Note for Fluid Dynamics Midterm Exam Project I

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Remark: This is an auxiliary note which you are free to choose use or not for the midterm exam project I.

1 Coordinate rotation

Explain a bit why we need to rotate the coordinate.

$$\begin{pmatrix} x_r \\ y_r \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

where (x, y) are the original coordinates of the turbines, and (x_r, y_r) are the corresponding rotated coordinates.

2 Overlapping area

We denote the radius of rotor as r_2, and the radius of wake at turbine we are looking at as r_1. If the distance d between the center of turbine and wake is larger than $(r_1 + r_2)$, i.e.,

$$d \geq r_1 + r_2 \tag{2}$$

then we end up no overlap between the turbine and wake. When $d < r_1 + r_2$ there begins overlapping, but we need to distinguish between two different cases. If d satisfies the following equation,

$$r_1 - r_2 < d < r_1 + r_2 \tag{3}$$

we have the partially overlap case, where is overlapping area can be found as

$$area = r_1^2 \cos^{-1}\left(\frac{r_1^2 - r_2^2 + d^2}{2dr_1}\right) + r_2^2 \cos^{-1}\left(\frac{r_2^2 - r_1^2 + d^2}{2dr_2}\right) - \frac{1}{2}\sqrt{T} \tag{4}$$

where T can be calculated as

$$T = \left((r_1 + r_2)^2 - d^2\right)\left(d^2 - (r_1 - r_2)^2\right) \tag{5}$$

The last case is fairly straightforward, where the turbine is fully covered by the wake, i.e.,

$$\begin{aligned} d &\leq r_1 - r_2 \\ area &= \pi r_2^2 \end{aligned} \tag{6}$$

3 Wake deflection due to yaw

What do we need to add here?