46a

$$\left.\begin{aligned} V_x &= V_\infty \left(1 + \frac{R^2 (y^2 - x^2)}{(x^2+y^2)^2} \right) \\ V_y &= -V_\infty \frac{2xy\, R^2}{(x^2+y^2)^2} \end{aligned}\right\} \xrightarrow[\substack{y = r\sin\varphi \\ r = R}]{x = r\cos\varphi} \left\{\begin{aligned} V_x &= V_\infty \left(1 + \sin^2\varphi - \cos^2\varphi \right) \\ &= 2 V_\infty \sin^2\varphi \\ V_y &= -2 V_\infty \sin\varphi \cos\varphi \end{aligned}\right.$$

$$\Rightarrow V_x^2 (r=R,\varphi) + V_y^2 (r=R,\varphi) = 4 V_\infty^2 \left[\sin^4\varphi + \sin^2\varphi \cos^2\varphi \right]$$

$$= 4 V_\infty^2 \sin^2\varphi$$

4P

Bernoulli eq.: $\frac{\rho_0}{2} \vec{V}^2 + P = \underbrace{P_0}_{= \frac{\rho_0}{2} V_\infty^2} \Rightarrow P(r=R,\varphi) = P_0 - \frac{\rho_0}{2}(V_x^2 + V_y^2)$

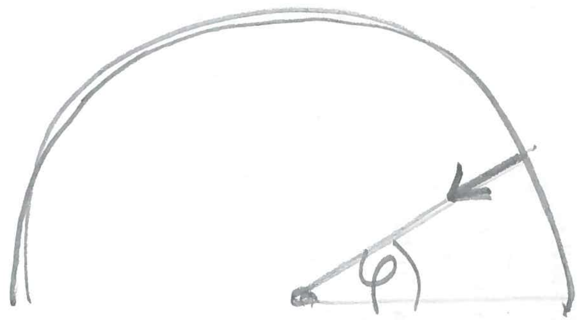
$$= P_0 - 2 \rho_0 V_\infty^2 \sin^2\varphi$$

$$= \frac{\rho_0}{2} V_\infty^2 \left(1 - 4 \sin^2\varphi \right)$$

$$= \frac{\rho_0 V_\infty^2}{2} \left(4 \cos^2\varphi - 3 \right)$$

4P

466



$$\vec{n} = \begin{pmatrix} -\cos\varphi \\ -\sin\varphi \end{pmatrix}$$

$$\vec{F} = \begin{pmatrix} D \\ L \end{pmatrix} = \int_0^{\pi} p(\varphi)\, \vec{n}\,(R\,d\varphi \cdot l_w) = -\vec{e}_y R l_w \int_0^{\pi} p(\varphi) \sin\varphi \, d\varphi$$

$$= -R l_w \frac{\rho_0 V_\infty^2}{2} \vec{e}_y \int_0^{\pi} \left(4\cos^2\varphi - 3\right) \sin\varphi \, d\varphi$$

<u>remark</u>: no component in x-direction!

$\Longrightarrow \vec{D} = 0.$ <u>(symmetry)</u>

46c

$$\int_0^{\pi} \cos^2\varphi \sin\varphi \, d\varphi = 2\int_0^{\pi/2} \cos^2\varphi \sin\varphi \, d\varphi$$

$$\begin{cases} x = \cos\varphi \\ dx = -\sin\varphi \, d\varphi \end{cases}$$

$$= -2\int_1^0 x^2 dx = \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3}$$

$$\int_0^{\pi} \sin\varphi \, d\varphi = -\cos\varphi \Big|_0^{\pi} = 2$$

$$\Rightarrow \vec{L} = - R l_w \frac{\rho_0 V_\infty^2}{2} \vec{e}_{\cancel{x}} \underbrace{\left[4 \cdot \frac{2}{3} - 3 \cdot 2 \right]}_{\frac{8}{3} - 6 = \frac{8}{3} - \frac{18}{3} = -\frac{10}{3}}$$

$$= \frac{5}{3} R l_w \rho_0 V_\infty^2 \vec{e}_{\cancel{x}}$$

6P

4bd

$$L > \tilde{m}_w g = \tilde{\rho}_w \pi R^2 l_w g$$

$$\tilde{\rho}_w = \rho_w - \rho_0 \quad \text{(buoyancy)}$$

6P

$$\Rightarrow \frac{5}{3} R l_w \rho_0 v_\infty^2 > (\rho_w - \rho_0) \pi R^2 l_w g$$

$$\Rightarrow v_\infty^2 > \frac{3\pi}{5} \frac{\rho_w - \rho_0}{\rho_0} R g$$

$$\Rightarrow v_\infty^{crit} = \sqrt{\frac{3\pi}{5} \frac{\rho_w - \rho_0}{\rho_0} R g}$$

numbers: $$v_\infty^{crit} = \left(\frac{3\pi}{5} \cdot 0{,}1 \cdot 0{,}003\,m \; 9{,}81 \frac{m}{sec^2}\right)^{1/2} = \left(\frac{3\pi}{5} \cdot 0{,}1 \cdot 0{,}3 \cdot 981\right)^{1/2} \frac{cm}{sec}$$

$$= \left(\frac{3\pi}{5} \cdot 3 \cdot 9{,}81\right)^{1/2} \frac{cm}{sec}$$

$$= \left(\frac{9 \cdot \pi \cdot 9{,}81}{5}\right)^{1/2} \frac{cm}{sec}$$

$$= 7{,}45 \frac{cm}{sec}$$