Note for Fluid Dynamics Midterm Exam Project I

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Remark: This is an auxiliary note which you are free to use or not for the midterm exam project I.

1 Coordinate rotation

The biggest challenge in this course project is to extend the wind farm modeling from one dimension (as you implemented in the first week's homework) to two dimensions. There are two ways to handle changing wind direction. One approach is to extend all the formulas you have already implemented to include the wind direction. Another approach, which we recommend, is to rotate the coordinates of the wind farm so that the x-axis is aligned with the wind direction. This makes all calculations simpler. For any given wind direction you first rotate the farm coordinates before calculating velocity deficits due to overlapping wakes. The rotation is calculated by applying a standard rotation matrix to the coordinates:

$$\begin{pmatrix} x_r \\ y_r \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

where (x, y) are the original coordinates of the turbines, and (x_r, y_r) are the corresponding rotated coordinates. When you change the wind direction the rotation should always be applied to the original coordinates and not previously rotated coordinates.

The formulas below assume you are rotating the wind farm coordinates.

2 Overlapping area

We denote the radius of rotor as r_2, and the radius of wake at turbine we are looking at as r_1. If the distance d between the center of turbine and wake is larger than $(r_1 + r_2)$, i.e.,

$$d \geq r_1 + r_2 \tag{2}$$

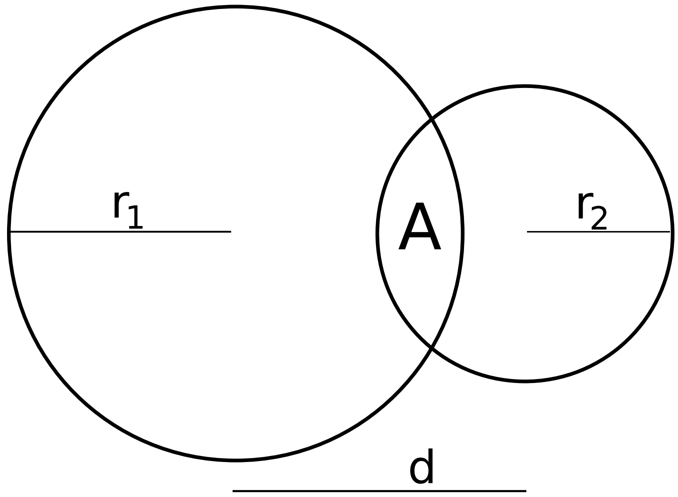

Figure 1: Overlapping area A of down wind turbine rotor disc with radius r_2 and a wake at the down wind turbine with radius r_1 caused by an upwind turbine. The distance between the centers of the two circles is denoted d.

then we end up no overlap between the turbine and wake. When $d < r_1 + r_2$ there begins overlapping, but we need to distinguish between two different cases. If d satisfies the following equation,

$$r_1 - r_2 < d < r_1 + r_2 \tag{3}$$

we have the partially overlap case, where is overlapping area can be found as

$$area = r_1^2 \cos^{-1}\left(\frac{r_1^2 - r_2^2 + d^2}{2dr_1}\right) + r_2^2 \cos^{-1}\left(\frac{r_2^2 - r_1^2 + d^2}{2dr_2}\right) - \frac{1}{2}\sqrt{T} \tag{4}$$

where T can be calculated as

$$T = \left((r_1 + r_2)^2 - d^2\right)\left(d^2 - (r_1 - r_2)^2\right) \tag{5}$$

The last case is fairly straightforward, where the turbine is fully covered by the wake, i.e.,

$$\begin{aligned} d &\leq r_1 - r_2 \\ area &= \pi r_2^2 \end{aligned} \tag{6}$$

3 Wake deflection due to yaw

In part b of the exam project you have to include wake deflection due to rotation and yaw. These effects should be excluded in part a. In this case the requirement for overlapping is now defined as

$$d = r_1 + r_2 + \delta_{\text{rot}} + \delta_{\text{yaw}} \tag{7}$$

where δ_{rot} is the wake deflection caused by the rotating blades and δ_{yaw} is the wake deflection due to the yaw angle of the upwind turbine. The expressions for the two deltas are found in the master's thesis of Emil Thøgersen, the wake conference paper, and the report uploaded together with the Nysted wind farm coordinates.