Code for Calculating the Spatial Response Component of a Spatiotemporal System Matrix

The following is documentation for code that calculates a spatial system matrix for SPECT image reconstruction that incorporates physical effects of attenuation and depth-dependent collimator response for a parallel-hole collimator. It implements a ray-casting method described by G. L. Zeng, G. T. Gullberg, B. M. W. Tsui, and J. A. Terry in "Three-Dimensional Iterative Reconstruction Algorithms with Attenuation and Geometric Point Response Correction" in IEEE Transactions on Nuclear Science, vol. 38, no. 2, pp. 693-702, 1991 (see "tns38_693.pdf").

Patient Coordinate System (PCS)

The PCS is defined by the geometric relationship between voxels in the attenuation map. When reading voxels sequentially from the file, it is assumed that:

- the x-coordinate changes most rapidly
- then the y-coordinate
- finally the z-coordinate.

For example, for the map in "std916.atn_ds2_half_crop" (see the figure in "atn_maps.pdf") this means that:

- the PCS origin lies at the patient's right anterior abdomen
- the x-axis points from the patient's right to their left
- the y-axis points from their chest to their back
- the z-axis points from their feet to their head.

The PCS axes are also shown in the figure in "patient.pdf".

When the system matrix is created, the columns of the matrix correspond one-to-one to the attenuation map voxels and are in the same sequential order as the voxels.

Detector Coordinate System (DCS)

The axial direction of the DCS, d_z, always aligns with the z-axis of the PCS (see the figure in "patient.pdf"). When the detector is horizontal and above the patient, the transverse direction of the DCS, d_t, aligns with the x-axis of the PCS. Similarly, when the detector is vertical and to the patient's left, d_t aligns with the y-axis of the PCS.

Note that at the gantry angle shown in the figure in "patient.pdf", the geometry is as follows:

- the DCS origin is in the detector corner behind the operator's head
- d_t points from the patient's right anterior to left posterior
- d_z points from the patient's feet to head.

The plane of the detector is assumed to be vertical and to the patient's left initially. The detector is then assumed to rotate counter-clockwise as viewed by a patient who is feet-first in the scanner. That is, the detector starts at their left and then rotates below, to their right, above, and finally back to their left.

If the assumed initial detector position and rotation direction do not correspond to the true position and direction, then either the system matrix rows or the projection data frames will need to be re-ordered so that they synchronize spatially.

The rows of the system matrix are organized according to the following sequence of projection data bins:

- the d_t-coordinate changes most rapidly
- then the d_z-coordinate
- finally the frame number (i.e., the gantry angle).

As mentioned before, if the true sequence of projection data bins varies from that described above, then either the system matrix rows or the bins will need to be re-ordered so that they synchronize spatially.

Modeling for Activity Detected from Outside the Axial Field of Interest

For each detector bin, the collimator provides a diverging field of view (see the left figure in "coll_resp.pdf"). This is because it is possible for an obliquely-incident photon to travel all of the way down a circular collimator hole, provided that the angle of incidence is less than arctan(2R/L), where R and L are the radius and length of the collimator hole, respectively (see the paper by Zeng, Gullberg, et al. in "tns38_693.pdf").

To deal with the diverging field of view, the axial extent of the detector is increased by two bins and the axial extent of image space is increased by the number of additional image slices that are "seen" by the extended detector (see the middle figure in "coll_resp.pdf"). The additional image slices are then treated as low-axial-resolution images on either side of the original images of interest (see the right figure in "coll_resp.pdf").

Although these additional images are of no interest, they must be reconstructed because they contribute "nuisance" counts to the detector bins that directly "see" the images of interest. The extra detector bins are needed to provide additional information to keep the larger inverse problem well-posed. The total number of additional image slices needed is:

$$2 * \text{ceiling}[(W + ((D/L)*2R)) / T]$$

where W is the width of the detector bins, D is the distance from the front of the crystal to the far side of the reconstructed field of view, and T is the thickness of the image slices.

Program for Calculating the Spatial Response

The C program in "FptrespRS.c" makes use of the code in the "cfi*.{h,c}" files. Simply typing "make" should compile and link all of the code. Parameters are input via #define commands and hard-wired file names in "FptrespRS.c". All distances are in cm.

#define commands are needed for:

DETECTOR_RADIUS - distance from center of rotation to front of crystal

FOCAL_LENGTH - parameter "D" mentioned above, i.e., largest perpendicular distance from front of crystal to far side of reconstructed field of view (this is along a diagonal line through a square image)

ANGLE_COUNT - number of geometric angles (i.e., gantry positions) at which projections are acquired

DETECTOR_XY_COUNT - number of detector bins along d_t axis

DETECTOR_Z_BIN - number of detector bins along d_z axis (includes two padded detector bins)

DETECTOR_BIN_WIDTH - parameter "W" mentioned above

COLLIMATOR_RADIUS - parameter "R" mentioned above

COLLIMATOR_LENGTH - parameter "L" mentioned above

COLLIMATOR_OFFSET - distance from front of crystal to back of collimator

VOXEL_{X,Y}_COUNT - numbers of voxels along X- and Y-axes of attenuation map and reconstructed images

VOXEL_Z_COUNT - number of image slices for reconstructed images only (includes two padded low-axial-resolution images in right figure in "coll_resp.pdf")

ATN_Z_COUNT - number of image slices in attenuation map only (includes padded high-axial-resolution images in middle figure in "coll_resp.pdf")

VOXEL_{X,Y,Z}_WIDTH - lengths of voxel edges in attenuation map and reconstructed images

(The next three parameters have to do with the ray-casting algorithm.)

RAY_SAMPLES_PER_VOXEL - minimum number of samples to obtain in each image space voxel while travelling along ray (recommend >= 2 a`la Nyquist)

RAYS_PER_VOXEL - minimum number of rays to intersect each image space voxel at far side of field of view (recommend >= 2 a`la Nyquist)

MAX_PTRESP_COUNT - resulting number of rays needed along each axis of "focal point plane" shown in Figure 10 of paper in "tns38_693.pdf" (i.e., 19x19 rays for example in code; setting this to one turns off point response calculation, as there is just one ray cast per detector bin) hard-wired file names - Search the code for two fopen() statements. There is one fopen() associated with reading the attenuation map binary data file. The other fopen() is in a loop associated with calculating and writing the binary data file for the system matrix for each gantry angle. One file is written for each gantry angle, so the entire system matrix is the concatenation of the output files.

binary data file formats - Values are type "cfiScalar", which is typedef-ed to "double" (64-bit real) in "cfiTypes.h". If single-precision (32-bit) values are preferred, change the typedef to "float".

other miscellanea - The byte order is big-endian for the double-precision 63x63x17 attenuation map in the file "std916.atn_ds2_half_crop". Attenuation map units are inverse centimeters (e.g., soft tissue attenuation coefficients are about 0.16 per cm, which corresponds to 140-keV photons from Tc-99m-labeled radiopharmaceuticals). The source code directory also contains an empty subdirectory "F916" for the output generated by running the program on the attenuation map in "std916.atn_ds2_half_crop".