tackles non-linear dynamics and measurement models
- Tackles non-differentiable models (no linearization)
- Tackles non-Gaussian uncertainties and multimodal state densities (ambiguities)
- Can start with huge initial uncertainty
- Relatively easy to implement

Limitations of the original version of the algorithm:

- Computationally demanding
- May converge to erroneous local state density maximum
- Can be hard to tune (number of particles, choice of noise modeling, resampling threshold)

More advanced Particle Filters

Original algorithm:

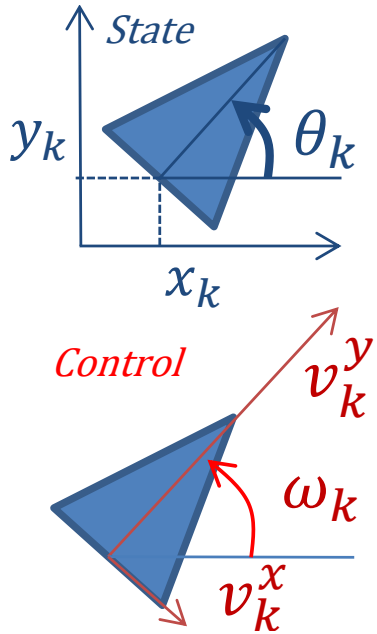
- Sequential Importance Resampling Particle Filter (SIR-PF) [2]

More advanced algorithms:

- Rao-Blackwellized Particle Filter (RBPF) [6]
- (Weighted) Ensemble Kalman Filter (WEnKF/EnKF) [3]
- Regularized Particle Filter (RPF) [4]
- Box Particle Filter (BPF) [7] and Box Regularized Particle Filter (BRPF) [8]
- Adaptive Approximate Bayesian Computational Particle Filter (A2BC-PF) [9]
- Non-Euclidean particle filters [10]
- Lie-Group particle filters [11]
- ...

Non-linear model: Prediction

State (ground frame) : $\mathbf{x}_k = [x_k, y_k, \theta_k]^T$,
Control (robot frame) : $\mathbf{u}_k = [v_k^x, v_k^y, \omega_k]^T$
Noised Odometry (input, robot frame) :
$\tilde{\mathbf{u}}_k = [\tilde{v}_k^x, \tilde{v}_k^y, \tilde{\omega}_k]^T \sim \mathcal{N}(\mathbf{u}_k, \mathbf{Q}_k)$
Process noise (robot frame) :
$\mathbf{w}_k = [w_k^{Vx}, w_k^{Vy}, w_k^{\omega}]^T \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k)$



Dynamics: $\mathbf{x}_k = f(\mathbf{x}_{k-1}, \tilde{\mathbf{u}}_k, \mathbf{w}_k)$

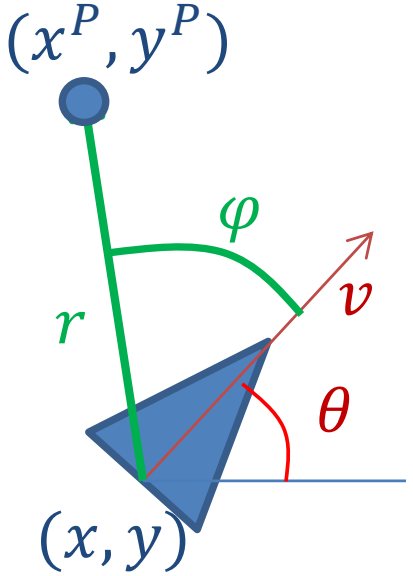
$$\mathbf{x}_k = \begin{bmatrix} x_{k-1} + \left((\tilde{v}_k^x + w_k^{Vx}) \cos\theta_{k-1} - (\tilde{v}_k^y + w_k^{Vy}) \sin\theta_{k-1} \right) \Delta t \\ y_{k-1} + \left((\tilde{v}_k^x + w_k^{Vx}) \sin\theta_{k-1} + (\tilde{v}_k^y + w_k^{Vy}) \cos\theta_{k-1} \right) \Delta t \\ \theta_{k-1} + (\tilde{\omega}_k + w_k^{\omega}) \Delta t \end{bmatrix}$$

Non-linear model: Correction

Measurements (e.g., radio beacon):

$$\mathbf{y}_k = h(\mathbf{x}_k) = \begin{bmatrix} r_k \\ \varphi_k \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{(x_k^P - x_k)^2 + (y_k^P - y_k)^2} \\ \operatorname{atan} \frac{y_k^P - y_k}{x_k^P - x_k} - \theta_k \end{bmatrix}$$



The map of landmarks is *a priori* known and is encoded in matrix *Map*. At each time-step k, one landmark is randomly selected to simulate a measurement (the current landmark index in the map is *iFeature*).

TP

1. Prendre en main la structure du code et repérer les différents paramètres du filtre. Expliquer comment s'agencent les grandes parties du code (simulation du véhicule, des capteurs, de l'odométrie, du Filtre Particulaire...).
2. Compléter le code avec les équations du filtre PF (prédiction, correction, ré-échantillonnage slide 25), le modèle dynamique (*motion_model*), le modèle de mesure (*observation_model*) décrits dans la slide 33 et commenter les résultats.
3. Faire varier le bruit de dynamique du filtre (matrice *QEst*) qu'observe-t-on ? Expliquer.
4. Faire varier le bruit de mesure du filtre (matrice *REst*), qu'observe-t-on ? Expliquer.
5. Faire varier le seuil de ré-échantillonnage *theta_eff* entre 0 et 1, tracer des histogrammes des poids en fonction de *theta_eff* et identifier le phénomène de dégénérescence et l'impact du ré-échantillonnage.
6. Simuler un trou de mesures entre $t = 250$ s et $t = 350$ s en utilisant la variable *notValidCondition* et expliquer les résultats.
7. Modifier la fréquence des mesures (passer à 0.1 Hz) en utilisant la variable *dt_meas* et expliquer les résultats.
8. Faire varier le nombre d'amers (variable *nLandmarks*) et étudier les performances du filtre en fonction du nombre d'amers. Régler le filtre pour obtenir les meilleures performances possibles et expliquer les résultats.
9. Proposer une autre façon de ré-échantillonner les poids, en remplacement de la fonction *re_sampling*, la coder puis commenter les résultats. Vous pourrez vous aider de [12] (référence slide 36, pdf fourni avec le code).

/!\ Les rapports de TP sont individuels. La moitié des points sera consacrée aux résultats, et l'autre moitié portera sur votre appropriation personnelle des concepts et l'interprétation des résultats.

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