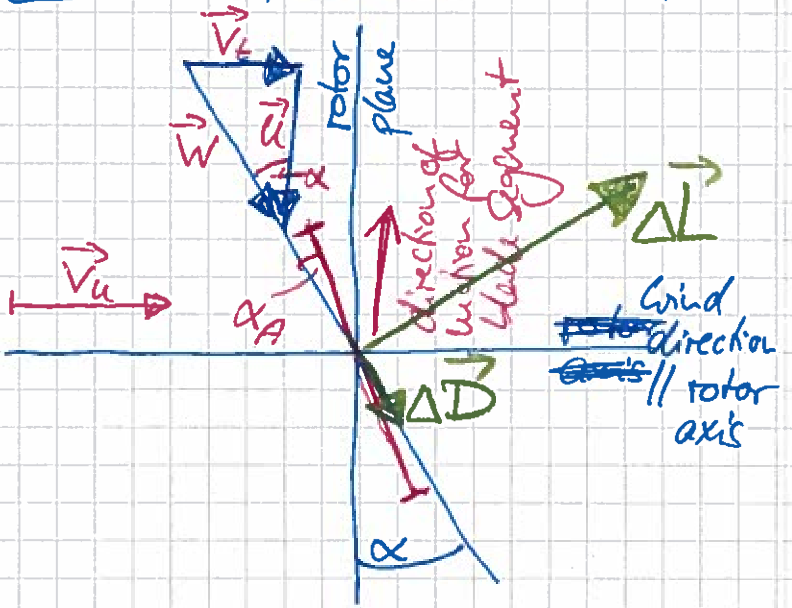
Summary (last session)



$$V_t = \underbrace{q}_{=2/3} V_u \,, \quad u = \Omega r \,, \quad \tan\alpha = \frac{V_t}{u}$$

$\Rightarrow$ Lift + drag forces:

$$\Delta L = \frac{\rho}{2} W^2(r)\, c(r)\, \Delta r \; C_L(\alpha_A)$$

$$\Delta D = \frac{\rho}{2} W^2(r)\, c(r)\, \Delta r \; C_D(\alpha_A)$$

$\Rightarrow$ projections:

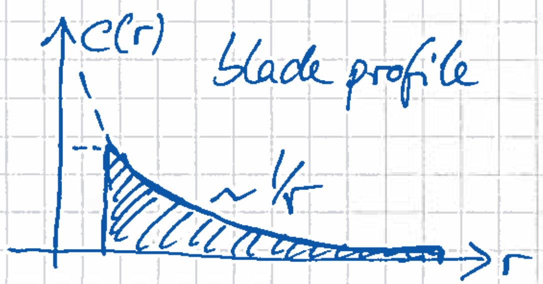
(on rotor plane) $\Delta U = \Delta L \sin\alpha - \Delta D \cos\alpha \approx \Delta L \sin\alpha$

(on axis direction) $\Delta T = \Delta L \cos\alpha + \Delta D \sin\alpha \approx \Delta L \cos\alpha$

$$P^{Betz}_{turbine} = \frac{16}{27} \frac{\rho}{2} V_u^3 \pi R^2 = \frac{16}{27} \frac{\rho}{2} V_u^3 \int_0^R 2\pi r \, dr$$

$$\Rightarrow \Delta P^{Betz}_{turbine}(r) = \frac{16}{27} \frac{\rho}{2} V_u^3 \, 2\pi r \, \Delta r \overset{!}{=} \Delta U(r) \cdot \Omega r \cdot N_{blades}$$

$\Rightarrow$



Projection of lift + drag forces onto rotor axis
$\Rightarrow$ thrust force:

$$\Delta T = \Delta L \cos\alpha + \Delta D \sin\alpha$$
$$\approx \Delta L \cos\alpha$$

$$\Rightarrow T = N \int_0^R dT$$
$$= N \int_0^R \frac{\rho}{2} W^2(r)\, c(r)\, C_L(\alpha_A) \cos\alpha(r)\, dr$$
$$= \int_0^R \frac{\cos\alpha}{\Omega r \sin\alpha} \underbrace{N \Omega r \frac{\rho}{2} C_L W^2 c \sin\alpha\, dr}_{= dP_{turbine}^{Betz}}$$

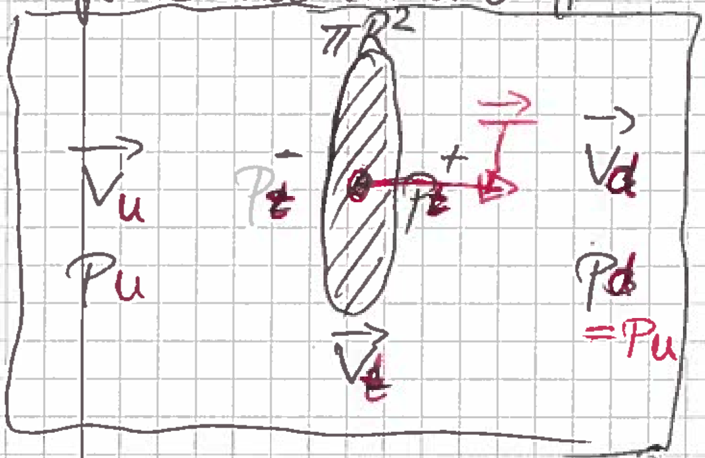
{ see page 28 !

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{V_t}{u} = \frac{V_t}{\Omega r}$$
$$\Rightarrow \frac{\cos\alpha}{\Omega r \sin\alpha} = \frac{1}{V_t}$$

$$= \frac{1}{V_t} \int_0^R dP_{turbine}^{Betz} = \frac{1}{V_t} P_{turbine}^{Betz}$$
$$= \frac{1}{\frac{2}{3} V_u} \underbrace{C_P^{Betz}}_{=16/27} \frac{\rho \pi R^2 V_u^3}{2}$$
$$= \underbrace{\frac{8}{9}}_{= C_T^{Betz} \text{ (thrust coefficient)}} \frac{\rho \pi R^2 V_u^2}{2}$$

{ thrust force is proportional to the kinetic energy contained in the wind.

We can derive this expression for the thrust force also in a different way:



thrust force =
= difference between the pressure forces in front and behind the rotor disc

$$T = \pi R^2 (p_t^- - p_t^+)$$

Bernoulli equation:

$$p_u + \frac{\rho}{2} V_u^2 = p_t^- + \frac{\rho}{2}(V_t^-)^2$$

$$p_d + \frac{\rho}{2} V_d^2 = p_t^+ + \frac{\rho}{2}(V_t^+)^2$$

$$V_t^- = V_t^+ = V_t \Longrightarrow p_t^- - p_t^+ = (p_u - p_d) + \frac{\rho}{2}(V_u^2 - V_d^2)$$

$$= \frac{\rho}{2}(V_u^2 - V_d^2)$$

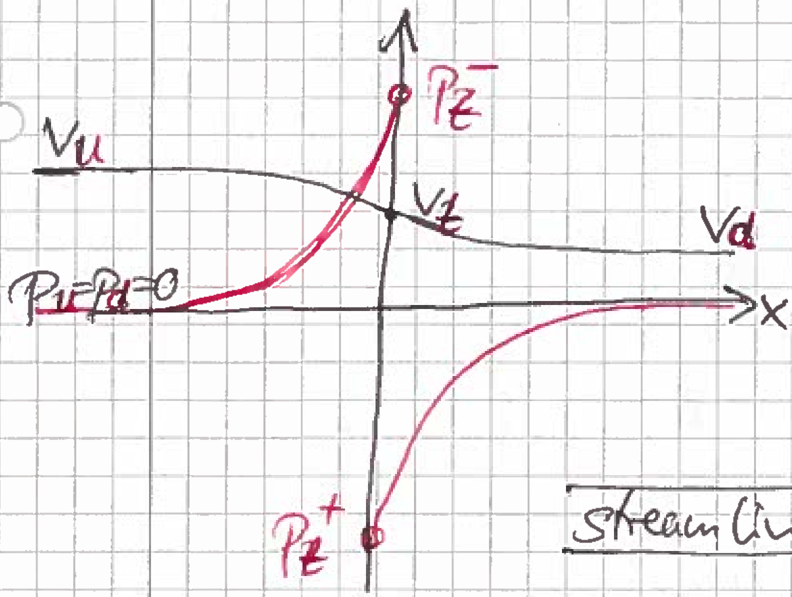
$p_u = p_d$
pressure p_u for upstream is equal to pressure p_d for downstream

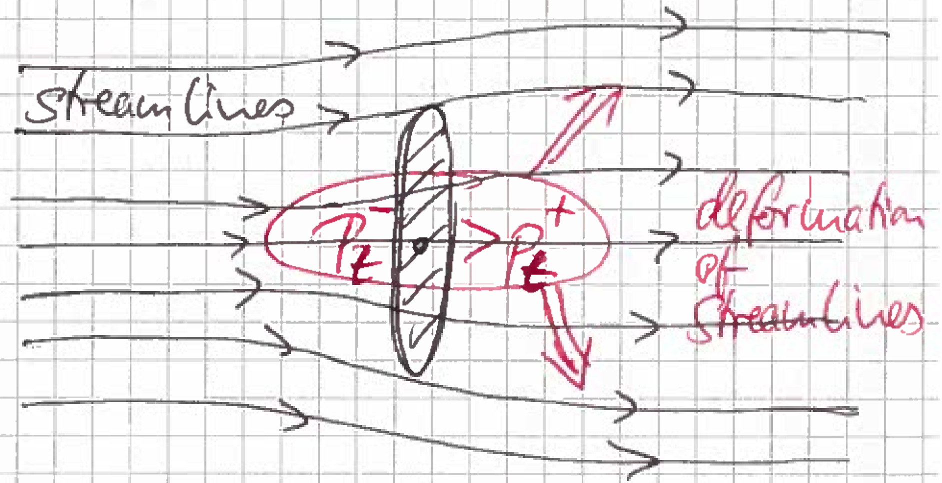
$$\Longrightarrow T = \pi R^2 \frac{\rho}{2}(V_u^2 - V_d^2)$$

$$= \frac{\rho \pi R^2 V_u^2}{2} \underbrace{\left(1 - \left(\frac{V_d}{V_u}\right)^2\right)}_{= C_T\left(\frac{V_d}{V_u}\right)}$$

$V_d = q V_u$

$$C_T\left(\frac{V_d}{V_u}\right) = 1 - q^2 \Longrightarrow C_T^{Betz}\left(\frac{V_d}{V_u} = \frac{1}{3}\right) = 1 - \frac{1}{9} = \frac{8}{9}$$

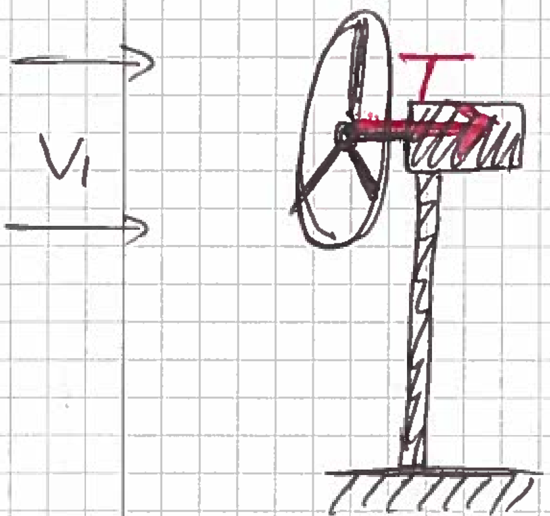




Wind-turbine interaction I:

tower + nacelle oscillations

⟹ 34a



Generic description of displacement oscillations (in longitudinal direction x):

$$m\ddot{x} + d\dot{x} + s x = T(t)$$

mass of nacelle

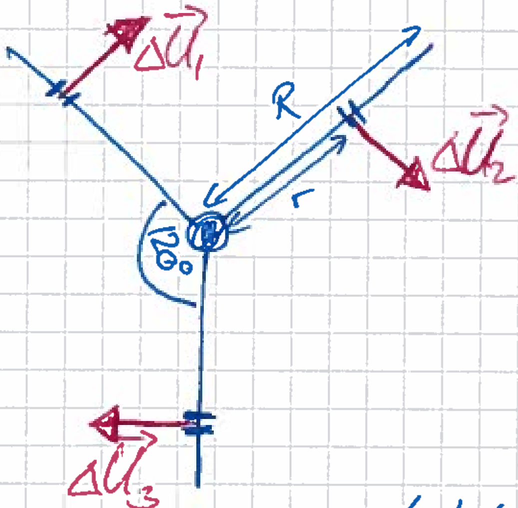
damping (tower + foundation properties) friction

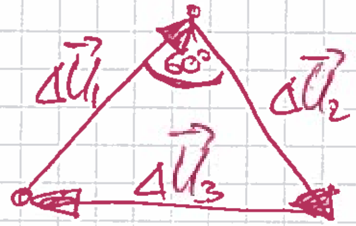
stiffness of tower

typically, damping is very small for wind turbines:

$\gamma^2 \ll \omega^2$ (weak damping) $\left\{ 2\gamma = d/m, \ \omega^2 = s/m \right.$

Opening remark: vector sum of lift forces
(projected on rotorplane)





$$\Delta\vec{L}_1 + \Delta\vec{L}_2 + \Delta\vec{L}_3 = 0$$

$\Longrightarrow$ total net force in rotor plane $= 0$

$\Longrightarrow$ does not pull the turbine to the side!

$\Longrightarrow$ no tower oscillations perpendicular to the wind direction

Possible real-world problems:

unbalanced masses of rotor blades

height-dependent velocity profile $v_w(\underline{\underline{z}})$

turbulence

$\Longrightarrow$ need: fast turbine controller!

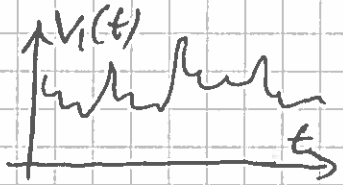
3 interesting examples:

(1) $T(t) = \begin{cases} T_0 & (t<0) \\ 0 & (t \geq 0) \end{cases}$ turbine shut down:

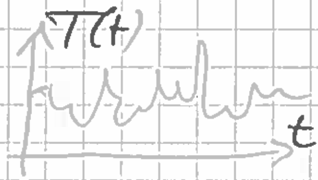
(2) tower shadow effect:

$$T(t) = T_0 + \Delta T \sin \alpha t \qquad \{ \alpha = 3\Omega$$

(3) turbulence:



$\Longrightarrow$



Example 1: turbine shut-down

$$\ddot{x} + 2\gamma \dot{x} + \omega^2 x = 0$$

$$2\gamma = d/m \qquad \omega^2 = S/m$$

$\Longrightarrow$ Solution: $x(t) = A e^{\lambda t}$

$\Longrightarrow \lambda^2 + 2\gamma\lambda + \omega^2 = 0 \Longrightarrow \lambda_{1/2} = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$

$\Longrightarrow x(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

$$= e^{-\gamma t}\left(A_1 e^{i\sqrt{\omega^2-\gamma^2}\,t} + A_2 e^{-i\sqrt{\omega^2-\gamma^2}\,t}\right)$$

$$\left\{ e^{i\varphi} = \cos\varphi + i\sin\varphi, \quad e^{-i\varphi} = \cos\varphi - i\sin\varphi \right.$$

$$= e^{-\gamma t}\left[(A_1+A_2)\cos\left(\sqrt{\omega^2-\gamma^2}\,t\right) + i(A_1-A_2)\sin\left(\sqrt{\omega^2-\gamma^2}\,t\right)\right]$$

initial values: $x(t=0)=x_0$, $\dot{x}(t=0)=0$

$$\Rightarrow x_0 = A_1 + A_2,$$

$$0 = (-\gamma)\underbrace{(A_1+A_2)}_{=x_0} + i(A_1-A_2)\sqrt{\omega^2-\gamma^2}$$

$$\Rightarrow i(A_1-A_2) = \frac{\gamma x_0}{\sqrt{\omega^2-\gamma^2}}$$

$$\Rightarrow x(t) = x_0 e^{-\gamma t}\left[\cos\left(\sqrt{\omega^2-\gamma^2}\,t\right) + \frac{\gamma}{\sqrt{\omega^2-\gamma^2}}\sin\left(\sqrt{\omega^2-\gamma^2}\,t\right)\right]$$

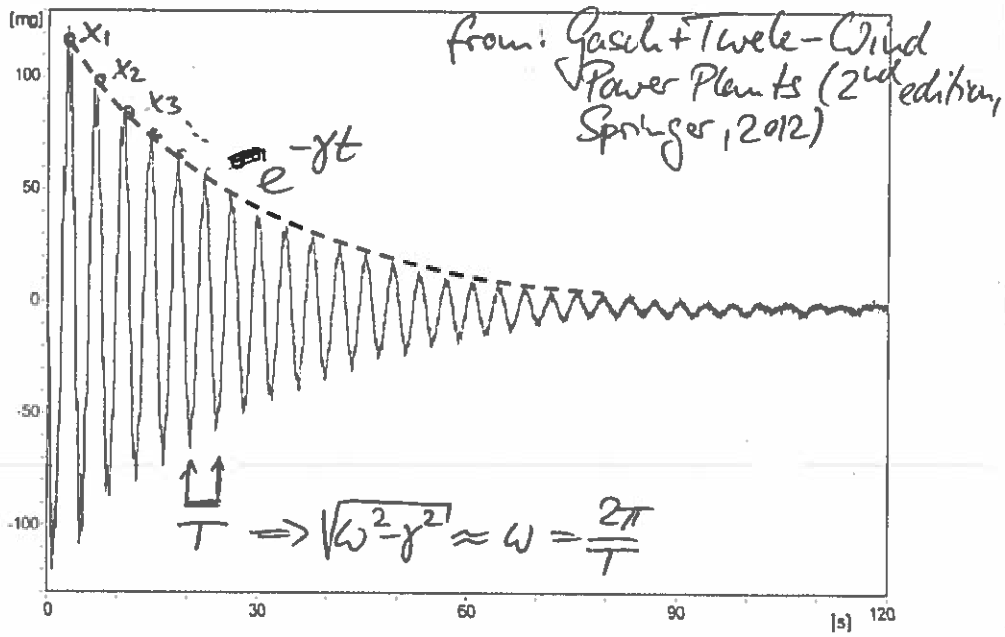


Fig. 8-11 Measured acceleration of the tower-nacelle system of a 2 MW wind turbine after an abrupt stop (1st natural bending frequency 0.26 Hz, damping factor including liquid damper approx. 2%), [8]

$$\frac{x_n}{x_{n+1}} = e^{\gamma T} \Rightarrow \ln\frac{x_n}{x_{n+1}} = \gamma T = \gamma\frac{2\pi}{\sqrt{\omega^2-\gamma^2}}$$

$\Rightarrow$ extraction of stiffness and damping parameters $\omega^2 = \frac{s}{m}$ and $2\gamma = \frac{d}{m}$!

Example 2: tower shadow effect

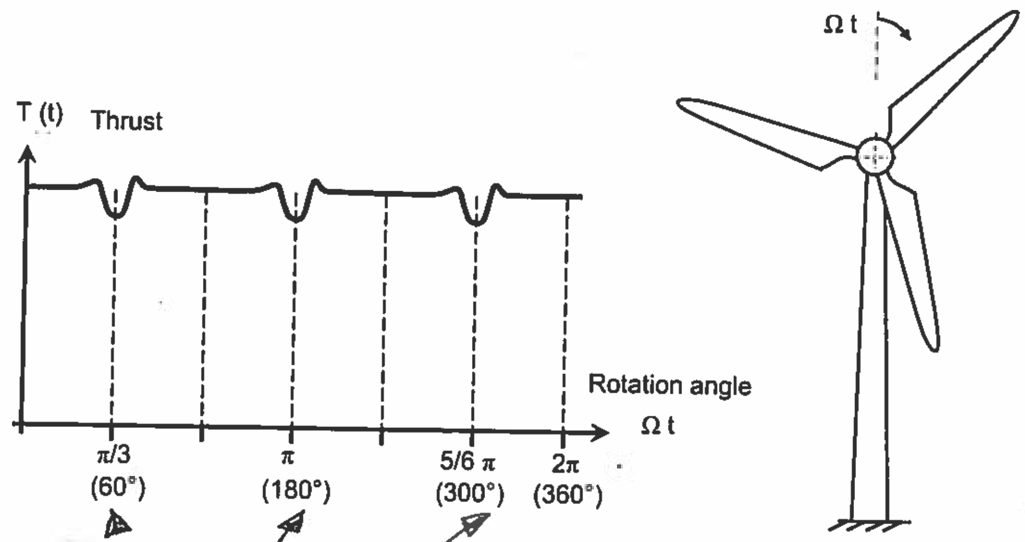


Fig. 8-7 Effects of tower dam: thrust versus rotation angle for a three-bladed wind turbine

less thrust when a blade passes the tower!

$$\Rightarrow T(t) = T_0 + \Delta T_3 \cos(3\Omega t + \varphi_3) + \Delta T_6 \cos(6\Omega t + \varphi_6) + \Delta T_9 \cos(9\Omega t + \varphi_9) + \dots$$

} higher-order harmonics

$\Rightarrow$ periodic forcing:

$\{ \alpha = 3\Omega$

$$m\ddot{x} + d\dot{x} + sx = T_0 + \Delta T \cos \alpha t$$

$$\left\{ y = x + \frac{T_0}{s} \right.$$

$$\Rightarrow m\ddot{y} + d\dot{y} + sy = \Delta T \cos \alpha t$$

$\{ f_0 = \frac{\Delta T}{m}$

$y \to x$

$$\Rightarrow \ddot{x} + 2\gamma\dot{x} + \omega^2 x = f_0 \cos \alpha t$$

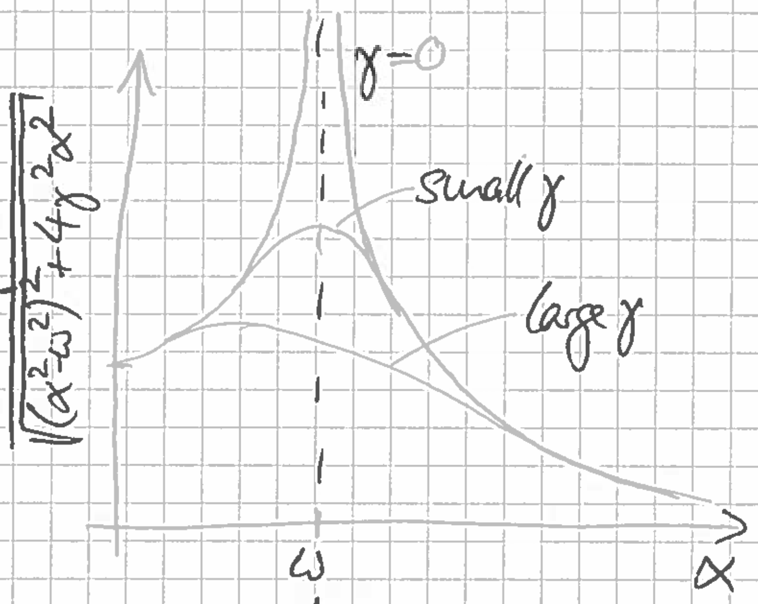
{ general solution =
= general solution of homogeneous differential equation
⊕ special solution of inhomogeneous differential equation

$$\left\{ \tan\varphi = \frac{-2\gamma\alpha}{\alpha^2-\omega^2} \right\}$$

$\Longrightarrow \ldots$

$$\Longrightarrow x(t) = \frac{f_0}{\sqrt{(\alpha^2-\omega^2)^2+4\gamma^2\alpha^2}}\cos(\alpha t-\varphi)$$

$$+\, e^{-\gamma t}\left(A\sin\left(\sqrt{\omega^2-\gamma^2}\,t\right)+B\cos\left(\sqrt{\omega^2-\gamma^2}\,t\right)\right)$$



$\alpha=\omega \Longrightarrow$ resonance catastrophe!

previous Example 1: $\omega \approx 0{,}26$ Hz

$\Omega \approx 15$ rpm $= 0{,}25$ Hz

$\Longrightarrow \alpha = 3\Omega \approx 0{,}75$ Hz $\approx 3\omega$

$\Longrightarrow$ no resonance catastrophe!

Further reading:

R. Gasch + J. Twele: Wind Power Plants (2nd edition, Springer 2012)

⇒ Section 8: Structural dynamics

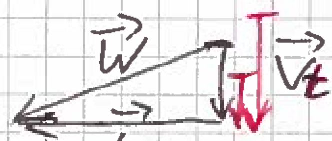
⇒ Course homepage: 150928_FurtherReading9.

(1) unbalanced mass + centrifugal force

⇒ oscillating lateral displacement

(2) aerodynamic loads

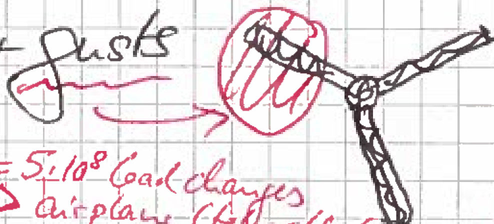
⇒ vertical wind profile:



⇒ periodic v_t fluctuations

⇒ $C_L \sim \sin(3\Omega t)$

⇒ turbulence + gusts



Comparison with airplane:

20 y turbine × 15 rpm = $1.6 \cdot 10^8$ rotations $\xRightarrow{\times 3}$ ≈ $5 \cdot 10^8$ load changes

⇒ airplane (take off, landing)

(3) blade vibrations

(flapwise, edgewise, torsion)

(4) drive-train oscillations

→ in particular: coupled blade-drive train system

required:

! ⇒ simulation of overall system dynamics:

multi-body simulation + modal analysis !

[redacted]