Fluid Dynamics + Turbulence (fall 2017)
Homework Problems V + voluntary Exercises

Posted:

Friday September 29, 2017.

Deadline for submission of homework problem:

Tuesday October 10 at 01.15 pm (on Blackboard).

Homework problem 5.1:
Laminar flow between two coaxial tubes

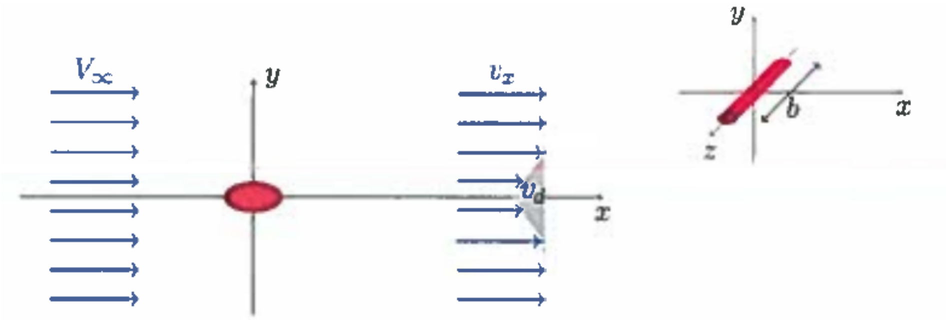
Consider a steady laminar flow through the annular space formed by two coaxial tubes aligned with the z-axis. The flow is along the axis of the tubes and is maintained by a pressure gradient dp/dz. Show that the axial velocity at any radius R is

$$u_z(R) = \frac{1}{4\mu}\frac{dp}{dz}\left[R^2 - a^2 - \frac{b^2 - a^2}{\ln(b/a)}\ln\frac{R}{a}\right], \tag{1}$$

where a is the radius of the inner tube and b is the radius of the outer tube. Find the radius at which the maximum velocity is reached, and the volume flow rate.

Homework problem 5.2: Laminar wake

Consider a long, slender object in "cross-wind", i.e. hit perpendicular to its axis (the z-axis) by a steady stream of an incompressible fluid:
 Far in front of the object, the velocity is uniform and in the x direction,



$v_y(-\infty, y) = 0$ and $v_x(-\infty, y) = V_\infty$. Far behind the object, a self-similar wake develops and we write the x component of the velocity as

$$v_x(x, y) = V_\infty - v_d(x, y) \tag{2}$$

where v_d is called the *velocity deficit*. We ignore any z-dependence because we assume the object to be very slender. We will focus on the region far behind the object where the pressure is simply a constant p_0 and where $v_d \ll V_\infty$ and $v_y \ll V_\infty$.

(a) Insert (2) in the Prandtl boundary layer equations for laminar flow with constant density and pressure. Show that by throwing away small terms, they can in our case be approximated by

$$\rho_0 V_\infty \frac{\partial v_d}{\partial x} = \mu \frac{\partial^2 v_d}{\partial y^2} \tag{3}$$

$$\frac{\partial v_y}{\partial y} = \frac{\partial v_d}{\partial x} \tag{4}$$

where ρ_0 is the fluid's density and μ its dynamic viscosity.

(b) Check that

$$v_d(x, y) = C x^{-\frac{1}{2}} \exp\left(-\frac{\rho_0 V_\infty y^2}{4\mu x}\right) \tag{5}$$

is a (self-similar) solution for v_d.

Exercise problem 5.1:
Steady laminar flow between parallel plates

(a) Read KCD section 9.2 (p. 413 - 415) [5th edition: 8.2, p. 312-314]

(b) Derive the result equation (9.5) [(8.5)]

(c) Discuss Figure 9.4 [8.4]

Exercise problem 5.2:
Steady laminar flow between concentric rotating cylinders

(a) Read KCD section 9.2 (p. 416 - 418) [8.2, p. 316-318]

(b) Derive the result equation (9.10) [(8.10)]

(c) Discuss the two limiting cases (9.11) and (9.12) [(8.11) and (8.12)]