



Parametric Reconstruction of Bent Tube Surfaces

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joint work with Konrad Polthier

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Mathematics for key technologies



- 1 Reconstruction of bent tubes
- 2 Overview of the algorithm

- Spine curve computation
- Curve reconstruction
- Spine curve segmentation
- Least squares distance optimization

- 3 Results

- Performance
- Examples



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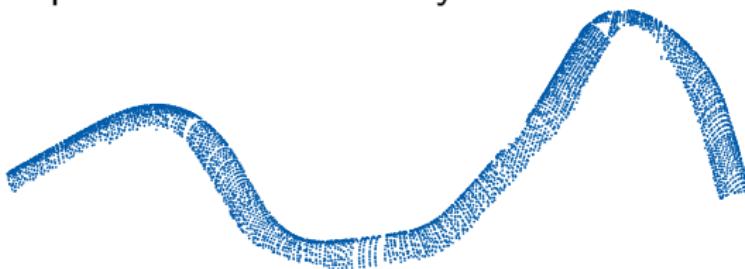
A problem from industry...



- ▷ Given: a bent metal tube



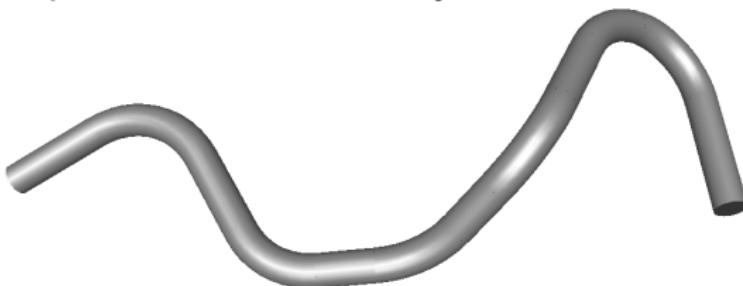
A problem from industry...



- ▷ Given: a bent metal tube
- ▷ Input: a point cloud of the tube surface (from laser scanner)

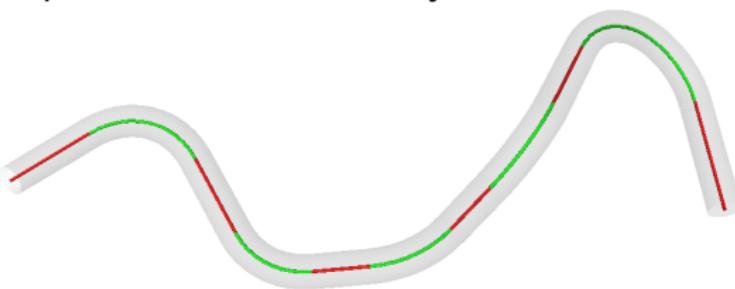


A problem from industry...



- ▷ Given: a bent metal tube
- ▷ Input: a point cloud of the tube surface (from laser scanner)
- ▷ Wanted: a parametric description of the tube surface

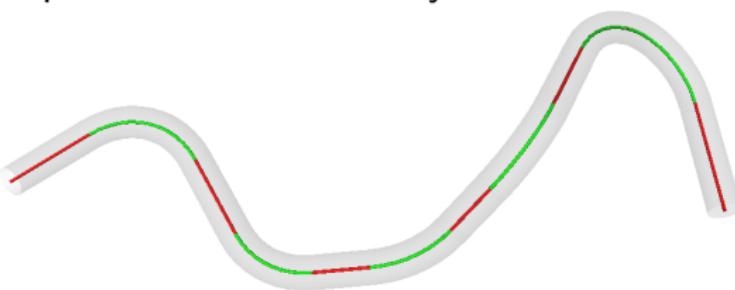
A problem from industry...



- ▷ Given: a bent metal tube
- ▷ Input: a point cloud of the tube surface (from laser scanner)
- ▷ Wanted: a parametric description of the tube surface
- ▷ Surface consists of G^1 continuous cylinder and torus segments



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- ▷ *Pipe surface* (envelope of a ball moving along the *spine curve*)

1 Reconstruction of bent tubes

2 Overview of the algorithm

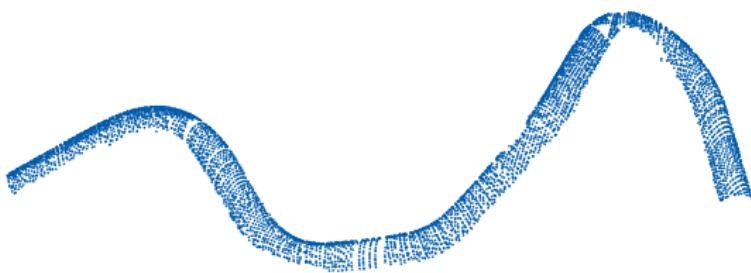
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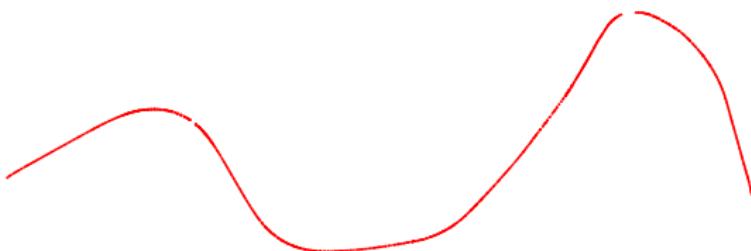
Overview of the algorithm



Decompose into subproblems:



Overview of the algorithm

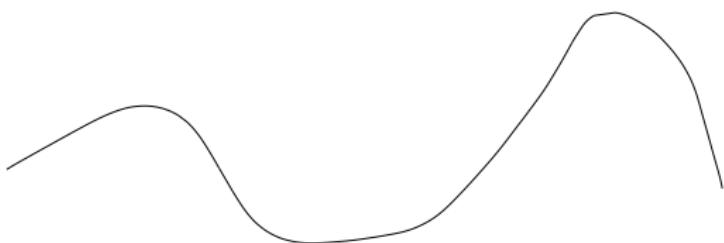


Decompose into subproblems:

- ▷ Project the surface points onto the spine curve



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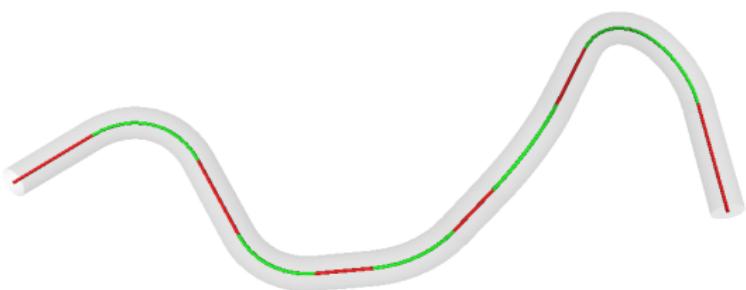


Decompose into subproblems:

- ▷ Project the surface points onto the spine curve
- ▷ Join and simplify the spine curve points

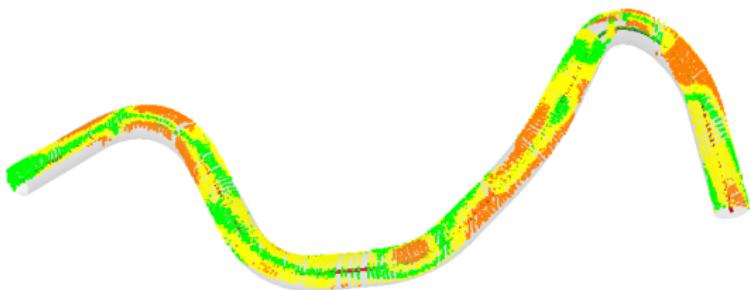


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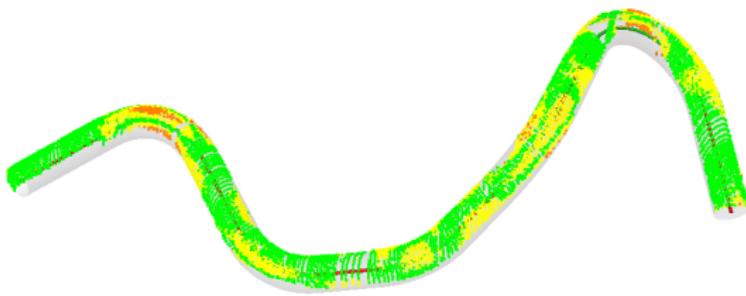
Decompose into subproblems:

- ▷ Project the surface points onto the spine curve
- ▷ Join and simplify the spine curve points
- ▷ Approximate the curve by G^1 continuous arcs and line segments



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- ▷ Project the surface points onto the spine curve
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Result: accurate parametric reconstruction of the tube surface

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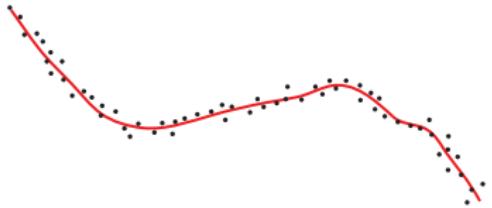


Moving least squares projection

To find the spine curve, we adapt the **moving least squares** method.

MLS curve (D. Levin, 1998)

Define a curve from a set of points





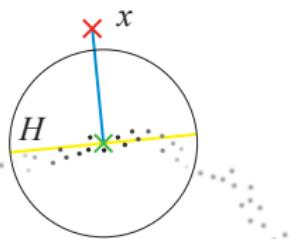
Moving least squares projection

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MLS curve (D. Levin, 1998)

Define a curve from a set of points

Let x be any point in the plane.



- ▷ Locally biased **least squares** fitting of a hyperplane H
- ▷ Center of bias **moves** with the projection of point x onto H
- ▷ x lies on optimal $H \Leftrightarrow x$ is a point on the MLS curve



Cylinder MLS projection

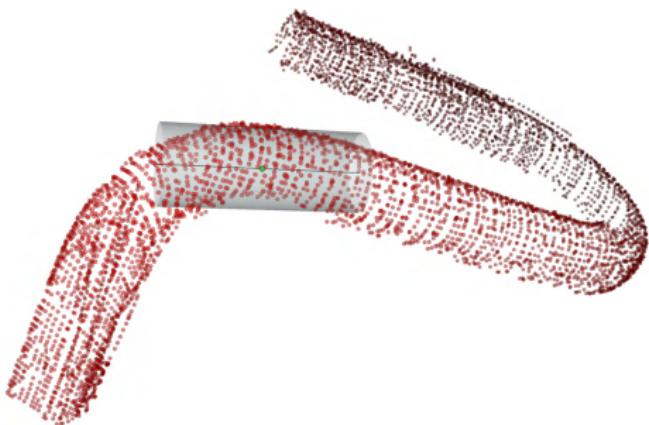
Idea: extend MLS projection to other primitives than hyperplanes



Cylinder MLS projection

Idea: extend MLS projection to other primitives than hyperplanes

- ▷ Locally fit cylinders to surface samples

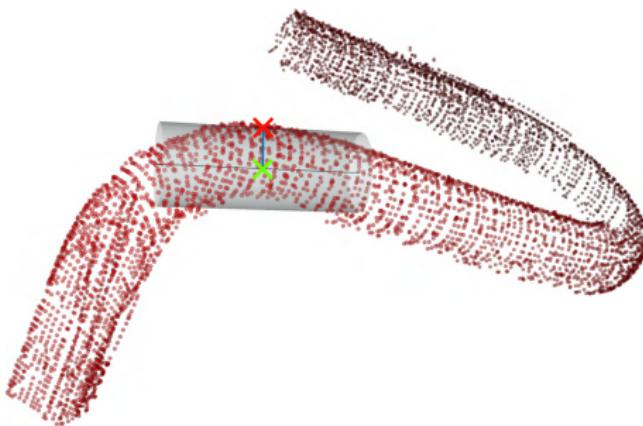




Cylinder MLS projection

Idea: extend MLS projection to other primitives than hyperplanes

- ▷ Locally fit cylinders to surface samples
- ▷ Project samples onto axis of cylinders

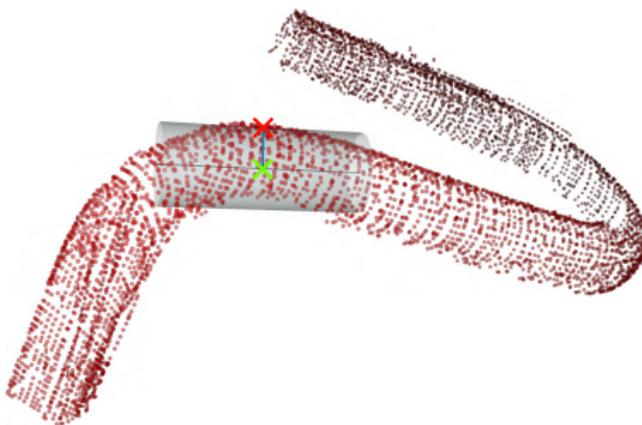




Cylinder MLS projection

Idea: extend MLS projection to other primitives than hyperplanes

- ▷ Locally fit cylinders to surface samples
- ▷ Project samples onto axis of cylinders
- ▷ Goal: approximate spine curve of pipe surface





Comparison to previous work



Our method



Comparison to previous work



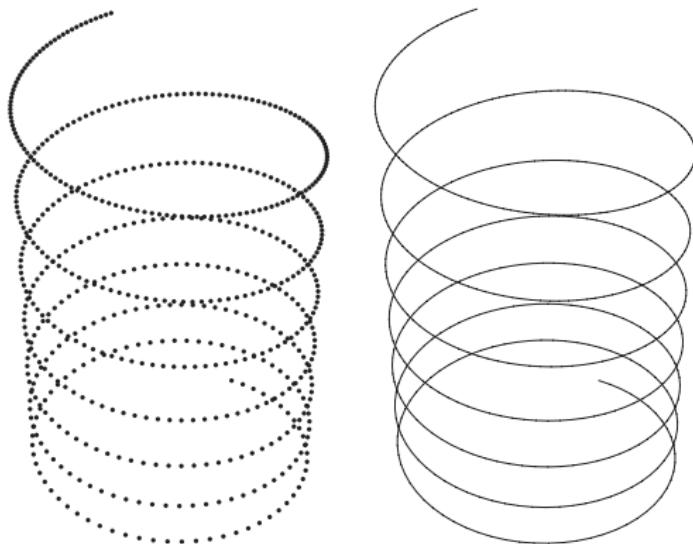
Our method

In-Kwon Lee, 2000

- ▷ Previous work: only use estimated normals and radius to shift samples
- ▷ Additional smoothing required

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Curve reconstruction: widely considered problem



We used the NN-Crust algorithm (Dey & Kumar, 1999)
Simplify polygonal curve for faster processing (Daescu, 2003)

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Curve approximation by arc-line splines

Find a G^1 continuous curve of arc and line segments
(arc-line spline)

- ▷ with distance $< \epsilon$ to the vertices of the input polygon
- ▷ with minimum number of segments

This problem is similar to a simpler problem...



Polygon simplification

Problem: Given a polygonal curve P .

Find another polygonal curve P' with distance $d(P, P') < \epsilon$
with minimum number of segments.



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Framework (H. Imai, M. Iri, 1986) used by many algorithms:

- ▷ Build a shortcut graph over vertices of polygon
- ▷ An edge e_{ij} is in the graph if the line segment $\overline{p_i p_j}$ approximates $[p_i, p_{i+1}, \dots, p_j]$
- ▷ Find a shortest path through the shortcut graph
- ▷ $\mathcal{O}(n^3)$ time, $\mathcal{O}(n)$ space (don't construct graph explicitly)



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Restriction to vertices of input polygon

For unrestricted vertex positions in \mathbb{R}^3 , no algorithm known

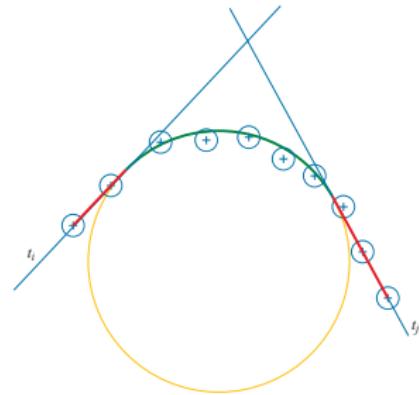


Arc-line spline simplification

Try something similar for arc-line splines

- ▷ Restrict solution set to something we can handle by a graph
- ▷ Find optimal solution for the restricted set using BFS

Estimated tangent lines as vertices of graph





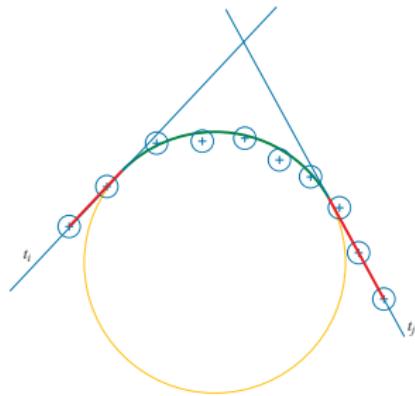
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Estimated tangent lines as vertices of graph

Problem: tangent lines are in general not coplanar

- ▷ To compute edge e_{ij} , adjust (tilt) tangent line t_j to make it coplanar with t_i
- ▷ Shortest path to j determines tangent line t_j for further computations



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Fine-tune parameters of arc-line spline

- ▷ Minimize least squares distance of surface to input samples
- ▷ Nonlinear optimization problem
- ▷ Levenberg-Marquardt, approximated Jacobian

Use LRA parametrization of arc-line splines

- ▷ Length between bends
- ▷ Rotation between bends
- ▷ Angle of bend
- ▷ Radius of bend (often fixed)

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Typical example

- ▷ 4 mm pipe radius
- ▷ 268 mm length
- ▷ 5 bends
- ▷ 6005 samples

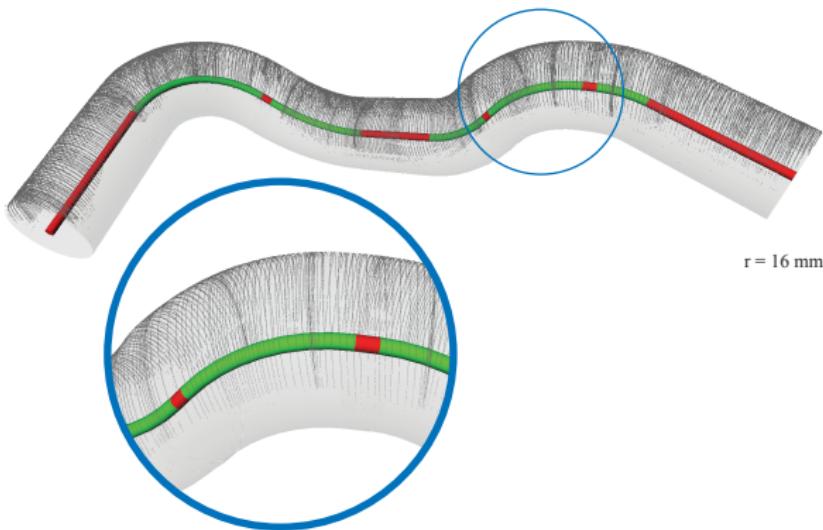
Total running time: 50 seconds

- ▷ Spine curve computation: 16 seconds
- ▷ Curve reconstruction: < 1 second
- ▷ Curve simplification: < 1 second
- ▷ Arc-line spline approximation: < 1 second
- ▷ Least-squares fitting: 34 seconds

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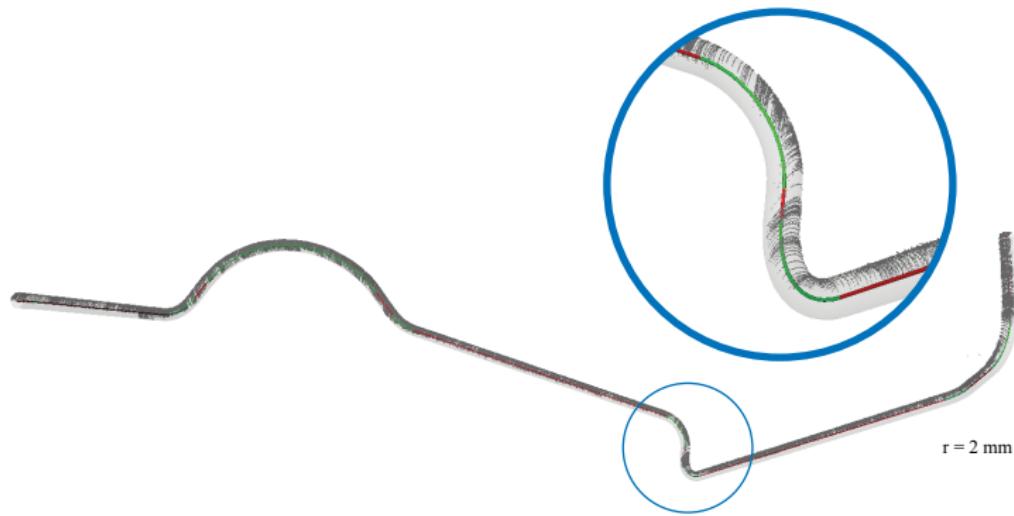


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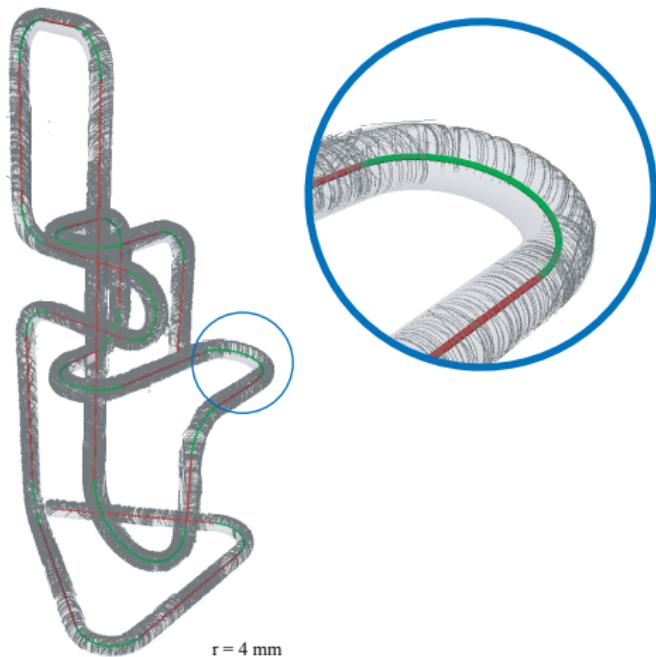


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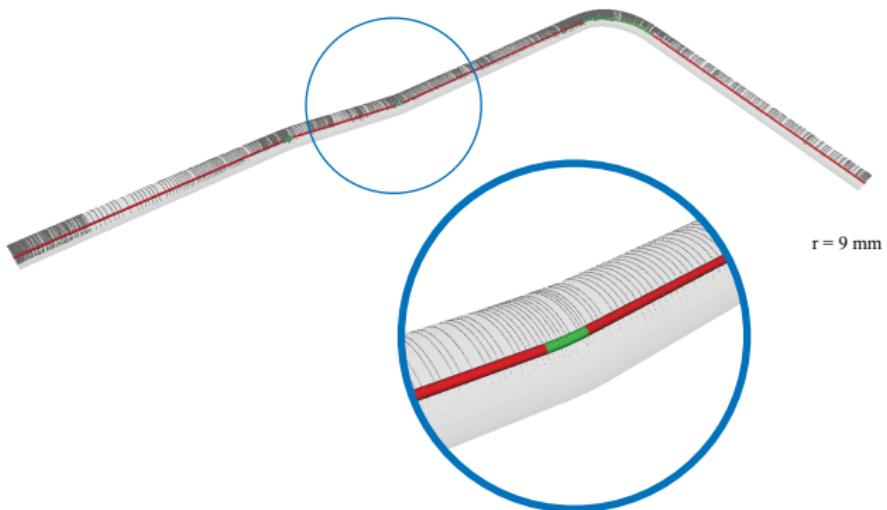


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Examples







Demonstration