Questions:

$\rho_0 \left[\vec{u}t + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right] +$
$\vec{\nabla}p -$
$\mu\left(\vec{\nabla}\cdot\vec{\nabla}\right)\vec{u}$
$=$
$\rho_0 \left[\frac{U}{T}\vec{u}'t' + \frac{U^2}{L}\left(\vec{u}'\cdot\vec{\nabla}'\right)\vec{u}'\right] +$
$\frac{\rho_0 U^2}{L}\vec{\nabla}'p' -$
$\mu\frac{U}{L^2}\left(\vec{\nabla}\cdot\vec{\nabla}\right)'\vec{u}'$
$=$
$\rho_0\frac{U^2}{L}\left\{\vec{u}'t' + \left(\vec{u}'\cdot\vec{\nabla}'\right)\vec{u}' + \vec{\nabla}'p' - \frac{\mu}{\rho_0 LV}\left(\vec{\nabla}\cdot\vec{\nabla}\right)'\vec{u}'\right\}$
L
U
$T =$
L/U
$P =$
$\rho_0 U^2$

$$Re = \frac{\rho_0 LU}{\mu} \tag{1}$$

Remark:

$$Re = \frac{\rho_0 LU}{\mu} = \frac{\rho_0 U^2/L}{\mu U/L}, \tag{2}$$

$u_z = u_z(r)$

$$\rho_0 \underbrace{\vec{u}t}_{=0} + \rho_0 \underbrace{\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}}_{=0} = \underbrace{\vec{f}_{\text{ext}}}_{=0} - \vec{\nabla}p + \mu\left(\vec{\nabla}\cdot\vec{\nabla}\right)\vec{u} \tag{3}$$

$$0 = \vec{\nabla}\cdot\vec{u} = \partial_x u_x + \partial_y u_y + \partial_z u_z = \partial_z u_z \tag{4}$$

$u_x \equiv u_y = 0$

$$\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = u_z \partial_z \left(0 \right) 0 u_z = 0. \tag{5}$$

$\vec{\nabla}p = \mu\left(x+y\right)u_z\vec{e}_z$ $\left(\vec{\nabla}p\right)_x = \left(\vec{\nabla}p\right)_y = 0$ $p = p(z)$

$\mu\left(\partial_x^2 + \partial_y^2\right)v_z(r) = \frac{\mu}{r}r\left(ru_z(r)r\right)$

$p(z)z\text{constant}\frac{}{Lz+p(z=0)=-\frac{\Delta p}{L}z+p(z=0)}$

$\mu\frac{}{rr(ru_z(r)r)=c=-\frac{\Delta p}{L}ru_z(r)r=-\frac{\Delta p}{\mu L}\frac{r^2}{2}+D_1u_z(r)=-\frac{\Delta p}{4\mu L}r^2+D_1\ln r+D_2}$

D_1 D_2 $u_z(r = R) = 0$ $|u_z(r = 0)| < \infty$

$$u_z(r) = \frac{\Delta p}{4\mu L}\left(R^2 - r^2\right) \tag{6}$$

$$Mt = \int_0^R \rho_0 u_z(r) 2\pi r dr = \frac{\pi\rho_0 R^4 \Delta p}{8\mu L} \tag{7}$$

Remark:

$$\left\{\underbrace{\rho_0, R, L}_{\text{known}}, \underbrace{\Delta p, Mt}_{\text{measured}}\right\} \Rightarrow \mu \tag{8}$$

Remark: $\Delta p = \Delta U, Mt = I$ $I = \frac{\pi\rho_0 R^4}{8L\mu}\Delta U$

$$R = R_0 \cdot f(Re) \tag{9}$$

$$f(Re \to 0) = 1. \tag{10}$$

$\delta = \delta(x)$
$\vec{\nabla} \cdot \vec{u} = u_x x + u_y y = 0$
$\mathcal{O}\left(\frac{u_\infty}{L}\right) + \mathcal{O}\left(\frac{u_y}{\delta}\right) = 0$
$\mathcal{O}(u_y) = \delta \frac{u_\infty}{L} = \frac{\delta}{L} u_\infty$

$$u_x u_x x + u_y u_x y = -\frac{1}{\rho_0} p x + \frac{\mu}{\rho_0} u_x x + \frac{\mu}{\rho_0} u_x y \quad (11)$$

$$\mathcal{O}\left(\frac{u_\infty^2}{L}\right) \mathcal{O}\left(\frac{\mu}{\rho_0} \frac{u_\infty}{\delta^2}\right) \quad (12)$$

$$\left(\frac{\delta}{L}\right)^2 \sim \frac{\mu}{\rho_0 L u_\infty} = \frac{1}{Re} \quad (13)$$

$Re \leq 1e5 - 1e6$

$$\delta(x) \sim \sqrt{\frac{\mu x}{\rho_0 u_\infty}} \quad (14)$$

$$u_x u_y x + u_y u_y y = -\frac{1}{\rho_0} p y + \frac{\mu}{\rho_0} u_y x + \frac{\mu}{\rho_0} u_y y \quad (15)$$

$$p y = 0 \Rightarrow p = p(x) \quad (16)$$

Prandtl equations:
$_x u_x x + u_y u_x y = -\frac{1}{\rho_0} p(x) x + \frac{\mu}{\rho_0} u_x y$
$u_x x + u_y y = 0$

Example:

$$p = p(x) \Rightarrow p(x)|_{\text{inside}} = p(x)|_{\text{outside}} \quad (17)$$

$_x|_{\text{outside}} = u_x = u_\infty = \text{constant}$
$u_y|_{\text{outside}} = 0$
$\rho_0 \frac{}{2u_x^2 = \text{constant} p(x) = \text{constant} p(x) x = 0}$
$_x \partial_x u_x + u_y \partial_y u_y = \frac{\mu}{\rho_0} \partial_y^2 u_x$
$\partial_x u_x + \partial_y u_y = 0$

Solution:

$$u_x(x, y) = u_\infty g\left(\frac{y}{\delta(x)}\right) \quad (18)$$

$\delta(x)$
$u_x(x, y)$
x

Question:
$_x x + u_y y = 0$
$u_x = \psi y, u_y = -\psi x$