

Questions:

$$\rho_0 \left[\vec{u} t + \left(\vec{u} \cdot \vec{\nabla} \right) \vec{u} \right] +$$

$$\vec{\nabla} p -$$

$$\mu \left(\vec{\nabla} \cdot \vec{\nabla} \right) \vec{u}$$

$$= \rho_0 \left[\frac{U}{T} \vec{u}' t' + \frac{U^2}{L} \left(\vec{u}' \cdot \vec{\nabla}' \right) \vec{u}' \right] +$$

$$\frac{\rho_0 U^2}{L} \vec{\nabla}' p' -$$

$$\mu \frac{U}{L^2} \left(\vec{\nabla} \cdot \vec{\nabla} \right)' \vec{u}'$$

$$= \rho_0 \frac{U^2}{L} \left\{ \vec{u}' t' + \left(\vec{u}' \cdot \vec{\nabla}' \right) \vec{u}' + \vec{\nabla}' p' - \frac{\mu}{\rho_0 L V} \left(\vec{\nabla} \cdot \vec{\nabla} \right)' \vec{u}' \right\}$$

$$\frac{L}{U}$$

$$\frac{L}{U} =$$

$$\frac{P}{\rho_0 U^2}$$

$$Re = \frac{\rho_0 L U}{\mu}$$

(1)

Remark:

$$Re = \frac{\rho_0 L U}{\mu} = \frac{\rho_0 U^2 / L}{\mu U / L},$$

(2)

$$\begin{array}{l} u_z = \\ u_z(r) \\ \rho_0 \underbrace{\vec{u}t}_{=0} + \rho_0 \underbrace{\left(\vec{u} \cdot \vec{\nabla}\right)}_{=0} \vec{u} = \underbrace{\vec{f}_{\text{ext}}}_{=0} - \vec{\nabla} p + \mu \left(\vec{\nabla} \cdot \vec{\nabla}\right) \vec{u} \end{array}$$

$$(3) \qquad 0 = \vec{\nabla} \cdot \vec{u} = \partial_x u_x + \partial_y u_y + \partial_z u_z = \partial_z u_z$$

$$(4) \qquad \begin{array}{l} u_x = \\ u_y = \\ 0 \\ \left(\vec{u} \cdot \vec{\nabla}\right) \vec{u} = u_z \partial_z \left(0\right) 0 u_z = 0. \end{array}$$

$$(5) \qquad \begin{array}{l} \vec{\nabla} p = \\ \mu \left(x+y\right) u_z \vec{e}_z \\ \left(\vec{\nabla} p\right)_x = \\ \left(\vec{\nabla} p\right)_y = \\ 0 \\ p = \\ p(z) \\ \mu \left(\partial_x^2 + \partial_y^2\right) v_z(r) = \\ \frac{\mu}{r} r \left(r u_z(r) r\right) \\ \frac{p(z) z \text{constant}}{L z + p(z=0) = -\frac{\Delta p}{L} z + p(z=0)} \\ \mu \frac{r r \left(r u_z(r) r\right) = c = -\frac{\Delta p}{L} r u_z(r) r = -\frac{\Delta p}{\mu L} \frac{r^2}{2} + D_1 u_z(r) = -\frac{\Delta p}{4 \mu L} r^2 + D_1 \ln r + D_2}{\end{array}$$

$$\begin{array}{l} D_1 \\ D_2 \\ u_z(r = \\ R) = \\ 0 \\ \left|u_z(r = \\ 0)\right| < \\ \infty \end{array}$$

$$u_z(r) = \frac{\Delta p}{4 \mu L} \left(R^2 - r^2\right)$$

$$(6) \qquad Mt = \int_0^R \rho_0 u_z(r) 2 \pi r dr = \frac{\pi \rho_0 R^4 \Delta p}{8 \mu L}$$

$$(7) \qquad \begin{array}{l} \textbf{Re-} \\ \textbf{mark:} \\ \left\{ \underbrace{\rho_0, R, L}_{\text{known}}, \underbrace{\Delta p, Mt}_{\text{measured}} \right\} \Rightarrow \mu \end{array}$$

$$(8) \qquad \begin{array}{l} \textbf{Remark:} \\ \Delta p = \\ \Delta U, Mt = \\ I \\ I = \\ \frac{\pi \rho_0 R^4}{8 L \mu} \Delta U \end{array}$$

$$R = R_0 \cdot f(Re)$$

$$(9) \qquad f(Re \rightarrow 0) = 1.$$

$$(10)$$

$$\begin{aligned}
 & \frac{\delta}{\delta(x)} = \\
 & \frac{\bar{u}}{\bar{u}} = \\
 & \frac{u_x x + u_y y}{u_x x + u_y y} = \\
 & \frac{0}{0} = \\
 & \mathcal{O}\left(\frac{u_\infty}{L}\right) + \\
 & \mathcal{O}\left(\frac{u_y}{\delta}\right) = \\
 & \frac{0}{0} = \\
 & \frac{\delta}{\delta} \frac{u_\infty}{L} = \\
 & \frac{\delta}{L} u_\infty \\
 & u_x u_x x + u_y u_x y = -\frac{1}{\rho_0} p x + \frac{\mu}{\rho_0} u_x x + \frac{\mu}{\rho_0} u_x y
 \end{aligned}$$

$$(11) \quad \mathcal{O}\left(\frac{u_\infty^2}{L}\right) \mathcal{O}\left(\frac{\mu}{\rho_0} \frac{u_\infty}{\delta^2}\right)$$

$$(13) \quad \left(\frac{\delta}{L}\right)^2 \sim \frac{\mu}{\rho_0 L u_\infty} = \frac{1}{Re}$$

$$(14) \quad \delta(x) \sim \sqrt{\frac{\mu x}{\rho_0 u_\infty}}$$

$$(15) \quad u_x u_y x + u_y u_y y = -\frac{1}{\rho_0} p y + \frac{\mu}{\rho_0} u_y x + \frac{\mu}{\rho_0} u_y y$$

$$(16) \quad p y = 0 \Rightarrow p = p(x)$$

Prandtl equations:

$$\begin{aligned}
 & \frac{u_x x + u_y y}{u_x x + u_y y} = \\
 & -\frac{1}{\rho_0} p(x) x + \\
 & \frac{\mu}{\rho_0} u_x y \\
 & \frac{u_x x + u_y y}{u_x x + u_y y} = \\
 & 0
 \end{aligned}$$

Example:

$$(17) \quad p = p(x) \Rightarrow p(x)|_{\text{inside}} = p(x)|_{\text{outside}}$$

$$\begin{aligned}
 & \frac{x}{u_x} \Big|_{\text{outside}} = \\
 & \frac{u_\infty}{u_\infty} = \\
 & \text{constant} \\
 & \frac{u_y}{u_y} \Big|_{\text{outside}} = \\
 & \frac{0}{\rho_0} \frac{2u_x^2 = \text{constant} p(x) = \text{constant} p(x) x = 0}{2u_x^2 = \text{constant} p(x) = \text{constant} p(x) x = 0} \\
 & \frac{x}{u_y} \frac{\partial_x u_x}{\partial_y u_y} + \\
 & \frac{\mu}{\rho_0} \frac{\partial_y^2 u_x}{\partial_y^2 u_x} \\
 & \frac{\partial_x u_x}{\partial_y u_y} + \\
 & \frac{\partial_y u_y}{\partial_y u_y} = \\
 & 0
 \end{aligned}$$

Solution:

$$(18) \quad u_x(x, y) = u_\infty g\left(\frac{y}{\delta(x)}\right)$$

$$\begin{aligned}
 & \frac{\delta(x)}{u_x(x, y)} \\
 & \frac{x}{x}
 \end{aligned}$$

Question:

$$\begin{aligned}
 & \frac{u_x x + u_y y}{u_x x + u_y y} = \\
 & \frac{0}{u_x} = \\
 & \psi y, u_y = \\
 & -\psi x
 \end{aligned}$$