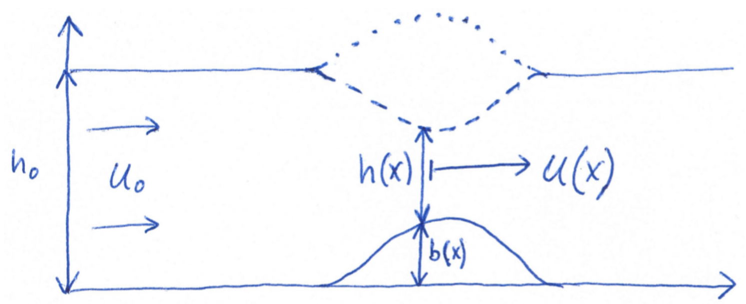
Fluid Dynamics + Turbulence (fall 2018)
Homework Problems II

Posted: September 6, 2018

Deadline: September 18 (Tuesday) at 08:30 am (on Blackboard).

Homework problem 2: Surface bump or dip in a shallow river flow?

Consider an ideal two-dimensional shallow river flow over a small bump at the river bed; see sketch.



(a) Explain the two conservation laws:

$$u_0 h_0 = u(x) h(x) \, , \tag{1}$$

$$\frac{u_0^2}{2} + g h_0 = \frac{u^2(x)}{2} + g \left(b(x) + h(x) \right) \, . \tag{2}$$

(b) Equations (1) and (2) are two equations for the two unknown functions $h(x)$ and $u(x)$. Eliminate $u(x)$ and derive an equation only for $h(x)$. Show that this equation is consistently solved by $h(x) = h_0$ when $b(x) = 0$. In order to solve the equation also for $b(x) > 0$, assume the bump $b(x) \ll h_0$ and the change of river surface height $\Delta h(x) = h(x) - h_0 \ll h_0$ to be small. Derive the solution

$$\Delta h(x) = \frac{b(x)}{\left(\frac{u_0^2}{g h_0} - 1 \right)} \tag{3}$$

by keeping first-order terms in small $b(x)/h_0$ and $\Delta h(x)/h_0$ in a consistent way.

(c) Use equation (3) in the two limits $u_0^2 \ll gh_0$ and $u_0^2 \gg gh_0$ to explain when a surface bump or dip occurs.

(d) What is the catch with equation (3) when $u_0^2 \approx gh_0$?

(e) Use the numbers $b_{\text{max}} = 0.1\,\text{m}$, $h_0 = 1\,\text{m}$, $g = 9.81\,\text{m/s}^2$ to calculate the extremum of $\Delta h(x)$ for $u_0 = 1\,\text{m/s}$ and $u_0 = 10\,\text{m/s}$.