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Control charts in financial applications: An overview

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Abstract

Statistical process control (SPC) methods have been widely popular not only in industrial applications but also in recent years in several nonindustrial scientific areas. The basic tools of SPC are control charts. SPC methods have been applied to various financial applications such as portfolio surveillance, stock trading, and interest rates. The purpose of this paper is to present the main applications of statistical monitoring process methods, specifically control charts, to the financial area. We focus on applications in stock markets such as technical trading rules, portfolio monitoring problems providing some directions for future research.

KEYWORDS

control charts, finance, portfolio monitoring, statistical process monitoring, stock markets

1 | INTRODUCTION

Statistical process control (SPC) has been used for many decades to monitor one or several quality characteristics of a process simultaneously Montgomery[1]. Control charts are one of the major tools of SPC. The main goal of a control chart is to monitor the underlying process, based on information observed from individual items or subgroups of items. The use of a control chart helps not only to monitor a process but also to improve its performance. The monitoring of several quality characteristics of a process simultaneously is called multivariate statistical process control (MSPC) or in case of one quality characteristic we have univariate SPC Montgomery[1], Woodall and Montgomery[2]. Bersimis et al[3] presented the basic procedures that use control charts for the implementation of MSPC.

A control chart statistic is computed from the quality characteristics that we observe and plotted against an upper control limit (UCL) and a lower control limit (LCL). If the control chart statistic exceeds the specified control limits then a signal is given that the process has been changed. Statistical processes are usually implemented in the following two phases: (1) Phase I, where control charts are used for retrospectively testing whether the process was in control when the first samples were being drawn. This phase includes the determination of the process being statistically in control. Also, the historical data of Phase I is used for estimating the parameters of the monitoring process. (2) Phase II, where control charts are used for monitoring the new observations of the process for any change from the in-control state (see, e.g., Bersimis et al[3] and Woodall[4]). In financial applications in contrast to industrial applications, the distinction between Phase I and Phase II control charts is difficult Golosnoy et al[5].

A basic component of a control chart procedure is the average run length (*ARL*), which is the average number of subgroups before a signal from the control chart is given in order to indicate that the process is out-of-control. The *ARL* is often used to compare the performance between control charts. Suppose that Z_t is the control chart statistic and h a control limit that determines the rejection area of the process. The run length, which is the number of observations before a signal is given, is denoted by:

$$N = \inf\{t \in \mathbb{N} : Z_t > h\}, \tag{1}$$

and the ARL is equal to $E(N)$. In the in-control state, the ARL (ARL_0) should be large but in the out-of-control-state it has to be small. Alternatively, someone can use the median run length (MRL), the median number of sample points before the first out-of-control signal is detected. Since the development of control charts by Shewhart,[6] various charts and procedures are being proposed and used in order to monitor processes. We remind that in the Shewhart procedure only the last observation is taken into consideration. Multivariate process control techniques were introduced by Hotelling in 1947.[7] Roberts[8] introduced the exponentially weighted moving average (EWMA) control chart. The EWMA control chart is a good alternative to the Shewhart control chart when we are interested in detecting small shifts, because these control charts are very effective against small process shifts Montgomery[1]. The control statistic in the univariate case is based on the EWMA defined as:

$$Z_t = (1 - \lambda)Z_{t-1} + \lambda X_t, \tag{2}$$

where $t = 1, 2, \dots, 0 < \lambda \leq 1$ is a smoothing constant parameter, and Z_0 is the target value with $Z_0 = E_0(X_t)$. The notation E_0 denotes the mean calculated when the process is in-control. The control chart gives a signal at time $t \geq 1$ if $Z_t \geq h$. The constant $h > 0$ is chosen such as ARL_0 to be equal to a certain value. Large values of the smoothing parameter λ give more weight to recent observations and small values give more weight to past observations. If $\lambda = 1$, then the EWMA chart reduces to the Shewhart chart.

Lowry et al[9] generalized the univariate EWMA control chart procedure to the multivariate case. Multivariate exponentially weighted moving average (MEWMA) control charts are constructed by applying a multivariate EWMA recursion directly to the components of the monitoring characteristic X_t. The advantage of this approach is that each characteristic element obtains its own smoothing factor and as a result allows more flexibility compared to the univariate EWMA Golosnoy and Schmid[10]. The MEWMA statistic is given by:

$$\mathbf{Z}_t = (\mathbf{I} - \mathbf{R})\mathbf{Z}_{t-1} + \mathbf{R}\mathbf{X}_t, \tag{3}$$

where $\mathbf{Z_0} = E_0(\mathbf{X_t})$, $\mathbf{I}$ is the $(k-1) \times (k-1)$ identity matrix, and $\mathbf{R} = \text{diag}(r_1, \dots, r_{k-1})$ is a diagonal matrix with elements $0 < r_i \leq 1$ for $i \in \{1, \dots, k-1\}$. A signal is given if:

$$\mathbf{Z}_t' \cdot \text{cov}_0(\mathbf{Z}_t)^{-1} \cdot \mathbf{Z}_t > h, \tag{4}$$

where the control limit $h > 0$ is chosen so as to achieve a specified ARL_0 and cov_0 is the covariance matrix when the process is in-control. The EWMA is used extensively in time-series modeling and forecasting and since it can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption.

The CUSUM chart was proposed by Page[11] for monitoring small shifts. The CUSUM control chart is used to monitor a process based on samples taken from the process at given time periods. The measurements of the samples at given times constitute a subgroup. The CUSUM chart shows the accumulated information of current and previous samples. CUSUM control charts are a good alternative when small shifts are important Montgomery[1]. CUSUM control charts can be constructed for individual observations or for groups of observations. Suppose μ_0 is the target of the process when the process is in-control for the quality characteristic X. The statistics C^+ and C^- are the one-sided upper and lower CUSUM limits, respectively. Defined by the iterative scheme:

$$C_t^+ = \max[0, C_{t-1}^+ + X_t - \mu_0 - k] \tag{5}$$

$$C_i^- = \max[0, C_{t-1}^+ + \mu_0 - k - X_t], \tag{6}$$

where k is the reference value or else the control chart constant parameter. A signal from the CUSUM scheme is given for upward shift if $C_t^+ > h$ and for downward shifts if $C_t^- < h$. The multivariate cumulative sum control chart (MCUSUM) is an extension of the univariate CUSUM control chart analysis. It is a procedure that uses the cumulative sum of deviations of each random vector previously observed compared to the nominal value to monitor the vector of means of a multivariate process Alves et al[12]. This chart was proposed by Crosier.[13] The scalar quantities of the univariate case are now replaced by vectors:

$$C_t = \sqrt{(\mathbf{S}_t + \mathbf{X}_t - \boldsymbol{\mu}_0)' \boldsymbol{\Sigma}^{-1} (\mathbf{S}_t + \mathbf{X}_t - \boldsymbol{\mu}_0)}, \tag{7}$$

where $\boldsymbol{\Sigma}$ is the variance matrix of the data and $\mathbf{S}_i$ are the cumulative sums defined as:

$$\mathbf{S}_i = \begin{cases} 0, & \text{if} \quad C_t \leq \kappa \\ (\mathbf{S}_{t-1} + \mathbf{X}_t - \boldsymbol{\mu}_0)\left(1 - \frac{k}{C_t}\right), & \text{if} \quad C_t > \kappa, \end{cases} \tag{8}$$

where the reference value $\kappa > 0$ is related to the magnitude of change and $S_0 = 0$. An out of control signal is given if $\mathbf{Z}_t > h$, with h being the control limit, $\mathbf{Z}_t = (\mathbf{S}_t' \boldsymbol{\Sigma}^{-1} \mathbf{S}_t)$. An alternative way to construct a vector accumulating multivariate CUSUM is given by Pignatiello and Runger.[14] The CUSUM and EWMA are control charts with memory which means that a parameter controls the impact of the past values. For more details about MEWMA and MCUSUM control charts, see, for example, Lowry et al[9] and Woodall and Ncube,[15] respectively.

SPC tools are used in various industrial and nonindustrial areas such as medicine, environment, chemical analysis, healthcare and public-health surveillance Tsui et al,[16] Frisén[17], network monitoring, and change-point problems. For a review in nonindustrial application of MSPC, see Bersimis et al.[18] In recent years, SPC methods have also gained popularity in many financial applications, such as stock trading and portfolio monitoring Frisén,[19] Golosnoy et al[20]. Jumah et al,[21] among other quality control techniques referred to SPC methods in the improvement of trading, banking, and service sectors. Bock et al[22] provided a comparison of surveillance methods and decision rules for finance. Various other applications of control charts include, for example, that of Schmid and Tzotchev[23] that applied multivariate EWMA control charts in order to detect a change in the parameters of the Cox–Ingersoll–Ross (CIR) model for the evolution of interest rates. Yousefi et al[24] implemented control chart techniques on nonnormal and autocorrelated data for monitoring the performance of a project. Berlemann et al[25] applied EWMA and CUSUM charts for the detection of U.S. house price bubbles and especially the estimation of their likely starting points. Freese[26] is focused on the detection of U.S. regional bubbles having data from different markets. Rębisz[27] applied control charts for country risk monitoring for various countries using the credit ratings. Golosnoy and Roestel[28] used CUSUM control charts for real-time monitoring of shifts in inflation expectations and specially to forward break-even inflation (FBI) series.

In financial applications, the underlying data are in most cases no longer independent and as a result control charts for dependent data have to be considered. Knoth and Schmid[29] presented a review for control charts in time series generally. Therein, the case of dependent data is taken into consideration and the new control chart schemes that are presented are based on the time-series structure, specially for autoregressive (AR) and autoregressive moving average processes (ARMA). Control charts for dependent data are usually residual-based or modified control charts. Residual-based charts are constructed from a transformation of the original data so that the resulting data are independent and standard control charts can now be applied (see, e.g., Alwan and Roberts,[30] Harris and Ross,[31] Pan and Jarret.[32]) Modified control charts use standard control chart procedures but the control limits are adjusted in order to account for the autocorrelation (see, e.g., Vassilopoulos and Stamboulis,[33] Schmid[34]). Another category of control charts, in order to overcome the problem of dependent data, is based on the difference between two subsequent values of the measured characteristic and it is known as difference control charts see Golosnoy and Schmid[10].

Okhrin and Schmid[35] presented a review of the methods used for monitoring univariate and multivariate linear time series. They discussed various modified and residual control charts with focus on the monitoring of the variance of financial series. Okhrin and Schmid[36] reviewed EWMA and CUSUM control charts for the surveillance of univariate and multivariate generalized autoregressive conditional heteroskedasticity (GARCH) processes. The authors considered a local measure of the variance based on the squared observations, the forecasts of the conditional variance, and the residuals. In a comparison study when the performance measure is the *ARL* both the EWMA and the CUSUM type charts based on the conditional volatility performed better. In contrast, in terms of maximum average delay the residual charts are preferred. Garthoff et al.[37] introduced control charts for simultaneous monitoring of the mean and the variance of multivariate nonlinear time series. Owlia et al[38] applied residual Shewhart control charts to monitor time-dependent GARCH financial processes in the presence of outliers in the data. The existence of outliers in the sample data can cause problems in the design of the control charts. A thorough discussion of control charts for dependent data in finance is given in the book of Frisén.[19]

The aim of this work is to present the basic economic and financial application fields of statistical process monitoring from the perspective of control charts with focus on portfolio monitoring and stock markets. The application of control chart schemes in finance can be seen as a three-step procedure. In the first step, the main purpose is the definition or the construction of the monitoring process. The second step refers to the choice of the appropriate control chart and depends on the data of the monitoring process. Finally, the third step is related with the interpretation of the signals obtained from the control charts. The challenge in this step is the economic interpretation of these signals specially in monitoring of optimal portfolio weights as we will see later. The transfer of sequential monitoring methods, such as control charts, from industrial to financial applications, it is not always obvious and many difficulties may arise. For example, in some applications of SPC in industry when the first false alarm signal appears, it is possible that the whole process is stopped. In applications such as portfolio monitoring as we will see later, the process cannot be stopped or the reasons of the change to be eliminated Golosnoy et al[39]. Another issue is the structure of the monitoring process which in most financial cases is more complicated than in the industrial applications.

The research papers reviewed in our work are presented thematically according to their application area in finance. Specifically, in Section 2 we review some specific applications of control charts in stock markets and stock trading. Section 3 is devoted to applications of SPC in portfolio monitoring with focus in multivariate control charts. In conclusion, we point out some issues for further research.

2 | CONTROL CHARTS AND STOCK MARKETS

Control charts have been applied in recent years in the decision process for stock trading and investigate the behavior of stock markets. In this section, we review several research papers with focus first on applications of control charts in the filter trading rule and later on applications of Shewhart and other procedures generally in stock markets.

2.1 | Filter trading rule and control charts

The use of SPC methods for the study of changes in stock market price levels was first proposed by Roberts.[40] Next, Hubbard[41] constructed control charts so as to determine the stock price trend and compare it with the gross national product (GNP) and personal income trends. Also, Hubbard[41] sets up decision rules for buying or holding stocks. The data used are logarithmic monthly values of Moody's Composite 200 Stock Average from 1950 to 1967.

Alexander[42,43] introduced filter trading rules followed by the work of Fama and Blume.[44] The filter trading rule is a mechanical trading rule, defined as a sequence of signals for buying and selling stocks. Briefly, the buy signal is given if, for example, the daily closing price of an observed stock moves up at least a certain percent x from a subsequent low. The investor sells the stock when a signal is given, that is, when the closing prices drop at least a certain percent y from a subsequent high. The values x and y are the filter sizes for the trading rule and represent the minimum acceptable percentage change of the stock value for the investor.

Lam and Yam[45] motivated by the filter trading rule used CUSUM techniques to create a trading strategy in the stock market equivalent to the filter trading rule. Starting from a sell signal at time $t = 0$ the filter trading rule is to generate a buy signal at day n if $\frac{r_t}{\min p_i} \geq x, i = 1, \dots, n$, where x is the filter size of the trading rule and p_t the closing stock price. The CUSUM procedure for the filter trading rule has as reference value $k = 0$, the control limit is $h = \log(1 + x)$, and has the following form:

$$S_n = \sum_{i=1}^{n} y_i = q_n - q_0, \tag{9}$$

with the difference of the current stock log price from a historical low $S_n^{'}$ defined recursively as

$$\begin{cases} S_0^{'} = 0 \\ S_n^{'} = \max(S_{n-1}^{'} + y_n,\ 0), \end{cases} \tag{10}$$

where $q_t = \log(p_t)$ is the logarithm of the closing stock prices p_t and $y_t = q_t - q_{t-1}, t = 1, 2, \ldots$ is the continuously compounded daily return from a stock investment. A signal is given if $S_n^{'} > h$. The filter size of the trading rule is $x = e^h - 1$. Lam and Yam[45] generalized the classical filter trading rule by setting the reference value $k \neq 0$. First they consider the general CUSUM procedure with $k > 0$ and $h = 0$ which means that such a general filter trading rule will give a buy signal to the investor when the one-day return exceeds k. This happens when:

$$\frac{p_t - p_{t-1}}{p_{t-1}} > e^{-k} - 1, \tag{11}$$

and the opposite when this general filter trading rule generates a sell signal to the investor. This procedure can be an investment strategy if we believe that a rising trend in the stock market starts with a large single-day rise and a downward trend that usually starts with a large drop in a single day. However, Lam and Yam[45] mentioned that a main drawback of this general filter trading rule (with $h = 0, k > 0$) is the absence of a stop-loss mechanism.

Yi et al[46] applied CUSUM techniques in predicting regime shifts in stock market indices but in contrast with Lam and Yam,[45] they take into account transaction fees. The same CUSUM technique is used in 30 different stock markets and its performance is compared. Suppose that x_i is the daily index of a certain stock market and r the logarithmic return with $r_i = \log(\frac{x_i}{x_{i-1}})$. Define $y_i = r_i - k$, where k is the reference value in the CUSUM procedure, then an upward or downward shift is detected by the following rule:

$$C_i \geq h, \text{ upward shift} \quad \text{and} \quad C_i^{'} \leq -h, \text{ downward shift}, \tag{12}$$

where h is the threshold value of the CUSUM procedure and $C_i = \max(C_{i-1} + y_i, 0),\ C_i^{'} = \max(C_{i-1}^{'} + y_i, 0),\ i = 1, \ldots, n$. The starting values are $C_0 = 0$ and $C_0^{'} = 0$. The result of different values for the parameters k, h is different trading cycles and CUSUM performances. We mention that the performance of each trading cycle is measured using the total profit (TP) or the daily profit (DP). The TP and DP of the CUSUM procedure that contains n trading cycles, the time between a buy and a sell signal, for the case of not taking into account transaction fees are given by:

$$TP = \frac{SP_1}{BP_1} \cdot \frac{SP_2}{BP_2} \cdots \cdot \frac{SP_n}{BP_n},\ DP = \frac{TP - 1}{D_1 + D_2 + \cdots + D_n}, \tag{13}$$

with SP the selling price and BP the buying price. After taking the transaction fees into consideration, the TP and the DP of the CUSUM procedure are:

$$TP^{'} = \frac{SP_1}{BP_1} \cdot \frac{SP_2}{BP_2} \cdot \cdots \cdot \frac{SP_n}{BP_n} \cdot (1 - \alpha)^{2n},\ DP^{'} = \frac{TP^{'} - 1}{D_1 + D_2 + \cdots + D_n}, \tag{14}$$

where $0 < \alpha < 1,\ TP^{'} < TP,\ DP^{'} < DP, \alpha$ is the proportion of the total trading amount charged as the trading fee for each stock buying or selling, and $D_i,\ i = 1, 2, \ldots, n$, are the days in which the stock is held in the ith trading cycle. The result of taking transaction fees into consideration is the deterioration of the performance of the CUSUM procedures. Yi et al[46] concluded that if transaction fees are included they find no acceptable values of k and h. In this situation, the performance of the CUSUM procedure is not so good as when the transaction fees are very small or excluded.

Žmuk[47] in the spirit of the work of Alexander[42,43] applied residual-based control charts to improve the decision-making process in short- and long-run stock trading. The empirical application included open and average prices of CROBEX10 index stocks on the Zagreb Stock Exchange and three types of control charts: individual units (I-chart), EWMA, and CUSUM. The possible presence of autocorrelation in open and average stock prices is dealt with the autoregressive integrated moving average (ARIMA) models. Like Lam and Yam,[45] no transaction fees are taken into consideration but also the simulation of the stock trading scenarios excluded the existence of outliers. In the long-term trading analysis based on opening prices, the stocks showed a higher variability level than in the short term. The use of the residual-based CUSUM control charts resulted in the highest investor trading score in most of the cases. The results of using average prices are almost similar to that of opening prices. Generally, higher profits for the investor are achieved with the use of opening prices than average prices in the short-run analysis. In addition, the total portfolio profit in the short run was achieved by using the residual-based CUSUM control chart in all possible stock trading cases. In the long run, the residual-based CUSUM control chart achieved the highest portfolio profit except for the case with the average trading stock prices and the use of 2-sigma control limits. In the short-run stock trading based on average prices and using different control limit levels, the overall and individually portfolio profits outperformed the profits from trading based on opening prices. In the long run, stock trading based on opening prices outperformed the trading based on average prices.

Xin et al[48] used CUSUM control charts under the spectrum of filter trading rule in a two-regime Markov switching model (MSM) for the returns of the underlying security. The parameters of the control scheme are the decision interval h and the reference value k, which are the filter size and the filter trading rule, respectively. The two-regime model has the regime I and regime II in which the security returns follow different distributions. Under the two-regime model, the market transits between bear and bull state. Generally, the transition probability matrix of a hidden Markov chain for two regimes has the following form:

$$M = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}. \tag{15}$$

This transition probability matrix has the constraints that the four probabilities should be all nonzero and it is invertible. The reference value is set equal to zero which means that the filter trading rule monitors whether the stock return series belongs to a bull or bear market. The four states of the proposed system are: long position under a bull market, long position under a bear market, short position under a bull market, and short position under a bear market. An extension of this scheme included the values of the filter trading rule $S_t^+,\ S_t^-$ which are the upsided and downsided CUSUM statistics, respectively, with stating values equal to zero. At the empirical application, the performance of the filter trading rule under the filter size that gives the largest expected unit time profit, is profitable. Also in many cases the annualized log-returns of filter trading rule outperforms the log-returns of the buy-and-hold strategy.

Cooper and Van Vliet[49] dealt with high-frequency trading (HFT) data and developed statistical techniques and tests alternative to traditional SPC that examine each trading event using the generalized lambda distribution (GLD). The need for their proposed control schemes is due to the fact that in the HFT systems large amount of trades per minute or per second are executed, something that makes difficult their real-time control. Also, traditional SPC methods usually assume normality in contrast with the HFT systems that produce skewed outputs with long tails which supports the selection of the GLD. The suggested statistical tests applied on the distribution of sample means and ranges like in traditional SPC and the distribution of the actual trading profits using the GLD. The distribution of the actual trading profits is called whole distribution of the SPC. The monitored procedure with the whole distribution SPC and the GLD refers to the actual data and each observation is tested rather than each sample mean or range, like in traditional SPC. Every observation now is compared to the presumed underlying distribution. As a result the whole distribution SPC method does not rely on the central limit theorem in order to generate the required statistics. The comparison analysis between traditional SPC and the whole distribution SPC is done through a simulation study. The results, according to Cooper and Van Vliet,[49] showed that the whole distribution SPC reacts quicker than traditional SPC to changes even if new single events differ from the reference distribution. However, there is a trade-off between the number of observations used in the tests and their sensitivity. A small number of observations can give more quickly the change detection in contrast with a large number which can lead to false signals.

Kumiega et al[50] following the work of Cooper and Van Vliet[49] used the GLD and SPC methods in the HFT so as to assess the performance of the investments. A basic issue in the HFT is if a trading system will generate sufficient profits so as to cover its costs. The traditional financial tools appear to have problems to correctly quantify the ability of an HFT system for algorithmic trading firms (ATFs) to cover that costs. The traditional risk measures compared with SPC methods are the Sharpe, Information, and Sortino ratios. Drawbacks for their use in assessing the performance of the system is, for example, that they ignore costs such as research and development (R&D) and operating expenses or fail to capture a series of operations of an ATF such as capital reallocation to other trading systems. The authors examined the performance of in-control HFT systems so as to meet the ATF specifications for profitability. After the definition of investor's trading strategy and the lower limit of capability of the system, a backtesting was performed and the capability of the system to meet returns on investment (ROI) requirements it was verified. We remind that the capability of a process measures the ability to satisfy some specifications. The simulation example and the use of X-bar chart for sample mean returns and an R chart of simulated HFT returns identify that the process is in-control. The capability study is necessary so as ATFs know if an HFT system will cover its own R&D costs. The capability of the trading system is measured with the quantity C_{pl} defined as:

$$C_{pl} = \frac{\mu_n - LSL}{3\sigma_n}, \tag{16}$$

where μ_n is the mean of all the samples, σ_n is the standard deviation of the sample means, and LSL is the lower specification limit of the costs that a capable firm must satisfy. Lower values of C_{pl} of a system may require the firm to reduce costs or reduce variation through trading process improvement.

Cooper et al[51] extended the work of Kumiega et al[50] and developed a new robust performance measurement methodology for algorithmic trading without any assumption for normality. Except for taking into account that returns may not necessarily follow the normal distribution they introduced the concept of multiscale capability of the system. The notion of multiscale capability refers to the fact that different time scales may be applied to the operation of the algorithm, the capital allocation decision to the trading strategy, and the funding decisions of investors and there is a need for a framework for unifying measurement of capability. The authors defined a set of conditions for the definition of which trading strategy is consider to be good for the investor and a methodology for ranking trading strategies according to a term structure of capability. The applied trading strategy at each time must generate a stable distribution of returns and HFT operate when the distribution of returns are in-control. An important part of the methodology is the definition of the expected loss when a left tail event happens when the process is in-control and the trading strategy needs to be changed. The purpose of the firm is not only to achieve profits in the long run but to perform in an acceptable level in the short run so as to cover its costs. The authors provided a framework for the relationship between the performance of the system in the short run and the distribution in the long run. The process in order to be capable uses a generalization of the C_{pl} value that must satisfy the following condition:

$$GC_{pl}(n) = \frac{\mu_n - c}{\mu_n - Q(a)_n} > 1, \tag{17}$$

where c is the allocated fixed and variable costs so as to research, build, and operate the trading system, the distance from the mean to the proxy for the left tail endpoint $3\sigma_n$ is replaced by the nonnormal $\mu_n - Q(a)_n$ for some level a. The mean μ_n follows the GLD and $Q(.)$ is the percentile function of the GLD. The acceptable value of n in order $GC_{pl}(n) > 1$ is the time for which the trading strategy is profitable and may vary across firms. Also the level of the percentile α is the risk tolerance of the trading firm. Low values mean that c is believed to be exceeded and high values mean that the desired profitability is not believed to be achieved. Generally, the values of n that equation (17) is valid are related with the financing decision that the firm face. Possible serial correlation in the time series of returns is dealt with using differences in an EWMA recursion of returns and the new control limits can be computed using the moving range (MR) method.

Dumičić and Žmuk[52] mentioned in their work difficulties for using statistical control charts for making decisions about trading on the stock market on short-term period. They applied univariate control schemes in opening and average prices of stocks from the CROBEX10 market index from the Zagreb Stock Exchange. No additional payments (such as dividends) for the investors are taken into consideration. The control schemes are the individual (I), the EWMA, and CUSUM control charts. For the case of taking open stock prices, they find for the various control schemes too many observations out of the control limits. An observation out-of-control will give a signal to the investor to perform a trading action. The authors indicate that many of these signals are probably false alarms. The same problem of many out-of-control limits observations that appears in the case of using average stock prices makes the use of control charts in portfolio analysis, according to the authors, dubious. Possible explanations for this problem, according to Dumičić and Žmuk[52] , may be the fact that stock prices show nonnormal distribution and exhibit autocorrealtion. More appropriate control chart procedures could be a solution to this problem. In recent years, many procedures have been developed for nonnormal and autocorrelated data.

An interest application of control charts is on the algorithm-controlled finance (ACF) trading machines. ACF trading machines consist not only of the interacting trade selection algorithms for taking positions in the financial markets but also with the technology required to automate some or all of the processes required for trade selection and execution Hassan et al[53]. Hassan et al[53] mentioned the problems of traditional risk measurement techniques and the need for SPC methods such as control charts in order to describe, monitor, and improve the performance of the ACF systems. Between the industrial systems and the ACF systems, there are some basic differences. In the ACF systems, risk managers can inspect the entire set of data, input and output, and more importantly in finance systems the process is assumed to be normal in contrast to industrial that normality is achieved through sampling methodology. Also in financial trading systems the process cannot be stopped in case of out-of-control situations but it can continue only to close existing open trading positions. The authors among others apply X-bar and R charts so as to monitor the performance in a trading machine. They use an X-bar on the returns and define appropriate criteria for the out-of-control situation. The stochastic variables of a trading machine are the mean and the variation of the inputs and outputs. It is obvious that any change to the algorithm of the trading machine applied in order to bring the machine into the in-control condition should lead to rebacktesting of the system and define the new benchmark values. Hassan et al[53] compared the results from classical risk control measures with SPC methodologies such as the control charts we previously mentioned. For the statistical arbitrage pairs trading investment example traditional risk measures such as average annual return, volatility, and Sharpe ratio for the in-sample period indicate that this trading machine could be acceptable for an investor. However, quality techniques applied on the outputs (specifically the returns), such as an X-bar chart, do not support this result and give signals where the trading algorithm is out-of-control. In addition, out-of-sample results show that the ACF system performed poorly in term of returns and more volatile in contrast with the in-sample backtesting of the system. The signals of the SPC tools helped the immediate correction and improvement of the trading system. Also, the authors dealt with serial correlation, a common problem in financial data and mention as a possible drawback of the SPC methods the sensitivity due to noisy financial data which led to false alarms. They removed the serial correlation from the return distribution applying EWMA techniques and re-performed the SPC methods. The purpose of designing an ACF trading system is the absence of autocorrelation and the repeatability in the results. The examined system generated a white noise and after testing on the error terms the monitoring results were found the same with the serially correlated data.

Bilson et al[54] examined the use of SPC methods in trading systems. They apply X-bar and R charts on the returns of two applications of trading systems, the first in a Long–Short pairs trading strategy using statistical arbitrage and the second in a foreign currency trading strategy. Their results are compared with those of traditional quantitative risk management methods trying to find differences in the decisions an investor makes based on these approaches. Another task is if the proposed SPC methods predict better the poor performance of the system and reduce the potential losses of the investor. The results are in accordance with that of Hassan et al[53] and SPC methods generate signals contrary to the signals generated by the traditional risk measures. Both applications referred to risk due to unknown probability distributions of the outputs. The first application referred to the risk of creating a finance trading model that overfits the data and the results measured with traditional methods show that came from a known probability distribution. The second application is the risk of running a trading model after the model faces uncertainty and stops working according to a market structure shift. SPC methods identified the change in the outputs of the trading model in contrast with the results using backtesting and traditional risk measures that the trading system produced acceptable returns.

2.2 Shewhart procedures and volatility in stock market returns

Govindaraju and Godfrey[55] explored the volatility of a stock market using Shewhart procedures. Following Shewhart,[6] they broke down volatility into common (C) causes and special (S) causes volatility. In financial applications, it may be difficult the distinction between special and common causes. Common causes are responsible for the controlled variation while special causes for the uncontrolled variation. Short-term variability mainly is due to common causes and usually can be estimated. In addition, long-term variability includes all the variation due to special causes. Govindaraju and Godfre[55] used rational subgrouping so as to check if a variation is common or special cause. The distribution function F of a variable of interest X can be written as a mixture of the distribution functions of X under common and special causes:

$$F(X) = (1-\alpha)F_C(X|\text{common causes}) + \alpha F_S(X|\text{special causes}), \tag{18}$$

where $F_C(.)$ and $F_S(.)$ are the distribution functions of X under common and special causes, respectively. In case that the process is in-control then the mixing proportion α is zero. The sample standard deviation of the entire data $\{x_t\},\ t = 1, \dots, n$ contains both common and special causes standard deviations and is given by

$$\hat{\sigma}_T = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}, \tag{19}$$

and suppose the sample standard deviation of the jth subgroup is given by

$$\hat{\sigma}_j = \sqrt{(x_{2j} - \bar{x}_j)^2 + (x_{2j-1} - \bar{x}_j)^2}, \tag{20}$$

where $\bar{x}_j = \frac{1}{2}(x_{2j} + x_{2j-1})$ is the mean of the jth subgroup, $j = 1, \dots, m$ and $m = \frac{n}{2}$ of the observations and adjusting for the bias using a correction factor c_4 we have:

$$\hat{\sigma}_C = \frac{1}{c_4 m} \sum_{j=1}^{m} \hat{\sigma}_j. \tag{21}$$

When the time-dependent effect of special causes is removed, the estimated standard deviation is an estimate of the persistent volatility. The results of the empirical application show that much of the volatility in stock returns is due to common causes and can be considered as the permanent risk. Also the concept of common cause variability can be applied to the portfolio selection. The trade-off between risk and return in a portfolio will depend on the choice of total risk or special cause variation used. Long-term investors are interested for special cause variation and analogously define their investment choices.

Premarathna et al[56] extended the work of Govindaraju and Godfre[55] and examined the risk/return and skewness/kurtosis trade-offs in a stock market using Shewhart methodology. The data are separated into rational subgroups and expected in each subgroup to be as homogeneous as possible. The subgroups are partitioned for common and special cause's variation.

The decision rule for determining variation subject to special causes is based on the trimmed mean of subgroup standard deviation:

$$\bar{S}_\alpha = \frac{1}{m - 2\alpha} \left\lceil \sum_{i=m\alpha+1}^{m-k\alpha} \bar{s}_i \right\rceil, \tag{22}$$

where α denotes the percentage of subgroup data that has to be trimmed, $\lceil . \rceil$ denotes the ceiling function, and $\bar{s}_i$ is the ith subgroup corrected average standard deviation. When a rational subgroup has a within-subgroup standard deviation that exceeds a certain limit, then in that time period a special cause of variation affects the volatility in the process and this subgroup should be removed from the calculation of $\bar{S}_\alpha$. The control limit using the new $\bar{S}_\alpha$ is recalculated. This procedure is repeated until they get a UCL that is based solely on subgroups whose variation is based only on common cause variation. Premarathna et al[56] in order to ensure the termination of the decision process set that if the number of subgroups is below the Poisson UCL then the subgroup removal process is stopped. From the empirical application, they found negative mean/standard deviation trade-off in periods of special cause variation and positive trade-off in common cause periods. As a result, the proposed method cleared up trade-offs that were not observed in the total periods of data. The negative trade-offs in the special cause periods are connected with increase in market volatility. The skewness/kurtosis trade-off is negative in both total and special cause periods and has not been observed before. Also the overall trade-off is mainly driven by events during the special cause periods.

2.3 | Other financial applications

Severin and Schmid[57] introduced and compared univariate modified and residual-based control schemes for monitoring GARCH processes applied to daily stock market returns. The modified control schemes are the modified Shewhart, EWMA, and CUSUM chart. The results from the simulation and empirical study favor the use of modified EWMA chart. Severin and Schmid[58] proposed control charts for GARCH processes in order to detect changes in the volatility of financial asset returns. They focused on modified Shewhart, EWMA, CUSUM, and residual control charts. An important prerequisite for the application of these control schemes is the existence of second moments. For the modified Shewhart and EWMA charts, various properties for the distribution of the run length are proved. These methods are compared in a simulation study with the target process to be an ARCH(1) process and an empirical study is made on stock market data.

Schipper and Schmid[59] mentioned that in the presence of variance changes, the opinion that EWMA and CUSUM control charts are suitable to detect small shifts rather than large is not always true. Schipper and Schmid[59] presented EWMA and CUSUM charts for detecting changes in the variance of a GARCH process and applied them to monitor stock market returns. Suppose that Y_t is the GARCH target process with mean μ_0 and variance γ_0 and X_t is the observed process of the data. The observed process in connection with the target process is modeled as follows:

$$X_t = \begin{cases} Y_t, & \text{for} \quad 1 \le t < \tau \\ \mu_0 + \Delta(Y_t - \mu_0), & \text{for} \quad t \ge \tau, \end{cases} \tag{23}$$

with $\Delta \ge 1$ and $\tau \in \mathbb{N}$. The distribution of Y_t is assumed to be known. The exponential weighted moving average and the cumulative sum, for the construction of the EWMA and CUSUM charts, respectively, are applied to the residuals of the process, the squared observations, the logarithm of the squared observations, and the conditional variance. Next we present the EWMA recursions for the four cases we mentioned previously:

$$\begin{aligned} &\text{Residual Chart}: Z_t = (1-\lambda)Z_{t-1} + \lambda \frac{X_t^2}{\hat{\sigma}_t^2}, \\ &\text{Squared Observations}: Z_t = (1-\lambda)Z_{t-1} + \lambda(X_t - \mu_0)^2, \\ &\text{Logarithm of the Squared Observations}: Z_t = (1-\lambda)Z_{t-1} + \lambda \ln \frac{(X_t - \mu_0)^2}{\gamma_0}, \\ &\text{Conditional Variance}: Z_t = (1-\lambda)Z_{t-1} + \lambda\hat{\sigma}_{t+1}^2 \end{aligned} \tag{24}$$

for $t \ge 1, \lambda \in (0,1]$, σ_t^2 is the conditional variance, and $\text{var}(X_t) = \gamma_0$ for $t < \tau$. In a comparison simulation study of these control schemes with the cases of the target processes to be a GARCH(1,1), the EWMA control chart based on the conditional variance outperforms the other schemes and provides in almost all cases the minimal out-of-control ARL (ARL_1). A suggested value for the smoothing parameter is $\lambda = 0.1$.

Śliwa and Schmid[60] were the first who applied control chart procedures for monitoring multivariate nonlinear time series and cross-covariances in particular. The underlying target process is assumed to be a GARCH(1,1) process. Two different types of MEWMA and univariate EWMA control charts are proposed for the surveillance of the multivariate GARCH processes. The first type is based on the exponential smoothing of each component for various examples of local measures for the covariances of the observed and the residual process. In the second type, the Mahalanobis distance between the local covariance measure and its in-control mean is calculated and then the univariate EWMA recursion is estimated. The proposed control schemes are applied to stock markets data.

Golosnoy et al[61] applied Shewahart and CUSUM control charts for monitoring the daily integrated volatility. The dynamics of daily integrated log-volatility are modeled through a linear state-space representation. This state-space representation links the observable volatility measure to the unobservable log-daily volatility. The daily integrated volatility σ_t^2 is not directly observable and the authors use three alternative estimators: the realized volatility RV_t measure, the bipower variations BV_t measure, and the staggered bipower variations SBV_t. The proposed state-space representation for the log-volatility $\omega_t = \log(\sigma_t^2)$ is

$$\omega_{t+1} - a_1 = \phi(\omega_t - a_1) + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, q) \tag{25}$$

$$s_t = \omega_t + \gamma_t, \quad \gamma_t \sim N(0, \upsilon_t), \tag{26}$$

where the measure s_t is $\log(RV_t)$, $\log(BV_t)$, or $\log(SBV_t)$ a_1, is the unconditional expectation of the log integrated volatility, q is the innovations variance, and $|\phi| < 1$. In addition, the innovations ϵ_t, γ_t are assumed to be uncorrelated with each other and not autocorrelated. The volatility modeled with this state-space representation assumes that there is no jumps in the price equation of the underlying asset. Jumps and changes in the parameters may have as a result changes in the distribution of volatility forecasting errors. The authors mentioned the need for differentiation of these two sources of change. The validity of this state-space model and its ability to provide proper volatility forecasts is tested via statistical monitoring techniques.These techniques are applied on the standardized volatility forecasting errors:

$$S_\eta = \frac{\eta}{(p_{t|t-1} + \upsilon_t)^{\frac{1}{2}}}, \tag{27}$$

where $\eta_t = s_t - s_{t|t-1}$ the forecasting errors and the conditional variance is $p_{t|t-1} = \text{var}(\omega_t - \omega_{t+1})$. The observable forecasting errors η_t follow a normal distribution with mean 0 and variance $p_{t|t-1} + v_t$. When the control chart on the forecasting errors or the standardized forecasting errors give a signal then the model described in the previous state-space representation does not provide proper volatility forecasts and action needs to be taken. In addition to the Shewhart chart, CUSUM-type control schemes are applied such as the CUSUM, the fluctuation sum, and the recursive residual chart (for more, see Andreou and Ghysels[62] and Horvath et al[63]). The detecting ability of the proposed control schemes is examined through a simulation and an empirical study. In the simulation study also, it is investigated the case of forecasting errors that do not follow normal distribution but t-distribution. The results in the simulation study showed that changes causing the largest average forecasting losses are detected with relative ease from all control charts. When detecting changes in the mean the CUSUM, the fluctuation sum and the recursive residual control schemes showed similar abilities and performed better than the Shewhart chart. The opposite happened when the authors detected increases in the variance q. The use of innovations that follow the t-distribution led to lower ARL_0. The empirical example consisted of daily data of four highly liquid stocks traded on the NYSE. The Shewhart charts based on all volatility measures provided similar number of signals for both in- and out-of-sample case. An interesting fact is that the majority of signals in the control charts occurred at the same days for all volatility measures. The authors mention that the signals could be categorized as isolated and clustered. The isolated signals can often be interpreted as outliers and clustered signals which are of the main interest indicate possibly problems with the model adequacy. The fact that obtained signals occur at different times from detected jumps may be an indicator to possible structural changes in the volatility model. This application of control chart techniques needs to be expanded for more complicated volatility models and to be examined in the case of reestimation of the model when a signal is detected and how this affects the number and time of signals.

The use of classic control charts has as a prerequisite that the data are known exactly. Kaya et al[64] examined the case in monitoring the volatility in a financial market when the data cannot be fully determined. They overcame this problem by applying the so-called fuzzy control charts for monitoring the variability of a process. The authors introduced two new fuzzy control charts: the fuzzy individual measurements control chart (FIMCC) and the fuzzy moving range control chart (FMRCC). For this purpose, the fuzzy set theory (FST) (see, e.g., Zadeh[65]) has been used along with control charts. For more about fuzzy control charts, see Raz and Wang,[66,67] Gülbay and Kahraman,[68] Erginel,[69] and Morabi et al.[70] In the empirical example, stock prices are forecasted using the exponential smoothing method for the BIST-30 Index. Next, the fuzzy values of stock prices are calculated. The proposed control schemes not only detect small shifts of stock prices but also increase the flexibility of control limits to analyze the variability of stock prices.

Doroudyan et al[71] used Shewhart control charts so as to monitor and detect changes in a financial processes modeled with ARMA-GARCH time-series structure and apply their method to monitor Tehran Stock Exchange price index (TEPIX). The control statistic is based on the residuals of the model. According to the type of shifts in Tehran Stock Exchange trends, Shewhart control charts were proposed for monitoring TEPIX. Simulation studies reveal the robustness and the change detection power of the proposed monitoring method. Suppose $X_1, X_2, \dots, X_n$ denote the observations of the financial process with ARMA(p,q)-GARCH (p,q) structure. They estimated the parameters of the model with maximum likelihood estimation (MLE) and subsequently the residuals ε_t and h_t. The control statistic is denoted as:

$$z_t = \frac{\varepsilon_t}{\sqrt{h_t}}. \tag{28}$$

The process is assumed to be in-control state until

$$z_t > UCL \text{ or } z_t < LCL. \tag{29}$$

The control limits UCL and LCL are determined such that the ARL_0 is equal to some predetermined values. According to Doroudyan et al,[71] the financial process goes to out-of-control state when at least one of the model parameters deviates from the in-control state. They found that the Shewhart method is almost symmetric in the changes in the parameters of the ARMA process which means that positive and negative shifts have almost the same ARL_1. For the GARCH part, only the positive shifts are considered.

Garthoff and Schmid[72] developed control chart procedures for simultaneously monitoring the mean and the covariance matrix of multivariate financial nonlinear time series with heavy tails. The examined financial time series are the constant conditional correlation (CCC) model, the extended constant conditional correlation (ECCC) model, the dynamic conditional correlation (DCC) model, and the generalized dynamic conditional correlation (GDCC) model. The proposed

EWMA or CUSUM type control charts are based on residuals that follow t-distribution.The data are daily logarithmic returns of the stock market indices Financial Times Stock Exchange and Cotation Assistée en Continu. The p-dimensional target process $\boldsymbol{Y_t}$ is assumed to be a conditional correlation model and has the following form:

$$\boldsymbol{Y}_t = \boldsymbol{\mu} + \boldsymbol{\Sigma}_t^{1/2}\boldsymbol{\epsilon}_t, \tag{30}$$

where $\boldsymbol{\mu}$ is the constant overall mean, $\boldsymbol{\Sigma}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$ is the covariance matrix with the diagonal matrix $\boldsymbol{D}_t = \mathrm{diag}(\sigma_{1t}, \dots, \sigma_{pt})$ that includes conditional standard deviations, and the conditional correlation matrix R_t of Y_t. The proposed EWMA and CUSUM control charts are applied on some characteristic quantities of nonlinear processes of the data. The two characteristic quantities are:

$$T_t^{(1)} = \begin{pmatrix} \boldsymbol{\eta}_t \\ vech(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') \end{pmatrix}, \tag{31}$$

and

$$T_t^{(2)} = \begin{pmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t\boldsymbol{\eta}_t' \end{pmatrix}, \tag{32}$$

where $\boldsymbol{\eta}_t$ is a transformation of the residuals and $vech$ the half-vectorization. In the in-control state, the variables η_t are independent and t-distributed and in the out-of-control state they are neither independent nor identically distributed. From the *ARL* point of view, the simulation study for the first characteristic quantity favors the MCUSUM based on the cumulative sum of the quantity. For the second characteristic quantity, the MEWMA chart based on the recursion of the characteristic quantity, has the best performance for detecting small changes in covariances. For larger changes in covariances, the Mahalanobis EWMA outperforms the other control charts. For larger changes in both the mean and covariance, the control schemes on the second characteristic quantity perform better. The use of maximum conditional expected delay (MCED) as a measure of performance has as a result the MEWMA chart based on the EWMA recursion outperforms the other control charts. Omitting covariances and monitoring the variances can improve the performance of the control schemes. The MCUSUM is appropriate for shifts in the mean. The empirical application confirms that neglecting covariances in the monitoring procedure reduces the number of full-sample signals. Li[73] among other methods for detection of structural breaks in multivariate normally distributed intraday stock data for a relative short time period use control charts. Specifically, a modification of the control statistic of a univariate EWMA chart that contains the singular value decomposition (SVD) of the covariance matrix. The results are compared with other methods for the detection of structural breaks and advantages and disadvantages of the proposed control chart procedure are presented. This is an area that more advanced control charts could be more useful.

3 | CONTROL CHARTS AND PORTFOLIO MONITORING

The modern portfolio theory (MPT) introduced by Markowitz[74] focuses on the trade-off between the expected return and the risk of an investment. The investors through the asset allocation try to maximize their returns every time by taking the best decisions. Portfolio optimization has attracted the interest of academia and practitioners alike, with the theory of stochastic optimal control in its various forms playing a dominant role. For example, in the area of continuous-time stochastic models (see, e.g., Korn and Korn[75]) we have various extensions along the lines of jump diffusion, MSMs, or hybrid systems (see, e.g., Azevedo et al[76] or Savku and Weber,[77,78]) as well as extensions using inside information or model uncertainty (see, e.g., Baltas and Yannacopoulos,[79] Baltas et al,[80] Papayiannis and Yannacopoulos[81]). Furthermore, in recent years SPC techniques, which are the main interest of this section, have been applied to the portfolio diversification problem as tool for decision making. Possible structural breaks in the distribution of the asset returns may result in changes in the optimal portfolio weights and action needs to be taken from the investor's point.

3.1 | Portfolio optimization framework

Consider n risky assets in the financial market and suppose that $\boldsymbol{X}_t$ is the k-dimensional vector of asset returns at a certain time t. Denote $E(\boldsymbol{X}) = \boldsymbol{\mu}$ and $\text{var}(\boldsymbol{X}) = \boldsymbol{\Sigma}$ the expected returns and the variance of the returns distribution, respectively. Also we assume that the asset returns are identically and independently distributed following a multivariate normal distribution. The Markowitz portfolio theory assumes that portfolios can be completely characterized by their expected return and variance. The formulation of mean–variance portfolio is

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} - \frac{1}{\delta}\boldsymbol{w}'\boldsymbol{\mu} \\ \text{subject to} \quad & \boldsymbol{w}'\mathbf{1} = 1, \end{aligned} \tag{33}$$

where $\boldsymbol{w}$ is the vector of portfolio weights, $\mathbf{1}$ is a vector of ones, and δ is the risk aversion coefficient.

The global minimum variance portfolio (GMVP) is the portfolio with the lowest possible variance given the assets covariance matrix. For its estimation, only the knowledge of the covariance matrix of the asset returns is required and as a result the GMVP weights do not suffer from estimation risk in the mean asset returns.The vector of optimal weights is the solution to the following minimization problem:

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \\ \text{subject to} \quad & \boldsymbol{w}'\mathbf{1} = 1. \end{aligned} \tag{34}$$

The vector of optimal weights $\mathbf{w}$ is given by

$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}. \tag{35}$$

The covariance matrix $\boldsymbol{\Sigma}$ is usually estimated from the sample covariance matrix. Suppose that $X_1, \dots, X_n$ are the n-period asset returns, then:

$$\hat{\boldsymbol{\Sigma}}_{t,n} = \frac{1}{n-1} \sum_{j=t-n+1}^{t} (\boldsymbol{X}_{\boldsymbol{j}} - \boldsymbol{\mu}_{\boldsymbol{t},\boldsymbol{n}})(\boldsymbol{X}_{\boldsymbol{j}} - \boldsymbol{\mu}_{\boldsymbol{t},\boldsymbol{n}})', \tag{36}$$

$$\hat{\boldsymbol{\mu}}_{t,n} = \frac{1}{n} \sum_{v=t-n+1}^{t} \boldsymbol{X}_v. \tag{37}$$

3.2 | Portfolio monitoring

Yashchin et al[82] used a three-step CUSUM procedure for monitoring and detecting changes in the performance of actively managed portfolios compared to a defined benchmark performance. In contrast to the performance measurement approach, the portfolio monitoring can identify regime changes or shifts in performance. Investors estimate their current portfolio performance every time and take action according to the results from the control schemes. The information ratio, the ratio of a portfolio's returns that exceed a particular benchmark to its tracking error (TE), is chosen as a measure of the portfolio performance and monitored from the CUSUM procedure. The generated sequence of excess returns is uncorrelated and follows approximately the normal distribution. The current information ratio of the portfolio is given as

$$\hat{IR} = \frac{12e_i}{\hat{\sigma}_{i-1}}, \tag{38}$$

where e_i and $\hat{\sigma}_i$ is the logarithmic excess return and the annualized TE of the portfolio in month i, respectively. Next, the log-likelihood ratio based on the k most recent observation is estimated. The estimation of the value of k that maximizes the log-likelihood ratio defines the optimum performance measurement interval. The log-likelihood ratio is defined as the natural logarithm of the ratio of the probability that the observed sequence of returns was generated by a bad portfolio manager to the probability that it was generated by a good portfolio manager. The maximization of the log-likelihood ratio is of great importance because it makes the CUSUM procedure robust to the distribution of portfolio returns and fast to detect a change in the portfolio performance. Finally, the log-likelihood ratio is compared to a threshold value and if it exceeds this value means that the performance has changed from good to bad and action is need to be taken. If the investigation shows that this is a false alarm the likelihood ratio is set to 0 and the procedure is restarted. The empirical findings of Yashchin et al[82] indicate that it takes on average 41 months to detect a bad performance, which is much faster than a t-test. For a good portfolio manager, the average time between false alarms is 84 months. The probability that it will outperform its benchmark over any specified horizon is simply related to its information ratio.

Gandy[83] monitored the performance of credit portfolios using survival analysis approach in CUSUM procedures. The credit portfolio changes either by the addition of new credits or when current credits leave the portfolio in case of default or full payment. Three scenarios are examined for the arrival of new customers and for the changes in the portfolio. Specifically, the customer's arrival rate follow a Poisson process or the arrival rate is doubled at time $t = 1$ or the arrival rate is reduced to half at time $t = 1$. The credit portfolio can have no change during the monitoring period (No-change condition), the default rate at $t = 1.5$ increases by 50% (Crisis condition), and the default rate from $t = 1.5$ and onward for all new customers is 50%. The proposed survival analysis CUSUM procedure is compared through a simulation study for the portfolio monitoring of default rate using sliding window with CUSUM control charts based on default rates and CUSUM control charts based on defaults within a given time after customer's arrival. The alternative strategies, with the exception of the CUSUM charts using a fixed follow-up time, are sensitive to changes in the portfolio population. This has as a result the increase of the default rates and the number of false alarms. The survival analysis of the CUSUM procedure detect faster the alarm times because they can use the information about credit defaults without any delay.

Golosnoy[84] proposed Shewhart and Hotelling control schemes for the surveillance of the portfolio characteristic beta from the one factor capital asset pricing model (CAPM). The Shewhart control chart is appropriate for the case of the univariate quantity beta when we have a single portfolio. Hotelling schemes are relevant when there is a set of portfolios and the monitoring quantity beta is a multivariate vector.

Riegel Sant'Anna et al[85] used EWMA procedures in order to monitor the rebalancing process of index tracking (IT) portfolio. When a signal is given then the portfolio composition is changed and the portfolio needs to be updated using a rebalancing strategy. The EWMA control charts are applied on portfolio's daily returns and daily volatility. The measure of daily returns is the TE, which is the difference between portfolio daily returns and index daily returns. The surveillance of IT portfolios is implemented on cointegration-based and optimization-based portfolios. The empirical study on data from the Brazilian Ibovespa stock index and the US S&P 100 index compares the SPC rebalancing approach with portfolios with the traditional fixed rebalancing windows. The results showed similar findings for both techniques in terms of returns and volatility. In markets with large volatility, the SPC approach is more consistent than the fixed rebalancing window approach.

3.3 | Monitoring optimal portfolio weights

Markowitz's portfolio theory is a single-period myopic portfolio allocation problem where the investor in every time period tries to maximize its quadratic utility function. Okhrin and Schmid[86] examined several distributional properties for optimal portfolio weights of four mean–variance portfolio strategies: expected quadratic utility optimal portfolio, GMVP, tangency portfolio, and Sharpe ratio portfolio. The assumption is that asset returns follow a stationary normal distribution. The estimation of optimal weights depends on the mean and variance of the asset returns but because the true values are unknown instead of these quantities their sample counterparts are used. Okhrin and Schmid[86] derived the exact and asymptotic distribution along with the first two moments of the optimal portfolio weights for the various cases. The asymptotic distributions are estimated both when asset returns are uncorrelated and correlated. For example, in the case of uncorrelated k risky asset returns for n time periods, the vector of the first $k - 1$ optimal weights ($\boldsymbol{w}^*$) in the GMVP follows a multivariate t-distribution with $n - k + 1$ degrees of freedom. The choice of use of $k - 1$ elements instead of k in every vector of optimal weights is because the sums of the elements are equal to one, their covariance matrices are not regular, and the rank of all covariance matrices is equal to $k-1$. The mean and the variance are given by:

$$E(\hat{\boldsymbol{w}}_{t,n}) = \boldsymbol{w} \text{ and } \operatorname{var}(\hat{\boldsymbol{w}}_{t,n}) = \frac{1}{n-k-1}\frac{\boldsymbol{Q}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}, \tag{39}$$

respectively, where $\boldsymbol{Q} = \boldsymbol{\Sigma}^{-1} - \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}\mathbf{1}'\boldsymbol{\Sigma}'}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}$ (see Bodnar and Schmid[87] and Golosnoy and Schmid[88]).

Bodnar and Schmid[89] investigated the distributional properties of the expected return and the variance of various portfolio strategies. The knowledge of the distributional properties and the first two moments is of crucial interest for the construction of control charts for the surveillance of optimal portfolio weights. Caution should be taken when somebody relies on asymptotic results of the asymptotic counterparts of the exact moments because they can differ significantly from the exact moments and the results to be inaccurate. The observed process is considered to be in-control if $E(\hat{\boldsymbol{w}}_{t,n}) = \boldsymbol{w}$ holds for all $t \geq 1$, otherwise the observed process is denoted to be out-of-control. The first approach is directly based on the process of the estimated weights $\hat{\boldsymbol{w}}_{t,n}$. The second one considers the process of the first differences $\{\boldsymbol{\Delta}_{t,n}\}$, defined as $\boldsymbol{\Delta}_{t,n} = \hat{\boldsymbol{w}}_{t,n} - \hat{\boldsymbol{w}}_{t-1,n}$ and its disadvantage is that depends on the estimation window length n.

Under the assumption that asset returns $\{\boldsymbol{X}_t\}$ are independent and identically normally distributed, with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, Golosnoy and Schmid[10] proposed several EWMA control charts for monitoring the weights of the GMVP. The estimation of optimal weights require the knowledge of the covariance matrix of asset returns. Since the true covariance matrix is unknown, the sample covariance is used for the estimation. The estimated sample weights are highly autocorrelated and for this reason the proposed control schemes are the modified EWMA charts as well as control charts based on the differences of one time lag of the sample weights. The use of differences has as a result to reduce the high autocorrelation within the original portfolio weight series. For each control scheme, control charts based on the multivariate EWMA recursion and Mahalanobis distance are constructed. For $t \leq 0$, it is assumed that there are no changes in the underlying process and for $t \geq 1$ the observed process is in-control if it is equal to the target process, otherwise it is said to be out-of-control. The out-of-control situation is modeled by changing the main diagonal or the off-diagonal elements of the covariance matrix or both. Golosnoy and Schmid[10] examined two types of changes in the covariance matrix of asset returns. The first type affects only the mean of the optimal portfolio weights. The second type which is appropriate for financial applications the changes are modeled in a way that represent the change of a bull market to a bear market. The first category of control charts monitors the process of the estimated weights $\hat{\boldsymbol{w}}_{t,n}$ and the second the process of the first differences of the portfolio weights: $\boldsymbol{\Delta}_{t,n} = \hat{\boldsymbol{w}}_{t,n} - \hat{\boldsymbol{w}}_{t-1,n}$. For example, in the case of modified control charts, the distance between the estimated GMVP weights $\hat{\boldsymbol{w}}^*_{t,n}$ and the target weights $\boldsymbol{w}^* = E_0(\hat{\boldsymbol{w}}^*_{t,n})$ are measured by the Mahalanobis distance. E_0 is the mean of optimal portfolio weights estimated when the process is in-control. This leads to:

$$T_{t,n} = (\hat{\boldsymbol{w}}^*_{t,n} - \boldsymbol{w}^*)'\boldsymbol{\Omega}^{*-1}(\hat{\boldsymbol{w}}^*_{t,n} - \boldsymbol{w}^*), t \geq 1. \tag{40}$$

The EWMA recursion is given by

$$Z_{t,n} = (1-\lambda)Z_{t-1,n} + \lambda T_{t,n} \tag{41}$$

for $t \geq 1$. The starting value $Z_{0,n}$ is set equal to $E_0(T_t) = k-1$. In the case of the multivariate EWMA control chart, the vector $\boldsymbol{Z}_{t,n}$ can be presented as

$$\boldsymbol{Z}_{t,n} = (\boldsymbol{I}-\boldsymbol{R})^t \boldsymbol{Z}_{0,n} + \boldsymbol{R}\sum_{\upsilon=0}^{t-1}(\boldsymbol{I}-\boldsymbol{R})^{\upsilon}\hat{\boldsymbol{w}}^*_{t-\upsilon,n}, \tag{42}$$

where $\boldsymbol{R} = \operatorname{diag}(r_1, \dots, r_{k-1})$ is a $(k-1)\times(k-1)$ diagonal matrix with diagonal elements $0 < r_i \leq 1$, $i \in \{1, \dots, k-1\}$. Consequently, it holds that $E_0(\boldsymbol{Z}_{t,n}) = \boldsymbol{w}^*$. The covariance matrix of the multivariate EWMA statistic $\boldsymbol{Z}_{t,n}$ in the in-control state is given by

$$\operatorname{cov}_0(\boldsymbol{Z}_{\boldsymbol{t},\boldsymbol{n}}) = \boldsymbol{R}\left(\sum_{i,j=0}^{t-1}(\boldsymbol{I}-\boldsymbol{R})^i \operatorname{cov}_0(\hat{\boldsymbol{w}}^*_{t-i,n}, \hat{\boldsymbol{w}}^*_{t-j,n})(\boldsymbol{I}-\boldsymbol{R})^J\right)\boldsymbol{R}. \tag{43}$$

A signal is given if:

$$(\boldsymbol{Z}_{t,n} - E_0(\boldsymbol{Z}_{t,n}))' \mathrm{cov}_0(\boldsymbol{Z}_t)^{-1}(\boldsymbol{Z}_{t,n} - E_0(\boldsymbol{Z}_{t,n})) > c. \tag{44}$$

The control limit c which defines the rejection area in every control scheme is estimated through a simulation study for a predetermined value of the *ARL* (usually in financial applications is equal to 120 days or 1/2 year of daily observations, see Golosnoy et al[5]). After a signal is given, the financial analyst should examine it and decide for further actions in concern with the portfolio allocation. The estimation of the covariance of the control statistic $\boldsymbol{Z}_{t,n}$ when the process is in-control requires the estimation of the covariance matrix between the weights. Golosnoy and Schmid[10] studied and approximated the limit behavior of $\mathrm{cov}_0(\boldsymbol{Z}_{t,n})$ as n tends to infinity. An alternative method is through Monte Carlo simulation study. The empirical application study of a portfolio favored the use of difference control charts in practice because they are able to give an alarm almost immediately with high probability for large changes. However, the proposed control schemes showed poor detection ability for some out-of-control situations such as in the case of modified charts when the variances but not the covariances are changed. Due to these poor detection abilities, Golosnoy et al[90] proposed some new characteristics for monitoring optimal portfolio weights in a GMVP. They suggested an alternative process $\{\boldsymbol{q}_t\}$ to the optimal weight process and a process $\{\boldsymbol{p}_{t,n}\}$ alternative to the difference process. For a sequence of independent and normally distributed k-dimensional random vectors $\boldsymbol{X_t}$ it holds that in the in-control state as $n \to \infty$, $n\boldsymbol{\Delta}_{t,n} - \boldsymbol{p}_{t,n} \xrightarrow{p} 0$, where

$$\boldsymbol{p}_{t,n} = -\boldsymbol{Q}((\boldsymbol{X}_t - \boldsymbol{\mu})(\boldsymbol{X}_t - \boldsymbol{\mu})' - (\boldsymbol{X}_{t-n} - \boldsymbol{\mu})(\boldsymbol{X}_{t-n} - \boldsymbol{\mu})')\boldsymbol{w}, \tag{45}$$

and $\boldsymbol{Q} = \boldsymbol{\Sigma}^{-1} - \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{1}\boldsymbol{1}'\boldsymbol{\Sigma}'}{\boldsymbol{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{1}}$. An alternative quantity for monitoring the process of optimal weights is

$$\boldsymbol{q}_t = -\boldsymbol{Q}((\boldsymbol{X}_t - \boldsymbol{\mu})(\boldsymbol{X}_t - \boldsymbol{\mu})' - \boldsymbol{\Sigma})\boldsymbol{w} = -\boldsymbol{Q}(\boldsymbol{X}_t - \boldsymbol{\mu})(\boldsymbol{X}_t - \boldsymbol{\mu})', \tag{46}$$

and $E_0(\boldsymbol{q}_t) = 0,\ Cov_0(\boldsymbol{q}_t) = \boldsymbol{Q}$ with $\boldsymbol{p}_{t,n} = 0$ and $\boldsymbol{1}'\boldsymbol{q_t} = 0$.

Suppose that the process $\{\boldsymbol{X}_t\}$ of asset returns for $t \geq 1$ when it is out-of-control, then the out-of-control mean of the characteristic $\boldsymbol{p}_{t,n}$ for $t \geq 1$ is given by

$$E_1(\boldsymbol{p}_{t,n}) = \begin{cases} -\boldsymbol{Q}\boldsymbol{\Sigma}_1\boldsymbol{w}, & 1 \leq t \leq n \\ \boldsymbol{0}, & t \geq n+1, \end{cases} \tag{47}$$

and the covariance matrix is defined as

$$\mathrm{cov}_1(\boldsymbol{p}_{t,n}) = \begin{cases} 2\boldsymbol{Q}(\boldsymbol{\Sigma}_1\boldsymbol{w}\boldsymbol{w}'\boldsymbol{\Sigma}_1 + (\boldsymbol{w}'\boldsymbol{\Sigma}_1\boldsymbol{w})\boldsymbol{\Sigma}_1)\boldsymbol{Q}, & t \geq n+1 \\ -\boldsymbol{Q}(\boldsymbol{\Sigma}_1\boldsymbol{w}\boldsymbol{w}'\boldsymbol{\Sigma}_1 + (\boldsymbol{w}'\boldsymbol{\Sigma}_1\boldsymbol{w})\boldsymbol{\Sigma}_1)\boldsymbol{Q} + \dfrac{\boldsymbol{Q}}{(\boldsymbol{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{1})^{-1}}, & 1 \leq t \leq n. \end{cases} \tag{48}$$

Control charts for the characteristic processes $\boldsymbol{p}_t$ and $\boldsymbol{q}_t$ are constructed for both univariate and multivariate EWMA recursion. The recursion equations are applied to the first $k-1$ components of the respective characteristic in each case. In a simulation study, the performance of the charts is compared in various out-of-control situations with performance measures the *ARL* and the worst-case conditional expected delay (WED). The results favored the opinion that for the detection of the changes a combination of control charts should be applied. Control charts based on the first differences are those with the worst overall performance. Also control procedures based on the characteristic quantity $\boldsymbol{q}_t$ performed better than those based on $\boldsymbol{p}_t$. If the type of change in the GMVP optimal weights is an increasing variance then the charts based on the characteristic $\boldsymbol{q}_t$ performed better than any other. A disadvantage of these control schemes is that they fail to detect changes because of a decreasing variance. In addition, changes caused by an increase in correlation, the charts that monitor the quantity $\hat{\boldsymbol{w}}_{t,n}$ outperformed the other control schemes. For changes in both the variances and the correlations, the charts for the characteristic $\boldsymbol{q}_t$ had the best performance.

Golosnoy et al[39] developed directionally invariant CUSUM control charts for monitoring the GMVP estimated optimal weights $\hat{\boldsymbol{w}}^*_{t,n}$ and the characteristic process $\boldsymbol{q}^*_t$, the multivariate CUSUM-w and CUSUM-q charts, respectively. Changes in the GMVP composition are due to changes in the covariance matrix of asset returns. The MCUSUM1 and MCUSUM2 charts of Pignatiello and Runger[14] and the projection pursuit (PPCUSUM) scheme of Ngai and Zhang[91] are applied for monitoring these processes. Simulation and empirical study compared the detection ability of the CUSUM schemes with the EWMA schemes for two types of changes. First, only in the variance matrix, which are responsible for large changes in optimal weights, and second in both the variance and the correlation matrix of asset returns, which had as a result small changes in optimal weights. The performance measures of the control charts are the ARL_1 and the WED. The results supported the opinion that the simultaneous use of both w-charts and q-charts is appropriate for the detection of the different types of changes. In the case of changes in the variance, the best performance can be observed for the MCUSUM1-q and the MCUSUM2-w control charts. If the variances of the asset returns are increasing, then CUSUM-q charts perform better than the CUSUM-w charts. If the variances are decreasing then the w-charts are more appropriate for the detection of changes in the weights. For changes in both the variances and the correlations, the best control scheme is the MEWMA-q control chart. The MCUSUM2-q chart performs better among the CUSUM-q control schemes. The CUSUM-w charts are appropriate for detecting changes which are the result of increasing correlation.

Bodnar[92] monitored optimal weights of a GMVP following a different approach by using the distribution of the estimator of the covariance matrix of asset returns in order to construct multivariate and simultaneous control charts. A significant benefit of this approach is that the proposed multivariate and simultaneous control schemes are independent of the covariance matrix of asset returns. The covariance matrix of asset returns is monitored for possible changes that may affect the mean and the covariance of a transformation of the vector of optimal weights. Suppose that $X_1, \dots, X_n$ the vector of asset returns, Bodnar and Schmid[93] showed that linear combinations of the components of the GMVP weights, $\boldsymbol{L}\hat{\boldsymbol{w}}$, follow a multivariate t-distribution with mean $\boldsymbol{L}\boldsymbol{w}$ and covariance $\frac{1}{n-p+1}\frac{\boldsymbol{LRL}'}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}$, where $\boldsymbol{L}$ is the $(q \times p)$-dimensional matrix of constants. Bodnar[92] proposed the following transformation of the vector of the optimal weights:

$$\hat{\boldsymbol{\nu}} = \sqrt{n-p}\sqrt{\mathbf{1}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{1}}(\boldsymbol{L}\hat{\boldsymbol{\Sigma}}^{-1}\boldsymbol{L}' - \frac{\boldsymbol{L}\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{1}\mathbf{1}'\hat{\boldsymbol{\Sigma}}^{-1}L'}{\mathbf{1}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{1}\mathbf{1}})^{-\frac{1}{2}}\boldsymbol{L}(\hat{\boldsymbol{w}} - \boldsymbol{w}), \tag{49}$$

and $\hat{\boldsymbol{\nu}} \sim t_{n-p}(\mathbf{0}, \frac{n-p}{n-p-2}I)$. Structural breaks in the covariance matrix of asset returns have as a result changes in the mean vector and the covariance matrix of the vector $\hat{\boldsymbol{\nu}}$. If a change in the covariance matrix happens then the composition of the optimal weights changes and a new vector $\hat{\boldsymbol{\nu}}$ is estimated with known mean and covariance.

Five types of control charts for multivariate surveillance are constructed: The multivariate Shewhart control chart, the MC1 control chart of Pignatiello and Runger,[14] the multivariate CUSUM control chart, the PPCUSUM control chart of Pignatiello and Runger[14] , and the MEWMA control chart. The proposed control schemes monitor changes in the covariance matrix by testing if the mean of the vector $\boldsymbol{\eta}_\mathbf{i}$ differs significantly from the target value $\boldsymbol{\mu}_\eta$. The quantity η_i is the $q + q(q+1)/2$ dimensional vector of the sequence of the independent covariance matrix estimators of the subsamples that occur if we divide the entire sample of asset returns in m subsets of size n_1. The expected value of the vectors $\boldsymbol{\eta}_\mathbf{i}$ in case of no structural breaks in the covariances of the asset returns has at positions $q+1, 2q+1, 3q, 4q-2, 5q-3, \dots, q+q(q+1)2$ value equal to $\frac{n_1-p}{n_1-p-2}$ and otherwise is zero. In addition, simultaneous monitoring procedures for detecting shifts in the mean and variance for each component of the vector of optimal weights are constructed. The simulation study supports the opinion that the MEWMA and the simultaneous MEWMA control charts outperform the other control schemes because they have the smallest out-of-control ARLs.

Golosnoy[94] monitored the change in the optimal weights of a GMVP by monitoring the unconditional covariance matrix of the k assets returns under the assumption of the locally constant volatility approach. The locally constant volatility approach Hsu et al[95] suggests that the covariance matrix between sudden changes remains constant. The covariance matrix is estimated through a time-varying estimation window. If the control charts give a signal then we choose a shorter estimation window or if there is no signal we increase the length of the window. The result of this approach is the reduction of the out-of-sample variance of the GMVP which is used as a performance measure. The two applied control schemes are proposed by Golosnoy and Schmid[10] for the vector of $k-1$ returns. The modified EWMA based on Mahalanobis distance and the EWMA difference control chart based on Mahalanobis distance, designed for detection of mean changes in the weights. The in-control mean $E_0(\hat{\boldsymbol{w}})$ and covariance matrix $\text{cov}_0(\hat{\boldsymbol{w}})$ of optimal weights are estimated with a time-varying approach. The time-varying length $m_t \geq n$ is defined from the signal of the control charts. At time $t = 1$, the estimation window m_t is chosen to have a small value m days. In the next time period $t = 2$, if no alarm has occurred, the length of the window increases by one observation $m_{t=2} = m + 1$. If now an alarm occurs at time $t = t + \tau$ then we restart the control charts setting $m_{t+\tau} = m$. For the evaluation of portfolio performance, the out-of-sample variance of realized portfolio returns $V(R^p)$ is used. The proposed portfolio monitoring method based on the estimation of the covariance matrix is compared with strategies that contain five alternative covariance matrix estimators: sample estimator, single-index model, shrinkage estimation, exponential smoothing estimator, and estimation with GARCH approach. The empirical study on stocks from the German stock market index DAX shows that the portfolio monitoring strategies with time-varying estimation window achieve smaller out-of- sample GMVP variance than the alternatives in most of cases.

Golosnoy et al[96] estimated the optimal GMVP weights following a different approach than previously described with the sample volatility estimators. They estimated the so-called realized GMVP weights by using the realized volatility measures computed from intraday data. The benefit of using this approach according to the authors is the better incorporation of the new daily information in the markets to the estimation of the covariance matrix of asset returns. The realized covariance matrix is given by

$$\mathbf{R_{r,t}} = \sum_{j=1}^{m} \mathbf{r_{t,j}} \mathbf{r}'_{\mathbf{t,j}}, \tag{50}$$

where $r_{t,j}$ are the m uniformly spaced intraday return vectors for day t. The vector of the realized GMVP weights $\mathbf{w_{r,t}}$ is given by

$$\mathbf{w_{r,t}} = \frac{\mathbf{R_{r,t}}^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{R_{r,t}}^{-1}\mathbf{1}}. \tag{51}$$

Similar to the work of Okhrin and Schmid,[86] the authors derived both finite sample and asymptotic distributional properties of the realized GMVP weights. Suppose that the realized covariance matrix $\mathbf{R_{r,t}}$ follows the conditional Wishart distribution with degrees of freedom m_t and covariance matrix $\mathbf{\Sigma_t}/m_t$, $\mathbf{\Sigma_t}$ is the true daily covariance matrix of asset returns. The result is that the vector $\mathbf{w_{r,t}}$ follows an $(k-1)$-elliptical t-distribution. In addition to the finite sample properties under certain conditions as $m_t \to \infty$, the vector $\mathbf{w_{r,t}}$ is asymptotically normally distributed. The proposed control chart is the univariate EWMA for $\lambda = 1$ which is the Shewhart control chart and it is applied to the differences between the target portfolio weights $\theta_{\mathbf{r,t}}$ from the GMVP weights of the current day, $\mathbf{\Delta r}, \mathbf{t} = \mathbf{w_{r,t}} - \theta_{\mathbf{r,t}}$. The vector of the target weights could be either deterministic or stochastic. As a consequence, the Mahalanobis distance of these differences is

$$T_{r,t} = (\mathbf{\Delta_{r,t}})' \operatorname{cov}(\mathbf{\Delta_{r,t}})^{-1}(\mathbf{\Delta_{r,t}}), \tag{52}$$

where $\operatorname{cov}(\mathbf{\Delta_{r,t}})$ is the $(k-1) \times (k-1)$-dimensional positive definite covariance matrix of $\mathbf{\Delta_{r,t}}$. The performance of the control chart is tested through a simulation and empirical example study. Taking into advantage the distributional properties of the optimal weights for this approach of portfolio monitoring, further work needs to be done and additional control chart procedures to be applied.

4 | CONCLUDING REMARKS AND OPEN PROBLEMS

In recent years, SPC techniques such as control charts, originating from industrial production, have found application to various fields such as finance. Monitoring of a financial time series for detection of changes can be an important tool for decision making. Control charts procedures are used, for example, for support of the decision process of trading at stock markets and helping to the creation of a trading and investment strategies. Also, control charts can be used as an auxiliary tool for portfolio surveillance by detecting possible structural breaks. The problem of identification of true structural breaks becomes more complicated when more than one monitoring signals are considered. The sequential surveillance of portfolios can be seen as a forward-looking approach for the detection of structural breaks. The signals obtained from the control charts should indicate the need for further investigation for the financial analyst in order to examine if a change has happened.

Further investigation should be given in decision strategies for stock trading when stock returns are modeled via a Hidden Markov Model (HMM). An alternative to the Markov regime switching model is the change point model proposed by Chib.[97] This change point model is appropriate when the economic conditions of each regime are not repeated and this area needs further attention. In addition to the filter trading rule, control charts may find application in trend-following strategies such as the moving average trading rule. Also in algorithmic trading machines more advanced control chart procedure could be used to explore cases such as limited market liquidity. Another interesting area of applications is the use of control charts in High-Frequency Algorithmic Trading strategies on cryptocurrency markets. Fuzzy control charts is an area that requires further study in topics such as decision making in stock trading through FST. For example, assuming that the expected rate of the asset returns is a fuzzy number.

Future research in portfolio surveillance should concentrate on extending existing control schemes for the GMVP or establishing new techniques. This requires thorough study not only of SPC methods but also of MPT. The majority of the work in monitoring portfolio weights is concentrated in GMVPs and should be extended for portfolio categories other than the GMVPs such as tangency or Sharpe ratio. This suggests exploring techniques in order to monitor simultaneously the mean and the variance of the portfolio weights. On a wider level, the application of control charts in areas different from the MPT such as the post-modern portfolio theory (PMPT) should be investigated. In addition, the assumption of the distribution of the asset returns for cases other than the Normal distribution needs further work. Most of the literature in the Markowitz's portfolio theory is focused on the equity portfolios and less on the class of fixed-income portfolios. The monitoring of both mean and variance in a mean-variance portfolio framework points out the need for applying more advanced SPC techniques. In recent years, high-dimensional models became increasingly popular in many financial applications. The extension of control chart procedures in these areas, such as studying the interaction between macroeconomics variables or in home price data, could be an interesting and challenging problem. One more possible application of control charts need to be considered is monitoring large-scale portfolio allocations. Control charts on portfolio based on realized covariance matrix is a new research area that future investigations are necessary in topics such as in high-dimensional portfolios using a parametric factor structure for the estimation of the covariance.

In many financial applications, the sequence of the data is less frequent than daily data, for example, monthly or quarterly data, specially in term structure models. Future studies should target on control schemes that take into consideration low-frequency data and affect distributional properties of estimators. For example, the use of low-frequency data for the asset returns may affect the estimation of the covariance matrix of the control statistic and the distributional properties of the optimal portfolio weights. In that case, future work should aim at Monte Carlo simulation techniques. Additional investigation is needed for control charts that assist the decision-making process in stock trading for stocks that have low-trading frequency.

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