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CUSUM Method in Predicting Regime Shifts and its Performance in Different Stock Markets Allowing for Transaction Fees

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ABSTRACT *Statistical Process Control (SPC) is a scientific approach to quality improvement in which data are collected and used as evidence of the performance of a process, organisation or set of equipment. One of the SPC techniques, the cumulative sum (CUSUM) method, first developed by E.S. Page (1961), uses a series of cumulative sums of sample data for online process control. This paper reviews CUSUM techniques applied to financial markets in several different ways. The performance of the CUSUM method in predicting regime shifts in stock market indices is then studied in detail. Research in this field so far does not take the transaction fees of buying and selling into consideration. As the study in this paper shows, the performances of the CUSUM when taking account of transaction fees are quite different to those not taking transaction fees into account. The CUSUM plan is defined by parameters h and k. Choosing the parameters of the method should be based on studies that take transaction fees into account. The performances of the CUSUM in different stock markets are also compared in this paper. The results show that the same CUSUM plan has remarkably different performances in different stock markets.*

KEY WORDS: SPC, CUSUM, regime shifts, financial markets, transaction fees

Introduction

SPC techniques were developed in the USA in the 1930s. In the 1950s, Japanese industry applied SPC widely and proved it to be an excellent method to save money and attract customers. SPC techniques have since been used extensively in industry in the UK and in the USA. SPC is a scientific approach to quality improvement in which data are collected and used as evidence of the performance of a process, organization or set of equipment. One example is that SPC has now been applied to many different parts of Transco, the main provider of gas transportation in mainland Britain and a number of different types of data have been encountered (Coleman *et al*., 2001a, b).

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Control charts play an important role in the methodology of SPC. The charts include Shewhart (1939) charts, the cumulative sum (CUSUM) charts (Page, 1961), the moving average charts and the 'Exponentially Weighted Moving Average' (EWMA) charts. These charts provide a focus for discussion, especially when limits are added to the charts to show the expected range of future measurements. The chart can show whether performance is typical or whether it has changed. By investigating the changes, special causes of variation in performance can be identified.

Having started in manufacturing industry, SPC has now developed as a useful tool to be used in promoting and maintaining the health of a commercial or industrial company. Wheeler (1993) has successfully applied SPC in management even though the data and circumstances differ considerably from those in production. A business measurement framework was created by Coleman *et al.* (2001b), using SPC techniques. SPC is also widely used in surveillance of changing points in medicine (see, for example Coleman *et al.*, 2004), meteorology, and so on.

CUSUM Techniques Used in the Financial Market

The cumulative sum (CUSUM) method was first developed by E.S. Page (1961) and G.A. Barnard (1959), using a series of cumulative sums of sample data for online process control. It is based on a method using the sequential likelihood ratio test, where, as each new sample becomes available, a test is conducted to determine whether or not the process mean deviates by at least a specified amount from a target value (see DeVor *et al.*, 1992). The CUSUM chart detects changes in the process mean by accumulating the sum of deviations of the observed sample means $\overline{X}_1, \ldots, \overline{X}_t$ from a reference value (see Quesenberry, 1997). The cumulative sum function about a reference value k at time t is

$$S_t = \sum_{i=1}^{t} (\overline{X}_i - k) = S_{t-1} + (\overline{X}_i - k)$$

with $S_0 = 0$.

An upward shift is detected if $S_t \geq h$. Or vice versa, a downward shift is detected if $S_t \leq -h$. There are two parameters in the CUSUM method, k and h, which are often referred to as a 'CUSUM plan'. These values are often selected from ARL (Average Run Length) tables (for example, Lucas & Crosier, 1982). In order to determine the parameters of a CUSUM plan, the acceptable and rejectable quality levels along with the desired respective ARLs are usually specified. The design parameters can then be obtained in a number of ways. Unfortunately, the calculations of the ARL for CUSUM charts are quite involved. The reference value k is often set to be μ, the process mean, but k can be set at any level (Lam & Yam, 1997). In Lam & Yam's study, the decision limit h is set to $10k$, $20k$, $30k$, $40k$, $50k$ and so on. We found that some of these settings incurred too many cycles (described in the next section), which is meaningless if the transaction fees are taken into consideration. In this paper, h is set to $h = 10\,k$.

CUSUM is an ideal tool of SPC. It can be used to detect a regime change in time series data. First developed to control the quality of production, the method is now widely used in financial markets. Kahya & Theodossiou's (1999) financial distress model uses time-series CUSUM methodology to monitor certain financial ratios of companies. In the hope of detecting a permanent change as early as possible, it aims to give managers prompt alerts to undertake actions to do investigations and assessment of the company. One desirable feature of the CUSUM method is that it has a very short 'memory' with respect to a firm's good performances over the years, but a long 'memory' in case of poor performances. The model's memory feature makes it sensitive to negative changes in a firm's financial condition.

Blondell *et al.*'s (2002) paper presents a new Cumulative Sum control scheme approach for the detection of regime shifts in financial time series that are serially correlated and are subject to independent mean level and volatility regime shifts of varying magnitudes.

Philips *et al.* (2003) used the CUSUM to dynamically monitor active portfolio managers' performance. Traditional performance measurement algorithms measure the performance of portfolios over a fixed period of time, which results in good performance in some years masking poor performance in others, making it difficult to estimate the portfolio's current performance, and harder still to identify transitions from good performance to bad. It is often claimed that it takes 40 years to determine whether an active portfolio outperforms its benchmark. In Philips *et al.*'s paper, they show that with an appropriate choice of parameters, the CUSUM can detect a transition from out performance (which means delivering better returns over the long term) to flat-to-the-benchmark performance in 40 months, and a transition to underperformance faster still. The following study will focus on using CUSUM to predict regime shifts in stock markets. Lam & Yam (1997) discovered that the CUSUM techniques could be adapted to yield a trading strategy in the financial market. In this paper, the performance of CUSUM including transaction fees will be employed and shown to give rather different results. The same CUSUM plan is used in 30 different stock markets to compare the performance.

Performance of CUSUM Taking Transaction Fees into Consideration

In studies of the stock market, return is an important indicator of the performance of the market. Let x_i denote the daily index of a certain stock market. The continuous return r is defined using natural logarithms as

$$r_i = \log\left(\frac{x_i}{x_{i-1}}\right)$$

The advantage of the continuous return is that it can be meaningfully added. For example, for a certain period n, the sum of the continuous return S_A can be expressed as

$$S_A = r_1 + r_2 \cdots + r_n = \log\left(\frac{x_1}{x_0}\right) + \log\left(\frac{x_2}{x_1}\right) + \cdots + \log\left(\frac{x_n}{x_{n-1}}\right) = \log\left(\frac{x_n}{x_0}\right)$$

In the CUSUM procedure, we take r_i as observations. Let $y_i = r_{\mathrm{i}} - k$, where k is the reference value in the CUSUM procedure.

Define one sided CUSUM C_i as follows:

$$C_i = \max(C_{i-1} + y_i, 0), \quad \text{where} \quad i = 1, 2, \ldots n \text{ and } C_0 = 0$$

An upward shift is detected as soon as the following inequality holds:

$$C_i \geq h$$

where h is the threshold value in the CUSUM procedure.

Similarly, define C_i' as

$$C_i' = \min(C_{i-1}' + y_i, 0), \text{ where } i = 1, 2, \ldots n \text{ and } C_0' = 0$$

A downward shift is detected as soon as $C_i' \leq -h$

The rationale behind the CUSUM used in the stock market is the filter-trading rule (Lam & Yam, 1997). The filter trading rule was defined from the study of Alexander (1961), which shows that if the stock has moved up $x\%$ (or moved down $y\%$), it is likely to move up more than $x\%$ further (or move down more than $y\%$ further) before it moves down by $x\%$ (or moves up by $y\%$). So a sell signal is prompted by a downward shift and a buy signal is prompted by an upward shift.

A trading cycle is the time between a buy signal and a sell signal, this means the time between when you buy a stock at a certain price and you sell it at a different price. The performance of this cycle can be measured by daily profit (DP) or total profit (TP). Total profit is the multiple of the original investment that you have earned from the cycle, and daily profit is the mean of every day's profit in the trading cycle.

If the trading cycle begins at day n with buying price BP and finishes at day m with selling price SP, and if we define $D = m - n$, then the TP and DP are defined as follows respectively

$$TP = SP/BP$$

$$DP = \frac{TP - 1}{D}$$

The total profit and daily profit of the CUSUM procedure that contains n cycles are defined as

$$TP = \frac{SP_1}{BP_1} \cdot \frac{SP_2}{BP_2} \cdot \cdots\cdots \cdot \frac{SP_n}{BP_n}$$

$$DP = \frac{TP - 1}{D_1 + D_2 + \cdots\cdots + D_n}$$

where D_i, $i = 1, 2, \ldots n$, means the days in which the stock is held in the ith cycle.

Different values of the parameters k and h lead to different trading cycles and therefore will result in different performances of CUSUM in the stock market, as shown in the study of Lam & Yam (1997). However, as the study did not take transaction fees into consideration, we cannot just employ the value of h and k as the trading strategy to make real money. In practice, no one can ignore transaction fees and it is an important factor that affects a buying or a selling decision. In the UK, for instance, the government would charge 0.5% on UK equity purchases. And the stockbroker would charge commission on both buying and selling as well. Transaction fees include brokers' commissions and spreads (the difference between the price the dealer paid for a stock and the price for which they can sell it). So fees are incurred when buying or selling securities, which increase with the number of trading cycles.

After taking the transaction fees into consideration, the total profit and the daily profit of the whole procedure are as follows, where a is the proportion of the total trading amount charged as the trading fee for each buying or selling trade.

$$TP' = \frac{SP_1}{BP_1} \cdot \frac{SP_2}{BP_2} \cdot \cdots \cdot \frac{SP_n}{BP_n} \cdot (1 - a)^{2n}$$

$$DP' = \frac{TP' - 1}{D_1 + D_2 + \cdots + D_n}$$

because $0 < a < 1$, $TP' < TP$ and $DP' < DP$.

Experiment

Different CUSUM Plans with FTSE100

In Lam & Yam's study, they use the closing Hang Seng Index as a proxy for prices of a portfolio of stocks. An index of market prices is a particular group of stocks. For instance, the S&P 500 is a basket of 500 stocks that are considered to be widely held. A portfolio is a collection of investments all owned by the same individual or organisation. In stock market terms, a portfolio means a list of stocks the investor has chosen. When he or she buys or sells the stocks, he or she needs to pay the transaction fees, so it makes sense to consider the transaction fees when studying the portfolio, and the index.

The data used in this paper are FTSE 100, namely the Financial Times Stock Index 100, from 2 April 1984 to 29 April 2004. This is 20 years' worth of data yielding approximately 4000 values from days when stocks were traded.

In this experiment, k is chosen as from 0.0001 to 0.006, and h is chosen as $h = 10$ k. For each value of k and h, a CUSUM is set up for the continuous return of the FTSE 100 index. A trading cycle starts when the CUSUM, C_i, goes above the upper limit, h, and a buy is signalled. The cycle stops when the CUSUM, C_i', goes below the lower limit, $-h$, and a sell is signalled. The next trading cycle does not start until the CUSUM, C_i, again goes above the upper limit, h.

Using the FTSE100 index from 23 July to 24 August in 1984, as shown in Figure 1, an example of a trading cycle using the CUSUM method is illustrated in Figure 2, with the data shown in Appendix 1. In this example, $k = 0.003$ and $h = 0.03$.

The trading cycle shown lasts from day 10 to day 20 and is of length 10 days. The next trading cycle starts sometime after day 20, where the CUSUM exceeds the upper control limit of $h = 0.03$.

Appendix 2 gives the potential profit made from using the CUSUM regime and the number of trading cycles with and without transaction fees for 21 different values of k. Total profit, number of cycles and daily profit are illustrated in Figures 3, 4 and 5 respectively.

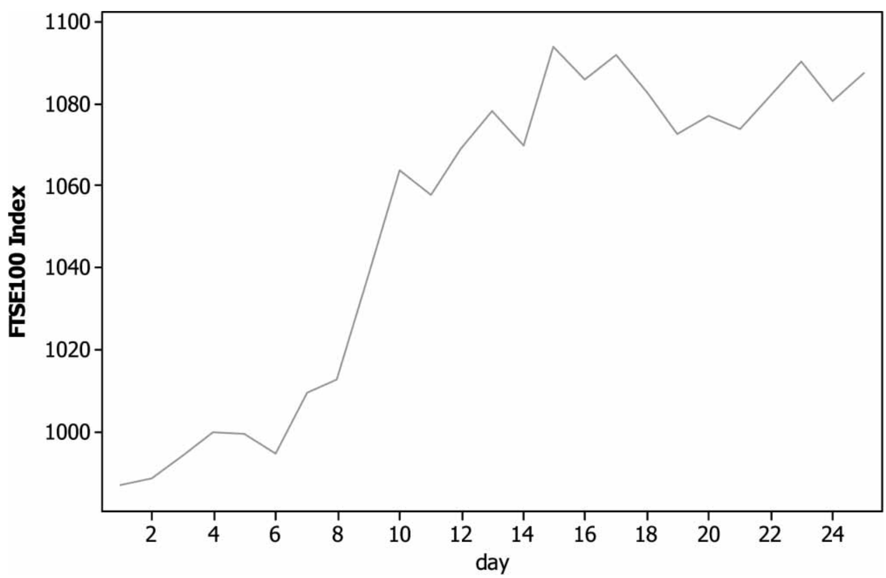

Figure 1. FTSE100 Index from 23 July 1984 (day 1) to 24 August 1984 (day 25)

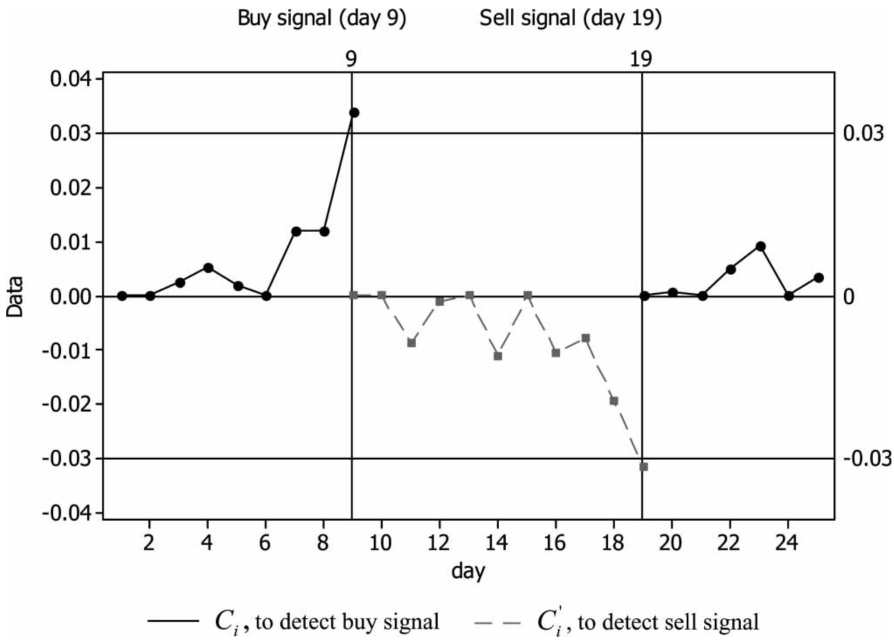

Figure 2. Detection of a buy signal and a sell signal

The CUSUM method is good at detecting small changes in the process (see Quesenberry, 1997). We can see from Appendix 2 that the smaller the value of h and k, the better the performance, without taking transaction fees into consideration. That is because, to detect small changes in the mean, the threshold value h could not be chosen too big. But the smaller values of h and k result in more trading cycles, which sometimes is meaningless in practice because of the high transaction fees in consequence. If k is very small, then so is h and there are likely to be a lot of false alarms, i.e. a sell when you should have held or a buy when you should have ignored it.

Total profit (*TP*) is the multiple of the original investment that you have achieved. If $TP < 1$, that means that you have lost money. Without considering transaction fees, the most profitable CUSUM plan is $k = 0.0004$ and $h = 0.004$, and the total profit is 5.91454, which means that the money you got from the stock market is nearly six times the investment. This, at least, is an acceptable outcome. While, after taking transaction fees into consideration, the total is only 2.08×10^{-5}. That is because with this CUSUM plan, the number of cycles is 834, and most of the profit is consumed by the transaction fees. In fact, study shows that, after taking transaction fees into consideration, the total profit is always less than 1.

There is a dilemma in how to choose the parameters for the CUSUM plan. From Figure 3, we can see that the total profit is satisfied when $k < 0.001$. But small k values incur too many buying and selling points. The changing of the number of cycles with k is shown in Figure 4.

Daily profit is the mean of the profit for every day when you are holding the stock and can be expressed as a percentage. $DP > 0$ means that, on average, you are earning money and $DP < 0$ means that you are on average losing money. Similar to Figure 3, Figure 5 for daily profit shows that taking transaction fees into consideration, the daily profit is never positive.

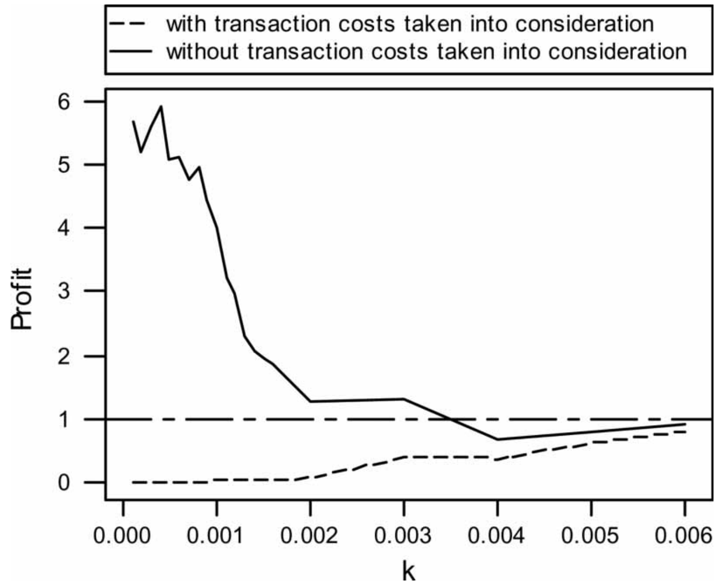

Figure 3. Total profit, with and without transaction fees into consideration

Comparing across Countries

Previous studies used one stock index as the example. Indexes from 30 different stock markets were experimented with in this study. The same values of h and k were used for each index, $k = 0.0003$, 0.003 and 0.006 with $h = 0.003$, 0.03 and 0.06 respectively. The reason to choose these values is because smaller values of h and k result in a better performance than do bigger values.

The results for profit with and without transaction fees and the number of cycles are given in Appendix 3. Daily profits are illustrated in Figure 6. Our study shows that different markets had different performances, even with the same parameters h and k. Different countries have different transaction fees. For the convenience of this study, we chose only one trading fee, $a = 0.75\%$ for all markets.

To study the difference between the performance of the CUSUM in the different stock markets, we compare the results with the overall profit of the market (MP). Generally

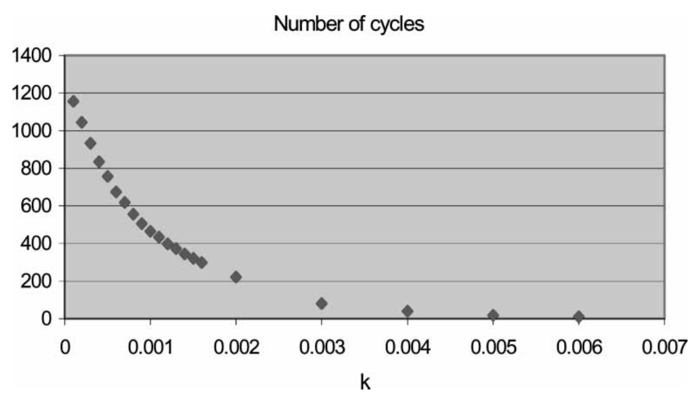

Figure 4. Number of cycles and k

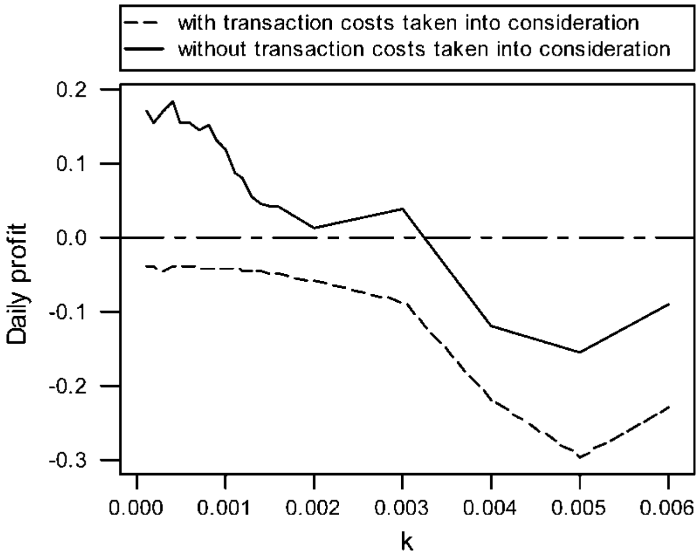

Figure 5. Daily profit, with and without transaction fees taken into consideration

speaking, any stock in a market should be growing from a long-term point of view, because all the money is invested in other industries for the purpose of making money. In this paper, the overall profit of a market is defined as the index of the market at the end of the study period divided by the index of the market at the beginning of the study period, so MP is expressed as the ending index/beginning index. Figure 6 shows the comparison of MP and total profit without transaction fees (*TP*), with $k = 0.0003$ and $h = 0.003$. As Figure 6 shows, without taking transaction fees into consideration, the performance of the CUSUM method is acceptable, because in most markets, *TP* is larger than *MP*. But, after taking transaction fees into consideration, the performance of the CUSUM becomes worse. As no TP' is bigger than 1, this means that investors will lose their money using the CUSUM plan. The values of TP' are not shown in Figure 6 because they are too small to be recognised (see Appendix 4).

A scatter plot of total profit and market profit for different countries is shown in Figure 7. It can be seen that there is a pattern of increasing *TP* corresponding to increasing *MP*.

Figure 6 shows that, except for some markets such as Indonesia and Hong Kong, total profit and daily profit without transaction fees are closely correlated to market profit, which is reasonable. Correlation coefficients are given in Table 1. In contrast, total profit and daily profit with transaction fees (TP' and DP') are not closely correlated with market profit. The scatter plot in Figure 7 shows an outlier, which is Brazil. Note that Brazil is a fast growing market with MP nearly 60. Excluding Brazil there is still a significant correlation of 0.47 for *TP* and 0.56 for *DP* with market profit (MP).

Table 1 tells us again that after taking transaction fees into consideration, the CUSUM method is not suitable for making real money in the stock market.

Discussion

The performance of the CUSUM method used in stock markets deteriorates when taking transaction fees into consideration. But the transaction fees can not be ignored in practice as they exist in all stock trading. Some acceptable CUSUM plans may become

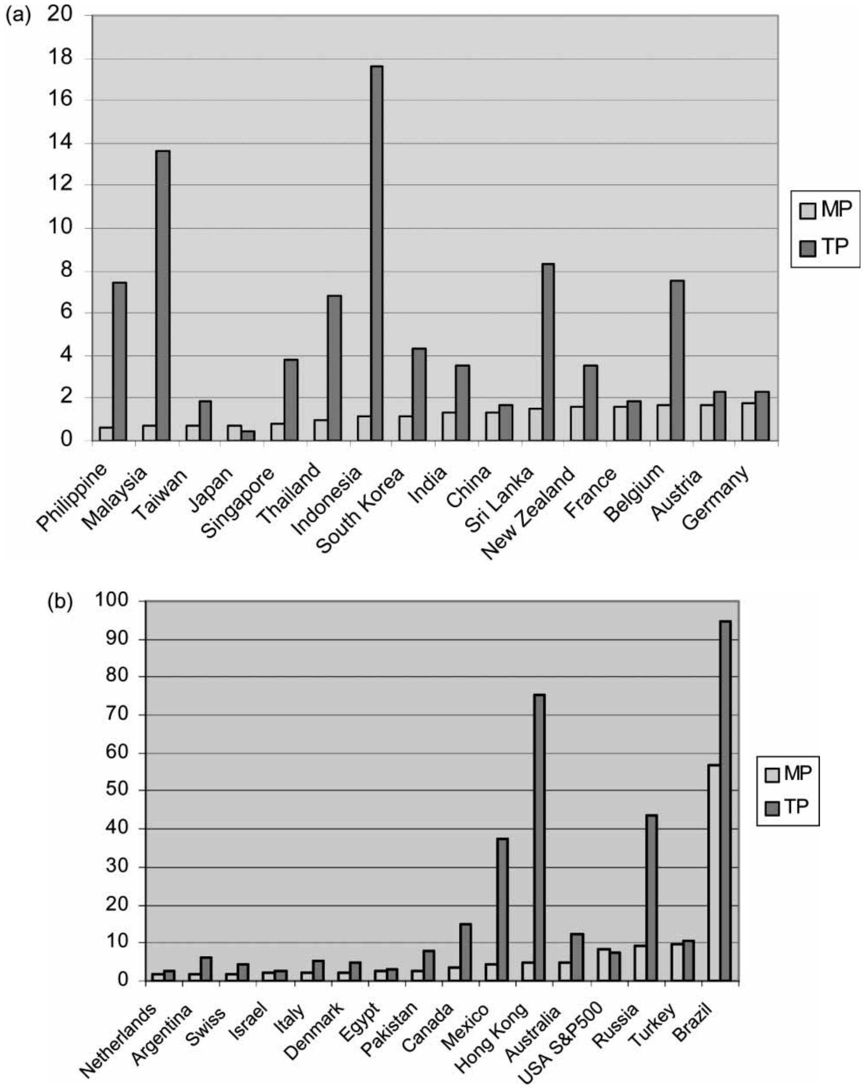

Figure 6. A comparison of profit of market (*MP*) and total profit without transaction fees (*TP*)

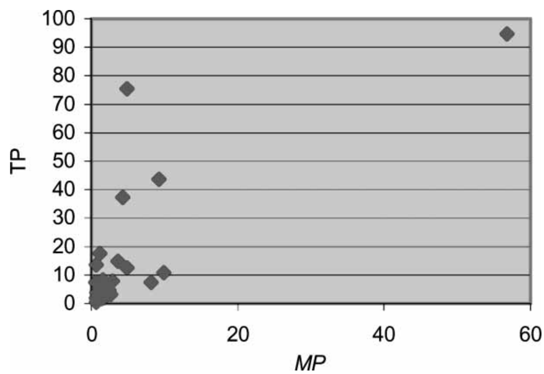

Figure 7. Scatter plot of *MP* and *TP*

Table 1. Correlation of *TP*, *DP*, *TP′* and *DP′* with *MP*

	TP	*DP*	*TP′*	*DP′*
Correlation with MP	0.761544	0.774836	−0.03849	0.118447

unacceptable if they result in too many cycles. So the choice of h and k is a balance between the profit and the cost. In our study, no values of k and h are acceptable if transaction fees are included.

Transaction fees vary greatly with different transaction amounts and trading times. Usually, the larger the amount of the transaction and the more frequent your trading, the less the transaction fee. Here we use only one constant value, 0.75%. In practice, with small values of h and k, the transactions are conducted frequently, so the performance of the CUSUM could be improved by reducing the transaction fees.

Further research in this field could focus on the markets where transaction fees are tiny or where there is no transaction fee at all, such as in foreign exchange markets.

The same CUSUM plan performs differently in different stock markets. Without taking transaction fees into consideration, for most stock markets, the total profits and the daily profits are closely correlated to the market profits, except for some markets such as Indonesia and Hong Kong. In contrast, after taking transaction fees into consideration, the correlations are tiny, which means transaction fees do affect the performance of the CUSUM method in the financial market. To take advantage of the CUSUM method in practice, further research is needed.

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Appendix 1. Example of a trading cycle using CUSUM method

	Date	FTSE100 Index	y_i	C_i	C_i'
Day 1	23-Jul-84	986.9		0	*
Day 2	24-Jul-84	988.7	−0.00118	0	*
Day 3	25-Jul-84	994.3	0.002648	0.002648	*
Day 4	26-Jul-84	999.9	0.002616	0.005264	*
Day 5	27-Jul-84	999.5	−0.0034	0.001864	*
Day 6	30-Jul-84	994.4	−0.00812	0	*
Day 7	31-Jul-84	1009.4	0.011972	0.011972	*
Day 8	1-Aug-84	1012.5	6.64E-05	0.012038	*
Day 9	2-Aug-84	1038.2	0.022066	0.034104	0
Day 10	3-Aug-84	1063.9	0.021453	*	0
Day 11	6-Aug-84	1058.0	−0.00856	*	−0.00856
Day 12	7-Aug-84	1069.3	0.007624	*	−0.00094
Day 13	8-Aug-84	1078.5	0.005567	*	0
Day 14	9-Aug-84	1069.9	−0.01101	*	−0.01101
Day 15	10-Aug-84	1094.1	0.019367	*	0
Day 16	13-Aug-84	1085.9	−0.01052	*	−0.01052
Day 17	14-Aug-84	1092.1	0.002693	*	−0.00783
Day 18	15-Aug-84	1082.9	−0.01146	*	−0.01929
Day 19	16-Aug-84	1072.8	−0.01237	0	−0.03166
Day 20	17-Aug-84	1077.0	0.000907	0.000907	*
Day 21	20-Aug-84	1073.9	−0.00588	0	*
Day 22	21-Aug-84	1082.5	0.004976	0.004976	*
Day 23	22-Aug-84	1090.5	0.004363	0.009339	*
Day 24	23-Aug-84	1080.7	−0.01203	0	*
Day 25	24-Aug-84	1087.6	0.003364	0.003364	*

Appendix 2. Potential Profit using CUSUM with FTSE100 Index

k	Daily Profit without considering transaction fees (%)	Total Profit without considering transaction fees	Daily Profit with considering transaction fees (%)	Total Profit with considering transaction fees	Number of cycles
0.0001	0.175162	5.67856	−0.0374	2E-07	1155
0.0002	0.157204	5.19891	−0.0374	8E-07	1044
0.0003	0.174531	5.6146	−0.0444	4.5E-06	933
0.0004	0.186016	5.91454	−0.0385	2.08E-05	834
0.0005	0.158285	5.10273	−0.0379	5.81E-05	756
0.0006	0.158349	5.13607	−0.0383	0.000201	674
0.0007	0.146787	4.7871	−0.0387	0.000436	618
0.0008	0.154886	4.96507	−0.0391	0.001166	555
0.0009	0.135334	4.43748	−0.0393	0.00218	506
0.0010	0.121396	4.02154	−0.0399	0.003718	464
0.0011	0.090862	3.21159	−0.0410	0.004665	434
0.0012	0.083887	2.97219	−0.0423	0.007423	398
0.0013	0.056598	2.27968	−0.0439	0.008421	372
0.0014	0.048748	2.06904	−0.0450	0.011651	344
0.0015	0.044789	1.95625	−0.0461	0.015575	321
0.0016	0.04274	1.86762	−0.0482	0.021022	298
…					
0.002	0.015097	1.25334	−0.0569	0.044973	221
0.003	0.042274	1.30521	−0.0851	0.385499	81
0.004	−0.11783	0.65476	−0.2190	0.358526	40
0.005	−0.15294	0.79812	−0.2960	0.608648	18
0.006	−0.08902	0.91899	−0.2300	0.790532	10

Appendix 3. Performance of CUSUM method in stock markets in different countries

	k	Daily profit without considering transaction fees (%)	Profit without considering transaction fees	Daily profit with considering transaction fees (%)	Profit with considering transaction fees	Number of cycles
Argentina	0.0003	0.590133	6.37611	−0.10246	0.06657	303
	0.003	0.287737	2.78109	0.043801	1.27113	52
	0.006	0.204992	1.34849	0.073874	1.12559	12
Brazil	0.0003	6.66754	94.6789	−0.06573	0.07644	473
	0.003	2.02144	22.9528	0.48689	6.28759	86
	0.006	1.9585	8.6186	1.38241	6.37759	20
Canada	0.0003	0.520947	14.8468	−0.03722	0.010788	480
	0.003	−0.023134	0.9392	−0.08661	0.772204	13
	0.006					0
Mexico	0.0003	2.74287	37.3705	−0.06576	0.12804	377
	0.003	0.27246	3.2205	0.069123	1.56336	48
	0.006	0.12844	1.2774	0.053492	1.11554	9
USA S&P500	0.0003	0.21991	7.51385	−0.03375	0.000235	689
	0.003	0.03269	1.12813	−0.03891	0.847457	19
	0.006	−3.963	0.88111	−4.4019	0.867943	1
Australia	0.0003	0.427226	12.4283	−0.03724	0.00389	536
	0.003	0.193414	1.4681	0.124573	1.30147	8
	0.006					0
China	0.0003	0.080028	1.66423	−0.11474	0.04764	236
	0.003	0.066841	1.23394	−0.02483	0.91309	20
	0.006	0.865988	1.2165	0.793272	1.19832	1
Hong Kong	0.0003	3.27327	75.467	−0.04364	0.007183	615
	0.003	0.04741	1.5423	−0.0425	0.51385	73
	0.006	−0.13056	0.8342	−0.2051	0.739519	8
India	0.0003	0.296872	3.52045	−0.10964	0.069171	261
	0.003	0.031481	1.1678	−0.05431	0.710528	33
	0.006	−0.603761	0.8068	−0.71511	0.771165	3
Indonesia	0.0003	2.06851	17.5688	−0.05294	0.57596	227
	0.003	0.21912	2.043	0.047157	1.22447	34
	0.006	0.25402	1.3023	0.144548	1.17201	7
Japan	0.0003	−0.0465227	0.41428	−0.07938	0.00055	440
	0.003	0.0279815	1.07583	−0.06626	0.820431	18
	0.006					0
Malaysia	0.0003	1.01528	13.6402	−0.07259	0.09627	329
	0.003	0.182	1.8099	0.022743	1.10121	33
	0.006	0.28699	1.4477	0.181676	1.28341	8
New Zealand	0.0003	0.15933	3.5445	−0.06168	0.014994	363
	0.003	−0.030362	0.95264	−0.11572	0.819477	10
	0.006					0
Pakistan	0.0003	0.751422	7.77783	−0.08898	0.1974	244
	0.003	0.661197	4.72915	0.372758	3.10236	28
	0.006	0.028136	1.02898	−0.0718	0.92605	7
Philippine	0.0003	0.836661	7.43392	−0.09482	0.270798	220
	0.003	0.113158	1.35645	−0.02634	0.917044	26
	0.006	0.051102	1.04497	−0.05147	0.954706	6
Singapore	0.0003	0.229355	3.81419	−0.08026	0.01519	367

(*Table continued*)

Appendix 3. Continued

	k	Daily profit without considering transaction fees (%)	Profit without considering transaction fees	Daily profit with considering transaction fees (%)	Profit with considering transaction fees	Number of cycles
	0.003	0.173896	1.85035	0.040021	1.1957	29
	0.006	0.358328	1.31891	0.271712	1.24182	4
South Korea	0.0003	0.397148	4.37576	−0.11071	0.05901	286
	0.003	0.100587	1.60956	−0.05411	0.67212	58
	0.006	0.101086	1.24362	0.003012	1.00726	14
Sri Lanka	0.0003	0.931571	8.27557	−0.01392	0.89131	148
	0.003	0.427873	2.60025	0.262823	1.98296	18
	0.006	0.101043	1.05254	0.011649	1.00606	3
Thailand	0.0003	0.739847	6.8004	−0.11174	0.12393	266
	0.003	0.314815	2.52056	0.091917	1.44396	37
	0.006	0.492744	1.43361	0.352024	1.30978	6
Taiwan	0.0003	0.099133	1.81884	−0.11791	0.02605	282
	0.003	0.251662	2.27089	0.092567	1.46746	29
	0.006	−0.114995	0.90685	−0.19619	0.84109	5
Israel	0.0003	0.227704	2.62353	−0.12558	0.104604	214
	0.003	0.01335	1.051	−0.06718	0.74337	23
	0.006	−0.535631	0.93572	−0.65216	0.921741	1
Egypt	0.0003	0.307879	3.13668	−0.03433	0.761742	94
	0.003	0.035646	1.03921	−0.04596	0.949445	6
	0.006					0
Turkey	0.0003	1.17897	10.7619	−0.11058	0.0844	322
	0.003	3.16271	23.5501	1.04621	8.45947	68
	0.006	0.95862	5.8219	0.52694	3.65052	31
Swiss	0.0003	0.234084	4.25611	−0.07027	0.02257	348
	0.003	0.117541	1.46194	0.020815	1.0818	20
	0.006	−0.002864	0.99831	−0.07759	0.95422	3
Russia	0.0003	4.78621	43.7409	−0.0631	0.43648	306
	0.003	0.6108	6.3506	0.120208	2.05302	75
	0.006	0.01554	1.0706	−0.07679	0.65137	33
Netherlands	0.0003	0.130425	2.83769	−0.07039	0.008238	388
	0.003	−0.004878	0.98093	−0.08868	0.653257	27
	0.006	−0.268503	0.85232	−0.35908	0.802506	4
Italy	0.0003	0.312423	5.18334	−0.07347	0.01622	383
	0.003	0.300252	2.33312	0.157807	1.70066	21
	0.006	0.0886	1.06999	0.028782	1.02274	3
Germany	0.0003	0.091882	2.29278	−0.0708	0.003871	424
	0.003	−0.0158467	0.92806	−0.09589	0.564661	33
	0.006	−0.0945394	0.95462	−0.18237	0.912461	3
France	0.0003	0.06313	1.86172	−0.07302	0.003239	422
	0.003	−0.009432	0.96208	−0.0845	0.660295	25
	0.006	−0.298783	0.91634	−0.39586	0.889158	2
Denmark	0.0003	0.27698	4.73369	−0.07208	0.028311	340
	0.003	0.002446	1.00861	−0.07523	0.735199	21
	0.006					0
Belgium	0.0003	0.459337	7.54555	−0.0652	0.07089	310
	0.003	0.06287	1.13643	−0.043	0.90669	15
	0.006	0.367546	1.14334	0.323736	1.12626	1

(*Table continued*)

Appendix 3. Continued

	k	Daily profit without considering transaction fees (%)	Profit without considering transaction fees	Daily profit with considering transaction fees (%)	Profit with considering transaction fees	Number of cycles
Austria	0.0003	0.100864	2.32132	−0.07505	0.016885	327
	0.003	−0.033122	0.93508	−0.10594	0.792356	11
	0.006					0

Note: Missing data means that there are no results for that value of k and h

Appendix 4. Comparison of market profit with total profit, with and without transaction fees

	Profit of Market (MP)	Daily profit without considering transaction fees (DP)	Total Profit without considering transaction fees (TP)	Daily profit with considering transaction fees (DP′)	Total Profit with considering transaction fees (TP′)
Philippine	0.5776	0.83666	7.4339	−0.09482	0.270798
Malaysia	0.6685	1.01528	13.6402	−0.07259	0.09627
Taiwan	0.6878	0.09913	1.8188	−0.11791	0.02605
Japan	0.6934	−0.04652	0.4143	−0.07938	0.00055
Singapore	0.7592	0.22936	3.8142	−0.08026	0.01519
Thailand	0.9703	0.73985	6.8004	−0.11174	0.12393
Indonesia	1.1335	2.06851	17.5688	−0.05294	0.57596
South Korea	1.1897	0.39715	4.3758	−0.11071	0.05901
India	1.3142	0.29687	3.5204	−0.10964	0.069171
China	1.364	0.08003	1.6642	−0.11474	0.04764
Sri Lanka	1.5294	0.93157	8.2756	−0.01392	0.89131
New Zealand	1.604	0.15933	3.5445	−0.06168	0.014994
France	1.6156	0.06313	1.8617	−0.07302	0.003239
Belgium	1.6537	0.45934	7.5456	−0.0652	0.07089
Austria	1.7002	0.10086	2.3213	−0.07505	0.016885
Germany	1.7596	0.09188	2.2928	−0.0708	0.003871
Netherlands	1.7985	0.13043	2.8377	−0.07039	0.008238
Argentina	1.8165	0.59013	6.3761	−0.10246	0.06657
Swiss	1.9457	0.23408	4.2561	−0.07027	0.02257
Israel	1.9839	0.2277	2.6235	−0.12558	0.104604
Italy	2.1745	0.31242	5.1833	−0.07347	0.01622
Denmark	2.352	0.27698	4.7337	−0.07208	0.028311
Egypt	2.6652	0.30788	3.1367	−0.03433	0.761742
Pakistan	2.8445	0.75142	7.7778	−0.08898	0.1974
Canada	3.6512	0.52095	14.8468	−0.03722	0.010788
Mexico	4.2849	2.74287	37.3705	−0.06576	0.12804
Hong Kong	4.8218	3.27327	75.467	−0.04364	0.007183
Australia	4.8548	0.42723	12.4283	−0.03724	0.00389
USA S&P500	8.1922	0.21991	7.5138	−0.03375	0.000235
Russia	9.2381	4.78621	43.7409	−0.0631	0.43648
Turkey	9.8567	1.17897	10.7619	−0.11058	0.0844
Brazil	56.8008	6.66754	94.6789	−0.06573	0.07644