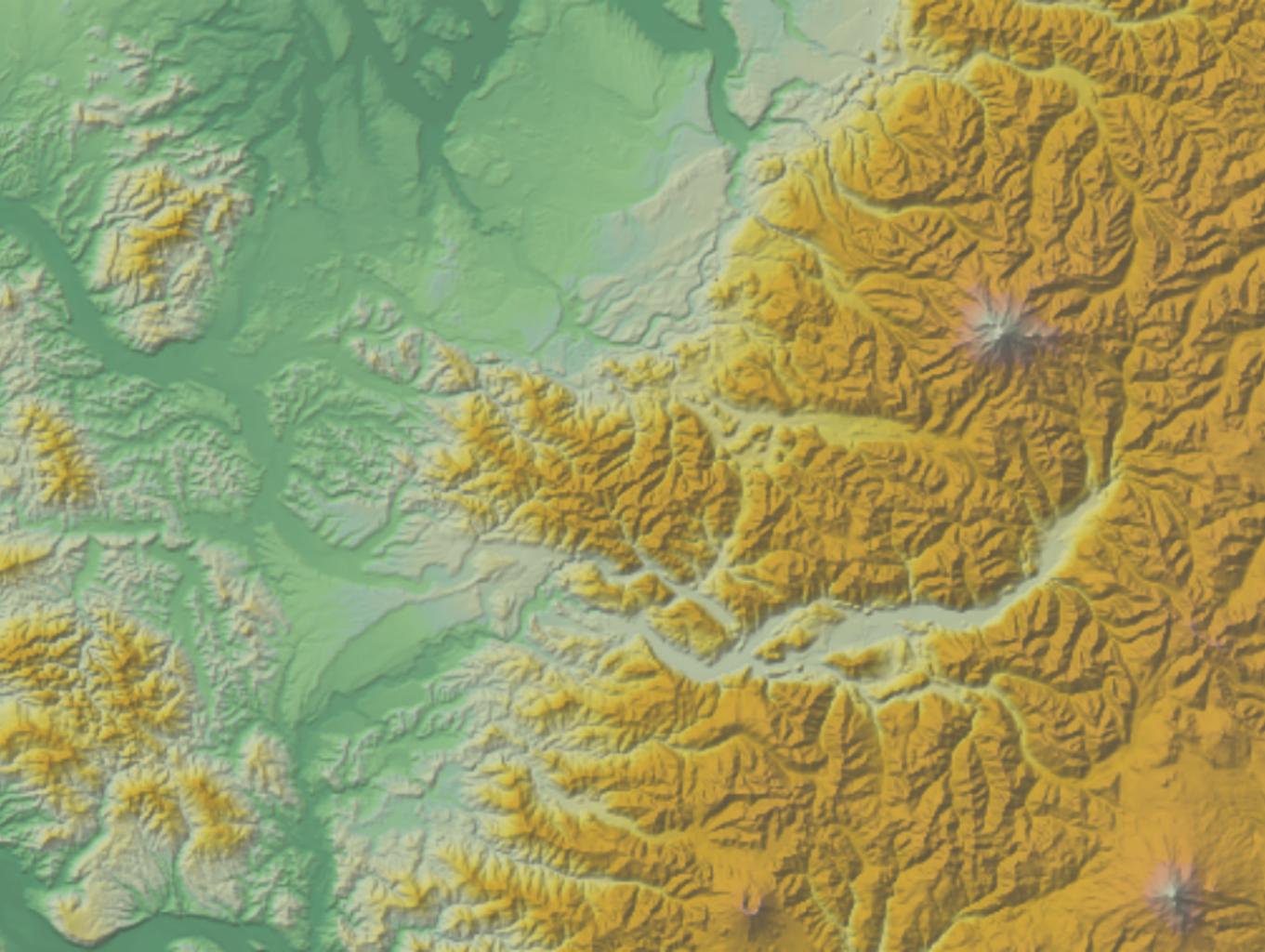


Simplifying the peaks of a function

Ulrich Bauer

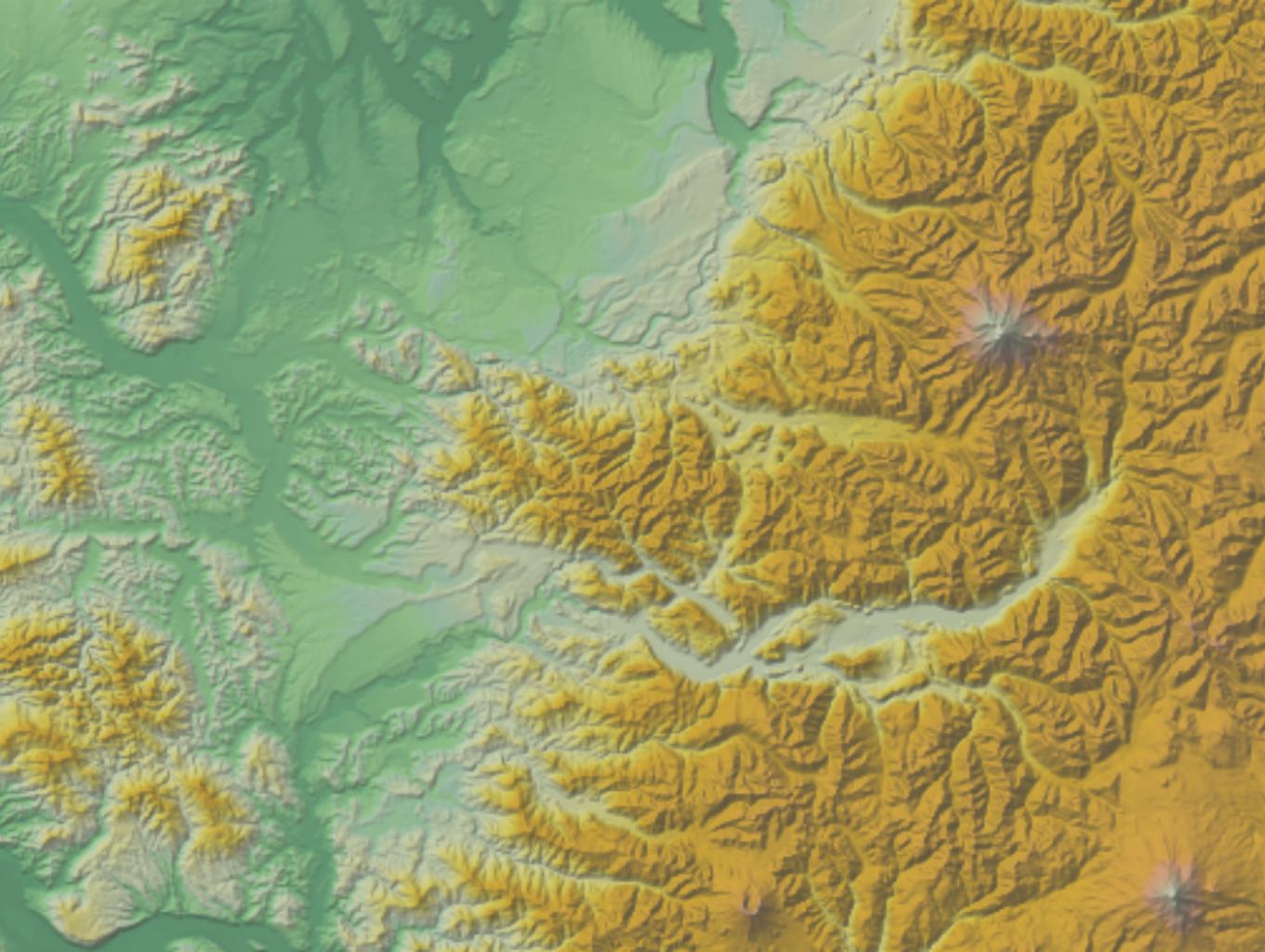
Jan 11, 2013





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Image State of Oregon

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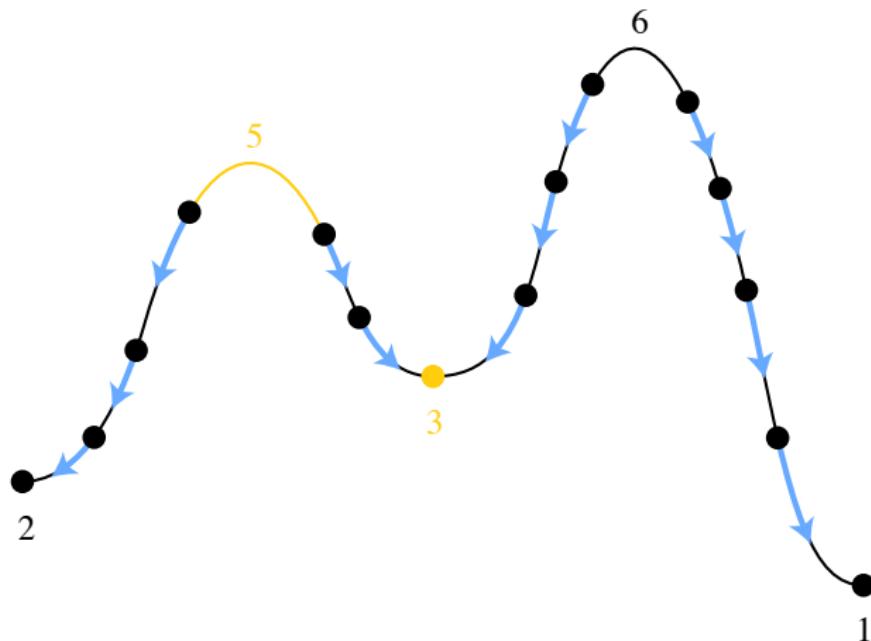
Goal

Given a height function f and a tolerance $\delta > 0$,
find a function g that:

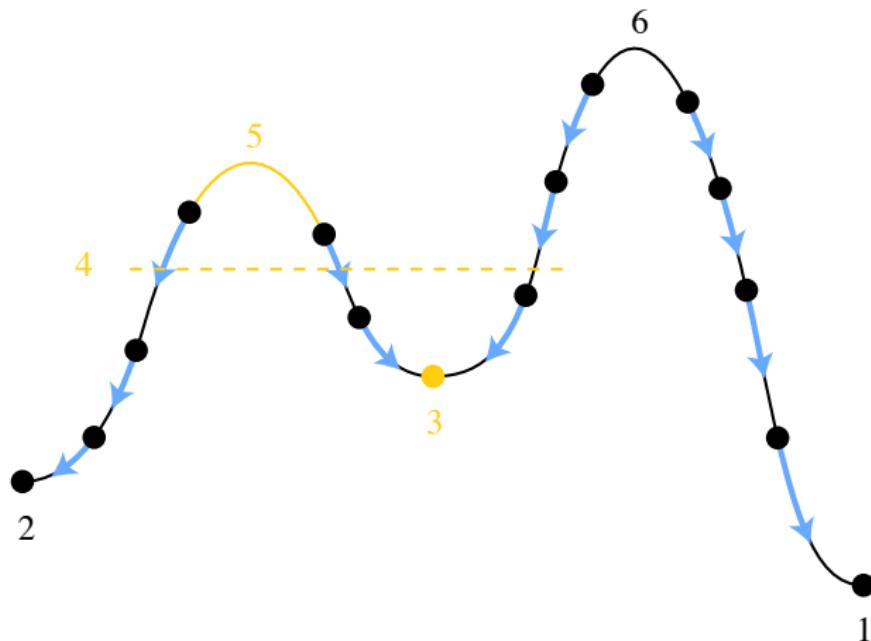
- ▶ minimizes number of critical points (local maxima, local minima, saddles)
- ▶ stays within the tolerance: $|g(x) - f(x)| \leq \delta$ for all x



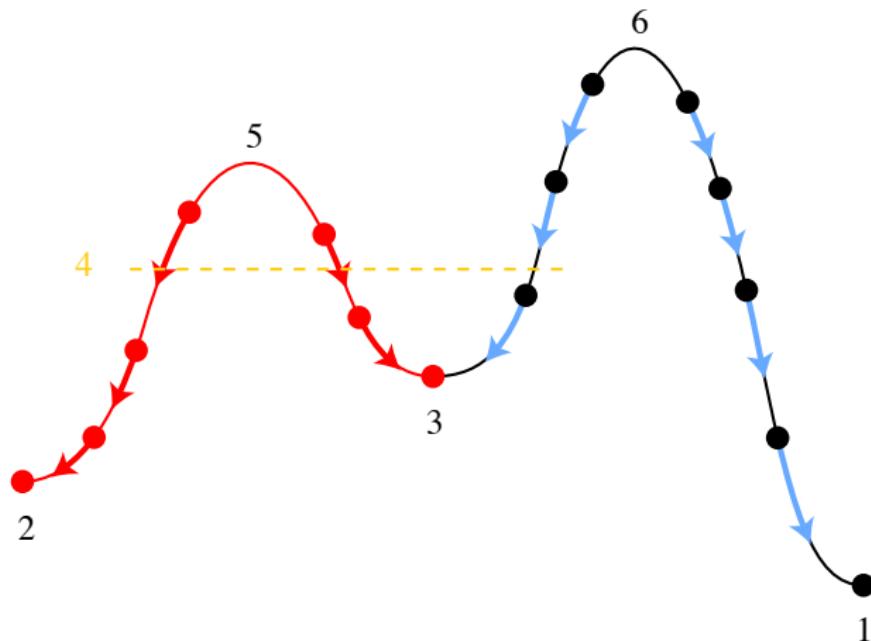
The bulldozer: eliminating a pair of critical points



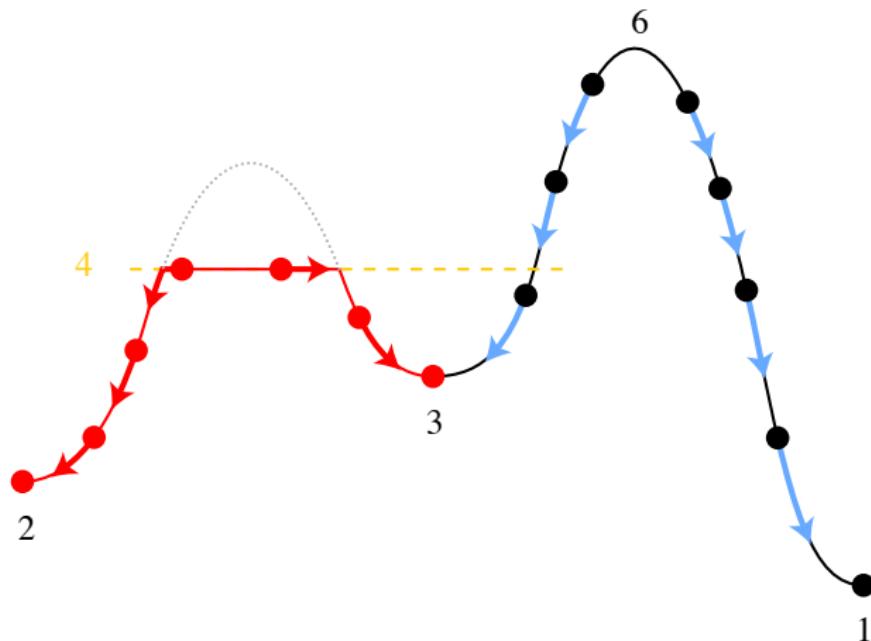
The bulldozer: eliminating a pair of critical points



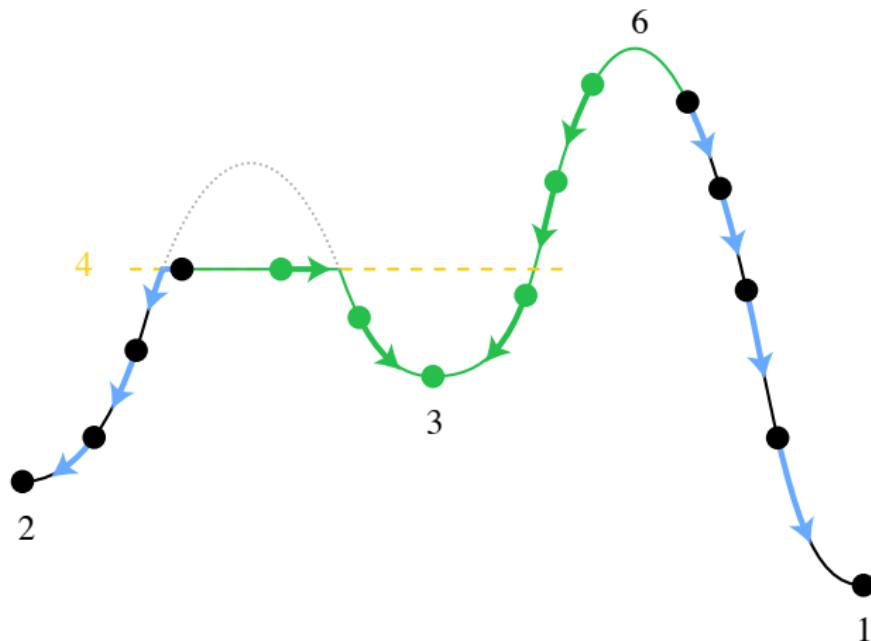
The bulldozer: eliminating a pair of critical points



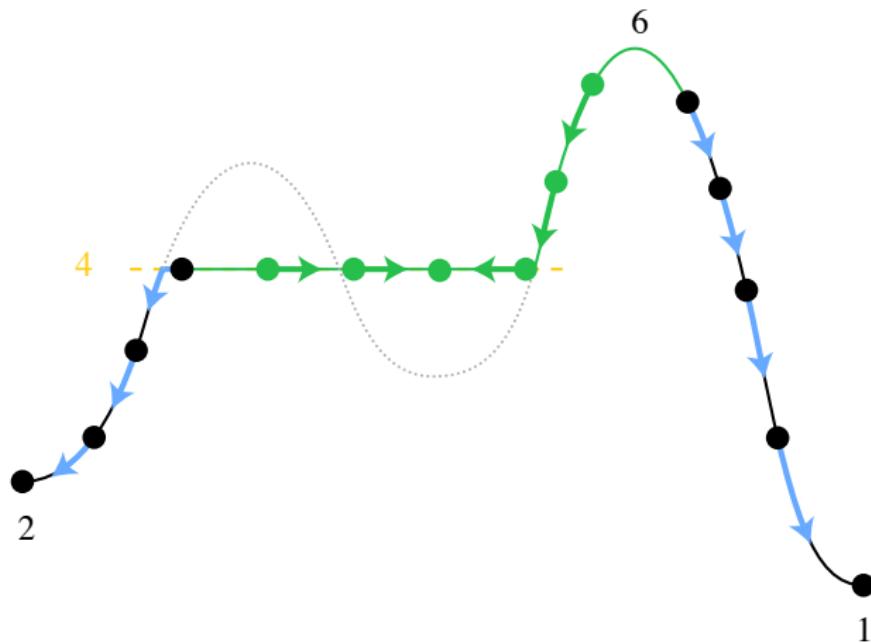
The bulldozer: eliminating a pair of critical points



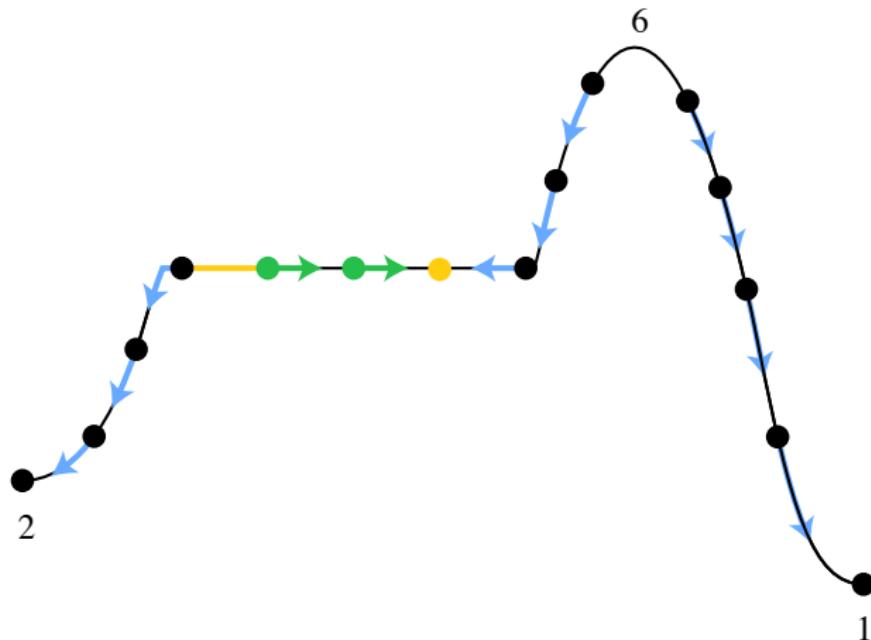
The bulldozer: eliminating a pair of critical points



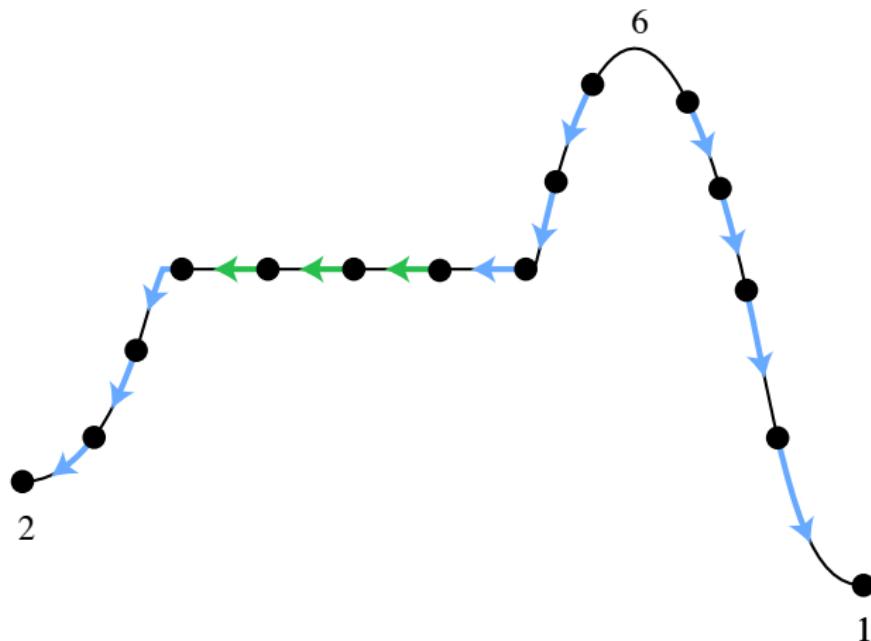
The bulldozer: eliminating a pair of critical points



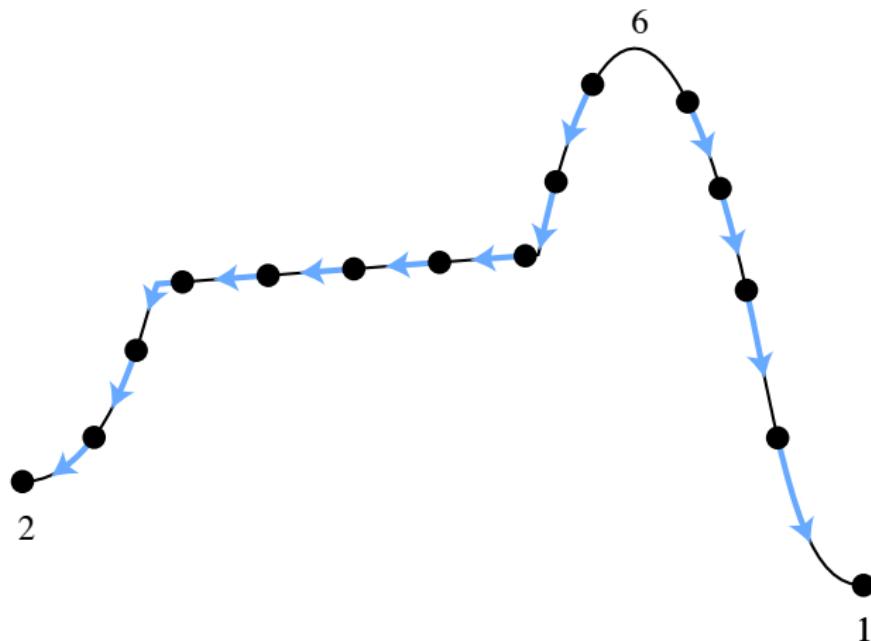
The bulldozer: eliminating a pair of critical points



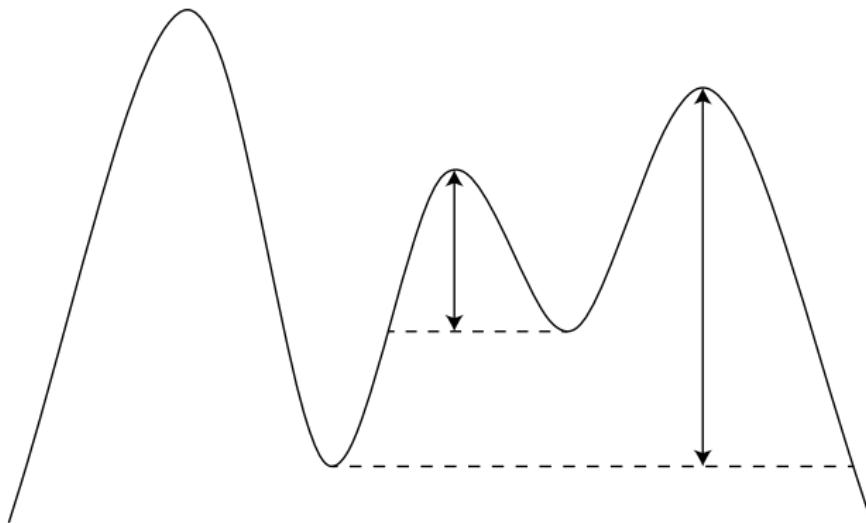
The bulldozer: eliminating a pair of critical points



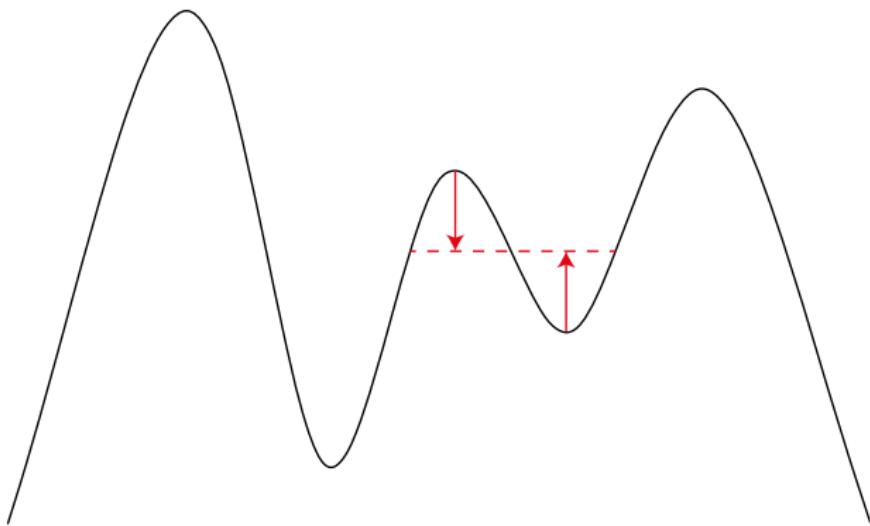
The bulldozer: eliminating a pair of critical points



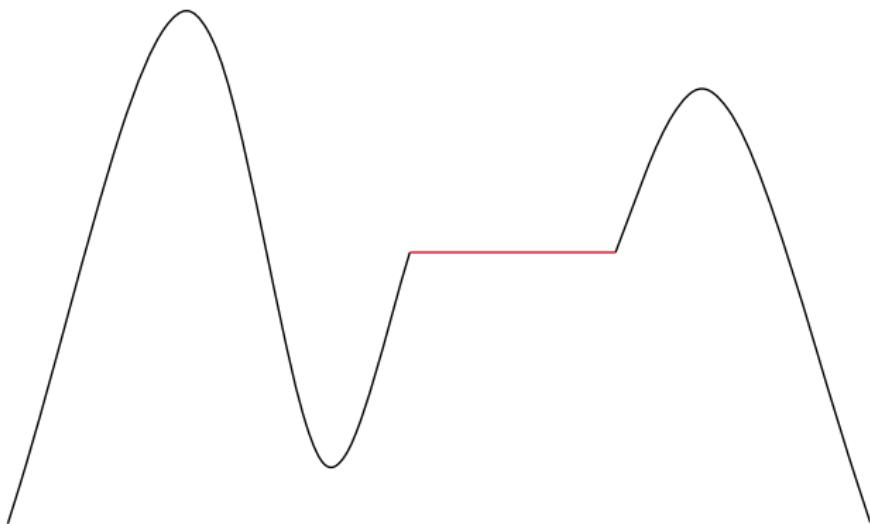
The persistence of a peak



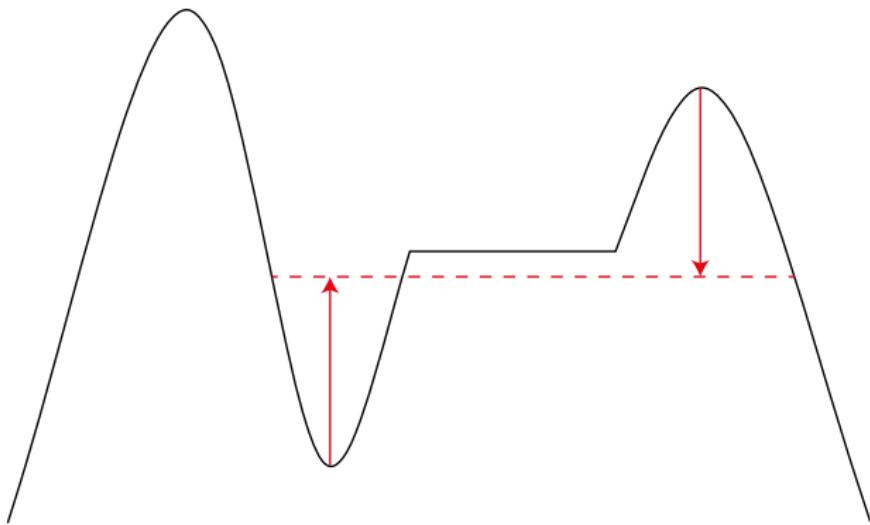
Eliminating persistence pairs



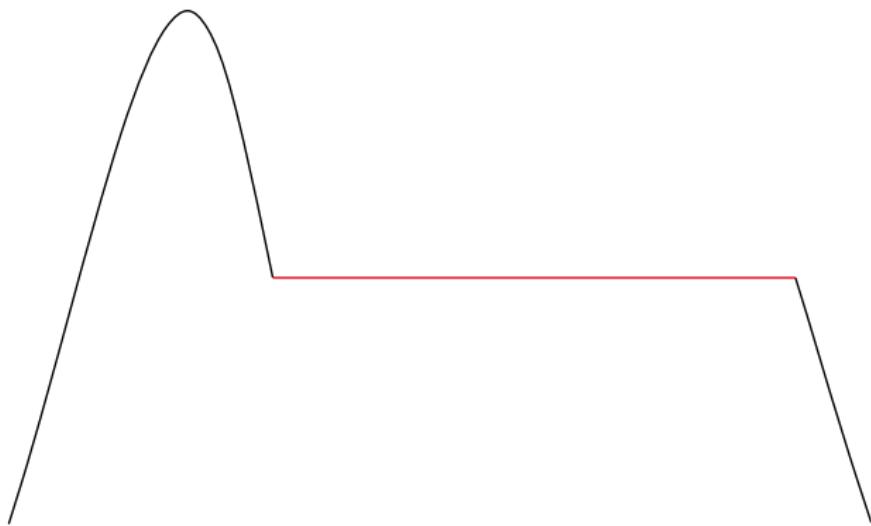
Eliminating persistence pairs



Eliminating persistence pairs



Eliminating persistence pairs



Minimizing critical points

In dimensions 1, 2:

- ▶ Eliminate all persistence pairs with persistence $\leq 2\delta$
- ▶ Result minimizes # critical points within tolerance δ

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But we can still do something nice...

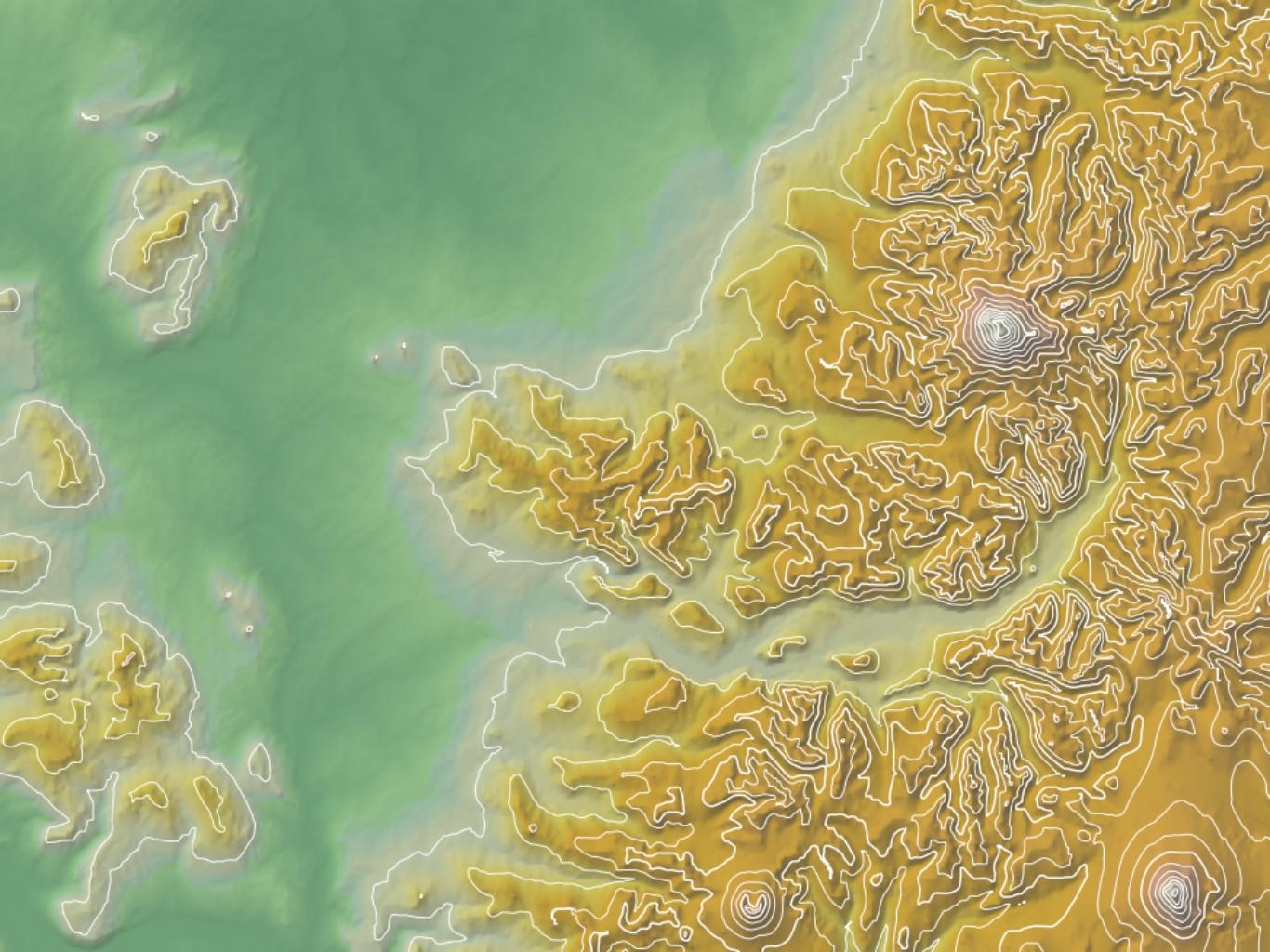
Minimizing local extrema

In any dimension:

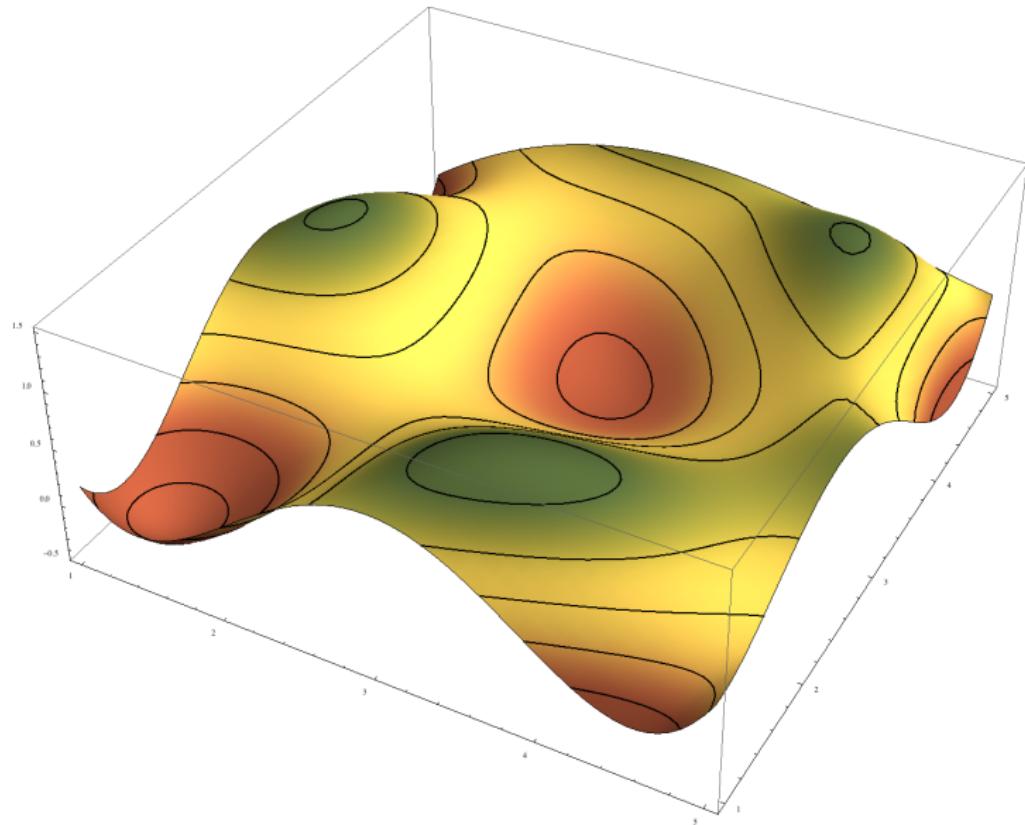
- ▶ Eliminate all local maxima with persistence $\leq 2\delta$
- ▶ Result minimizes # local maxima within tolerance δ



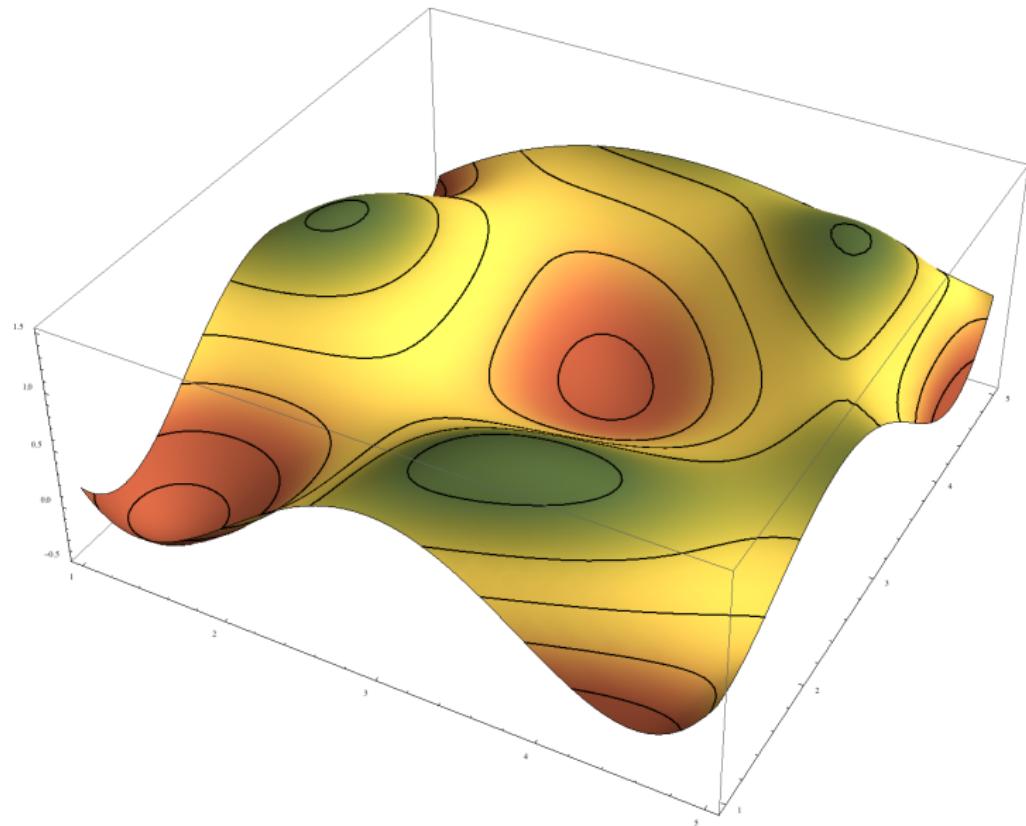




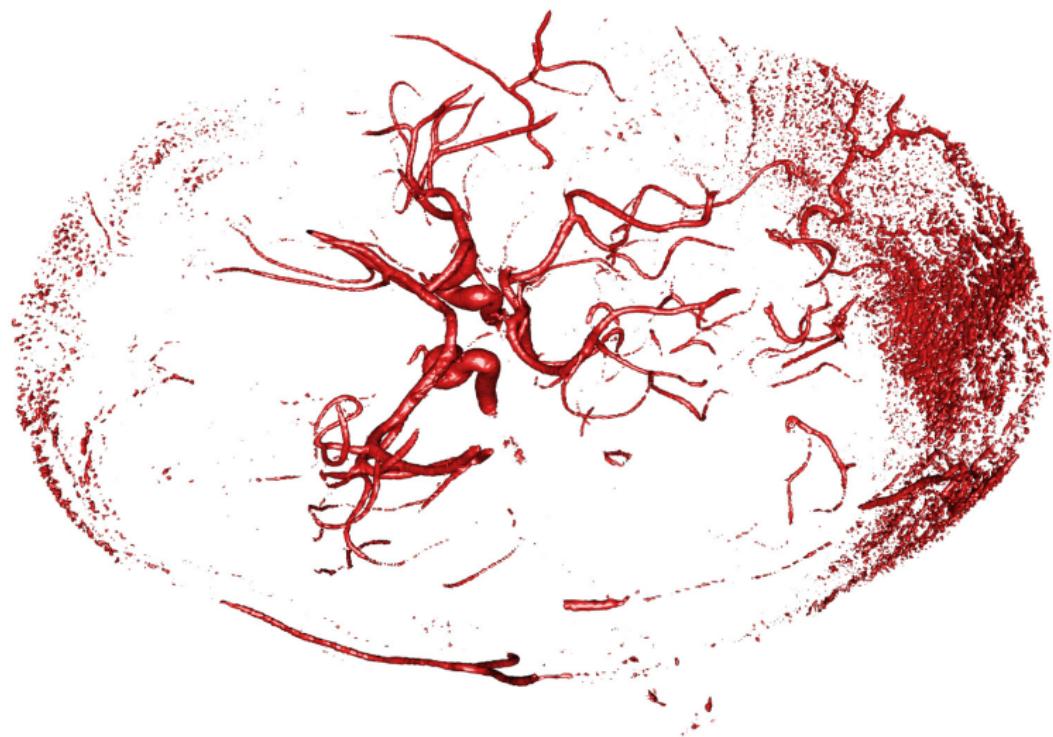
Critical points and topology of sublevel sets



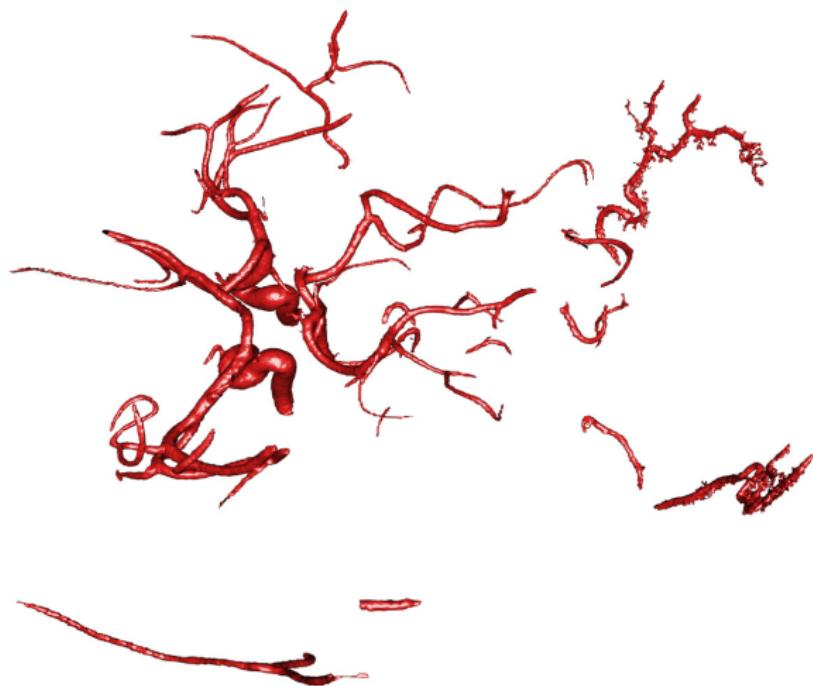
Critical points and topology of sublevel sets



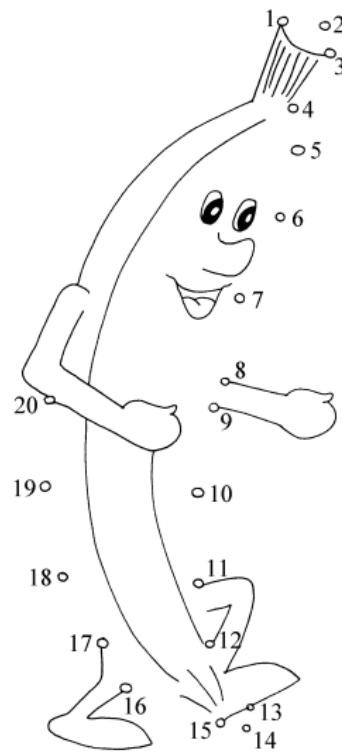
Removing local extrema from 3D data



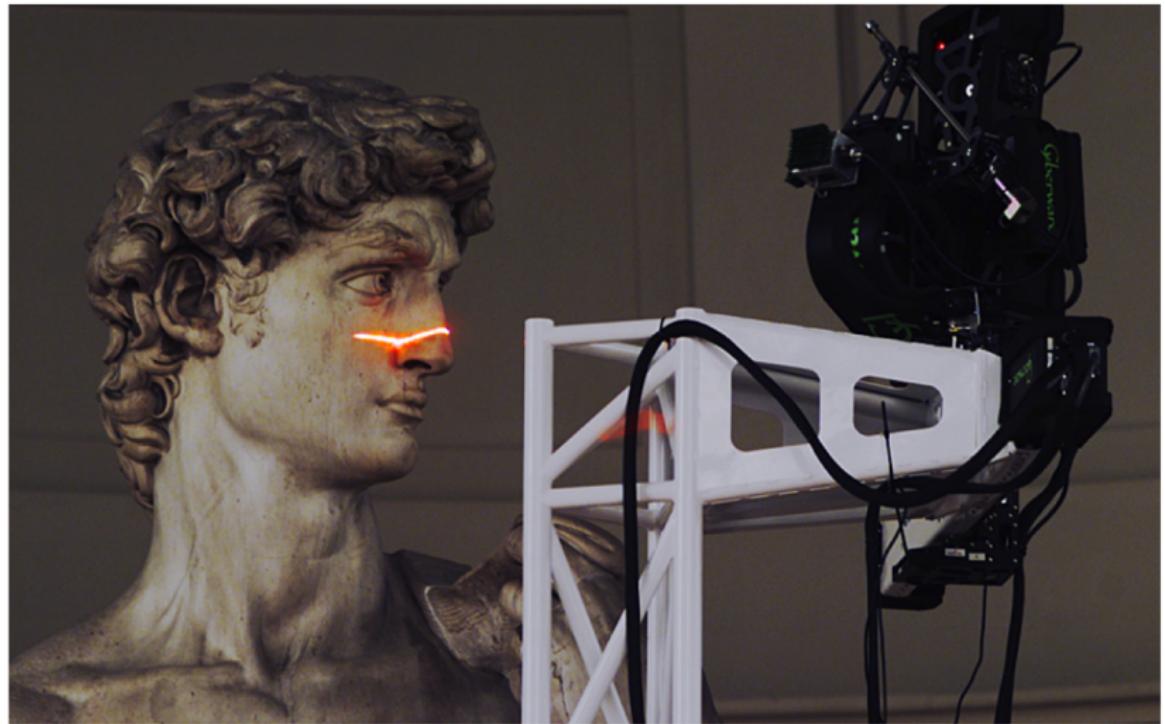
Removing local extrema from 3D data



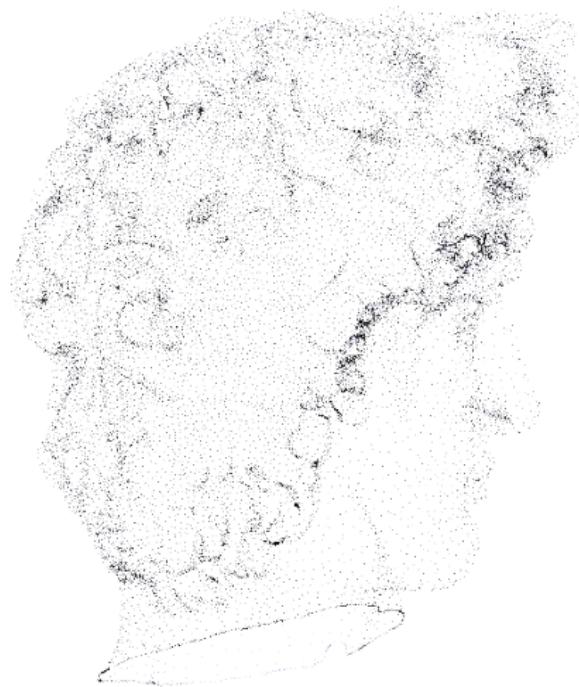
An old problem...



Connect the dots



Connect the dots



Connect the dots



Thanks for your attention!