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Time-Series Modeling for Statistical Process Control

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In statistical process control, a state of statistical control is identified with a process generating independent and identically distributed random variables. It is often difficult in practice to attain a state of statistical control in this strict sense; autocorrelations and other systematic time-series effects are often substantial. In the face of these effects, standard control-chart procedures can be seriously misleading. We propose and illustrate statistical modeling and fitting of time-series effects and the application of standard control-chart procedures to the residuals from these fits. The fitted values can be plotted separately to show estimates of the systematic effects.

KEY WORDS: Shewhart charts; CUSUM charts; EWMA charts; ARIMA models; Special cause; Common cause.

1. INTRODUCTION

In standard applications of statistical process control, a state of statistical control is identified with a random process—that is, a process generating independent and identically distributed (iid) random variables. Once a state of statistical control is attained, departures from statistical control may occur. These departures typically are reflected in extreme individual observations (outliers) or aberrant sequences of observations (runs above and below a level or runs up and down).

Departures from a state of statistical control are discovered by plotting and viewing data on a variety of control charts, such as Shewhart, cumulative sum (CUSUM), exponentially weighted moving average (EWMA), and moving-average charts. Having found departures, we hope to find explanations for them in terms of assignable or special causes. ["Assignable cause" is a term introduced by Shewhart (1931); "special cause" is an alternative term suggested by Deming (1982).] We then hope to move from "out of control" to "in control" by correcting or removing the special causes.

In practice, however, it may be difficult either to recognize a state of statistical control or to identify departures from one because *systematic nonrandom patterns*—reflecting common causes—may be present throughout the data. [The term "common cause" was suggested by Deming (1982).] When systematic nonrandom patterns are present, casual inspection makes it hard to separate special causes and common causes.

A natural solution to this difficulty is to model systematic nonrandom patterns by time-series models that go beyond the simple benchmark of iid random variables. One possibility, for example, is a first-order autoregressive model, in which each observation may be regarded as having arisen from a regression model for which the current observation on the process is the dependent variable and the previous observation is the independent variable.

If such a time-series model fits the data, leaving only residuals that are consistent with randomness, it is futile to search for departures from statistical control and their corresponding special causes. The practical emphasis would then shift to trying to gain better general understanding of the process. Process improvement would be sought by identification and understanding of the common causes making for autocorrelated behavior. Autocorrelated behavior means that there are carryover effects from earlier observations. The mechanism of these carryover effects must be sought.

Whether or not we achieve full understanding of the common causes underlying autoregressive behavior, fitting of the autoregressive model makes it possible, by study of its residuals, to isolate the departures from control that may be traceable to special causes. Otherwise, these departures are confounded with the dominant autoregressive behavior of the data.

Hence, when the data suggest lack of statistical control, one should attempt to model systematic nonrandom behavior by time-series models—autoregressive or other—*before* searching for special causes. In particular, we suggest using the autoregressive integrated moving average (ARIMA) models of Box and Jenkins (1976), identified and estimated by standard techniques, to supplement the iid model as the guiding paradigm in practical work. This approach leads to two basic charts rather than one:

1. *Common-Cause Chart (a chart of fitted values based on ARIMA models).* This chart provides guidance in seeking better understanding of the process and in achieving real-time process control.
2. *Special-Cause Chart [or chart of residuals (or one-step prediction errors) from fitted ARIMA models].* This

chart can be used in traditional ways to detect any special causes, without the danger of confounding special causes with common causes. The iid model guides the interpretation of this second chart; all traditional tools of process control are applicable to it.

This strategy applies not only to industrial processes that manufacture items sequentially but also to quality control and statistical studies generally in all areas of businesses and other organizations.

In the light of the widespread use of ARIMA models in other areas of statistical application, we find it surprising that the practice of statistical process control has not moved in the direction here proposed. Although there have been many applications of time-series concepts in process control, *the thrust of these applications has been directed toward testing for randomness, not toward modeling of departures from randomness.* In writings on quality control, we have been able to locate only one suggestion along the lines of this article: Montgomery (1985, p. 265) outlined the strategy of basing control charts on residuals from time-series modeling. In addition, a referee has pointed out an application in which standard control-charting procedures were used by Berthouex, Hunter, and Pallesen (1978) to study the residuals from a time-series model.

The use of time-series models requires more statistical skill than the use of traditional Shewhart charts, because one must know something about modeling nonrandom time series. But when the Shewhart charts are not fully pertinent to an application, there is little choice. Moreover, the level of skill needed to work with time-series models is less demanding than at first appears. First, ARIMA modeling has to some extent been automated by computer programs. Second, a few simple special cases of ARIMA models, such as the first-order integrated moving average process—ARIMA(0,1,1)—may serve as good approximations for many or even most practical applications. [The EWMA chart is based on ARIMA(0,1,1).]

In addition, we believe that greater knowledge of time-series methods would lead to more skilled interpretation of control-chart data, even in the absence of formal time-series modeling.

The plan of the article is as follows: In Section 2, we discuss Shewhart's definition of statistical control and sketch its practical implementation. In Section 3, we explore limitations of the traditional implementation of process control. In Section 4, we outline our suggested extension of the traditional implementation. In Section 5, we fill in details of our proposals, offer an illustrative application, and briefly discuss typical applications. In Section 6, we consider possible ways of easing the demands for expertise in time-series analysis required by our proposals. In Section 7, we outline various procedures required for full exploitation of our proposals, particularly for the interpretation of what lies behind systematic variation through time, and what can be done to deal intelligently with this variation.

2. SHEWHART'S DEFINITION OF A STATE OF STATISTICAL CONTROL

Since the classic pioneering work of Shewhart (1931), the concept of a state of statistical control has been central to prediction and control of industrial and other processes. Shewhart defined a state of control as follows: "a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within the given limits" (p. 6).

As implemented by Shewhart and his colleagues and successors, this definition of a state of statistical control has been specialized from general probabilistic prediction to prediction based on iid or simple random behavior. Further, in many applications the conditional distribution of individual quality measures, or of statistics computed from subsamples of these measures, can be approximated by familiar distributions such as the normal, binomial, and Poisson. Shewhart control charts based on these distributions became, and have remained, simple and powerful tools.

For example, a process yielding a quantitative quality measure can be regarded as in a state of statistical control if means of subsamples of 5 drawn at fixed time intervals behave as iid normal variables and if ranges of the same subsamples behave as iid variables following the distribution of ranges in samples of 5 from a normal distribution. In modern terminology, investigation of whether or not a process is in a state of statistical control means "diagnostic checking" of data from the process to see if the observed behavior is compatible with these specifications about the process. A Shewhart chart for variables facilitates this diagnostic checking, as do other common control charts.

If the verdict of checking is that the process is in a state of statistical control, there is need for surveillance to assure that the state of control continues or to sound an alert if it does not. If an alert is sounded, investigation and, possibly, corrective action are called for. One great advantage of the Shewhart chart is that the investigation for special causes is facilitated if undertaken soon after the alert is sounded.

Checking for a state of statistical control is usually regarded as a test of a null hypothesis. An out-of-control state is any hypothesis alternative to this null hypothesis. Procedures for practical process control draw heavily on hypothesis-testing procedures suggested by statistical theory. For example, an inference of departure from control often is attached to a single subsample mean outside "three-sigma control limits" on a Shewhart control chart for means ($\bar{X}$-bar chart)—roughly the .003 level of significance.

This particular test is reasonable if one believes that an important alternative hypothesis to the hypothesis of statistical control is the possibility that sudden and substantial shocks may impinge on the "constant cause" system underlying a process that has previously been in control. Against such an alternative hypothesis, the three-sigma control limits provide sufficient power to detect major shocks without sounding frequent false alarms in the absence of such shocks.

Shewhart applied the expression "assignable cause" to sources of sudden and substantial shocks, but he envisaged other kinds of assignable causes. For example, for economic data he mentioned "such things as trends, cycles, and seasonals" (p. 146). Most of the extensions of Shewhart's control-chart procedures can be regarded as tests that are sensitive to particular departures from iid behavior. For example, there are criteria based on counts of runs above and below a given level, runs up and down, and runs of points within the three-sigma limits but more than, say, two standard deviations from the mean. Other tests are based on the CUSUM chart originally introduced by Page (1954). There are also control charts based on moving averages EWMA's. For discussion of the various approaches to control charts see, for example, Montgomery (1985) or Wadsworth, Stephens, and Godfrey (1986).

3. LIMITATIONS OF THE TRADITIONAL IMPLEMENTATION OF PROCESS CONTROL

Underlying standard control-chart procedures, there is a view of reality that envisages just two possibilities, a state of statistical control versus everything else. "A state of statistical control" is a sharp concept; "everything else" is not, although in particular applications it is sometimes made concrete in terms of specific alternatives against which one hopes tests to be powerful, such as sudden and substantial shocks or gradual trends.

Although this dichotomy of statistical control versus everything else often serves well in practice, it is not required by Shewhart's basic idea of a state of control, which requires only that "we can predict, at least within limits, how the phenomenon may be expected to vary in the future" (p. 6), and it may needlessly limit our perspective in applied work:

1. If one looks closely, "everything else" is likely to be the rule rather than the exception. A state of statistical control is often hard to attain; indeed, in many applications it appears that this state is never achieved, except possibly as a very crude approximation. (A reader who doubts this will find it interesting—as one of us has done—to try to achieve a state of statistical control for body weight.) Our examination of published and unpublished data from actual applications suggests that distinctly nonrandom behavior will often be found in claimed examples of statistical control. In particular, substantial autoregressive behavior is very common.

An interesting discussion of this point was provided by Eisenhart (1963, p. 167), who commented: "Experience shows that in the case of measurement processes the ideal of strict statistical control that Shewhart prescribes is usually very difficult to attain, just as in the case of industrial production processes. Indeed, many measurement processes simply do not and, it would seem, cannot be made to conform to this ideal . . . " (p. 167). Eisenhart went on to cite an earlier comment of Student along the same lines.

2. An out-of-control process can be nonetheless a predictable process, one that is not affected by special causes. The failure to realize this, however, may lead people to view such a process as a sequence of isolated episodes, each with its own special cause and associated hint as to appropriate intervention. Preoccupation with nonexistent special causes diverts attention from common causes. Thus, if the data autocorrelated, one may fail to look for factors that make for lagged effects.

Consider an application that we have examined recently, one that is far removed from industrial practice. The time-series behavior of monthly closing of the Dow–Jones Industrial Index from August 1968 through March 1986 is far from random. Technical analysts of the stock market often suggest special causes for variations of this index:—for example, "profit taking," "concern about the budget deficit," or "lowering of the discount rate."

Yet the first differences of logs of the Dow–Jones Index are nearly iid normal to a closer approximation than we have typically seen in industrial data from processes alleged to be in control. For the stock market, then, a very simple application of time-series modeling suggests that there were no special causes for market behavior in the sense of sudden shocks that are incompatible with a state of statistical control. Rather, we are observing a simple time-series model, namely a *random walk* on the logs of the Dow–Jones Index.

3. Emphasis on, say, a normal iid process as the model for a state of statistical control tends to encourage the development of a complex set of decision or testing procedures for detecting out-of-control situations. These rules may be based on combined consideration of individual extreme points, checks of runs above and below a given point and runs up and down, CUSUM charts, and so forth. In testing language, we have multiple tests, so the correct risks of Type I error must be calculated by some kind of compounding of probabilities, and this calculation, if possible at all, is very difficult.

The focus on normal iid random variables also tends to lead people to place undue emphasis on normality, since control limits are sometimes calculated without detailed scrutiny of the sequence plots of individual observations or even of control charts on means. Moreover, approximate normality of a histogram is often erroneously assumed to imply a state of statistical control for the process. For an example, see Deming (1982, p. 114).

4. SUGGESTED EXTENSION OF TRADITIONAL PROCESS CONTROL

To call attention to common causes when they are present, we suggest an alternative to the dichotomy of "a state of statistical control" versus "out of control." The alternative is based on the familiar decomposition of regression analysis: Actual = Fitted + Residual. (When the regression fit is used for forecasting, the parallel dichotomy is Actual = Predicted + Error.)

Our experience suggests that in a wide range of applications in which processes are not in control in the sense of iid random variables, one can use relatively elementary regression techniques to identify and fit appropriate time-series models. If we succeed in finding such a model, we have reached a negative verdict about statistical control in the sense of iid and we obtain fitted values and residuals along with probabilistic assessments of uncertainty. We can regard the process as "in control in a broader sense," a sense that is entirely consistent with Shewhart's conception of a state of statistical control.

Since successful time-series modeling decomposes the actual series into fitted values and residuals, the traditional purposes of process control can be served by the two basic charts mentioned in Section 1:

1. *A Time-Series Plot of the Fitted Values* (*without computation of control limits*). This plot can be regarded as a series of point estimates of the conditional mean of a process—our best current guess based on past data of the location of the underlying process.
2. *Standard Control Charts* (*Shewhart, CUSUM, or other*) *for the Residuals*. Control limits are based on the time-series model itself; for example, limits for prediction errors would be based on the standard errors of one-step-ahead forecasts.

This two-step approach appears to be an obvious union of time-series modeling with traditional ideas of process control, but the possibility of the union appears not to have been widely exploited. The closest approaches that we have seen are those of Hoadley (1981) and Hunter (1986). So far as we can tell, other applications of time-series concepts to process control have been oriented toward more sophisticated testing of the traditional null hypothesis of iid behavior. The power of proposed tests may be assessed against specific time-series alternatives, but these alternatives are not explicitly modeled.

In the regression decomposition here proposed, each fitted value is conditioned only on past data rather than on all of the data, as in signal-extraction theory. We propose the regression decomposition because it articulates well with current statistical practice and can be implemented easily with standard computing packages.

5. BOX–JENKINS MODELING FOR PROCESS CONTROL

A useful set of tools for implementing these decompositions is provided by the modern principles of time-series modeling, illustrated, for example, by another classic pioneering work, that of Box and Jenkins (1976). They stated their goals as follows:

> In this book we describe a statistical approach to forecasting time series and to the design of feed forward and feedback control schemes. . . . The control techniques discussed are closer to those of the control engineer than the standard quality control procedures developed by statisticians. This does not mean we believe that the traditional quality control chart is unimportant but rather that it performs a different function from that with which we are here concerned. An important function of standard control charts is to supply a continuous screening mechanism for detecting assignable causes of variation. Appropriate display of plant data ensures that changes that occur are quickly brought to the attention of those responsible for running the process. Knowing the answer to the question, "*when* did a change of this particular kind occur?" we can then ask "*why* did it occur?" Hence, a continuous incentive for process improvement, often leading to new thinking about the process, can be achieved.
>
> In contrast, the control schemes we discuss in this book are appropriate for the periodic, optimal adjustment of a manipulated variable, whose effect on some output quality characteristic is already known, so as to minimize the variation of that quality characteristic about some target value.
>
> The reason control is necessary is that there are inherent *disturbances* or *noise* in the process. . . . (p. 4)

Our view is that Box–Jenkins time-series methods—and their many extensions—are applicable both to the control schemes mentioned in this quotation and to the traditional process-control objective of continuous process surveillance to detect special causes of variation. We believe that the basic philosophy of diagnostic checking espoused by Box and Jenkins leads almost inevitably to the approach to process surveillance outlined in this article.

To illustrate time-series modeling, we shall use a special case of Box–Jenkins "ARIMA" models known in statistical quality control as the EWMA, explained by Hunter (1986). In Hunter's use of EWMA, it was a procedure for testing a state of statistical control, although he did emphasize the predictive quality of the EWMA and the need for prediction for control and went on to relate the EWMA to the control engineer's modeling approach. In our approach, the EWMA emerges as a flexible time-series model that, for many but not all processes, may be a satisfactory approximation.

One appealing interpretation of EWMA is that the process being studied can be decomposed into two components:

1. iid random disturbances, with mean 0
2. A *random walk,* which is the sum of a fraction a of all past iid random disturbances.

To bring out our main point, we shall consider an application in which individual observations rather than subsample means are used. We consider Series A of Box and Jenkins (1976), 197 concentration readings on a chemical process, single readings taken every two hours. (We have subtracted a constant 10 from the actual readings.) The following computer output was produced by Minitab.

First, we show in Figure 1 the sequence plot of the series itself, called *Y*. From visual examination alone, we see that the series is obviously out of control, with strong evidence of positively autocorrelated behavior (see also Box and Jenkins 1976, pp. 178–187). The sample mean is 7.06 and the sample standard deviation is .40, so conventional three-sigma control limits would extend from 5.86 to 8.26, limits that are shown in Figure 1 only for illustration. All points are within these limits (the maximum is 8.2 and the minimum is 6.1). (If the data are viewed without regard to time sequence, they conform very closely to the normal model, providing another illustration of nonrandom but approximately normal data.)

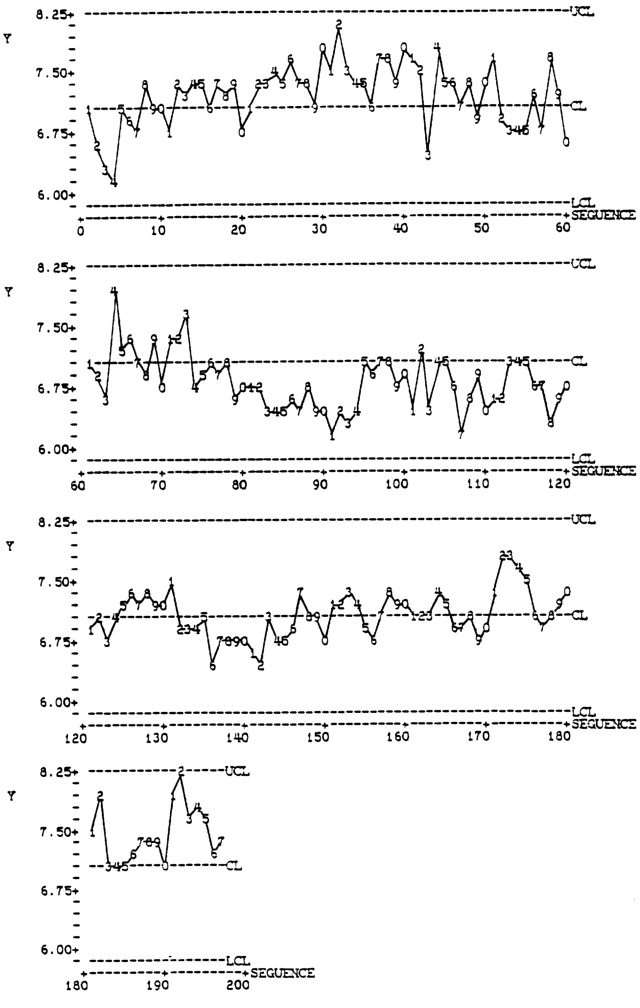

Figure 1. Sequence Plot of Series A, Box and Jenkins (1976): 197 Concentration Readings on a Chemical Process, Single Readings Taken Every Two Hours. Data are coded by subtraction of a constant 10 from each observation. CL is the center line at a height equal to the sample mean, and UCL and LCL show the location of conventional three-sigma control limits.

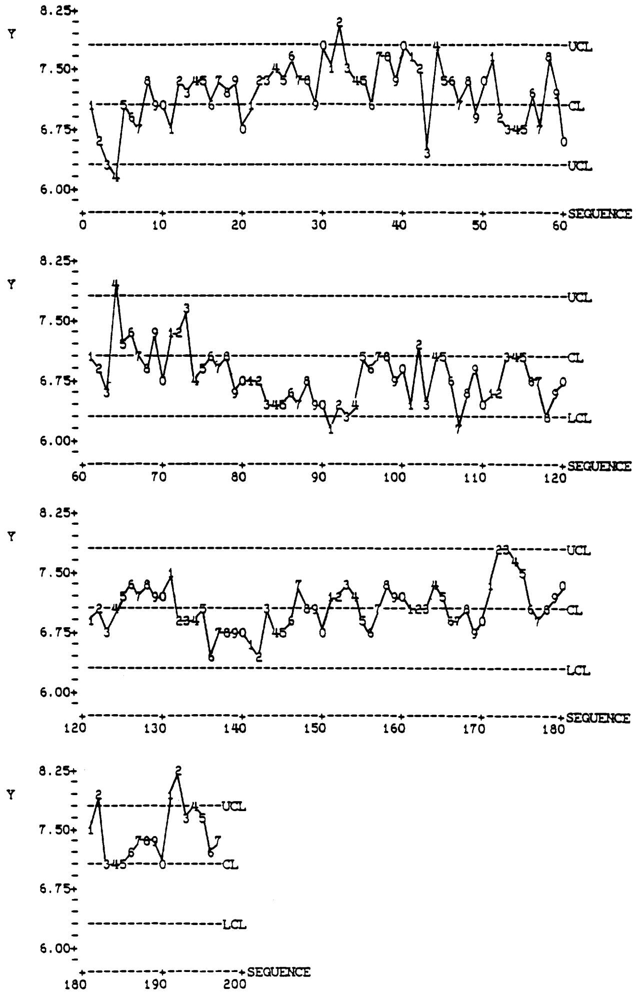

Figure 2. Sequence Plot of the Same Data as in Figure 1. UCL and LCL are limits computed from the mean of moving ranges of successive observations.

It is seen not only that the data are positively autocorrelated, but that it is not even obvious that the data should be regarded as coming from a stationary process. Even if the process were deemed stationary, however, three-sigma limits should logically be based on the *marginal* rather than the *conditional* standard deviation. Any concern with control limits, however, diverts attention from the basic observation that *the conditional mean is changing constantly*. These changes of the conditional mean are essential to understanding and control of the process.

A common approach for dealing with nonrandom data like these is to base control limits on the mean of moving ranges of successive observations (Wadsworth et al. 1986). We see in Figure 2 that these limits are much tighter and show many points out of control, occurring mainly at the peaks and troughs of the waves of the data.

These control limits based on moving ranges alert the user that the process is not close to being random, but they provide no real guidance in understanding what is happening.

For many users, data like these suggest a series of loosely connected episodes, each inviting ad hoc explanations. (This parallels the after-the-fact "explanations" offered by stock-market analysts to "explain" changes of the Dow–Jones Industrial Index!) The process of achieving a state of statistical control is sometimes pictured as finding special explanations for each episode, making a correction, observing the process further, finding further special explanations for further episodes, and so forth, until control is finally attained.

Here, however, as in the example of the Dow–Jones Industrial Index, simple time-series modeling unifies the picture immediately.

Box and Jenkins fitted two models, each of which describes the data about equally well. We consider one of these models (the model underlying EWMA), namely a first-order integrated moving average, called "ARIMA(0,1,1)," which, as pointed out previously, can be interpreted as a random-walk trend plus a random deviation from trend. This model specifies that the observed Y_t is the sum of an unobserved random shock A_t plus a (proper) fraction a of the sum of all past random shocks $A_{t-1}, A_{t-2}, \ldots$. In the current application we have assumed that the mean of changes of Y_t is 0. As estimated by the Minitab ARIMA procedure, we have

$$\text{Fitted } Y_t = Y_{t-1} - .705A_{t-1}.$$

The quantity .705 is an estimate of what is called θ by Box and Jenkins; $1 - .705 = .295$ is an estimate of what is often called a in discussions of the EWMA. The standard error of the estimated θ is .05, and the standard deviation of residuals is .318. (We have suppressed the constant term, for two reasons: We have judged such a term unlikely a priori, and the sample evidence suggests an estimated constant near 0.)

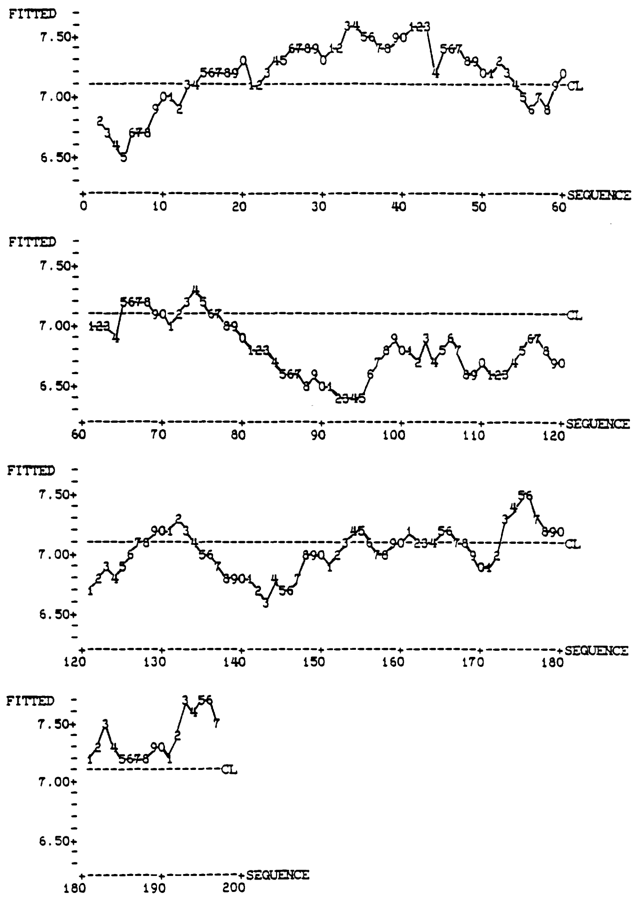

Figure 3. Common-Cause Chart: Sequence Plot of Fitted Values for the Data of Figures 1 and 2. Estimates are based on the ARIMA(0,1,1) model with 0 constant.

5.1 Common-Cause Chart: Fitted Values

It is useful to display the fitted values or estimated conditional means of the process, called "FITTED" in Figure 3, on a time-sequence plot. This plot gives a view of the level of the process (estimated conditional mean) and of the evolution of that level through time. We see the systematic behavior of the process that pervades the entire period of observation. This behavior may aid in real-time control or in better understanding how the process is working.

The model just fitted can be interpreted as follows: Each observation can be thought of as a random disturbance plus a random-walk trend or drift that reflects a certain fraction of the sum of all past random disturbances. Thus a part of each disturbance continues to affect the process into the indefinite future. The fitted values in Figure 3—FITTED—are estimates of the underlying random-walk trend; they follow a random walk without drift. The chart of FITTED represents the first of the two charts proposed by us. Each point is an estimate of the local level of the process itself, as distinguished from the observed readings.

To illustrate the type of control decisions that could be based on this plot, suppose that the most desirable level of the process is 7.0, and that increasing deviations from that level entail increasing economic loss from less-than-optimal product. Suppose further that at a certain known cost it is possible at any time to recenter the process 7.0. Then one can make an economic calculation to balance the expected loss of bad product over some specified period of time against the cost of recentering. This calculation will define action limits both below and above 7.0 at which the process should be recentered.

Note that these action limits are conceptually different from traditional control limits. They are not signals that it is time to look for special causes; rather, they are signals that a specific corrective action is needed.

5.2 Special-Cause Chart: Residuals

The second graph is essentially a standard control chart for residuals. To facilitate visualization of control limits, we plot in Figure 4 the standardized residuals

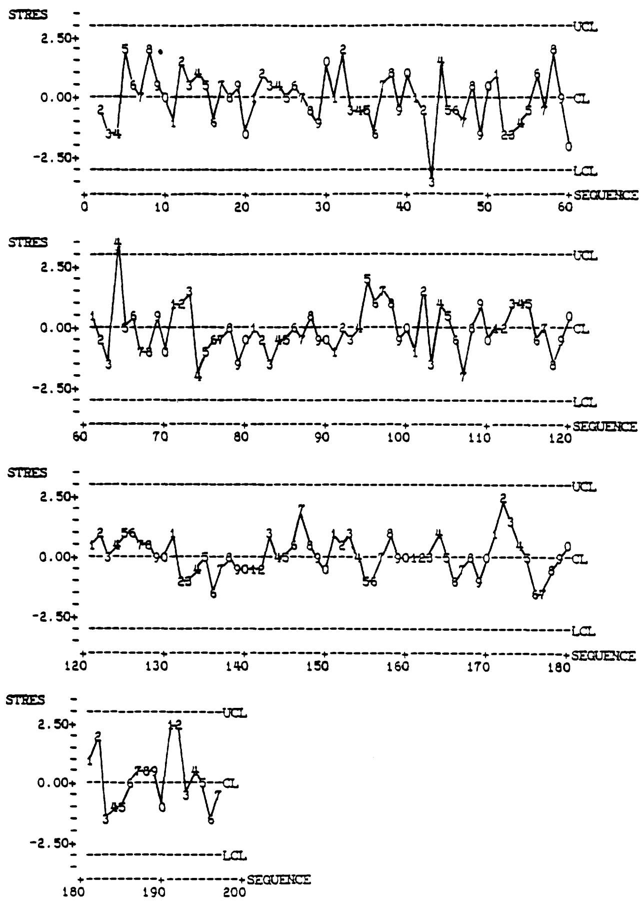

Figure 4. Special-Cause Chart: Sequence Plot of Residuals for the Data of Figures 1 and 2. Estimates are based on the ARIMA(0,1,1) model with 0 constant.

(STRES) with approximate center line and upper and lower three-sigma limits.

It is seen that two individual points—observations 43 and 64—breach the three-sigma limits. By contrast, the first set of control limits calculated previously suggested no points out of control, but the second set, based on mean ranges, suggested many individual points out of control. These out-of-control points did include observation 64, which was at the top of a wave, but not observation 43.

In addition, the preceding plot reveals perhaps two or three short intervals for which run counts would suggest some suspicion of lack of control. Note that run counts are applied only after the residuals from the time-series model have been isolated. As applied to the original series, run counts would suggest that the process was nearly continually out of control, sometimes on the high side and sometimes on the low side.

5.3 Other Simple Models

The ARIMA(0,1,1) model fitted previously is not the only reasonable model for the data. Box and Jenkins also fit ARIMA(1,0,1) with a nonzero constant. The two models give nearly identical fitted values and residuals and hence about the same degree of overall fit. For many purposes of control, whether real-time control or process surveillance, either would be suitable.

From a theoretical point of view, the two models have very different implications for the long-run behavior of the process, assuming continuation of the basic conditions now being observed. ARIMA(0,1,1) is nonstationary, but ARIMA(1,0,1) is stationary. Given the ARIMA(0,1,1) fitted previously, there is no tendency of the process to revert to its mean level as it wanders away. Given the ARIMA(1,0,1) for the same process, there is a relatively weak tendency to mean reversion. Moreover, prospective control limits as we look farther into the future are constant for ARIMA(1,0,1) but ever widening for ARIMA(0,1,1).

At one time we contemplated the concept of a *stationary* process as a possible extension of the concept of iid in defining a state of statistical control. The present example illustrates, however, that data may not permit a sharp distinction between stationary and nonstationary. Hence we propose only that *some* time-series model be used to define statistical control in an extended sense. Even a nonstationary model permits probabilistic prediction.

Of course, if the weight of statistical evidence and background knowledge suggests that the process should be regarded as nonstationary, the problem confronting management is inherently more difficult, since a nonstationary process has no tendency toward mean reversion. For example, a stationary ARIMA model about, say, a linear trend suggests disaster in the absence of intervention. At the same time, the chart of residuals could suggest that special causes are occurring along the road to ruin of the process itself.

5.4 Applications in Business and Economics

We have used a "public" data set to illustrate the mechanics of our basic proposals in an application that will be familiar to many readers. Even here, however, it results in identification of apparent out-of-control points that are easy to miss given the generally satisfactory fit by ARIMA models.

We have had substantial experience in applications to business and economics, and these have confirmed the fruitfulness of time-series modeling. One interesting illustration is based on a report by BaRon (1978), who studied the number of air tourist arrivals in Israel by months for 20 years starting in January 1956. Except for an upward trend and seasonality, BaRon's data conveyed the same visual general impression as the series presented previously. Based on observation of the data and knowledge of the historical background, BaRon classified the 240 months into 26 segments of varying lengths; for example, segment 9 extended from February 1961 to July 1962. Within these segments, he used three subclassifications: "regular," "short monotonic," and "unusual." February–December 1961, for example, was identified as "unusual," and labeled "Eichman trial: disturbed growth," whereas January–July 1962 was "regular." BaRon also picked out nearly 40 "extreme" months.

In a discussion of BaRon's paper, Roberts (1978) reported that the series could be well fitted by a multiplicative seasonal model (0,1,1) by (0,1,1)12 on the logs, with a constant term. Only one residual turned up as an unmistakable outlier, and that was for the month of October 1973—the Yom Kippur War. The model itself is the so-called "airline" model used by Box and Jenkins (1976) to fit a monthly time series of international airline passenger travel from 1949 to 1960. This suggests that many common causes underlying BaRon's data were similar to those underlying foreign travel worldwide and that special causes were rare.

Our approach has also been used by several hundred students in the Executive Program and other programs of the Graduate School of Business at the University of Chicago who have applied it to data arising in various areas of organizations for which they worked, ranging from finance and marketing to production and research and development. These applications typically find systematic time-series variation, including autocorrelation, seasonality, and trend. Many of them also find at least one outlying residual, and most of these students are able to identify plausible special causes for these residuals.

Typical examples are a sudden decrease in mean of length of stay in a hospital, which was traced to the imposition of the diagnostically related groups program; an extreme negative bank stock return related to a public announcement of a write-off of nonperforming loans; a sharp decline in company sales of a capital-equipment manufacturer in the third quarter of 1980, subsequently traced to the lagged effects of the economic recession that had occurred earlier in the year; a time-of-day effect in the occurrence of computer crashes; and an extremely high reading of hardness of steel coils, traced to a scheduling error by which a billet of mixed steel was inadvertently processed into a coil.

One interesting economic illustration encountered by several students from a variety of perspectives was the great increase in volatility of interest rates starting in the fall of 1979, for which the special cause appears to be a change of policy by the Federal Reserve System, a switch in emphasis from targeting interest rates to targeting monetary aggregates.

6. TIME-SERIES MODELING IN PRACTICE

A practical limitation on the use of the approach advocated in this article is that implementation requires some skill in analysis of time series, whereas the implementation of the standard Shewhart procedures entails only the most elementary statistical knowledge. We believe that in many applications, the ability better to sort out special causes from common causes justifies the use of the more elaborate machinery required by our approach.

Although there can be no completely satisfactory substitute for statistical skills in time-series analysis, we believe that much can be accomplished by people with limited skills. For example, modern computational tools make possible relatively automated implementation of time-series modeling; automatic fitting of Box–Jenkins models has for some time been available in software of general purpose software packages like Minitab, Statgraphics, SCA, or Systat now available on the IBM PC and compatibles. [There are even programs that attempt to automate model identification, as well as fitting! See, e.g., Shumway (1986).]

It is reassuring to know that precise model identification may not be essential to effective process control; several alternative models may fit the past data, at least, about equally well. In particular, the two models mentioned in our example in Section 5—ARIMA(1,0,1) and ARIMA(0,1,1)—offer reasonably good fit for a wide range of applications. (As one referee has suggested, the EWMA model may be a good all-purpose starting point.)

If a process can be modeled, then the traditional objectives of quality control—or surveillance—can be better served. Moreover, a control mechanism can be designed, and if a principal disturbance to the process can also be modeled, we have the adaptive control situation described by Box and Jenkins (1976).

7. ENGINEERING AND ECONOMIC ISSUES

More difficult than model identification and fitting is the interpretation and exploitation of fitted values for real-time process control, especially when process recentering is costly, and for better understanding of underlying common causes that make the process behave as it does. Real-time process control has received considerable attention, for example, by Box and Jenkins (1970) and Box, Jenkins, and MacGregor (1974). Systematic study of common causes has attracted little attention. We hope, in future work, to examine questions like these in some detail.

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