The Coherence Criterion: A Unified Framework for Stability in Hierarchical Systems

Temporal Coupling, Spectral Constraints, and Cross-Domain Failure Modes

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Abstract

The **Coherence Criterion** proposes a unified mathematical framework for system stability in hierarchical systems operating across multiple temporal scales (Δt). System viability depends on maintaining bounded temporal divergence between coupled layers, formalized via **spectral analysis of coupled linear operators** [5].

The stability condition is $\rho(M) < 1$ (where ρ is the spectral radius of the coupling matrix M). Violation of this criterion leads to two pathological regimes: **Rigidity Runaway** ($\lambda \to +1$) and **Acceleration Runaway** ($\lambda \to -1$) [5].

The paper argues this represents **structural correspondence** across autonomous vehicles, LLMs, institutions, governance, and financial markets, with failures exhibiting qualitatively similar bifurcation geometry [5].

1 The Coherence Criterion - Theory

1.1 The Central Claim

The fundamental constraint for system survival is that internal tempos must stay coupled tightly enough to maintain coherence [3]. The central claim is that **temporal structure is the organizing principle of complex systems** [3].

- **Failure Mechanism:** Failure occurs when a **fast layer decouples from a slower integration layer** because the temporal gap exceeds the system's coupling bandwidth [6].

Table 1: Cross-Domain Examples of Temporal Layering Failure

Example	**Fast Layer (τ_1)**	**Slow Layer (τ_2)**	**Resulting Failure**
Autonomous Vehicle	Perception (∼100ms)	Tactical Planning (∼1s)	No consistent world model, Fatal Collision (2018)
2010 Flash Crash	HFT Algorithms (ms)	Human Traders (s-min)	Market lost $1 trillion
LLM Hallucination	Token Generation (ms)	Semantic Verification (External)	Locally plausible, globally false text [8]

1.2 Mathematical Framework

The evolution of two coupled layers (z_1, z_2) is modeled by a discrete-time system $z_{t+1} = Mz_t + \eta_t$, where M is the coupling matrix:

$$M = \begin{pmatrix} a_1 & c_{12} \\ b_{21} & a_2 \end{pmatrix} \tag{1}$$

- $a_k \in [0, 1)$: Layer persistence/damping [5].
- c_{12}: Fast layer response to slow layer (strategic guidance) [5].
- b_{21}: Slow layer integration of fast layer (evidence accumulation) [5].

Theorem 1 (Coherence Criterion): The system maintains bounded trajectories if and only if the spectral radius $\rho(M) < 1$.

The Eigenvalues (λ) are given by:

$$\lambda = \frac{a_1 + a_2}{2} \pm \sqrt{\frac{(a_1 - a_2)^2}{4} + b_{21}c_{12}} \tag{2}$$

Critical Boundaries:

1. **Divergence Boundary ($\lambda \to +1$):** Coupling exceeds damping, $b_{21}c_{12} > (1 - a_1)(1 - a_2)$ [5].
   - **Failure Mode: Rigidity Runaway** (positive feedback, locked-in states) [5].
2. **Flip Boundary ($\lambda \to -1$):** Negative feedback dominates, $b_{21}c_{12} < -(1 + a_1)(1 + a_2)$ [5].
   - **Failure Mode: Acceleration Runaway** (oscillatory instability, over-correction) [5].

1.3 The Coherence Metric (χ)

This metric measures coupling bandwidth relative to temporal divergence:

$$\chi = \frac{b_{21}c_{12}}{|a_1 - a_2|} \tag{3}$$

- Coherence requires $0 < \chi < \chi_{critical}$ AND $\rho(M) < 1$ [5].

1.4 The Principle of Temporal Adjacency

Hierarchy emerges as the evolutionary solution to the Δt problem [3].

- **Principle:** A coordination system can maintain coherence only when interacting layers differ by no more than **O(10^2) in characteristic timescale ($\kappa < 100$)** [3].
- When $\kappa > 100$, the coupling strength required to maintain stability pushes eigenvalues toward the critical boundaries [3].
- This limit appears consistently across control systems, neural systems, economic systems, and social coordination [3].

2 The Coherence Map - Cross-Domain Validation

The framework is validated by **Stress Equivalence Mapping**, which tests if different systems exhibit **Qualitatively Equivalent Eigenvalue Trajectories** and **Phenomenological Correspondence** at critical boundaries under stress [5].

Table 2: Cross-Domain Failure Mode Correspondence

Boundary	Failure Mode	AV Example	Institutional Example	Financial Example
$\lambda \to +1$	**Rigidity Runaway** (Slow Dominance)	Wrong object classification persists (Uber)	Policy paralysis, Institutional lockup (Governance) [7]	Market rigidity before crashes (2008 Crisis) [7]
$\lambda \to -1$	**Acceleration Runaway** (Fast Decoupling)	Phantom braking on transient signals [1]	Policy whiplash, Overreaction (Governance) [7]	Flash crash, Algorithmic oscillation (2010 Crash) [2]

2.1 Case Study Highlights

- **Academic Publishing:** The Replication Crisis is a Δt failure. Research (τ_1, weeks) is decoupled from legitimacy verification (τ_3, years), a gap $\approx 10^2$ [4]. Increasing publication velocity ($b_{21} \uparrow$) without accelerating verification (τ_3 constant) pushes the system toward instability [4].
  - **Interventions validated:** Pre-registration (reduces fast-layer gain b_{21}) and replication studies (strengthens τ_3 verification) [4].
- **Governance Collapse:** The temporal divergence between social media (τ_1, hours) and constitutional norms (τ_3, decades) increased from $\approx 10^3$ to $\approx 10^5$ between 1970 and 2020, while coupling remained constant [7]. This puts the system near the coherence envelope, causing policy oscillation and rigidity (e.g., January 6th, policy whiplash) [7].
- **Financial Markets:** The shift to High-Frequency Trading (HFT) increased temporal divergence between trading (τ_1) and human oversight (τ_2) by **six orders of magnitude** [6].
  - **2010 Flash Crash** was **Acceleration Runaway** ($\lambda \to -1$): fast layer decoupled, oscillating without damping [2].
  - **2008 Financial Crisis** was **Rigidity Runaway** ($\lambda \to +1$): slow regulatory frameworks persisted in outdated models despite fast-layer evidence of danger [7].

3 Implications and Predictions

3.1 Practical Implications for Design

The Coherence Criterion provides clear design principles for multi-layer systems:

1. **Respect Adjacency:** Do not couple layers separated by more than $O(10^2)$ in timescale without explicit integration [3].
2. **Monitor Stress:** Track gain amplification, latency spikes, and oscillation as early warning signs [5].
3. **Intervention Hierarchy:** When coherence fails, the fix is always structural [6]:
   - Reduce Δt between layers.
   - Strengthen coupling mechanisms.
   - Add mid-layer integration where missing (e.g., Retrieval-Augmented Generation in LLMs, which the framework correctly predicts will reduce hallucination) [8].

3.2 Falsifiable Prediction

The framework generates testable predictions [5]:

- **General:** Measures that reduce Δt or adjust coupling should restore stability **regardless of domain** [5].
- **Academia:** Fields with higher publication velocity (larger b_{21}) and weaker verification (τ_3 years) should exhibit higher replication failure rates [4].
- **Governance:** Continued temporal divergence will require either adaptation of slow-layer mechanisms (compress τ_3) or fragmentation (accept decoupling) [7].

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