# An Application of CUSUM Chart on Financial Trading

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***Abstract*—The applications of CUSUM quality control chart to financial markets is not new in literature. It has been shown that the filter trading rule is mathematically equivalent to the CUSUM quality control test as both are based on change point detection theory. Filter trading rule has been extensively studied in the field of testing the financial market efficiency. In this paper, we studied the filter trading rule under a model assumption of Markov switching model (MSM) which has become very popular in financial applications. From our studies, it is found that the filter trading rule may beat the buy-and-hold strategy when the two-regime MSM fit the asset returns well.**

***Index Terms*—CUSUM chart, filter trading rule, Markov switching model**

## I. Introduction

Let $x_t, t = 1, 2, ...$ be the observation series from a production line. [1]'s one-sided CUSUM chart used to control the up-shift of the observations $x_t$ has test statistics $S_t^+$ with $S_0^+ = 0$ and

$$S_t^+ = \max(S_{t-1}^+ + x_t - k, 0). \tag{1}$$

It is also called the up-sided CUSUM or $S_t^+$ CUSUM. $k$ is the reference value denoting the required standard quality of the observations. Let $\delta$ be the decision interval of this CUSUM chart, then it will send an up "out-of-control" alarm once $S_t^+ \geq \delta$. It tells that the mean path of observations has shifted up away from the standard quantity significantly and a check is needed. Similarly, the down-sided CUSUM chart has the test statistics $S_t^-$ with $S_0^- = 0$ and

$$S_t^- = \min(S_{t-1}^- + x_t - k, 0). \tag{2}$$

It is also called $S_t^-$ CUSUM. With decision interval $\delta$, it will send a down "out-of-control" alarm once $S_t^- \leq -\delta$. It tells that the mean path of observations has shifted down away from the standard quantity significantly and a check is needed. [1] has pointed out that this CUSUM test can be interpreted as a repeated Wald's sequential probability ratio tests (SPRT) which gives CUSUM test a theoretical basis and lead to its many applications in different areas. One of the many applications of CUSUM chart to financial markets is in technical trading. [2] showed that there is a mathematical equivalence between the filter trading rule and CUSUM test. The observations $x_t$, $t = 1, 2, \ldots$, in CUSUM quality chart are corresponding to the return series of a trading security in filter trading rule. When CUSUM chart sends out an up "out-of-control" warning, it is equivalent to the case where the filter trading rule generates a long signal. The intuition is that a long position should be suggested when an up-shift of the return series is detected. While when CUSUM chart sends out an down "out-of-control" warning, it is equivalent to the case where the filter trading rule generate a short signal. The decision interval $\delta$ in CUSUM chart is called filter size and the reference value $k$ is zero in filter trading rule. $k = 0$ in financial market means the filter trading rule monitors whether the drift of the return series is significantly larger than zero (bull market) or significantly less than zero (bear market).

In Alexander's earliest empirical work [3], it was reported that the filter trading rule yielded substantial profits significantly greater than those of the buy-and-hold policy. This wrongly led him to conclude that the market is inefficient. However, he later clarified that the price discontinuities, the dividend's effect and the transaction costs all introduced serious biases. Further studies by [4], [5], [6] and [7] also empirically showed that after taking transaction costs into account, the filter trading rule cannot outperform the buy-and-hold strategy, with the conclusion that the weak form of EMH cannot be rejected for their data. These findings form a basis for a wide range of research in the field of market efficiency. More recently, [8] [9] discussed some new filter rule tests. It is known that just like any other technical trading rule, the filter trading rule is not profitable when the asset price follows a random walk. In this paper, we study the profitability of filter trading rule under a two-regime Markov switching model. We first derive the analytical calculation of the filter trading profit under a regime switching model. We then fit a two-regime model to the real world data. With the estimated parameters, we choose the optimal filter size which gives the largest theoretical trading profit. Finally, we apply the filter trading rule with the chosen filter size to the real data trading. We discuss the profitability as well as the model fitting of the two-regime model in different index futures markets over the world. Based on our results, when the two-regime model fits reasonably well for the data, then the filter trading rule based on the fitted model could beat the buy-and-hold strategy.

978-1-4799-2548-3/13 $31.00 © 2013 IEEE
DOI 10.1109/CIS.2013.44

## II. Filter Trading Rule under A Two-regime Model

### A. Filter Trading Signals in Terms of Draw-up (Draw-down) of Asset Prices

Consider a financial asset whose closing price at $t$ is $p_t$ ($t = 0, 1, 2 \ldots$). Let $\log(p_t)$ be the log price of the asset and let $x_t = \log(p_t) - \log(p_{t-1})$ be the continuously compounding return when the asset is held from a time point $t-1$ to the time point $t$. For a long time, there has been a great interest in devising a mechanical trading rule which dictates when to hold the asset and when to sell short the asset so as to achieve profitable trading. According to [10], a mechanical trading rule consists of a time series of buy/sell signals $B_t$, $t = 0, 1, 2, \ldots$ where $B_0$ can start with a zero value. However, once $B_t$ takes on a nonzero value, it will only be equal to $+1$ or $-1$. When $B_t = +1(-1)$, an investor will automatically take a long (short) position from time point $t$ to time point $t+1$. In order to be a practical trading rule, a mechanical trading rule has to satisfy the property that $B_t$ is a function of $p_0, p_1, p_2, \ldots, p_t$. In filter trading, suppose $B_0 = +1$ which means we are holding a long position, then we monitor the $S_t^-$CUSUM value and when the price moves down at least $100\delta\%$ from a subsequent high, that is the $S_t^-$ CUSUM exceed $-\delta$ at time $t$, we then close the long position and go short, that is $B_1 = \cdots = B_{t-1} = +1$ and $B_{t+1} = -1$. Suppose $B_0 = -1$ which means we are holding a short position, then we monitor the $S_t^+$CUSUM value and a buy signal will be generated when the asset price moves up at least $100\delta\%$ from a subsequent low, that is the $S_t^+$CUSUM exceed $\delta$ at time $t$, we then close the short position and go long, that is $B_1 = \cdots = B_{t-1} = -1$ and $B_{t+1} = +1$.

### B. Two-regime Models in Financial Market

Many financial market practitioners believe that the market is sometimes bullish and sometimes bearish. Driven by this belief, a two-regime model consisting of a random walk with positive drift as well as a random walk with negative drift can be a good description of the market. To complete the asset return model, [11]'s approach using a hidden Markov chain to govern the transition between the bullish and bearish states can be adopted. Regime switching models are not new and can be dated back at least to [12], who investigated regime regression models. During its long history, one example of the famous work on the application of regime switching model is [11]. He introduces dynamic Markov switching models to deal with the endogenous structural breaks. After that, lots of empirical works and applications on regime switching structure were done in many fields such as [13] on business cycle, [14] on labor market recruitment, [15] on option pricing, and so on. Here we assume a regime switching structure for the security return process and based on that, we study the profit of the filter trading rule. Two-regime model means a Markov regime switching model with two regimes: regime I and regime II. Let us assume when the market is in regime I, the asset return follows a distribution $F_1$ (with density function $f_1$) with mean $\mu_1$ and standard deviation $\sigma_1$. When the market is in regime II, the asset return follows another distribution $F_2$ (with density function $f_2$) with mean $\mu_2$ and standard deviation $\sigma_2$. There exists a hidden Markov chain to control the market to switch between the two regimes. The possibility of staying at current regime or switching to the other are decided by the transition probability matrix (TPM) of the hidden Markov chain. The TPM is a two-by-two matrix and we define it as

$$\mathbf{M}_2 = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}. \tag{3}$$

Given the market is in regime I at time $t$, $p_{11}$ is the probability that the market will stay in regime I at time $t+1$ and $p_{12}$ is the probability that the market will switch to regime II at time $t+1$. Given the market is in regime II at time $t$, $p_{22}$ is the probability that the market will stay in regime II at time $t+1$ and $p_{21}$ is the probability that the market will switch to regime I at time $t+1$. This matrix has two constraints that the four probabilities should be all nonzero and $p_{11} \neq p_{21}$. The first constraint ensures that there is always some chance for the Markov chain to jump from one regime to the other. The second constraint ensures that $\mathbf{M}_2$ is invertible.

### C. Markov Chain State Space and Transition Probabilities for Long Term Filter Trading

Let the filter trading rule gives a trading signal $B_{t-1}$ at the time point $t-1$ and the asset return from $t-1$ to $t$ is $x_t$. Then the profit derived by the filter trading rule in the one-step period from time $t-1$ to $t$ is given by $G_t = B_{t-1}x_t$ and the expected unit time profit can be denoted by $\mathrm{E}(G_t)$. We are interested in computing $\mathrm{E}(G_t)$ when the system reaches an equilibrium as trading time tends to infinity. We propose to use a Markov Chain approach to compute the profit of the filter trading rule. Under a two-regime model, the market transits between a bearish state and a bullish state. When the filter trading rule is in operation, the trading position of the filter trading rule also transits between two positions, the long position and the short position. Jointly speaking, the system as a whole will have four states: long position under a bull market, long position under a bear market, short position under a bull market, short position under a bear market. Interestingly, when we incorporate also the $S_t^+$ or $S_t^-$ values of the filter trading rule, the whole system enjoys a bigger Markov structure on top of the two-regime Markov structure. To be more rigorous, the system under investigation follows a Markov process $M_t = \{B_t, D_t, I_t\}, t = 0, 1, 2, \ldots$ where each element is explained as follows.

$$\begin{cases} B_t = -1 & \text{when filter trading is taking a short position,} \\ B_t = +1 & \text{when filter trading is taking a long position.} \end{cases}$$

$$\begin{cases} D_t = \left|S_t^-\right| & \text{if } I_t = +1, \\ D_t = S_t^+ & \text{if } I_t = -1. \end{cases}$$

$$\begin{cases} I_t = \mathrm{I} & \text{if the market is in regime 1,} \\ I_t = \mathrm{II} & \text{if the market is in regime 2.} \end{cases}$$

Table I
STATES OF MARKOV CHAIN FOR LONG TERM FILTER TRADING

| | | | |
|---|---|---|---|
| $(S,0,\mathrm{I})$ | $(S,0,\mathrm{II})$ | $(L,0,\mathrm{I})$ | $(L,0,\mathrm{II})$ |
| $(S,1,\mathrm{I})$ | $(S,1,\mathrm{II})$ | $(L,1,\mathrm{I})$ | $(L,1,\mathrm{II})$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $(S,H-1,\mathrm{I})$ | $(S,H-1,\mathrm{II})$ | $(L,H-1,\mathrm{I})$ | $(L,H-1,\mathrm{II})$ |

Note that $D_t$ can be any real number from 0 to $\delta$, the filter size. In order to have finite number of states. We divide the interval $[0,\delta]$ into $H$ sub-intervals in which each interval has length $c=\delta/H$. When $D_t$ lies between $c(v-1)/H$ and $cv/H$ where $v$ is a positive integer, we collapse the states into a single state $D_t=v$. After collapsing, the Markov process becomes a Markov chain with $2\times H\times 2$ states and the transition probabilities from one state to the other can be computed as follows.

Without loss of generality, suppose we start with a state in short position with $D_t=v$. There are three cases depending on the magnitude of the asset return $x_t$. Case one: $x_t>(H-v)c$, the system will end up with $D_{t+1}=0$, $B_{t+1}=+1$ and the probability is $\Pr(x_t\geq(H-v)c)$ . Case two: $x_t\leq -cv+c$, the system will end up with $D_{t+1}=0$, $B_{t+1}=-1$ and the probability is $\Pr(x_t<-vc+c)$. Case three: $-vc+c\leq x_t<(H-v)c$, the system will end up with $B_{t+1}=-1$ and $D_{t+1}=w$ with $\Pr((w-v)c\leq x_t<(w-v+1)c)$ where $w$ is an integer between 0 and $H$. The above transition probability computation is based on approximation, however, the approximation will become better and better when $c$ tends to zero.

*D. The Expected Unit Time Profit of Long Term Filter Trading under A Two-regime Model*

To calculate the expected unit time profit of long term filter trading, we study the whole states space of the Markov chain built above. The Markov chain has altogether $4H$ states and the state space is $\{(B,D,I):B=\pm1;D=0,1,...,H-1;I=\mathrm{I}\text{ or }\mathrm{II}\}$. For clear presentation, we use $S$ (short) to denote $B=-1$ and $L$ (long) to denote $B=+1$ when quoting the states. The stationary probabilities for the $4H$ states can be solved by the Markov chain approach as long as we provide the transition probability matrix of the Markov chain. In Table I, we list all the states of the large Markov chain for long term filter trading.

The transition probability matrix of this Markov chain is a $4H$-dimensional matrix describing the transitions between each pair of the above states. The transition probabilities of regime from state $I_t$ to $I_{t+1}$ are given in $\mathbf{M}_2$ and under the definition of MSM model, the switching of regimes does not depend on the value of $(B_t,D_t)$. Therefore, the joint probabilities $\Pr((B_t,D_t,I_t)\to(B_{t+1},D_{t+1},I_{t+1}))$ are $\Pr(I_t\to I_{t+1})\Pr((B_t,D_t)\to(B_{t+1},D_{t+1})|I_{t+1})$. The transition probability of each pair of $(B_t,D_t)$ has been described in Section 2.3 in three different cases. It can be easy calculated when given the distribution of $x_t$. In our model setting, we assume the distribution functions of different regimes are known. That is, when $I_{t+1}$ is given, $F(x_{t+1})$ and $f(x_{t+1})$ are known. Therefore, the transition probability matrix of this large Markov chain can be calculated without difficulty. With the transition probability matrix, we can calculate its stationary distribution and denote it by $\pi$ as

$$\pi=\begin{pmatrix}\pi(S,0,\mathrm{I})\\ \pi(S,0,\mathrm{II})\\ \vdots\\ \pi(S,H-1,\mathrm{I})\\ \pi(S,H-1,\mathrm{II})\\ \vdots\\ \pi(L,0,\mathrm{I})\\ \pi(L,0,\mathrm{II})\\ \vdots\\ \pi(L,H-1,\mathrm{I})\\ \pi(L,H-1,\mathrm{II})\end{pmatrix}\tag{4}$$

When the long term filter trading is going on sufficiently long, the Markov chain will converge to the stationary distribution $\pi$ and we have the expected unit time log-return as

$$\begin{aligned}\mathrm{E}(\mathbf{G}_t) &= \left(\sum_{j=0}^{H_L-1}\pi_{(L,j,\mathrm{I})},\sum_{j=0}^{H_L-1}\pi_{(L,j,\mathrm{II})}\right)\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}\\ &\quad-\left(\sum_{j=0}^{H_S-1}\pi_{(S,j,\mathrm{I})},\sum_{j=0}^{H_S-1}\pi_{(S,j,\mathrm{II})}\right)\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}\end{aligned}\tag{5}$$

We define the theoretical optimal filter size as $\delta^*$ which gives the largest expected unit time profit $\mathrm{E}(\mathbf{G}_t)$. Although we can not write out the analytical formula for this optimal filter size, it can be obtained by tabulating or plotting the calculations of $\mathrm{E}(\mathbf{G}_t)$ for numbers of filter sizes and allocating the maximum point.

## III. EMPIRICAL STUDIES

In this section, we want to test whether the filter trading rule with the optimal filter size chosen in last section will be profitable or not under real world data. We first fit a two-regime model to the markets we want to study. We consider four different index futures markets over the world. They are (1) Standard&Poor Index (SPX) futures, (2) FTSE 100 Index (UKX) futures, (3) Nikkei 225 Index (NKY) futures (4) DAX Index (DAX) futures. We use a common period which is from July 1, 1998 to July 1, 2013 to conduct the empirical studies. The first step is to estimate the parameter $\theta$ of the two-regime model for each of the market where $\theta=\{\mu_1,\sigma_1,\mu_2,\sigma_2,p_{11},p_{22}\}$. The estimation results are presented in Table II. The t-values for $\mu_1$ and $\mu_2$ are the test statistics for testing significance of the bullish regime ( $\mu_1=0$ versus $\mu_1>0$ ) and bearish regime ($\mu_2=0$ versus $\mu_2<0$) respectively. The asterisks for the t-values signify the level of significance of the t-test. A single, double and triple asterisks correspond to 10%, 5% and 1% significance level

Table II
Markov Switching Model Estimation for Six Global Markets

| | | SPX | UKX | NKY | DAX |
|---|---|---|---|---|---|
| $\mu_1$ | Estimate | 0.0022 | 0.0018 | 0.0015 | 0.0033 |
| | SE | (0.0008) | (0.0008) | (0.0011) | (0.0011) |
| | t-value | [2.87]*** | [2.29]*** | [1.45]* | [2.82]*** |
| $\mu_2$ | Estimate | -0.0021 | -0.0039 | -0.0068 | -0.0058 |
| | SE | (0.0020) | (0.0026) | (0.0055) | (0.0034) |
| | t-value | [-1.05] | [-1.50]* | [-1.85]** | [-1.71]** |
| $\sigma_1$ | Estimate | 0.015 | 0.0162 | 0.0249 | 0.0228 |
| | SE | (0.0039 ) | (0.0043) | (0.0064) | (0.0058) |
| $\sigma_2$ | Estimate | 0.0350 | 0.0379 | 0.0506 | 0.0517 |
| | SE | (0.0095) | (0.0114) | (0.0193) | (0.0151) |
| $p_{11}$ | Estimate | 0.9748 | 0.9751 | 0.9601 | 0.9773 |
| $p_{22}$ | Estimate | 0.9635 | 0.9480 | 0.8528 | 0.9497 |

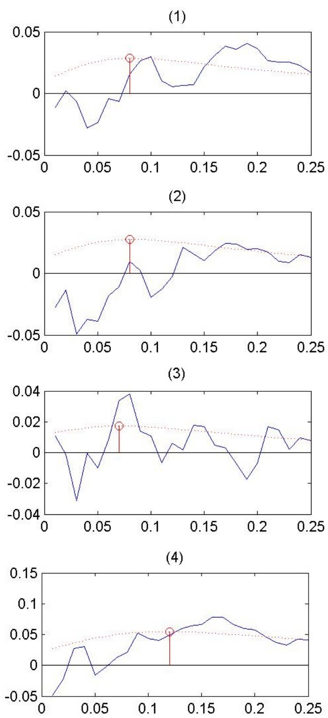


Figure 1. Filter Trading Profits with Different Filter Sizes in Six Markets

respectively. The t-values without asterisk mean they are not significant even in 10% significance level.

The theoretical optimal filter sizes are chosen and listed in Table III and also shown by a vertical bar in each graph of Figure 1. The x-axis of these graphs is the filter size and the y-axis is the annualized log-return of the filter trading. In each graph, solid curve gives the analytical annualized log-return while the dotted curved gives the log-return from trading on the real data. We find out the real trading returns at the chosen filter size in real markets and list them in Table III.

We observe that the performance of the filter trading rule with the theoretical optimal filter size are all profitable. The annualized log-returns of filter trading rule for UKX, NKY and DAX all outperform the buy-and-hold strategy. However,

Table III
The Performances of Filter Trading Rule and Buy-and-Hold Strategy

| | SPX | UKX | NKY | DAX |
|---|---|---|---|---|
| Theoretical Optimal Filter Size $\delta^*$ | 0.08 | 0.08 | 0.07 | 0.12 |
| Annualized Log-Return under $\delta^*$ | 0.0155 | 0.0098 | 0.0338 | 0.0491 |
| Annualized Buy-and-hold Return | 0.0212 | 0.0012 | -0.0163 | 0.0192 |

for SPX, it is slightly less than that of buy-and-hold strategy. From Table II, we found that values for SPX have very small t-statistics when testing whether the regime II is significantly bearish ($\mu_2 < 0$). In other word, the two-regime model with bull and bear markets specification may not be suitable for SPX data. This is consistent with other literature that the filter trading rule can not beat buy-and-hold strategy when the market follows one-regime random walk model.

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