Parametric Reconstruction of Bent Tube Surfaces

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joint work with Konrad Polthier

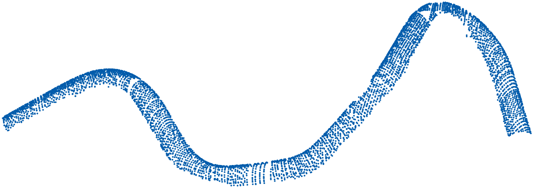
DFG Research Center MATHEON
Mathematics for key technologies

Industry Challenges in Geometric Modeling, CAD and Simulation, 14.3.2008

A problem from industry...

- Given: a bent metal tube

A problem from industry...



- Given: a bent metal tube
- Input: a point cloud of the tube surface (from laser scanner)

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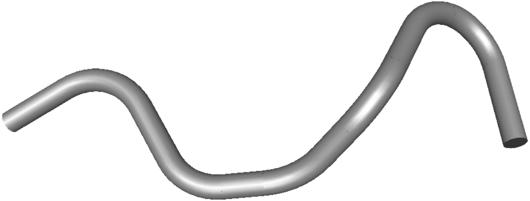
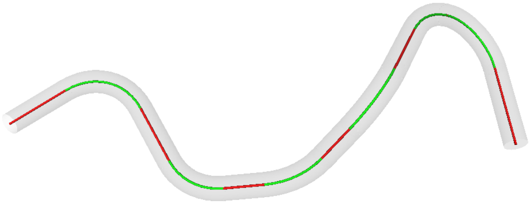


- ▷ Given: a bent metal tube
- ▷ Input: a point cloud of the tube surface (from laser scanner)
- ▷ Wanted: a parametric description of the tube surface

Problem description

A problem from industry. . .



- Given: a bent metal tube
- Input: a point cloud of the tube surface (from laser scanner)
- Wanted: a parametric description of the tube surface
- Surface consists of G^1 continuous cylinder and torus segments

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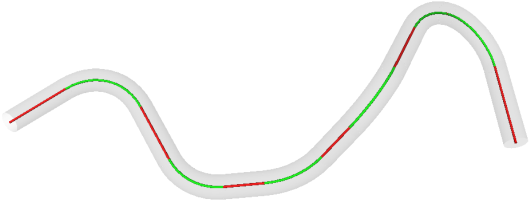

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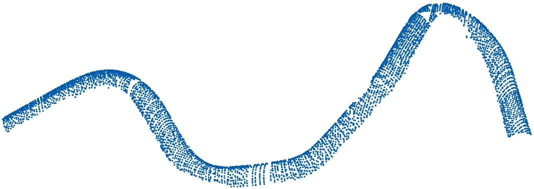
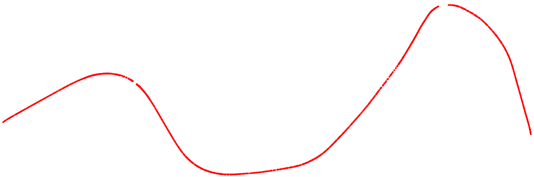
- Input: a point cloud of the tube surface (from laser scanner)
- Wanted: a parametric description of the tube surface
- Surface consists of G^1 continuous cylinder and torus segments
- *Pipe surface* (envelope of a ball moving along the *spine curve*)

1. Reconstruction of bent tubes
2. Overview of the algorithm
   - Spine curve computation
   - Curve reconstruction
   - Spine curve segmentation
   - Least squares distance optimization
3. Results
   - Performance
   - Examples

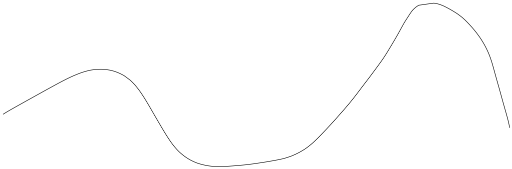


Decompose into subproblems:



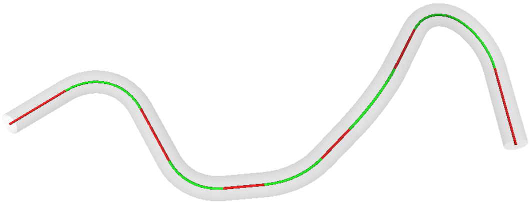
Decompose into subproblems:

- Project the surface points onto the spine curve



Decompose into subproblems:

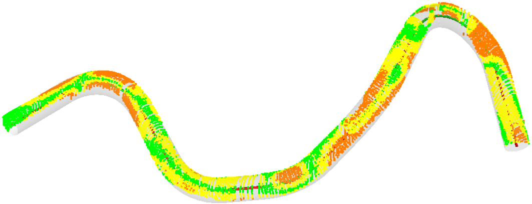
- Project the surface points onto the spine curve
- Join and simplify the spine curve points



Decompose into subproblems:

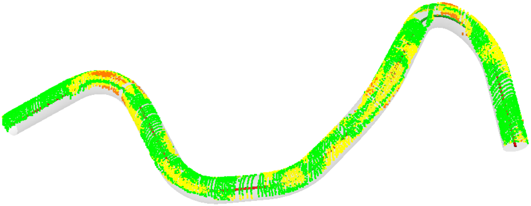
- ▷ Project the surface points onto the spine curve
- ▷ Join and simplify the spine curve points
- ▷ Approximate the curve by G^1 continuous arcs and line segments

Overview of the algorithm



Decompose into subproblems:

- ▷ Project the surface points onto the spine curve
- ▷ Join and simplify the spine curve points
- ▷ Approximate the curve by G^1 continuous arcs and line segments
- ▷ Fit the surface to the input samples



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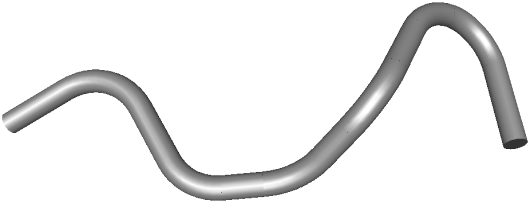

Decompose into subproblems:

- Project the surface points onto the spine curve
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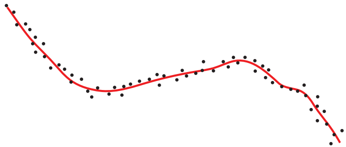
Result: accurate parametric reconstruction of the tube surface

Outline

To find the spine curve, we adapt the moving least squares method.



MLS curve (D. Levin, 1998)

Define a curve from a set of points

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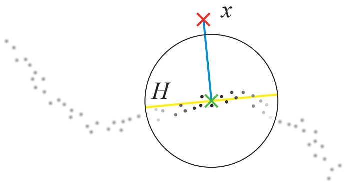

MLS curve (D. Levin, 1998)

Define a curve from a set of points

Let x be any point in the plane.

- Locally biased least squares fitting of a hyperplane H
- Center of bias moves with the projection of point x onto H
- x lies on optimal $H \Leftrightarrow x$ is a point on the MLS curve

Idea: extend MLS projection to other primitives than hyperplanes

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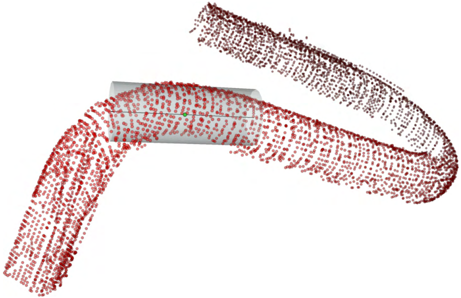
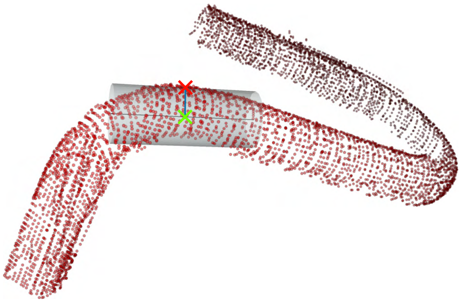
- ▷ Locally fit cylinders to surface samples



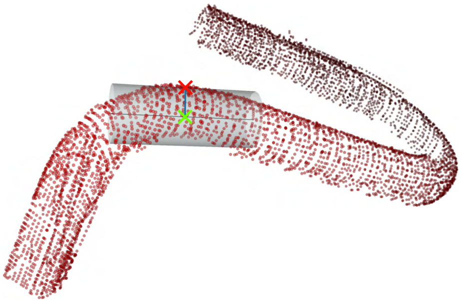
Idea: extend MLS projection to other primitives than hyperplanes

- Locally fit cylinders to surface samples
- Project samples onto axis of cylinders

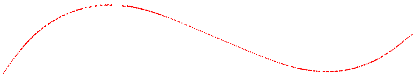


Idea: extend MLS projection to other primitives than hyperplanes

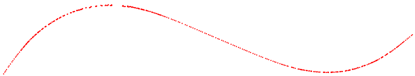
- ▷ Locally fit cylinders to surface samples
- ▷ Project samples onto axis of cylinders
- ▷ Goal: approximate spine curve of pipe surface



Comparison to previous work



Our method



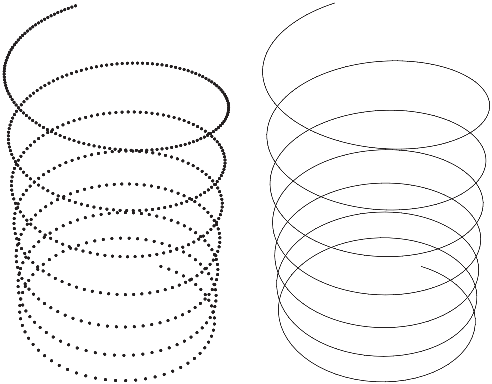
Our method

In-Kwon Lee, 2000

- Previous work: only use estimated normals and radius to shift samples
- Additional smoothing required

Outline

Curve reconstruction: well-investigated problem



We used the NN-Crust algorithm (Dey & Kumar, 1999)
Simplify polygonal curve for faster processing (Daescu, 2003)

Outline

Find a G^1 continuous curve of arc and line segments *(arc-line spline)*

- with distance $< \epsilon$ to the vertices of the input polygon
- with minimum number of segments

This problem is similar to a simpler problem...

Problem: Given a polygonal curve P.
Find another polygonal curve P' with distance $d(P, P') < \epsilon$
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Framework (H. Imai, M. Iri, 1986) used by many algorithms:

- Build a shortcut graph over vertices of polygon
- An edge e_{ij} is in the graph if the line segment $\overline{p_i p_j}$ approximates $[p_i, p_{i+1}, \ldots, p_j]$
- Find a shortest path through the shortcut graph
- $\mathcal{O}(n^3)$ time, $\mathcal{O}(n)$ space (don't construct graph explicitly)

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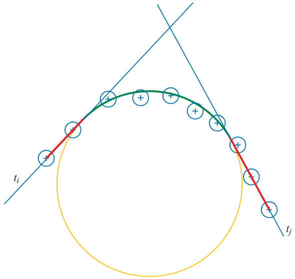
Restriction to vertices of input polygon
For unrestricted vertex positions in $\mathbb{R}^3$, no algorithm known

Arc-line spline simplification

Try something similar for arc-line splines

- Restrict solution set to something we can handle by a graph
- Find optimal solution for the restricted set using BFS

Estimated tangent lines as vertices of graph



Arc-line spline simplification

Try something similar for arc-line splines

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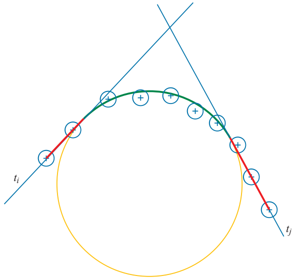
Estimated tangent lines as vertices of graph

Problem: tangent lines are in general not coplanar

- To compute edge e_{ij}, adjust (tilt) tangent line t_j to make it coplanar with t_i
- Shortest path to j determines tangent line t_j for further computations



Least squares distance optimization

Fine-tune parameters of arc-line spline

- Minimize least squares distance of surface to input samples
- Nonlinear optimization problem
- Levenberg-Marquardt, approximated Jacobian

Use LRA parametrization of arc-line splines

- Length between bends
- Rotation between bends
- Angle of bend
- Radius of bend (usually fixed)

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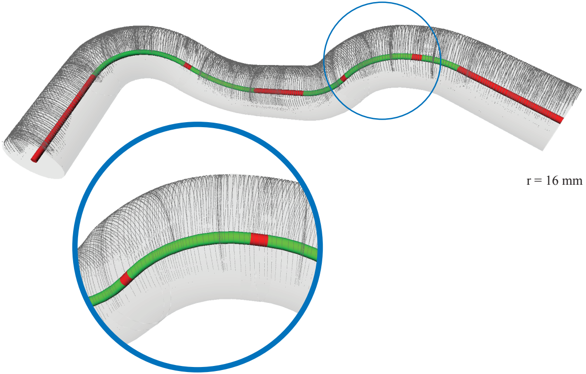
Running time

Typical example

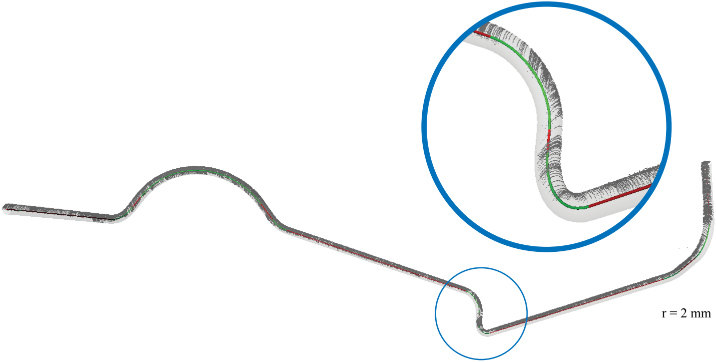
- 4 mm pipe radius
- 268 mm length
- 5 bends
- 6005 samples

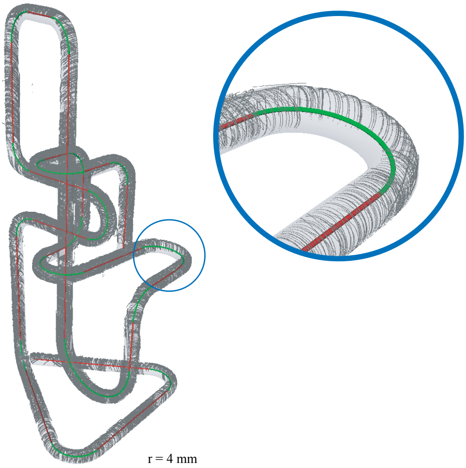
Total running time: 50 seconds

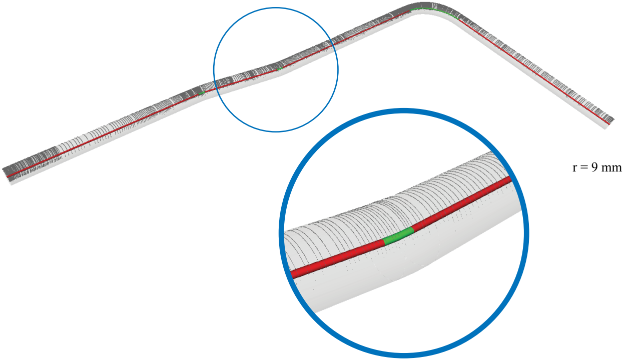
- Spine curve computation: 16 seconds
- Curve reconstruction: < 1 second
- Curve simplification: < 1 second
- Arc-line spline approximation: < 1 second
- Least-squares fitting: 34 seconds


r = 16 mm

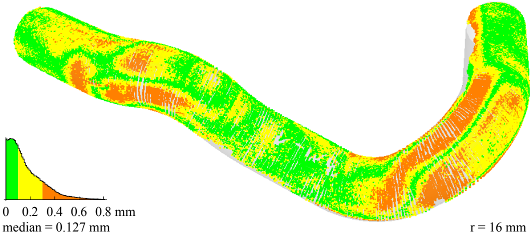
Examples




r = 4 mm



r = 9 mm


0 0.2 0.4 0.6 0.8 mm
median = 0.127 mm
r = 16 mm

Demonstration