ATOMIC STRUCTURE

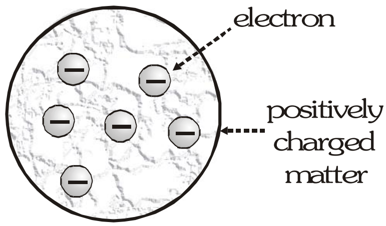
VARIOUS MODELS FOR STRUCTURE OF ATOM

- **Dalton's Theory**

  Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed. All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties. The atoms of different elements are made up of hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen). The atom is stable and electrically neutral.

- **Thomson's Atom Model**

  The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere. Atom is a positively charged sphere of radius 10^{-10} m in which electrons are embedded in between. The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.



- **Shortcomings of Thomson's model**

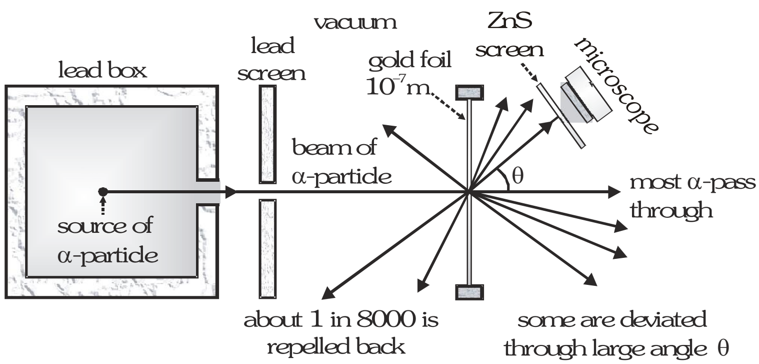
  (i) The spectrum of atoms cannot be explained with the help of this model

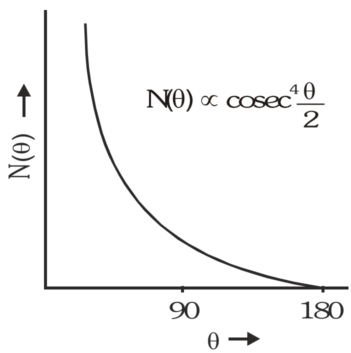
  (ii) Scattering of α–particles cannot be explained with the help of this model

RUTHERFORD ATOM MODEL

- **Rutherford experiments on scattering of α – particles by thin gold foil**

  The experimental arrangement is shown in figure. α–particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of α–particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α–particles are scattered through different angles. A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the α–particles, scattered from the gold foil at angles varying from 0 to 180 . When an α–particle strikes the screen, it produces a flash of light and it is observed by the microscope. It was found that :

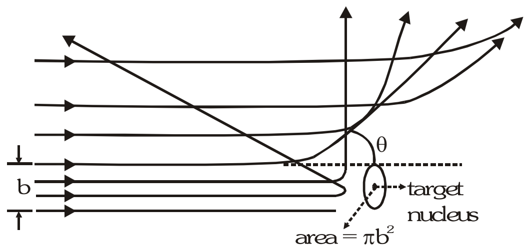


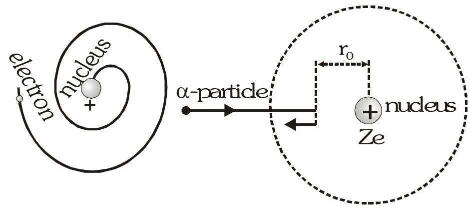


- Most of the α – particles went straight through the gold foil and produced flashes on the screen as if there were nothing inside gold foil. Thus the atom is hollow.
- Few particles collided with the atoms of the foil which have scattered or deflected through considerable large angles. Few particles even turned back towards source itself.
- The entire positive charge and almost whole mass of the atom is concentrated in small centre called a nucleus.
- The electrons could not deflected the path of a α – particles i.e. electrons are very light.
- Electrons revolve round the nucleus in circular orbits. So, Rutherford 1911, proposed a new type of model of the atom. According to this model, the positive charge of the atom, instead of being uniformly distributed throughout a sphere of atomic dimension is concentrated in a very small volume (Less than 10^{-13}m is diameter) at it centre. This central core, now called nucleus, is surrounded by clouds of electron makes. The entire atom electrically neutral. According to Rutherford scattering formula, the number of α – particle scattered at an angle θ by a target are given by $N_\theta = \dfrac{N_0 nt(2Ze^2)^2}{16(4\pi\varepsilon_0)^2 r^2 (mv_0^2)^2} \times \dfrac{1}{\sin^4\frac{\theta}{2}}$

Where N_0 = number of α – particles that strike the unit area of the scatter
n = Number of target atom per m^3
t = Thickness of target
Ze = Charge on the target nucleus
$2e$ = Charge on α – particle
r = Distance of the screen from target
v_0 = Velocity of α – particles at nearer distance of approach the size of a nucleus or the distance of nearer approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{(2Ze)^2}{\left[\frac{1}{2}mv_0^2\right]} = \frac{1}{4\pi\epsilon_0}\frac{(2Ze)^2}{E_K} \quad \text{where } E_K = \text{K.E. of } \alpha\text{–particle}$$





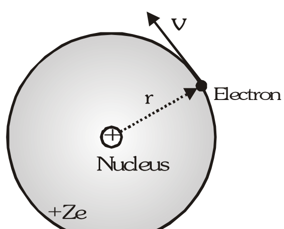
Bohr's Atomic Model

In 1913 Neils Bohr, a Danish Physicist, introduced a revolutionary concept i.e., the quantum concept to explain the stability of an atom. He made a simple but bold statement that "The old classical laws which are applicable to bigger bodies cannot be directly applied to the sub–atomic particles such as electrons or protons.

Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

1. The centripetal force required for an encircling electron is provided by the electrostatic attraction between the nucleus and the electron i.e. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)e}{r^2} = \dfrac{mv^2}{r}$ ...(i)

ε_0 = Absolute permittivity of free space = 8.85 10^{-12} C^2 N^{-1} m^{-2}
m = Mass of electron
v = Velocity (linear) of electron
r = Radius of the orbit in which electron is revolving.



2. Electrons can revolve only in those orbits in which angular momentum of electron about nucleus is an integral multiple of $\frac{h}{2\pi}$. i.e., $mvr = \frac{nh}{2\pi}$ ...(ii)

n = Principal quantum number of the orbit in which electron is revolving.

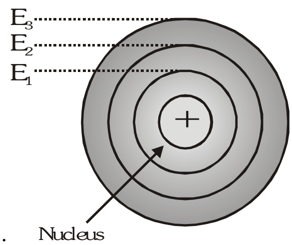
3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in a shell is characterised by a definite energy, angular momentum and orbit number. While in any of these orbits, an electron does not radiate energy although it is accelerated.

4. Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from an outer orbit (higher energy states) to an inner orbit (lower energy states) and also absorbs energy when it jumps from an inner orbit to an outer orbit. $E_n - E_m = h\nu_{n,m}$

where, E_n = Outer energy state

E_m = Inner energy state

$\nu_{n,m}$ = Frequency of radiation



5. The energy absorbed or released is always in the form of electromagnetic radiations.

MATHEMATICAL ANALYSIS OF BOHR'S THEORY

From above equation (i) and (ii) i.e., $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ and $mvr = \frac{nh}{2\pi}$ ...(ii)

We get the following results.

1. **Velocity of electron in nth orbit :** By putting the value of mvr in equation (i) from (ii) we get

$$\frac{1}{4\pi\varepsilon_0}Ze^2 = \left(\frac{nh}{2\pi}\right)\times v \Rightarrow v = \frac{Z}{n}\left[\frac{e^2}{2\varepsilon_0 h}\right] = \frac{Z}{n}v_0 \text{ ....(iii)}$$

Where, $v_0 = \frac{(1.6\times10^{-19})^2}{2\times8.85\times10^{-12}\times6.625\times10^{-34}}$ = 2.189 $\quad 10^6$ ms^{-1} = $\frac{c}{137}$ = 2.2 $\quad 10^6$ m/s

where c = 3 $\quad 10^8$ m/s = speed of light in vacuum

2. **Radius of the nth orbit :** From equation (iii), putting the value of v in equation (ii), we get

$$m\left(\frac{Z}{n}\times\frac{e^2}{2\varepsilon_0 h}\right)r = \frac{nh}{2\pi} \Rightarrow r = \frac{n^2}{Z}\left[\frac{\varepsilon_0 h^2}{\pi m e^2}\right] = \frac{n^2}{Z}r_0 \text{ ...(iv)}$$

where $\quad r_0 = \frac{8.85\times10^{-12}\times(6.625\times10^{-34})^2}{3.14\times9.11\times10^{-31}\times(1.6\times10^{-19})^2}$ = 0.529 $\quad 10^{-10}$ m ≈ 0.53Å

3. **Total energy of electron in n^{th} orbit :**

From equation (i) $KE = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$ and $PE = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -2K.E.$ $\quad \therefore\ |PE| = 2\ KE$

Total energy of the system $E = KE + PE = -2KE + KE = -\ KE = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

By putting the value of r from the equation (iv), we get $E = \frac{Z^2}{n^2}\left(-\frac{me^4}{8\varepsilon_0^2h^2}\right) = \frac{Z^2}{n^2}\cdot E_0$ ...(v)

where $E_0 = \frac{-(9.11\times10^{-3})(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2\times(6.625\times10^{-34})^2} = -13.6\ eV$

4. **Time period of revolution of electron in nth orbit :** $T = \frac{2\pi r}{v}$

By putting the values of r and v, from (iii) and (iv) $T = \frac{n^3}{Z^2}\times\left(\frac{4\ \epsilon_0^2\ h^3}{me^4}\right) = \frac{n^3}{Z^2}\cdot T_0$

where, $T_0 = \frac{4\times(8.85\times10^{-12})^2\times(6.625\times10^{-34})^3}{9.11\times10^{-31}\times(1.6\times10^{-19})^4} = 1.51 \quad 10^{-16}$ second

5. **Frequency of revolution in nth orbit :**

$f = \frac{1}{T} = \frac{Z^2}{n^3}\times\frac{me^4}{4\varepsilon_0^2h^3} = \frac{Z^2}{n^3}\cdot f_0$ where, $f_0 = \frac{9.11\times10^{-31}\times(1.6\times10^{-19})^4}{4\times(8.85\times10^{-12})^2(6.625\times10^{-34})^3} = 6.6 \quad 10^{15}$ Hz

6. **Wavelength of photon**

$\Delta E = E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2 = 13.6\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2 \Rightarrow \Delta E = \frac{hc}{\lambda} \Rightarrow \frac{1}{\lambda} = \bar{v} = \frac{me^4}{8\varepsilon_0^2h^3c}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$

$= R_\infty\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]Z^2$ where, $\bar{v}$ is called wave number.

$R_\infty = R_H =$ Rydberg constant

$= \frac{9.11\times10^{-31}\times(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2(6.625\times10^{-34})^3\times3\times10^8} = 1.097 \quad 10^7\ m^{-1} = 1.097 \quad 10^{-3}$ Å^{-1} (for stationary nucleus).

If nucleus is not stationary (i.e., mass of nucleus is not much greater than the mass of the revolving particle like electron), then $R = \frac{R_\infty}{1+m/M}$ where, m = mass of revolving particle and M = mass of nucleus

SPECTRAL SERIES OF HYDROGEN ATOM

It has been shown that the energy of the outer orbit is greater than the energy of the inner ones. When the Hydrogen atom is subjected to external energy, the electron jumps from lower energy state i.e. the hydrogen atom is excited. The excited state is unstable hence the electron return to its ground state in about 10^{-8} sec. The excess of energy is now radiated in the form of radiations of different wavelength. The different wavelength constitute spectral series. Which are characteristic of atom emitting, then the wave length of different members of series can be found from the following relations

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This relation explains the complete spectrum of hydrogen. A detailed account of the important radiations are listed below.

- **Lyman Series** : The series consist of wavelength which are emitted when electron jumps from an outer orbits to the first orbit i. e., the electronic jumps to K orbit give rise to lyman series. Here $n_1 = 1$ & $n_2 = 2, 3, 4, ...\infty$
  The wavelengths of different members of Lyman series are :

- **First member** : In this case $n_1 = 1$ and $n_2 = 2$ hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$

  $$\Rightarrow \lambda = \frac{4}{3R} \Rightarrow \lambda = \frac{4}{3 \times 10.97 \times 10^6} = 1216 \quad 10^{-10} \text{ m} = 1216Å$$

- **Second member** : In this case $n_1 = 1$ and $n_2 = 3$ hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$

  $$\Rightarrow \lambda = \frac{9}{8R} \Rightarrow \lambda = \frac{9}{8 \times 10.97 \times 10^6} = 1026 \quad 10^{-10} \text{ m} = 1026Å$$

  Similarly the wavelength of the other members can be calculated.

- **Limiting members** : In this case $n_1 = 1$ and $n_2 = \infty$, hence $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R$

  $$\Rightarrow \lambda = \frac{1}{R} \Rightarrow \lambda = \frac{1}{10.97 \times 10^6} = 912 \quad 10^{-10}\text{m} = 912Å$$

  This series lies in ultraviolet region.

- **Balmer Series** : This series is consist of all wave lengths which are emitted when an electron jumps from an outer orbit to the second orbit i. e., the electron jumps to L orbit give rise to Balmer series.
  Here $n_1 = 2$ and $n_2 = 3, 4, 5 ....\infty$ The wavelength of different members of Balmer series.

- **First member** : In this case $n_1 = 2$ and $n_2 = 3$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$

  $$\Rightarrow \lambda = \frac{36}{5R} \Rightarrow \lambda = \frac{36}{5 \times 10.97 \times 10^6} = 6563 \quad 10^{-10}\text{m} = 6563Å$$

- **Second member** : In this case $n_1 = 2$ and $n_2 = 4$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{16}$

  $$\Rightarrow \lambda = \frac{16}{3R} \Rightarrow \lambda = \frac{16}{3 \times 10.97 \times 10^6} = 4861 \quad 10^{-10}\text{m} = 4861Å$$

- **Limiting members**: In this case $n_1 = 2$ and $n_2 = \infty$, hence $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R} = 3646Å$

- **Paschen Series :** This series consist of all wavelength are emitted when an electron jumps from an outer orbit to the third orbit i. e., the electron jumps to M orbit give rise to paschen series. Here n_1=3 & n_2= 4, 5, 6 .....∞.

  The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$

  where n_2 = 4, 5, 6 ...∞

  For the first member, the wavelengths is 18750Å. This series lies in infra–red region.
- **Bracket Series :** This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbits to the fourth orbit i. e., the electron jumps to N orbit give rise to Brackett series.

  Here n_1 = 4 & n_2 = 5, 6, 7, ...∞.

  The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$

  where n_2 = 5, 6, 7 ....∞

  This series lies in infra–red region of spectrum.
- **Pfund series :** The series consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the fifth orbit i. e., the electron jumps to O orbit give right to Pfund series.

  Here n_1 = 5 and n_2 = 6, 7, 8 ....∞.

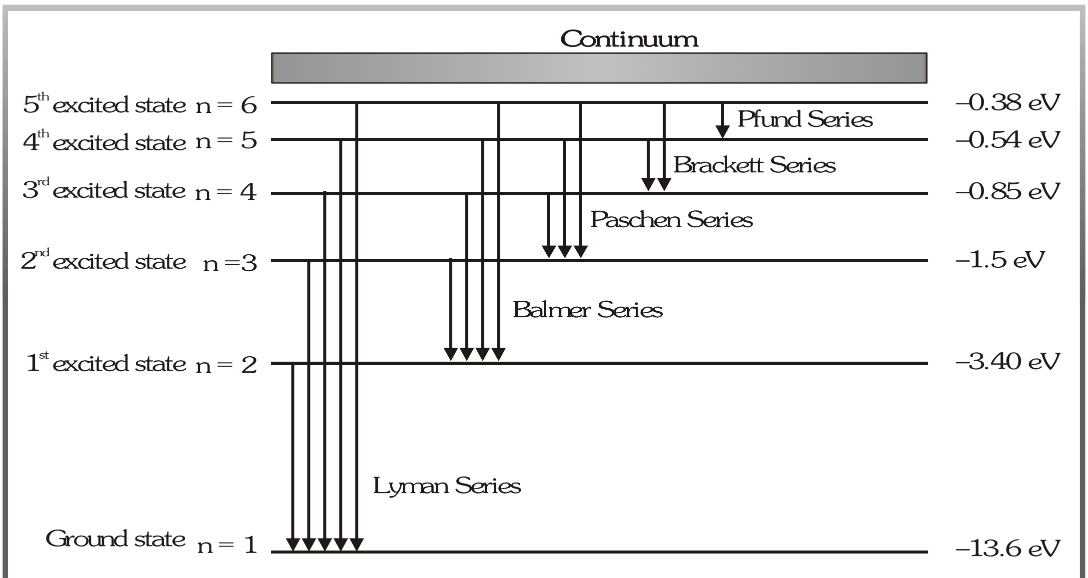
  The different wavelengths of this series can be obtained from the formula $\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$

  where n_2 = 6, 7, 8 ....∞

  This series lies in infra–red region of spectrum.

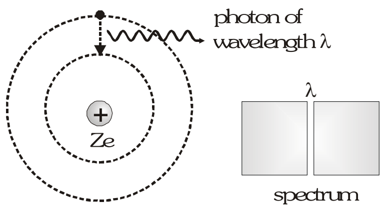
The result are tabulated below

S. No.	Series Observed	Value of n_1	Value of n_2	Position in the Spectrum
1.	Lyman Series	1	2,3,4...∞	Ultra Violet
2.	Balmer Series	2	3,4,5...∞	Visible
3.	Paschen Series	3	4,5,6....∞	Infra–red
4.	Brackett Series	4	5,6,7....∞	Infra–red
5.	Pfund Series	5	6,7,8....∞	Infra–red

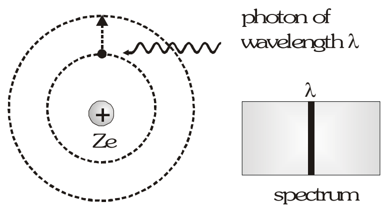


EXCITATION AND IONISATION OF ATOMS

Consider the case of simplest atom i. e., hydrogen atom, this has one electron in the innermost orbit i.e., (n = 1) and is said to be in the unexcited or normal state. If by some means, sufficient energy is supplied to the electron. It moves to higher energy states. When the atom is in a state of a high energy it is said to be excited. The process of raising or transferring the electron from lower energy state is called excitation. When by the process of excitation, the electron is completely removed from the atom. The atom is said to be ionized. Now the atom has left with a positive charge. Thus the process of raising the atom from the normal state to the ionized state is called ionisation. The process of excitation and ionisation both are absorption phenomena. The excited state is not stationary state and lasts in a very short interval of time (10^{-8} sec) because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. This is accompanied by the emission of radiation according to BOHR'S frequency condition.



origin of emission spectra



origin of absorption spectra

The energy necessary to excite an atom can be supplied in a number of ways. The most commonly kinetic energy (Wholly or partly) of the electrons is transferred to the atom. The atom is now in a excited state. The minimum potential V required to accelerate the bombarding electrons to cause excitation from the ground state is called the resonance potential. The various values of potential to cause excitation of higher state called **excitation potential**. The potential necessary to accelerate the bombarding electrons to cause ionisation is called the **ionization potential**. The term critical potential is used to include the resonance, excitation and ionisation potential. We have seen that the energy required to excite the electron from first to second state is 13.6 – 3.4 = 10.2 eV from first to third state is 13.6 – 1.5 = 12.1 eV., and so on. The energy required to ionise hydrogen atom is 0 – (– 13.6) = 13.6 eV. Hence ionization potential of hydrogen atom is 13.6 volt.

SUCCESSES AND LIMITATIONS

Bohr showed that Planck's quantum ideas were a necessary element of the atomic theory. He introduced the idea of quantized energy levels and explained the emission or absorption of radiations as being due to the transition of an electron from one level to another. As a model for even multielectron atoms, the Bohr picture is still useful. It leads to a good, simple, rational ordering of the electrons in larger atoms and quantitatively helps to predict a greater deal about chemical behavior and spectral detail.

Bohr's theory is unable to explain the following facts :

- The spectral lines of hydrogen atom are not single lines but each one is a collection of several closely spaced lines.
- The structure of multielectron atoms is not explained.
- No explanation for using the principles of quantization of angular momentum.
- No explanation for Zeeman effect. If a substance which gives a line emission spectrum is placed in a magnetic field, the lines of the spectrum get splitted up into a number of closely spaced lines. This phenomenon is known

Example

A hydrogen like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de–excitation. Ground state energy of hydrogen atom is –13.6 eV.

Solution

The energy released during de–excitation in hydrogen like atoms is given by : $E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2$

Energy released in de–excitation will be maximum if transition takes place from nth energy level to ground state i.e.,

$$E_{2n} - E_1 = 13.6\left[\frac{1}{1^2} - \frac{1}{(2n)^2}\right]Z^2 = 204 \text{ eV} \ldots\text{(i)} \quad \& \quad \text{also } E_{2n} - E_n = 13.6\left[\frac{1}{n^2} - \frac{1}{(2n)^2}\right]Z^2 = 40.8 \text{ eV}\ldots\text{(ii)}$$

Taking ratio of (i) to (ii), we will get $\frac{4n^2 - 1}{3} = 5 \Rightarrow n^2 = 4 \Rightarrow n=2$

Putting n=2 in equation (i) we get $Z^2 = \frac{204 \times 16}{13.6 \times 15} \Rightarrow Z=4$

$$\because E_n = -13.6\frac{Z^2}{n^2} \Rightarrow E_1 = -13.6\frac{4^2}{1^2} = -217.6 \text{ eV} = \text{ground state energy}$$

ΔE is minimum if transition will be from 2n to 2n–1 i.e. between last two adjacent energy levels.

$$\therefore \Delta E_{min} = E_{2n} - E_{2n-1} = 13.6\left[\frac{1}{3^2} - \frac{1}{4^2}\right]4^2 = 10.57 \text{ eV}$$

is the minimum amount of energy released during de–excitation.

Example

A single electron orbits around a stationary nucleus of charge +Ze where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third orbit. Find

(i) The value of Z.

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) The wavelength of electronic radiation required to remove the electron from first Bohr orbit to infinity.

(iv) Find the K.E., P.E. and angular momentum of electron in the 1st Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3 10^{-11} m,
Velocity of light = 3 10^{8} m/s, Planck's constant = 6.6 10^{-34} J–s]

Solution

The energy required to excite the electron from n_1 to n_2 orbit revolving around the nucleus with charge +Ze is given by $E_{n_2} - E_{n_1} = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]Z^2 \Rightarrow E_{n_2} - E_{n_1} = Z^2 \times (13.6)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

(i) Since 47.2 eV energy is required to excite the electron from $n_1 = 2$ to $n_2 = 3$ orbit

$$47.2 = Z^2 \quad 13.6 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.988 \approx 25 \Rightarrow Z=5$$

(ii) The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ is given by

$$E_4 - E_3 = 13.6 \; Z^2 = 5 \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25 \times 13.6 \times 7}{144} = 16.53 \text{ eV}$$

(iii) The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$$E_\infty - E_3 = 13.6 \times Z^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25 \text{eV} = 340 \text{eV}$$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$$hf = \frac{hc}{\lambda} = 13.6 \quad 25 \quad (1.6 \quad 10^{-19})J \Rightarrow \lambda = \frac{\left(6.6 \times 10^{-34}\right) \times 10^8 \times 3}{13.6 \times 25 \times \left(1.6 \times 10^{-19}\right)} = 36.397 \text{Å}$$

(iv) $\text{K.E.} = \frac{1}{2}mv_1^2 = \frac{1}{2} \times \frac{Ze^2}{4\pi g \in_0 r_1} = 543.4 \quad 10^{-19}$ J $\quad$ P.E. = – 2 K.E. = – 1086.8 $\quad 10^{-19}$ J

Angular momentum $= mv_1r_1 = \frac{h}{2\pi} = 1.05 \quad 10^{-34}$ Js

The radius r_1 of the first Bohr orbit is given by

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2} \cdot \frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5} \left(\because \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10} \text{m}\right) = 1.106 \quad 10^{-10} \text{ m} = 0.106 \text{ Å}$$

Example

An isolated hydrogen atom emits a photon of 10.2 eV.

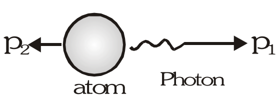
(i) Determine the momentum of photon emitted (ii) Calculate the recoil momentum of the atom

(iii) Find the kinetic energy of the recoil atom [Mass of proton= $m_p = 1.67 \quad 10^{-27}$ kg]

Solution

(i) Momentum of the photon is $p_1 = \frac{E}{c} = \frac{10.2 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 5.44 \quad 10^{-27}$ kg–m/s

(ii) Applying the momentum conservation $p_2 = p_1 = 5.44 \quad 10^{-27}$ kg–m/s



(iii) $K = \frac{1}{2}mv^2$ (v = recoil speed of atom, m = mass of hydrogen atom) $K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$

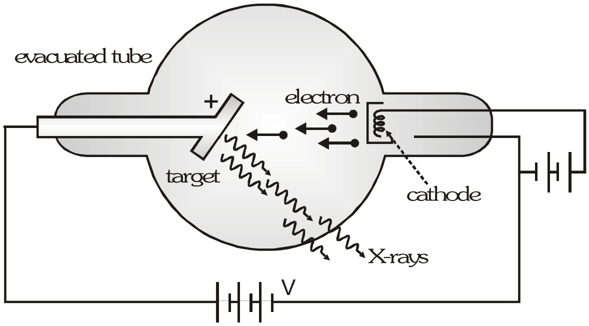
Substituting the value of the momentum of atom, we get $K = \frac{\left(5.44 \times 10^{-27}\right)^2}{2 \times 1.67 \times 10^{-27}} = 8.86 \quad 10^{-27}$J

Physical quantity	Formula	Ratio Formulae of hydrogen atom
Radius of Bohr orbit (r_n)	$r_n = \frac{n^2h^2}{4\pi^2 mkZe^2}$; $r_n = 0.53\frac{n^2}{Z}$ Å	$r_1 : r_2 : r_3 \ldots r_n = 1:4:9\ldots n^2$
Velocity of electron in n^{th} Bohr orbit (v_n)	$v_n = \frac{2\pi kZe^2}{nh}$; $v_n = 2.2\quad 10^6\frac{Z}{n}$	$v_1 : v_2 : v_3 \ldots v_n = 1 : \frac{1}{2} : \frac{1}{3} \ldots \frac{1}{n}$
Momentum of electron (p_n)	$p_n = \frac{2\pi mke^2z}{nh}$; $p_n \propto \frac{Z}{n}$	$p_1 : p_2 : p_3 \ldots p_n = 1 : \frac{1}{2} : \frac{1}{3} \ldots \frac{1}{n}$
Angular velocity of electron(ω_n)	$\omega_n = \frac{8\pi^3k^2Z^2mc^4}{n^3h^3}$; $\omega_n \propto \frac{Z^2}{n^3}$	$\omega_1 : \omega_2 : \omega_3 \ldots \omega_n = 1 : \frac{1}{8} : \frac{1}{27} \ldots \frac{1}{n^3}$
Time Period of electron (T_n)	$T_n = \frac{n^3h^3}{4\pi^2k^2Z^2me^4}$; $T_n \propto \frac{n^3}{Z^2}$	$T_1 : T_2 : T_3 \ldots T_n = 1:8:27: \ldots :n^3$
Frequency (f_n)	$f_n = \frac{4\pi^2k^2Z^2e^4m}{n^3h^3}$; $f_n \propto \frac{Z^2}{n^3}$	$f_1 : f_2 : f_3 \ldots f_n = 1 : \frac{1}{8} : \frac{1}{27} \ldots \frac{1}{n^3}$
Orbital current (I_n)	$I_n = \frac{4\pi^2k^2Z^2me^5}{n^3h^3}$; $I_n \propto \frac{Z^2}{n^3}$	$I_1 : I_2 : I_3 \ldots I_n = 1 : \frac{1}{8} : \frac{1}{27} \ldots \frac{1}{n^3}$
Angular momentum (J_n)	$J_n = \frac{nh}{2\pi}$; $J_n \propto n$	$J_1 : J_2 : J_3 \ldots J_n = 1 : 2 : 3 \ldots n$
Centripetal acceleration (a_n)	$a_n = \frac{16\pi^4k^3Z^3me^6}{n^4h^4}$; $a_n \propto \frac{Z^3}{n^4}$	$a_1 : a_2 : a_3 \ldots a_n = 1 : \frac{1}{16} : \frac{1}{81} \ldots \frac{1}{n^4}$
Kinetic energy (E_{k_n})	$E_{K_n} = \frac{RchZ^2}{n^2}$; $E_{K_n} \propto \frac{Z^2}{n^2}$	$E_{K_1} : E_{K_2} \ldots E_{K_n} = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$
Potential energy (U_n)	$U_n = \frac{-2RchZ^2}{n^2}$; $U_n \propto \frac{Z^2}{n^2}$	$U_1 : U_2 : U_3 \ldots U_n = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$
Total energy (E_n)	$E_n = \frac{-RchZ^2}{n^2}$; $E_n \propto \frac{Z^2}{n^2}$	$E_1 : E_2 : E_3 \ldots E_n = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$

X–RAYS

ROENTGEN EXPERIMENT

Roentgen discovered X–ray. While performing experiment on electric discharge tube Roentgen observed that when pressure inside the tube is 10^{-3}mm of Hg and applied potential is kept 25 kV then some unknown radiation are emitted by anode. These are known as X–ray. X–rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.



To Produce X–ray Three Things are Required

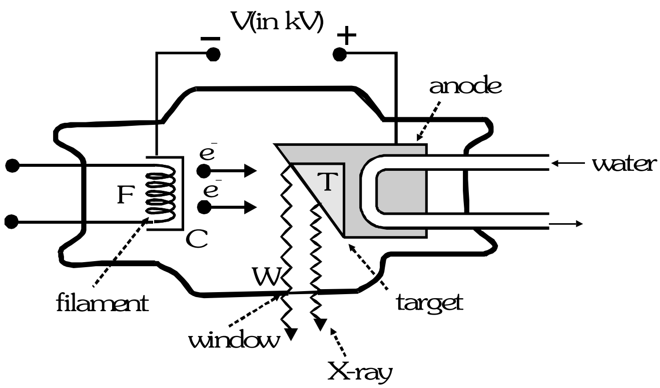
(i) Source of electron

(ii) Means of accelerating these electron to high speed

(iii) Target on which these high speed electron strike

COOLIDGE METHOD

Coolidge developed thermoionic vacuum X–ray tube in which electron are produced by thermoionic emission method. Due to high potential difference electrons (emitted due to thermoionic method) move towards the target and strike from the atom of target due to which X–ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X–ray. Rest of energy is wasted in form of heat.



Characteristics of target

(a) Must have high atomic number to produce hard X–rays.

(b) High melting point to withstand high temperature produced.

(c) High thermal conductivity to remove the heat produced

(d) Tantalum, platinum, molybdenum and tungsten serve as target materials

• **Control of intensity** : The intensity of X–ray depends on number of electrons striking the target and number of electron depend on temperature of filament which can be controlled by filament current.

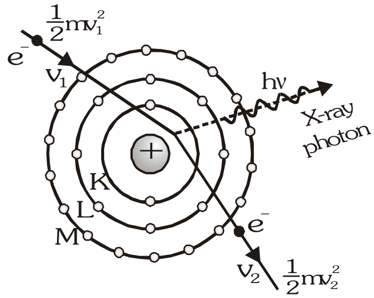
Thus intensity of X–ray depends on current flowing through filament.

- **Control of Penetrating Power**: The Penetrating power of X–ray depends on the energy of incident electron. The energy of electron can be controlled by applied potential difference. Thus penetrating power of X–ray depend on applied potential difference. Thus the intensity of X–ray depends on current flowing through filament while penetrating power depends on applied potential difference

	Soft X–ray	Hard X–ray
Wavelength	10 Å to 100 Å	0.1 Å – 10 Å
Energy	$\frac{12400}{\lambda}$ eV–Å	$\frac{12400}{\lambda}$ eV–Å
Penetrating power	Less	More
Use	Radio photography	Radio therapy

- **Continuous spectrum of X–ray** :

When high speed electron collides from the atom of target and passes close to the nucleus. There is coulomb attractive force due to this electron is deaccelerated i.e. energy is decreased. The loss of energy during deacceleration is emitted in the form of X–rays. X–ray produced in this way are called Braking or Bremstralung radiation and form continuous spectrum. In continuous spectrum of X–ray all the wavelength of X–ray are present but below a minimum value of wavelength there is no X–ray. It is called cut off or threshold or minimum wavelength of X–ray. The minimum wavelength depends on applied potential.

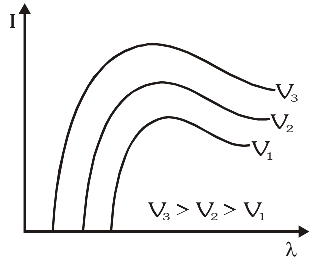


- **Loss in Kinetic Energy**

$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = h\nu$ + heat energy if $v_2 = 0$, $v_1 = v$ (In first collision, heat = 0)

$\frac{1}{2}mv^2 = h\nu_{max}$ ...(i)

$\frac{1}{2}mv^2 = eV$ ...(ii) [here V is applied potential]

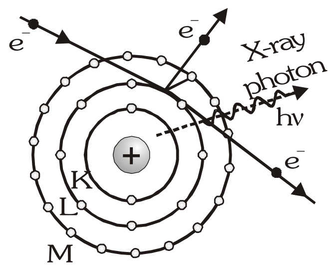


from (i) and (ii) $h\nu_{max} = eV \Rightarrow \frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} = \frac{12400}{V}$ volt $= \frac{12400}{V}$ 10^{-10} m volt

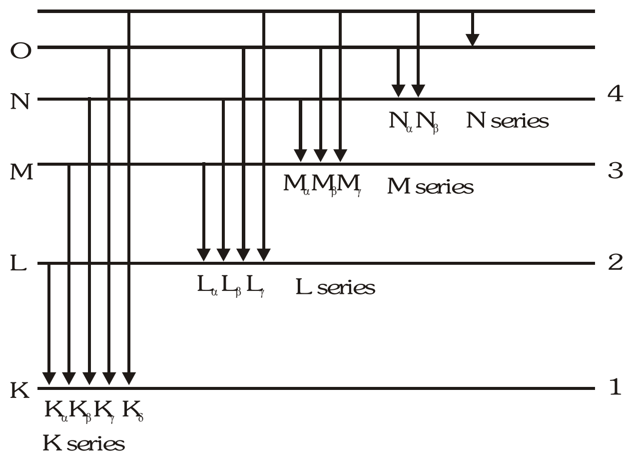
Continuous X–rays also known as white X–ray. Minimum wavelength of these spectrum only depends on applied potential and doesn' t depend on atomic number.

- **Characteristic Spectrum of X–ray**

When the target of X–ray tube is collide by energetic electron it emits two type of X–ray radiation. One of them has a continuous spectrum whose wavelength depend on applied potential while other consists of spectral lines whose wavelength depend on nature of target. The radiation forming the line spectrum is called characteristic X–rays. When highly accelerated electron strikes with the atom of target then it knockout the electron of orbit, due to this a vacancy is created. To fill this vacancy electron jumps from higher energy level and electromagnetic radiation are emitted which form characteristic spectrum of X–ray. Whose wavelength depends on nature of



From Bohr Model



$n_1 = 1, \quad n_2 = 2, 3, 4.......$K series

$n_1 = 2, \quad n_2 = 3, 4, 5.......$L series

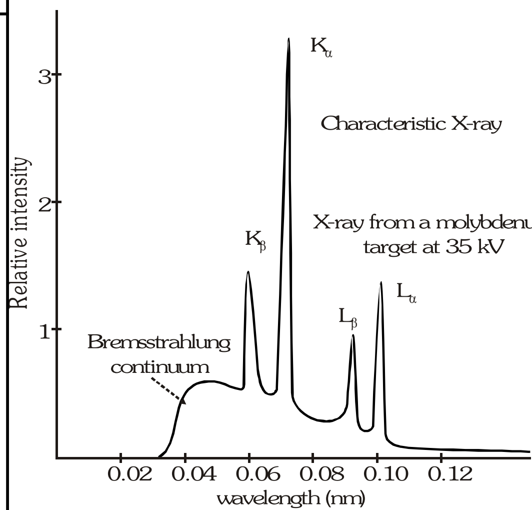
$n_1 = 3, \quad n_2 = 4, 5, 6.......$M series

First line of series = α

Second line of series = β

Third line of series = γ

Transition	Wave-length	Energy	Energy difference	Wavelength
$L \rightarrow K$	$\lambda_{K\alpha}$	$h\nu_{K\alpha}$	$-(E_K - E_L)$	$\lambda_{K\alpha} = \dfrac{hc}{(E_K - E_L)}$
$(2 \rightarrow 1)$			$= h\nu_{K\alpha}$	$= \dfrac{12400}{(E_K - E_L)}$ eVÅ
$M \rightarrow K$	$\lambda_{K\beta}$	$h\nu_{K\beta}$	$-(E_K - E_M)$	$\lambda_{K\beta} = \dfrac{hc}{(E_K - E_M)}$
$(3 \rightarrow 1)$			$= h\nu_{K\beta}$	$= \dfrac{12400}{(E_K - E_M)}$ eVÅ
$M \rightarrow L$	$\lambda_{L\alpha}$	$h\nu_{L\alpha}$	$-(E_L - E_M)$	$\lambda_{L\alpha} = \dfrac{hc}{(E_L - E_M)}$
$(3 \rightarrow 2)$			$= h\nu_{L\alpha}$	$= \dfrac{12400}{(E_L - E_M)}$ eVÅ



MOSELEY'S LAW

Moseley studied the characteristic spectrum of number of many elements and observed that the square root of the frequency of a K– line is closely proportional to atomic number of the element. This is called Moseley's law.

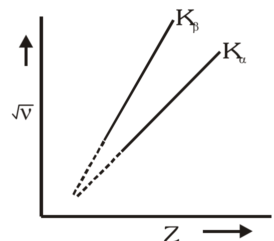
$$\sqrt{\nu} \propto (Z - b) \Rightarrow \nu \propto (Z - b)^2 \Rightarrow \nu = a\,(Z - b)^2 \quad ...(i)$$

Z = atomic number of target

ν = frequency of characteristic spectrum

b = screening constant (for K– series b=1, L series b=7.4)

a = proportionality constant



From Bohr Model $\qquad \nu = RcZ^2 \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right] \quad ...(ii)$

Comparing (i) and (ii) $\qquad a = Rc\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

- Thus proportionality constant 'a' does not depend on the nature of target but depend on transition.

	Bohr model		Moseley's correction
1.	For single electron species	1.	For many electron species
2.	$\Delta E = 13.6Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ eV	2.	$\Delta E = 13.6\ (Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ eV
3.	$\nu = RcZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	3.	$\nu = Rc(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	4.	$\frac{1}{\lambda} = R\ (Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

- For X–ray production, Moseley formulae are used because heavy metal are used.

When target is same $\lambda \propto \dfrac{1}{\frac{1}{n_1^2}-\frac{1}{n_2^2}}$ **When transition is same** $\lambda \propto \dfrac{1}{(Z-b)^2}$

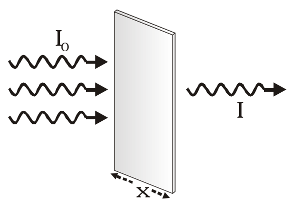
ABSORPTION OF X–RAY

When X–ray passes through x thickness then its intensity $I = I_0e^{-\mu x}$

I_0 = Intensity of incident X–ray

I = Intensity of X–ray after passing through x distance

μ = absorption coefficient of material



- Intensity of X–ray decrease exponentially.
- Maximum absorption of X–ray $\longrightarrow$ Lead
- Minimum absorption of X–ray $\longrightarrow$ Air

Half thickness ($x_{1/2}$)

The distance travelled by X–ray when intensity become half the original value $x_{1/2} = \dfrac{\ell n_2}{\mu}$

Example

When X–rays of wavelength 0.5Å pass through 10 mm thick Al sheet then their intensity is reduced to one sixth. Find the absorption coefficient for Aluminium .

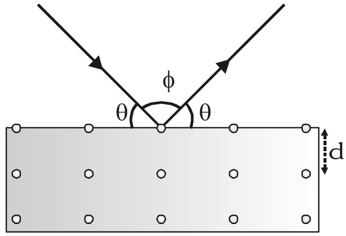
Solution

$$\mu = \frac{2.303}{x}\ \log\left(\frac{I_o}{I}\right) = \frac{2.303}{10}\log_{10}6 = \frac{2.303\times 0.7781}{10} = 0.1752/\text{mm}$$

DIFFRACTION OF X–RAY

Diffraction of X–ray is possible by crystals because the interatomic spacing in a crystal lattice is order of wavelength of X–rays it was first verified by Lauve.

Diffraction of X–ray take place according to Bragg's law $2d \sin\theta = n\lambda$



d = spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle n = 1, 2, 3 ....

For Maximum Wavelength

$\sin\theta = 1$, $n = 1 \Rightarrow \lambda_{max} = 2d$

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

PROPERTIES OF X–RAY

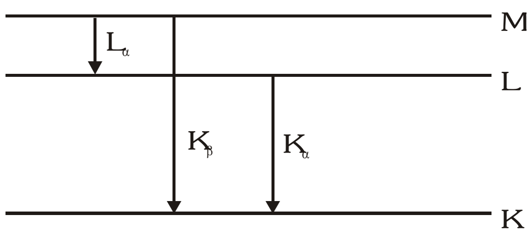
- X–ray always travel with the velocity of light in straight line because X–rays are em waves
- X–ray is electromagnetic radiation it show particle and wave both nature
- In reflection, diffraction, interference, refraction X–ray shows wave nature while in photoelectric effect it shows particle nature.
- There is no charge on X–ray thus these are not deflected by electric field and magnetic field.
- X–ray are invisible.
- X–ray affects the photographic plate
- When X–ray incidents on the surface of substance it exerts force and pressure and transfer energy and momentum
- Characteristic X–ray can not obtained from hydrogen because the difference of energy level in hydrogen is very small.

Example

Show that the frequency of K_β X–ray of a material is equal to the sum of frequencies of K_α and L_α X–rays of the same material.

Solution

The energy level diagram of an atom with one electron knocked out is shown above.



Energy of K_α X–ray is $E_{K\alpha} = E_L - E_K$

and of K_β X–ray is $E_{K\beta} = E_M - E_K$

and of L_α X–rays is $E_{L\alpha} = E_M - E_L$

thus $E_{K\beta} = E_{K\alpha} + E_{L\alpha}$ or $\nu_{K\beta} = \nu_{K\alpha} + \nu_{L\alpha}$

PHOTO ELECTRIC EFFECT

PHOTOELECTRIC EFFECT

It was discovered by Hertz in 1887. He found that when the negative plate of an electric discharge tube was illuminated with ultraviolet light, the electric discharge took place more readily. Further experiments carried out by Hallwachs confirmed that certain negatively charged particles are emitted, when a Zn plate is illuminated with ultraviolet light. These particles were identified as electrons.

The phenomenon of emission of electrons from the surface of certain substances, when suitable radiations of certain frequency or wavelength are incident upon it is called photoelectric effect.

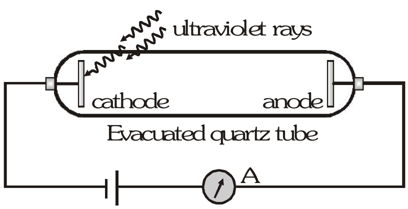
EXPLANATION OF PHOTOELECTRIC EFFECT

- **On the basis of wave theory** :According to wave theory, light is an electromagnetic radiation consisting of oscillating electric field vectors and magnetic field vectors. When electromagnetic radiations are incident on a metal surface, the free electrons [free electrons means the electrons which are loosely bound and free to move inside the metal] absorb energy from the radiation. This occurs by the oscillations of electron under the action of electric field vector of electromagnetic radiation. When an electron acquires sufficiently high energy so that it can overcome its binding energy, it comes out from the metal.

- **On the basis of photon theory:** According to photon theory of light, light consists of particles (called photons). Each particle carries a certain amount of energy with it. This energy is given by $E=h\nu$, where h is the Plank's constant and ν is the frequency. When the photons are incident on a metal surface, they collide with electrons. In some of the collisions, a photon is absorbed by an electron. Thus an electron gets energy $h\nu$. If this energy is greater than the binding energy of the electron, it comes out of the metal surface. The extra energy given to the electron becomes its kinetic energy.

EXPERIMENTS

- **Hertz Experiment :** Hertz observed that when ultraviolet rays are incident on negative plate of electric discharge tube then conduction takes place easily in the tube.

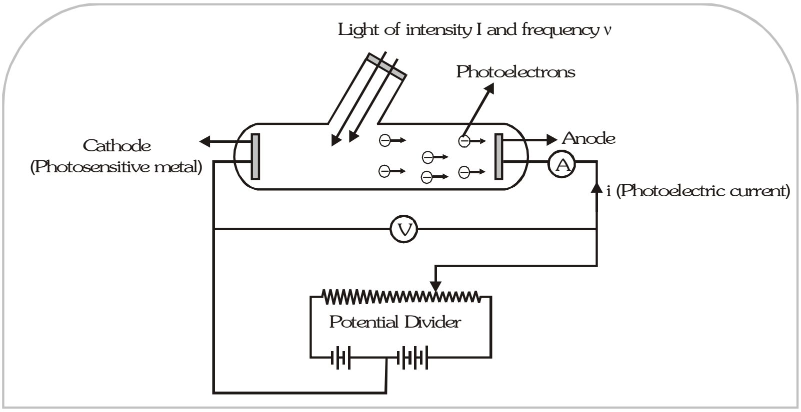


- **Hallwach experiment** : Hallwach observed that if negatively charged Zn plate is illuminated by U.V. light, its negative charge decreases and it becomes neutral and after some time it gains positive charge. It means in the effect of light, some negative charged particles are emitted from the metal.

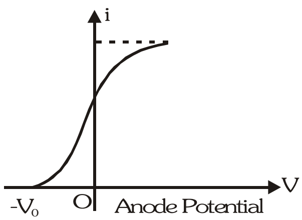
- **Lenard Explanation** : He told that when ultraviolet rays are incident on cathode, electrons are ejected. These electrons are attracted by anode and circuit is completed due to flow of electrons and current flows. When U.V. rays incident on anode, electrons are ejected but current does not flow.

  For the photo electric effect the light of short wavelength (or high frequency) is more effective than the light of long wavelength (or low frequency)

• **Experimental study of photoelectric Effect** : When light of frequency ν and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference between the anode and cathode.



(i) Variation of photoelectric current with potential difference : With the help of the above experimental setup, a graph is obtained between current and potential difference. The potential difference is varied with the help of a potential divider. The graph obtained is shown below.



The main points of observation are :

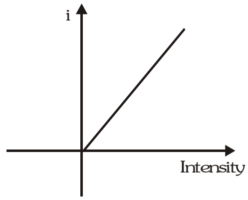
(a) At zero anode potential, a current exists. It means that electrons are emitted from cathode with some kinetic energy.

(b) As anode potential is increased, current increases. This implies that different electrons are emitted with different kinetic energies.

(c) After a certain anode potential, current acquires a constant value called saturation current. Current acquires a saturation value because the number of electrons emitted 7 per second from the cathode are fixed.

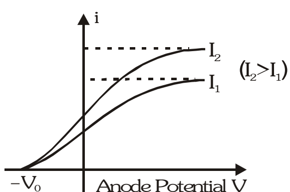
(d) At a certain negative potential, the photoelectric current becomes zero. This is called stopping potential (V_0). Stopping potential is a measure of maximum kinetic energy of the emitted electrons. Let KE_{max} be the maximum kinetic energy of an emitted electron, then $KE_{max} = eV_0$.

(ii) Variation of current with intensity



The photoelectric current is found to be directly proportional to intensity of incident radiation.

(iii) Effect of intensity on saturation current and stopping potential



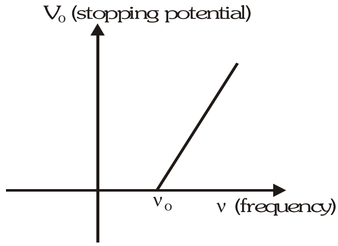
(a) Saturation current increases with increase in intensity.

(b) Stopping potential (and therefore maximum kinetic energy) is independent

(iv) **Effect of frequency**

(a) Stopping potential is found to vary with frequency of incident light linearly. Greater the frequency of incident light, greater the stopping potential.

(b) There exists a certain minimum frequency ν_0 below which no stopping potential is required as no emission of electrons takes place. This frequency is called threshold frequency. For photoelectric emission to take place, $\nu > \nu_0$.



GOLDEN KEY POINTS

- Photo electric effect is an instantaneous process, as soon as light is incident on the metal, photo electrons are emitted.
- Stopping potential does not depend on the distance between cathode and anode.
- The work function represented the energy needed to remove the least tightly bounded electrons from the surface. It depends only on nature of the metal and independent of any other factors.

- **Failure of wave theory of light**

(i) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron has to wait to gain sufficient energy to come out. But in experiment there is no time lag. Emission of electrons takes place in less than 10^{-9} s. This means, electron does not absorb energy. They get all the energy once.

(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

- **Einstein's Explanation of Photoelectric Effect**

Einstein explained photoelectric effect on the basis of photon–electron interaction. The energy transfer takes place due to collisions between an electrons and a photon. The electrons within the target material are held there by electric force. The electron needs a certain minimum energy to escape from this pull. This minimum energy is the property of target material and it is called the work function. When a photon of energy E=hν collides with and transfers its energy to an electron, and this energy is greater than the work function, the electron can escape through the surface.

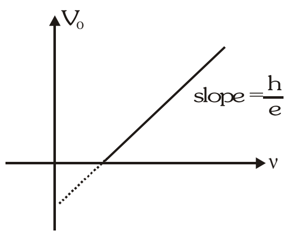
- **Einstein's Photoelectric Equation** $h\nu = \phi + KE_{max}$

Here hν is the energy transferred to the electron. Out of this, ϕ is the energy needed to escape.

The remaining energy appears as kinetic energy of the electron.

Now $KE_{max} = eV_0$ (where V_0 is stopping potential)

$\therefore\ h\nu = \phi + eV_0 \Rightarrow\ V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$



Thus, the stopping potential varies linearly with the frequency of incident radiation.

GOLDEN KEY POINTS

- Einstein's Photo Electric equation is based on conservation of energy.
- Einstein explained P.E.E. on the basis of quantum theory, for which he was awarded noble prize.
- According to Einstein one photon can eject one e^- only. But here the energy of incident photon should greater or equal to work function.
- In photoelectric effect all photoelectrons do not have same kinetic energy. Their KE range from zero to E_{max} which depends on frequency of incident radiation and nature of cathode.
- The photo electric effect takes place only when photons strike bound electrons because for free electrons energy and momentum conservations do not hold together.

Example

Calculate the possible velocity of a photoelectron if the work function of the target material is 1.24 eV and wavelength of light is 4.36 10^{-7} m. What retarding potential is necessary to stop the emission of electrons?

Solution

As $KE_{max} = h\nu - \phi \Rightarrow \frac{1}{2}mv_{max}^2 = h\nu - \phi = \frac{hc}{\lambda} - \phi$

$$v_{max} = \sqrt{\frac{2\left(\frac{hc}{\lambda} - \phi\right)}{m}} = \sqrt{\frac{2\left(\frac{6.63\times10^{-34}\times3\times10^{8}}{4.36\times10^{-7}} - 1.24\times1.6\times10^{-19}\right)}{9.11\times10^{-31}}} = 7.523 \quad 10^5 \text{ m/s}$$

$\therefore$ The speed of a photoelectron can be any value between 0 and 7.43 10^5 m/s

If V_0 is the stopping potential, then $eV_0 = \frac{1}{2}mv_{max}^2$

$$\Rightarrow V_0 = \frac{1}{2}\frac{mv_{max}^2}{e} = \frac{hc}{e\lambda} - \frac{\phi}{e} = \frac{12400}{4360} - 1.24 = 1.60 \text{ V} \quad \left[\because \frac{hc}{e} = 12400\times10^{-10}\,\text{V}-\text{m}\right]$$

Example

The surface of a metal of work function ϕ is illuminated by light whose electric field component varies with time as $E = E_0 [1 + \cos\omega t]\sin\omega_0 t$. Find the maximum kinetic energy of photoelectrons emitted from the surface.

Solution

The given electric field component is $E = E_0 \sin\omega_0 t + E_0 \sin\omega_0 t \cos\omega t = E_0 \sin\omega_0 t + \frac{E_0}{2}[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$

$\therefore$ The given light comprises three different frequencies viz. ω, $\omega_0 + \omega$, $\omega_0 - \omega$

The maximum kinetic energy will be due to most energetic photon.

$$\therefore KE_{max} = h\nu - \phi = \frac{h(\omega + \omega_0)}{2\pi} - \phi \qquad \left(\because \omega = 2\pi\nu \text{ or } \nu = \frac{\omega}{2\pi}\right)$$

Example

When light of wavelength λ is incident on a metal surface, stopping potential is found to be x. When light of wavelength $n\lambda$ is incident on the same metal surface, stopping potential is found to be $\frac{x}{n+1}$. Find the threshold wavelength of the metal.

Solution

Let λ_0 is the threshold wavelength. The work function is $\phi = \frac{hc}{\lambda_0}$.

Now, by photoelectric equation $ex = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ...(i) $\quad \frac{ex}{n+1} = \frac{hc}{n\lambda} - \frac{hc}{\lambda_0}$ ...(ii)

From (i) and (ii) $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = (n+1)\frac{hc}{n\lambda} - (n+1)\frac{hc}{\lambda_0} \Rightarrow \frac{nhc}{\lambda_0} = \frac{hc}{n\lambda} \Rightarrow \lambda_0 = n^2\lambda$

PHOTON THEORY OF LIGHT

- A photon is a particle of light moving with speed 299792458 m/s in vacuum.
- The speed of a photon is independent of frame of reference. This is the basic postulate of theory of relativity.
- The rest mass of a photon is zero. i.e. photons do not exist at rest.
- Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. $m \propto \frac{1}{\lambda}$

  So mass of violet light photon is greater than the mass of red light photon. $(\because \lambda_R > \lambda_V)$
- According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

  $E \propto \nu$ or $E = h\nu$

  $E = \frac{hc}{\lambda}$ joule $(\because c = \nu\lambda)$ or $E = \frac{hc}{\lambda e} = \frac{12400}{\lambda}$ eV – Å $\left[\because \frac{hc}{e} = 12400(\text{Å} - \text{eV})\right]$

  Here E = energy of photon, c = speed of light, h = Planck's constant, e = charge of electron

  h = 6.62 10^{-34} J–s, ν = frequency of photon, λ = wavelength of photon
- Linear momentum of photon $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$
- A photon can collide with material particles like electron. During these collisions, the total energy and total momentum remain constant.
- Energy of light passing through per unit area per unit time is known as intensity of light.

  Intensity of light $I = \frac{E}{At} = \frac{P}{A}$ ...(i)

  Here P = power of source, A = Area, t = time taken

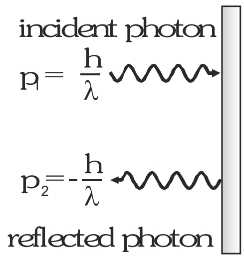
  E = energy incident in t time = $Nh\nu$, N = number of photon incident in t time

  Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii) $\left[\because n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

  From equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A} \Rightarrow n = \frac{P}{h\nu} = \frac{P\lambda}{hc} = 5 \quad 10^{24}\ J^{-1}\ m^{-1} \quad P \quad \lambda$
- When photons fall on a surface, they exert a force and pressure on the surface. This pressure is called radiation

- **Force exerted on perfectly reflecting surface**

Let 'N' photons are there in time t,



Momentum before striking the surface $(p_1) = \frac{Nh}{\lambda}$

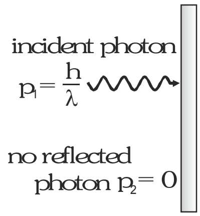
Momentum after striking the surface $(p_2) = -\frac{Nh}{\lambda}$

Change in momentum of photons $= p_2 - p_1 = \frac{-2Nh}{\lambda}$

But change in momentum of surface $= \Delta p = \frac{2Nh}{\lambda}$; So that force on surface $F = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

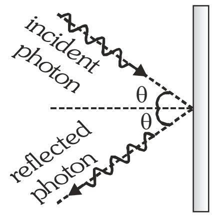
$\therefore F = \frac{2h}{\lambda} \times \frac{P\lambda}{hc} = \frac{2P}{c}$ and Pressure $= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$ $\left[\because I = \frac{P}{A}\right]$

- **Force exerted on perfectly absorbing surface**



$$F = \frac{p_1 - p_2}{t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{Nh}{t\lambda} = n\frac{h}{\lambda}\ ;\ F = \frac{P}{c}\ \left(\because n = \frac{P\lambda}{hc}\right) \text{ and Pressure } = \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$$

- **When a beam of light is incident at angle θ on perfectly reflector surface**



$$F = \frac{2P}{c}\cos\theta = n\left[\frac{2h}{\lambda}\right]\cos\theta = \frac{2IA\cos\theta}{c};\ \text{Pressure} = \frac{F}{A} = \frac{2I\cos\theta}{c}$$

Example

The intensity of sunlight on the surface of earth is 1400 W/m^2. Assuming the mean wavelength of sunlight to be 6000 Å, calculate:–

(a) The photon flux arriving at 1 m^2 area on earth perpendicular to light radiations and

(b) The number of photons emitted from the sun per second (Assuming the average radius of Earth's orbit to be 1.49 10^{11} m)

Solution

(a) Energy of a photon $E = \frac{hc}{\lambda} = \frac{12400}{6000}$ =2.06 eV = 3.3 10^{-19} J

Photon flux $= \frac{IA}{E} = \frac{1400 \times 1}{3.3 \times 10^{-19}}$ = 4.22 10^{21} photons/sec.

(b) Number of photons emitted per second $n = \frac{P}{E} = \frac{IA}{E} = \frac{1400 \times 4\pi \times (1.49 \times 10^{11})^2}{3.3 \times 10^{-19}}$ = 1.18 10^{45}

Example

In a photoelectric setup, a point source of light of power 3.2 10^{-3} W emits monochromatic photons of energy 5.0 eV. The source is located at a distance 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and radius 8 10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assuming that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission, Find (i) the number of photoelectrons emitted per second. (ii) the time t after light source is switched on, at which photoelectron emission stops.

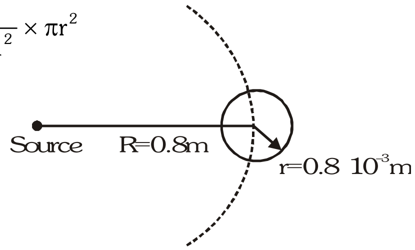
Solution

Energy of a single photon E=5.0 eV = $8 \quad 10^{-19}$ J

Power of source P = $3.2 \quad 10^{-3}$ W

$\therefore$ number of photons emitted per second $n = \frac{P}{E} = \frac{3.2\times10^{-3}}{8\times10^{-19}} = 4 \quad 10^{15}/s$

The number of photons incident per second on metal surface is $n_0 = \frac{n}{4\pi R^2}\times\pi r^2$



$$n_0 = \frac{4\times10^{15}}{4\pi(0.8)^2}\times\pi\left(8\times10^{-3}\right)^2 = 1.0\times10^{11} \text{ photon/s}$$

Number of electrons emitted = $\frac{1.0\times10^{11}}{10^6} = 10^5$ /s

$$KE_{max} = h\nu-\phi = 5.0 - 3.0 = 2.0 \text{ eV}$$

The photoelectron emission stops, when the metallic sphere acquires stopping potential.

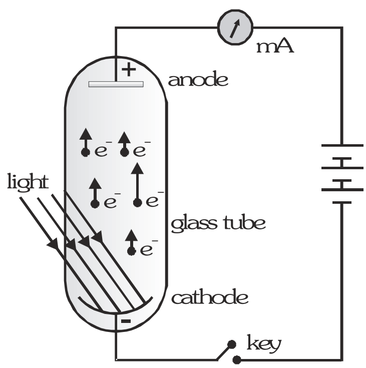
As KE_{max} = 2.0 eV $\Rightarrow$ Stopping potential V_0 = 2V $\Rightarrow 2 = \frac{q}{4\pi\varepsilon_0 r} \Rightarrow q = 1.78 \quad 10^{-12}$ C

Now charge q = (number of electrons/second) t e $\Rightarrow t = \frac{1.78\times10^{-12}}{10^5\times1.6\times10^{-19}} = 111$s

PHOTO CELL

A photo cell is a practical application of the phenomenon of photo electric effect, with the help of photo cell light energy is converted into electrical energy.

- **Construction** : A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).
- **Working:** When light of frequency greater than the threshold frequency of cathode material falls on the cathode, photoelectrons emitted are collected by the anode and an electric current starts flowing in the external circuit. The current increase with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.



- **Application**

(i) In television camera. (ii) In automatic door (iii) Burglar's alarm

(iv) Automatic switching of street light and traffic signals.

MATTER WAVES THEORY

DUAL NATURE OF LIGHT

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light. Experimental phenomena of light photoelectric effect and Crompton effect, pair production and positron inhalational can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light.

It is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both can not be possible simultaneously.

De-Broglie HYPOTHESIS

De Broglie imagined that as light possess both wave and particle nature, similarly matter must also posses both nature, particle as well as wave. De Broglie imagined that despite particle nature of matter, waves must also be associated with material particles. Wave associated with material particles, are defined as matter waves.

- **De Broglie wavelength associated with moving particles**

If a particle of mass m moving with velocity v

Kinetic energy of the particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ momentum of particle $p = mv = \sqrt{2mE}$ the wave length associated with the particles is $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ $\lambda \propto \frac{1}{p} \Rightarrow \lambda \propto \frac{1}{v} \Rightarrow \lambda \propto \frac{1}{\sqrt{E}}$

The order of magnitude of wave lengths associated with macroscopic particles is 10^{-24} Å.

The smallest wavelength whose measurement is possible is that of γ – rays ($\lambda \simeq 10^{-5}$ Å). This is the reason why the wave nature of macroscopic particles is not observable.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α – particle, atom, molecule etc. is of the order of 10^{-10} m, it is equal to the wavelength of X–rays, which is within the limit of measurement. Hence the wave nature of these particles is observable.

- **De Broglie wavelength associated with the charged particles**

Let a charged particle having charge q is accelerated by potential difference V.

Kinetic energy of this particle $E = \frac{1}{2}mv^2 = qV$ Momentum of particle $p = mv = \sqrt{2mE} = \sqrt{2mqV}$

The De Broglie wavelength associated with charged particle $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

- **For an Electron** $m_e = 9.1 \quad 10^{-31}$ kg, $q = 1.6 \quad 10^{-19}$ C, $h = 6.62 \quad 10^{-34}$ J–s

De Broglie wavelength associated with electron $\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{meter} = \frac{12.27}{\sqrt{V}} \text{Å} \quad \text{so} \quad \lambda \propto \frac{1}{\sqrt{V}}$$

150.6

- **For Proton** $m_p = 1.67 \times 10^{-27}$ kg

  De Broglie wavelength associated with proton

$$\lambda_p = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}V}} \; ; \; \lambda_p = \frac{0.286\times10^{-10}}{\sqrt{V}}\text{ meter } = \frac{0.286}{\sqrt{V}}\,\text{Å}$$

- **For Deuteron** $m_d = 2 \quad 1.67 \quad 10^{-27}$ kg, $q_d = 1.6 \quad 10^{-19}$ C

$$\lambda_d = \frac{6.62\times10^{-34}}{\sqrt{2\times2\times1.67\times10^{-27}\times1.6\times10^{-19}V}} = \frac{0.202}{\sqrt{V}}\,\text{Å}$$

- **For α Particles** $q_\alpha = 2 \quad 1.6 \quad 10^{-19}$ C, $m_\alpha = 4 \quad 1.67 \quad 10^{-27}$ kg

$$\therefore \lambda_\alpha = \frac{6.62\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{-27}\times2\times1.6\times10^{-19}V}} = \frac{0.101}{\sqrt{V}}\,\text{Å}$$

DE BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES

- **Kinetic energy of uncharged particle** $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

  m = mass of particle, v = velocity of particle, p = momentum of particle.

- **Velocity of uncharged particle** $v = \sqrt{\frac{2E}{m}}$

- **Momentum of particle** $p = mv = \sqrt{2mE}$

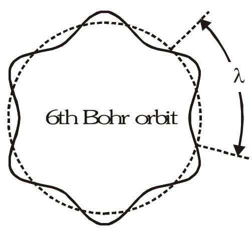
  wavelength associated with the particle $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

Kinetic energy of the particle in terms of its wavelength $E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m\lambda^2\times1.6\times10^{-19}}\,eV$

For a neutron $m_n = 1.67 \quad 10^{-27}$ kg $\therefore \lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times E}} = \frac{0.286\times10^{-10}}{\sqrt{E}}\text{meter}\sqrt{eV} = \frac{0.286}{\sqrt{E}}\,\text{Å}\sqrt{eV}$

EXPLANATION OF BOHR QUANTIZATION CONDITION

According to De Broglie electron revolves round the nucleus in the form of stationary waves (i. e. wave packet) in the similar fashion as stationary waves in a vibrating string. Electron can stay in those circular orbits whose circumference is an integral multiple of De–Broglie wavelength associated with the electron, $2\pi r = n\lambda$



$\because \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda \quad \therefore mvr = \frac{nh}{2\pi}$

This is the Bohr quantizations condition.

equivalent straightened orbit

Example

Find the initial momentum of electron if the momentum of electron is changed by p_m and the De Broglie wavelength associated with it changes by 0.50 %

Solution

$$\frac{d\lambda}{\lambda}\times 100 = 0.5 \Rightarrow \frac{d\lambda}{\lambda} = \frac{0.5}{100} = \frac{1}{200} \text{ and } \Delta p = p_m$$

$$\because p = \frac{h}{\lambda}, \text{ differentiating } \frac{dp}{d\lambda} = -\frac{h}{\lambda^2} = -\frac{h}{\lambda}\frac{1}{\lambda} = -\frac{p}{\lambda} \Rightarrow \frac{|dp|}{p} = \frac{d\lambda}{\lambda} \therefore \frac{p_m}{p} = \frac{1}{200} \Rightarrow p = 200\ p_m$$

Example

An α–particle moves in circular path of radius 0.83 cm in the presence of a magnetic field of 0.25 Wb/m^2. Find the De Broglie wavelength associated with the particle.

Solution

$$\lambda = \frac{h}{p} = \frac{h}{qBr} = \frac{6.62\times10^{-34}}{2\times1.6\times10^{-19}\times0.25\times83\times10^{-4}} \text{ meter} = 0.01\ \text{Å} \qquad \left[\because \frac{mv^2}{r} = qvB\right]$$

Example

A proton and an α–particle are accelerated through same potential difference. Find the ratio of their de- Broglie wavelength.

Solution

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}} \quad [\because E = qV]$$

For proton $m_p = m$, $q = e$

For α–particle $m_\alpha = 4\ m$, $q = 2e$, $\dfrac{\lambda_\alpha}{\lambda_p} = \sqrt{\dfrac{m_p q_p}{m_\alpha q_\alpha}} = \dfrac{1}{2\sqrt{2}}$

Example

A particle of mass m is confined to a narrow tube of length L.

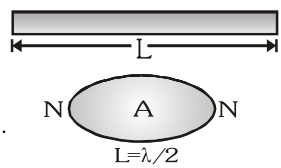
(a) Find the wavelengths of the de–Broglie wave which will resonate in the tube.

(b) Calculate the corresponding particle momenta, and

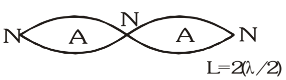
(c) Calculate the corresponding energies.

Solution

(a) The de–Broglie waves will resonate with a node at each end of the tube.

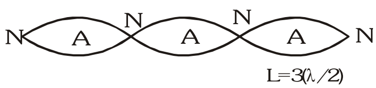


Few of the possible resonance forms are as follows : $\lambda_n = \dfrac{2L}{n}$, n=1,2,3.....

(b) 

Since de–Broglie wavelengths are $\lambda_n = \dfrac{h}{p_n}$

$p_n = \dfrac{h}{\lambda_n} = \dfrac{nh}{2L}$, $n = 1,2,3....$



(c) The kinetic energy of the particles are $K_n = \dfrac{p_n^2}{2m} = \dfrac{n^2h^2}{8L^2m}$, n = 1, 2, 3,....

NUCLEAR PHYSICS

ATOMIC NUCLEUS

The atomic nucleus consists of two types of elementary particles, viz. protons and neutrons. These particles are called nucleons. The proton (denoted by p) has a charge +e and a mass $m_p = 1.6726 \quad 10^{-27}$ kg, which is approximately 1840 times larger than the electron mass. The proton is the nucleus of the simplest atom with Z=1, viz the hydrogen atom.

The neutron (denoted by n) is an electrically neutral particle (its charge is zero). The neutron mass is $1.6749 \quad 10^{-27}$ kg. The fact that the mass of a neutron exceeds the mass of a proton by about 2.5 times the electronic masses is of essential importance. It follows from this that the neutron in free state (outside the nucleus) is unstable (radioactive). With half life equal to 12 min, the neutron spontaneously transforms into a proton by emitting an electron (e^-) and a particle called the antineutrino $(\overline{\nu})$.

This process can be schematically written as follows : ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \overline{\nu}$

The most important characteristics of the nucleus are the charge number Z (coinciding with atomic number of the element) and mass number A. The charge number Z is equal to the number of protons in the nucleus, and hence it determines the nuclear charge equal to Ze. The mass number A is equal to the number of nucleons in the nucleus (i.e., to the total number of protons and neutrons). Nuclei are symbolically designated as X_Z^A or ${}_ZX^A$ where X stands for the symbol of a chemical element.

For example, the nucleus of the oxygen atom is symbolically written as O_8^{16} or ${}_8O^{16}$.

The shape of nucleus is approximately spherical and its radius is approximately related to the mass number by $R = 1.2\ A^{1/3} \quad 10^{-15}\ m = 1.2 \quad 10^{-15} \quad A^{1/3}\ m$

Most of the chemical elements have several types of atoms differing in the number of neutrons in their nuclei. These varieties are called isotopes. For example carbon has three isotopes ${}_6C^{12}$, ${}_6C^{13}$, ${}_6C^{14}$. In addition to stable isotopes, there also exist unstable (radioactive) isotopes. Atomic masses are specified in terms of the atomic mass unit or unified mass unit (u). The mass of a neutral atom of the carbon ${}_6C^{12}$ is defined to be exactly 12 u. $1u = 1.66056 \quad 10^{-27}\ kg = 931.5\ MeV$.

BINDING ENERGY

The rest mass of the nucleus is smaller than the sum of the rest masses of nucleons constituting it. This is due to the fact that when nucleons combine to form a nucleus, some energy (binding energy) is liberated. The binding energy is equal to the work that must be done to split the nucleus into the particles constituting it.

The difference between the total mass of the nucleons and mass of the nucleus is called the mass defect of the nucleus represented by $\Delta m = [Zm_p + (A-Z)m_n] - m_{nuc}$

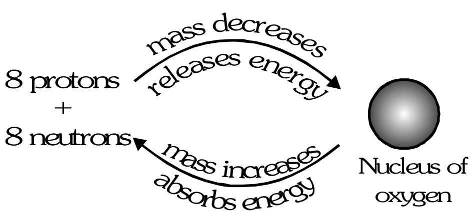
Multiplying the mass defect by the square of the velocity of light, we can find the binding energy of the nucleus.

$$BE = \Delta mc^2 = [(Zm_p + (A-Z)m_n) - m_{nuc}]c^2$$

If the masses are taken in atomic mass unit, the binding energy is given by

$$BE = [(Zm_p + (A-Z)m_n) - m_{nuc}]\ 931.5\ MeV$$

Let us take example of oxygen nucleus. It contains 8 protons and 8 neutrons. We can discuss concept of binding energy by following diagram.



$8m_p + 8m_n >$ mass of nucleus of oxygen

For nucleus we apply mass energy conservation, $8m_p + 8m_n = \text{mass of nucleus} + \frac{B.E.}{c^2}$

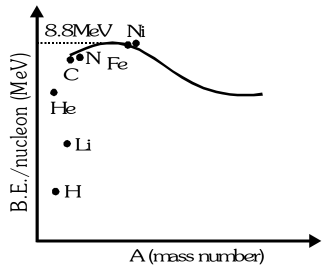
For general nucleus ${}^{A}_{Z}X$, mass defect = difference between total mass of nucleons and mass of the nucleus

$$\Delta m = [Zm_p + (A-Z)m_n] - M$$

$$B.E. = \Delta mc^2 \text{ (joules)} = (\Delta m)_{\text{in amu}} \quad 931.5 \text{ MeV}$$

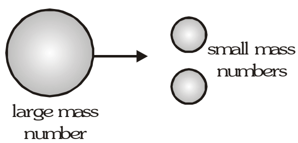
Binding Energy per Nucleon

Stability of a nucleus does not depend upon binding energy of a nucleus but it depends upon binding energy per nucleon $\quad B.E./\text{nucleon} = \frac{B.E.}{\text{mass number}} \qquad \text{Stability} \propto \frac{B.E.}{A}$

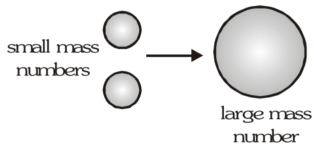


(i) B.E./A is maximum for A =62 (Ni), It is 8.79460 ± 0.00003 MeV/nucleon, means most stable nuclei are in the region of A=62.

(ii) Heavy nuclei achieve stability by breaking into two smaller nuclei and this reaction is called fission reaction.



(iii) Nuclei achieve stability by combining and resulting into heavy nucleus and this reaction is called fusion reaction.



(iv) In both reactions products are more stable in comparison to reactants and Q value is positive.

NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

We can apply :

(i) Conservation of momentum (ii) Conservation of charge (iii) Conservation of mass–energy

For any nuclear reaction $\quad \underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3}{Y} + \underset{K_4}{b}$

By mass energy conservation

(i) $K_1 + K_2 + (m_1 + m_2)c^2 = K_3 + K_4 + (m_3 + m_4)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma K_P - \Sigma K_R = (\Sigma m_R - \Sigma m_P)c^2$

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$$Q = \Sigma(B.E.)_{product} - \Sigma(B.E.)_{reactants}$$

Example

Let us find the Q value of fusion reaction

$$^4He + {}^4He \rightarrow {}^8Be, \quad \text{if } \frac{B.E.}{A} \text{ of He} = X \text{ and } \frac{B.E.}{A} \text{ of Be} = Y \Rightarrow Q = 8Y - 8X$$

Q value for α decay

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 \Rightarrow Q = K_\alpha + K_Y \qquad ....(i)$$

Momentum conservation, $\quad p_Y = p_\alpha \qquad ...(ii)$

$$K_\alpha = \frac{p^2}{2 \times m \times 4} \qquad K_Y = \frac{p^2}{2m(A-4)} = \frac{4K_\alpha}{A-4}$$

$$Q = K_\alpha + \frac{4K_\alpha}{A-4} = \frac{A}{A-4}K_\alpha \qquad K_\alpha = \frac{A-4}{A}Q$$

For α decay A > 210 which means maximum part of released energy is associated with K.E. of α. If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction is called Threshold energy E_{th}, given by

$$E_{th} = -Q\left(\frac{m_1}{m_2}+1\right) \quad \text{where } m_1 = \text{mass of the projectile.}$$

E_{th} = minimum kinetic energy of the projectile to initiate the nuclear reaction

m_2 = mass of the target

Example

How much energy must a bombarding proton possess to cause the reaction $_3Li^7 + {}_1H^1 \rightarrow {}_4Be^7 + {}_0n^1$

(Mass of $_3Li^7$ atom is 7.01600, mass of $_1H^1$ atom is 1.0783, mass of $_4Be^7$ atom is 7.01693)

Solution

Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Reactants : Total mass = $(7.01600 - 3\,m_e) + (1.0783 - 1\,m_e) = 8.0943 - 4m_e$

Products : Total mass = $(7.01693 - 4m_e) + 1.0087 = 8.02563 - 4m_e$

The energy is supplied as kinetic energy of the bombarding proton. The incident proton must have more than this energy because the system must possess some kinetic energy even after the reaction, so that momentum is conserved with momentum conservation taken into account, the minimum kinetic energy that the incident particle must possess can be found with the formula.

where, $Q = -[(8.02563 - 4m_e) - (8.0943 - 4\,m_e)]\ 931.5\ \text{MeV} = -63.96\ \text{MeV}$

$$E_{th} = -\left(1+\frac{m}{M}\right)Q = -\left(1+\frac{1}{7}\right)(-63.96) = 73.1\ \text{MeV}$$

NUCLEAR FISSION

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

Hahn and Strassmann done the first fision of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 190 MeV (≃ 200 MeV) energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are

$$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + 200\text{ MeV} \quad \text{or} \quad U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2{}_0n^1 + 200\text{ MeV}$$

and many other reactions are possible.

- The average number of secondary neutrons is 2.5.
- Nuclear fission can be explained by using "liquid drop model" also.
- The mass defect Δm is about 0.1% of mass of fissioned nucleus
- About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ – rays.

NUCLEAR CHAIN REACTION :

The equation of fission of U^{235} is $\quad U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + Q$.

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reaction' would start.

If there is no control on chain reaction then in a short time (≃10^{-6} sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb'). If chain is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) are generating electricity.

NATURAL URANIUM :

It is mixture of U^{235} (0.7%) and U^{238} (99.3%).

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV. But U^{238} is fissionable with fast neutrons.

Note : Chain reaction in natural uranium can't occur. To improve the quality, percentage of U^{235} is increased to 3%. The proposed uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

LOSSES OF SECONDARY NEUTRONS :

Leakage of neutrons from the system : Due to their maximum K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238} : Which is not fissionable by these secondary neutrons.

CRITICAL SIZE (OR MASS) :

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called **critical size**.

REPRODUCTION FACTOR :

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction)

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and will stop.

NUCLEAR REACTOR (K = 1) : Credit $\rightarrow$ To Enricho Fermi

Construction :

- **Nuclear Fuel :** Commonly used are U^{235} , Pu^{239}. Pu^{239} is the **best**. Its critical size is less than critical size of U^{235}. But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.
- **Moderator** : Its function is to slow down the fast secondary neutrons. Because only slow neutrons can bring the fission of U^{235}. The moderator should be light and it should not absorb the neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) Graphite etc. are used. These are rich of protons. Neutrons collide with the protons and interchange their energy. Thus neutrons get slow down.
- **Control rods** :They have the ability to capture the slow neutrons and can control the chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.
- **Coolant** : A substance which absorb the produced heat and transfers it to water for further use. Generally coolant is water at high pressure

FAST BREADER REACTORS

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy. The amount of produced fuel (Pu^{239}) is more than consumed fuel (U^{235})

- **Fuel :** Natural Uranium.
- **Process**: During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

$$_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

- **Moderator :** Are not used in these reactors.
- **Coolant :** Liquid sodium

NUCLEAR FUSION :

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) & $m_C < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m \; 931$ MeV

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus :

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\;{}_{+}\beta^0 + 2\,\nu + 26 \text{ MeV}$$

- Energy released per fission >> Energy released per fusion.
- Energy per nucleon in fission $\left[=\dfrac{200}{235} \simeq 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[=\dfrac{24}{4} \simeq 6\text{MeV}\right]$

REQUIRED CONDITION FOR NUCLEAR FUSION

- **High temperature** :

  Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.
- **High Pressure (or density)** :

  Which ensure frequent collision and increases the probability of fusion. The required temperature and pressure at earth (lab) are not possible. These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and there protons fuse to form helium nuclei.

HYDROGEN BOMB

It is based on nuclear fusion and produces more energy than an atom bomb.

Pair production	Pair Annihilation
A γ–photon of energy more than 1.02 MeV, when interact with a nucleus produces pair of electron (e^-) and positron (e^+). The energy equivalent to rest mass of e^- (or e^+)=0.51 MeV. The energy equivalent to rest mass of pair (e^-+e^+) = 1.02 MeV. For pair production Energy of photon $\geq$ 1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E–1.02) MeV divides approximately in equal amount to each particle as the kinetic energy or $(KE)_{e^- \text{ or } e^+} = \left[\dfrac{E_{Ph}-1.02}{2}\right]$ MeV If E < 1.02 MeV, pair will not produce.	When electron and positron combines they annihilates to each other and only energy is released in the form of two gama photons. If the energy of electron and positron are negligible then energy of each γ–photon is 0.5/MeV

Example

In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439) in 24 hours by slow neutrons (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost. (i) Write the complete reaction (ii) Calculate the total energy produced in kilowatt hour. Given 1u = 931 MeV.

Solution

The nuclear fission reaction is $_{92}U^{235} + {_0}n^1 \rightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3\,{_0}n^1$

Mass defect $\Delta m = [(m_u + m_n) - (m_{Ba} + m_{Kr} + 3m_n)] = 256.0526 - 235.8373 = 0.2153$ u

Energy released Q = $0.2153 \quad 931$ = 200 MeV. Number of atoms in 1 g = $\dfrac{6.02 \times 10^{23}}{235} = 2.56 \quad 10^{21}$

Energy released in fission of 1 g of U^{235} is E $= 200 \quad 2.56 \quad 10^{21} = 5.12 \quad 10^{23}$ MeV

$= 5.12 \quad 10^{23} \quad 1.6 \quad 10^{-13} = 8.2 \quad 10^{10}$ J

$= \dfrac{8.2 \times 10^{10}}{3.6 \times 10^{6}}$ kWh $= 2.28 \quad 10^{4}$ kWh

Example

It is proposed to use the nuclear fusion reaction : $_1H^2 + {_1}H^2 \rightarrow {_2}He^4$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of $_1H^2$ is 2.0141 u and mass of $_2He^4$ is 4.0026 u)

Solution

Energy released in the nuclear fusion is Q = $\Delta mc^2 = \Delta m(931)$ MeV (where Δm is in amu)

Q = $(2 \quad 2.0141 - 4.0026) \quad 931$ MeV = 23.834 MeV = $23.834 \quad 10^6$ eV

Since efficiency of reactor is 25%

So effective energy used $= \dfrac{25}{100} \times 23.834 \times 10^6 \times 1.6 \times 10^{-19}$ J $= 9.534 \quad 10^{-13}$ J

Since the two deuterium nuclei are involved in a fusion reaction,

therefore, energy released per deuterium is $\dfrac{9.534 \times 10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required $= \dfrac{200 \times 10^6 \times 86400}{\dfrac{9.534 \times 10^{-23}}{2}} = 3.624 \quad 10^{25}$

Since 2g of deuterium constitute $6 \quad 10^{23}$ nuclei, therefore amount of deuterium required is

$2 \times 3.624 \times 10^{25}$

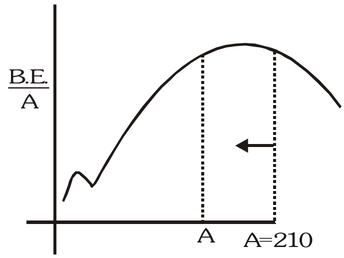
RADIOACTIVITY

The process of spontaneous disintegration shown by some unstable atomic nuclei is known as natural radioactivity. This property is associated with the emission of certain types of penetrating radiations, called radioactive rays, or Becquerel rays (α, β, γ–rays). The elements or compounds, whose atoms disintegrate and emit radiations are called radioactive elements. Radioactivity is a continuous, irreversible nuclear phenomenon.

Radioactive Decays

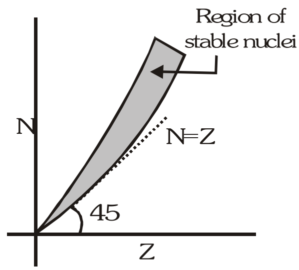
Generally, there are three types of radioactive decays (i) α decay (ii) β^- and β^+ decay (iii) γ decay

- ### α decay



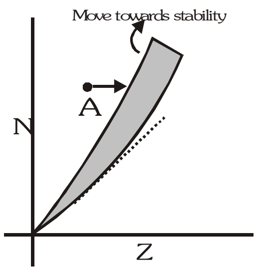
In α decay, the unstable nucleus emits an α particle. By emitting α particle, the nucleus decreases it's mass energy number and move towards stability. Nucleus having A>210 shows α decay.By releasing α particle, it can attain higher stability and Q value is positive.

- ### β decay



In beta decay (N/Z) ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding we discuss N/Z graph. There are two type of unstable nuclides

  - **A type**



For A type nuclides $(N/Z)_A > (N/Z)$ stable

To achieve stability, it increases Z by conversion of neutron into proton

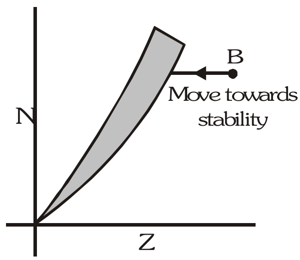
$${}_0n^1 \rightarrow {}_1p^1 + e^{-1} + \bar{\nu}, \quad {}_ZX^A \rightarrow {}_{Z+1}Y^A + \underset{(\beta\ \text{particle})}{e^{-1}} + \bar{\nu}$$

This decay is called β^{-1} decay.

Kinetic energy available for e^{-1} and $\bar{\nu}$ is, $Q = K_\beta + K_{\bar{\nu}}$

K.E. of β satisfies the condition $0 < K_\beta < Q$

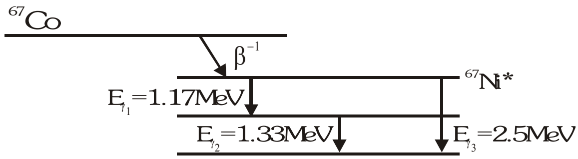
  - **B type**



For B type nuclides $(N/Z)_B < (N/Z)$ stable

To achieve stability it decreases Z by the conversion of a proton into neutron. That is, $p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}$, ${}_ZX^A \rightarrow {}_{Z-1}Y^A + \underset{(\beta^+\text{particle})}{e^+} + \nu$

- **γ decay** : when an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.



Order of energy of γ photon is 100 KeV e.g. $^{67}_{27}Co \rightarrow \underset{\text{(higher energy state)}}{^{67}_{28}Ni^*} + \beta^- + \bar{\nu}$, $^{67}_{28}Ni^* \rightarrow ^{67}_{28}Ni + \gamma$ photon

Properties of α, β and γ rays

Features	α–particles	β–particles	γ–rays
Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electrons ($_{-}\beta^0$ or β^-)	Electromagnetic wave (photons)
Charge	Twice of proton (+2e) $\approx 4m_p$	Electronic (– e)	Neutral
Mass	(rest mass of β) m_p–mass of proton	rest mass = 0 = (rest mass of electron)	
Speed	1.4 10^7 m/s. to 2.2 10^7 m/s. (Only certain value between this range). Their speed depends on nature of the nucleus. So that it is a characteristic speed.	1% of c to 99% of c (All possible values between this range) β–particles come out with different speeds from the same type of nucleus. So that it can not be a characteristic speed.	Only c = 3 10^8 m/s γ–photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
K.E.	≈ MeV	≈ MeV	≈ MeV
Energy spectrum	Line and discrete (or linear)	Continuous (or linear)	Line and discrete
Ionization power (α>β>γ)	10,000 times of γ–rays	100 times of γ–rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
Penetration power (γ>β>α)	$\frac{1}{10000}$ times of γ–rays	$\frac{1}{100}$ times of γ–rays (100 times of α)	1(100 times of β)
Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection
Explanation of emission	By Tunnel effect (or quantum mechanics)	By weak nuclear interactions	With the help of energy levels in nucleus

Laws of Radioactive Decay

1. The radioactive decay is a spontaneous process with the emission of α, β and γ rays. It is not influenced by external conditions such as temperature, pressure, electric and magnetic fields.

2. The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of nuclei) i.e. $\frac{dN}{dt} = -\lambda N$

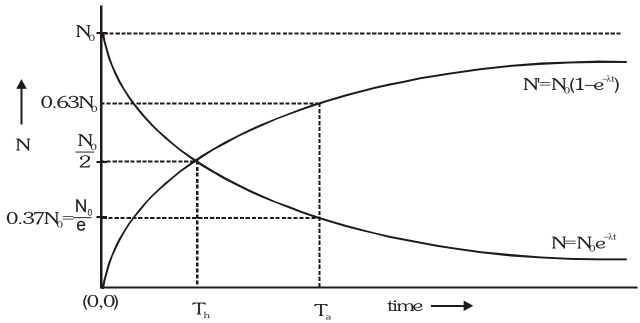
where λ is called the decay constant. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow \ell n\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t=0. The number that survives at time t is therefore

$N=N_0e^{-\lambda t}$ and $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N_t}\right)$ this function is plotted in figure.

Graph : Time versus N (or N')



- **Half life (T_h)** : It is the time during which number of active nuclei reduce to half of initial value.

If at t = 0 no. of active nuclei N_0 then at t = T_h number of active nuclei will be $\frac{N_0}{2}$

From decay equation $N = N_0e^{-\lambda t}$

$$\frac{N_0}{2} = N_0e^{-\lambda T_h} \Rightarrow T_h = \frac{\ell n(2)}{\lambda} = \frac{0.693}{\lambda} \approx \frac{0.7}{\lambda}$$

- **Mean or Average Life (T_a)** : It is the average of age of all active nuclei i.e.

$$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

(i) At t = 0, number of active nuclei = N_0 then number of active nuclei at

$t = T_a$ is $N = N_0e^{-\lambda T_a} = N_0e^{-1} = \frac{N_0}{e} = 0.37N_0 = 37\%$ of N_0

(ii) Number nuclei which have been disintegrated within duration T_a is

$N' = N_0 - N = N_0 - 0.37\ N_0 = 0.63\ N_0 = 63\%$ of N_0

- $T_a = \frac{1}{\lambda} = \frac{T_h}{\ell n(2)} = \frac{T_h}{0.693} = 1.44\ T_h$
- Within duration $T_h \Rightarrow 50\%$ of N_0 decayed and 50% of N_0 remains active
- Within duration $T_a \Rightarrow 63\%$ of N_0 decayed and 37% of N_0 remains active

ACTIVITY OF A SAMPLE (OR DECAY RATE)

It is the rate of decay of a radioactive sample $R = -\frac{dN}{dt} = N\lambda$ or $R = R_0 e^{-\lambda t}$

- Activity of a sample at any instant depends upon number of active nuclei at that instant.

  $R \propto N$ (or active mass) , $R \propto m$
- R also decreases exponentially w.r.t. time same as the number of active nuclei decreases.
- R is not a constant with N, m and time while λ, T_h and T_a are constant
- At $t = 0$, $R = R_0$ then at $t = T_h$ $\Rightarrow R = \frac{R_0}{2}$ and at $t = T_a \Rightarrow R = \frac{R_0}{e}$ or $0.37\ R_0$
- Similarly active mass of radioactive sample decreases exponentially. $m = m_0 e^{-\lambda t}$
- Activity of m gm active sample (molecular weight M_w) is $R = \lambda N = \frac{0.693}{T_h}\left[\frac{N_{AV}}{M_W}\right] m$

here N_{AV} = Avogadro number = 6.023 10^{23}

SI UNIT of R : 1 becquerel (1 Bq)= 1 decay/sec

Other Unit is curie : 1 Ci = 3.70 10^{10} decays/sec

1 Rutherford : (1 Rd) =10^6 decays/s

Specific activity : Activity of 1 gm sample of radioactive substance. Its unit is Ci/gm

e.g. specific activity of radium (226) is 1 Ci/gm.

Example

The half–life of cobalt–60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?

Solution

The activity is proportional to the number of undecayed atoms: In each half–life, the remaining sample decays to half of its initial value. Since $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) = \frac{1}{8}$, therefore, three half–lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

Example

A count rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.

(i) Find the decay constant. (ii) Also, find the half–life of the sample.

Solution

Initial activity $A_i = \left.\frac{-dN}{dt}\right|_{t=0} = \lambda N_0 = 4750$ ...(i) Final activity $A_f = \left.\frac{-dN}{dt}\right|_{t=5} = \lambda N = 2700$ ...(ii)

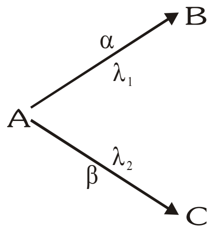
Dividing (i) by (ii), we get $\frac{4750}{2700} = \frac{N_0}{N_t}$

The decay constant is given by $\lambda = \frac{2.303}{t}\log\frac{N_0}{N_t} = \frac{2.303}{5}\log\frac{4750}{2700} = 0.113\ \text{min}^{-1}$

Half–life of the sample is $T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.14$ min

- **Parallel radioactive disintegration**

Let initial number of nuclei of A is N_0 then at any time number of nuclei of



A, B & C are given by $N_0 = N_A + N_B + N_C \Rightarrow \dfrac{dN_A}{dt} = \dfrac{-d}{dt}(N_B + N_C)$

A disintegrates into B and C by emitting α, β particle.

Now, $\dfrac{dN_B}{dt} = -\lambda_1 N_A$ and $\dfrac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \dfrac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2)N_A$

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

Example

The mean lives of a radioactive substances are 1620 and 405 years for α–emission and β–emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by α–emission and β–emission simultaneously.

Solution

When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by $\lambda_{av} = \lambda_\alpha + \lambda_\beta$ when λ_α = disintegration constant for α–emission only λ_β = disintegration constant for β–emission only

Mean life is given by $T_m = \dfrac{1}{\lambda}$, $\lambda_{av} = \lambda_\alpha + \lambda_\beta \Rightarrow \dfrac{1}{T_m} = \dfrac{1}{T_\alpha} + \dfrac{1}{T_\beta} = \dfrac{1}{1620} + \dfrac{1}{405} = \dfrac{1}{324}$

$\lambda_{av} \quad t = 2.303 \log \dfrac{N_0}{N_t}$, $\dfrac{1}{324} t = 2.303 \log \dfrac{100}{25} \Rightarrow t = 2.303 \quad 324 \log 4 = 449$ years.

Example

A radioactive decay is given by $A \xrightarrow[t_{1/2}=8\text{ yrs}]{} B$

Only A is present at t=0. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

Solution

		A	$\longrightarrow$	B
at	$t=0$	N_0		0
at	$t=t$	N		$N_0 - N$

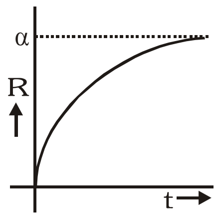
Probability of getting A, $P_A = \dfrac{N}{N_0}$

Probability of getting B, $P_B = \dfrac{N_0 - N}{N_0} \Rightarrow P_B = 15\, P_A \Rightarrow \dfrac{N_0 - N}{N_0} = 15\dfrac{N}{N_0} \Rightarrow N_0 = 16N \Rightarrow N = \dfrac{N_0}{16}$

Remaining nuclei are $\dfrac{1}{16}$ th of initial nuclei, hence required time t=4 half lives =32 years

Radioactive Disintegration with Successive Production

$$\xrightarrow[(\alpha=\text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$$



$$\frac{dN_A}{dt} = \alpha = \lambda N_A \text{ ....(i)}$$

when N_A in maximum $\frac{dN_A}{dt} = 0 = \alpha - \lambda N_A = 0$, N_A max $= \frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$

By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

Example

$$\xrightarrow{10^{21}\text{ per sec}} A \xrightarrow{\lambda=\frac{1}{30}} B$$

A shows radioactive disintegration and it is continuously produced at the rate of 10^{21} per sec. Find maximum number of nuclei of A.

Solution

At maximum, $r_{\text{production}} = r_{\text{decay}} \Rightarrow 10^{21} = \frac{1}{30} N \Rightarrow$ N=30 10^{21}

Soddy and Fajan's Group Displacement Laws :

(i) **α–decay** : The emission of one α–particle reduces the mass number by 4 units and atomic number by 2 units. If parent and daughter nuclei are represented by symbols X and Y respectively then,

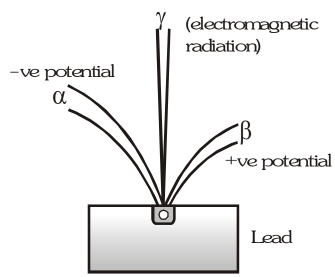
$${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}(\alpha)$$

(ii) **β–decay** : Beta particles are said to be fast moving electrons coming from the nucleus of a radioactive substance. Does it mean that a nucleus contains electrons? No, it is an established fact that nucleus does not contain any electrons. When a nucleus emits a beta particle, one of its neutrons breaks into a proton, an electron (i.e., β–particle) and an antineutrino $n \rightarrow p + e + \overline{\nu}$

where n= neutron p = proton e = β–particle

Thus emission of a beta particle is caused by the decay of a neutron into a proton. The daughter nucleus thus has an atomic number greater than one (due to one new proton in the nucleus) but same mass number as that of parent nucleus. Therefore, representing the parent and daughter nucleus by symbols X and Y respectively, we have ${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + \beta + \overline{\nu}$

(iii) **γ–decay** :When parent atoms emit gamma rays, no charge is involved as these are neutral rays. Thus there is no effect on the atomic number and mass number of the parent nucleus. However the emission of γ–rays represents energy. Hence the emission of these rays changes the nucleus from an excited (high energy) state to a less excited (lower energy) state.



SOME WORKED OUT EXAMPLES

Example#1

A photon of energy 12.09 eV is completely absorbed by a hydrogen atom initially in the ground state. The quantum number of the excited state is

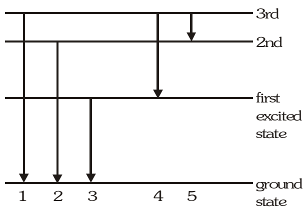
(A) 4 (B) 5 (C) 3 (D) 2

Solution **Ans. (C)**

Energy difference in hydrogen atom = $13.6 - \frac{13.6}{n^2} = 12.09 \Rightarrow n^2 \approx 9 \Rightarrow n=3$

Example#2

The figure indicates the energy level diagram of an atom and the origin of five spectral lines in emission spectra. Which of the spectral lines will also occur in the absorption spectra?



(A) 1, 2, 4 (B) 2, 3, 4, 5 (C) 1, 2, 3 (D) 1,2,3,4,5

Solution **Ans. C**

For absorpiton spectra, atom must be in ground state

Example#3

The ionization energy of Li^{++} is equal to

(A) 6hcR (B) 2hcR (C) 9hcR (D)hcR

Solution **Ans. (C)**

Ionization energy = $(Rhc)Z^2$ = 9hcR $\left(\text{Note} : E_n = -\frac{RhcZ^2}{n^2}\right)$

Example#4

The electron in a hydrogen atom makes a transition from $n=n_1$ to $n=n_2$ state. The time period of the electron in state n_1 is eight times that in state n_2. The possible values of n_1 and n_2 are

(A) $n_1 = 8$, $n_2=1$ (B) $n_1=4$, $n_2=1$ (C) $n_1=4$, $n_2=2$ (D) $n_1=2$, $n_2=4$

Solution **Ans. (C)**

Time period $T = \frac{2\pi r}{v} \propto n^3$ But $T_1 = 8T_2 \Rightarrow n_1^3 = 8n_2^3 \Rightarrow n_1=2n_2$

Example#5

When a hydrogen atom is excited from ground state to first excited state, then

(A) its kinetic energy increases by 10.2 eV (B) its kinetic energy decreases by 13.6 eV

(C) its potential energy increases by 10.2 eV (D) its angular momentum increases by $h/2\pi$

Solution **Ans. (D)**

In ground state , kinetic energy = 13.6 eV, Potential energy = – 27.2 eV

In first excited state, kinetic energy = 3.4 eV, Potential energy = – 6.8 eV

Angular momentum is $=\frac{nh}{2\pi}$; Difference of angular momentum for consecutive orbit $=\frac{h}{2\pi}$

Example#6

The energy of a tungsten atom with a vacancy in L shell is 11.3 KeV. Wavelength of K_α photon for tungsten is 21.3 pm. If a potential difference of 62 kV is applied across the X–rays tube following characteristic X–rays will be produced .

(A) K,L series (B) only K_α & L series (C) only L series (D) None of these

Solution **Ans. (C)**

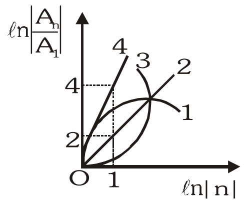
$$\Delta E = = \frac{hc}{\lambda} = \frac{1240}{\lambda(nm)} \Rightarrow \frac{1240}{21.3 \times 10^{-3}}$$

ΔE_2 = 58.21 KeV ; ΔE_1 = 11.3 KeV ; ΔE = 69.51 KeV

ΔE < 62 KeV Therefore only L series will be produced.

Example#7

The figure shows a graph between $\ell n\left|\frac{A_n}{A_1}\right|$ and $\ell n|n|$, where A_n is the area enclosed by the n^{th} orbit in a hydrogen like atom. The correct curve is-



(A) 4 (B) 3 (C) 2 (D) 1

Solution **Ans. (A)**

$$A_n = \pi r^2{}_n \Rightarrow \frac{A_n}{A_1} = \left(\frac{r_n}{r_1}\right)^2 = \left(\frac{n}{1}\right)^4 \Rightarrow \log_e \frac{A_n}{A_1} = 4 \log_e(n)$$

Example#8

Consider the radiation emitted by large number of singly charged positive ions of a certain element. The sample emit fifteen types of spectral lines, one of which is same as the first line of lyman series. What is the binding energy in the highest energy state of this configuration ?

(A) 13.6 eV (B) 54.4 eV (C) 10.2eV (D) 1.6 eV

Solution **Ans. (D)**

$\frac{n(n-1)}{2} = 15 \Rightarrow n = 6$ Element should be hydrogen like atoms so ion will be He^+

Binding energy $= \frac{(13.6)(Z^2)}{n^2} = \frac{(13.6)(4)}{36} = \frac{13.6}{9} = 1.6eV$

Example#9

The probability that a certain radioactive atom would get disintegrated in a time equal to the mean life of the radioactive sample is-

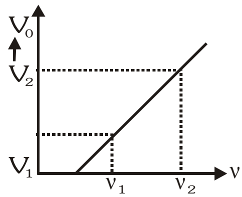
(A) 0.37 (B) 0.63 (C) 0.50 (D) 0.67

Solution **Ans. (B)**

Required probability $P(t) = \frac{N_0(1-e^{-\lambda t})}{N_0} = 1 - e^{-\lambda\left(\frac{1}{\lambda}\right)} = 1 - e^{-1} \simeq 0.63$

Example#10

Figure shows the graph of stopping potential versus the frequency of a photosensitive metal. The plank's constant and work function of the metal are



(A) $\left(\frac{V_2+V_1}{\nu_2+\nu_2}\right)e, \left(\frac{V_2\nu_1+V_1\nu_2}{\nu_2+\nu_1}\right)e$

(B) $\left(\frac{V_2+V_1}{\nu_2-\nu_1}\right)e, \left(\frac{V_2\nu_1+V_1\nu_2}{\nu_2-\nu_1}\right)e$

(C) $\left(\frac{V_2-V_1}{\nu_2+\nu_1}\right)e, \left(\frac{V_2\nu_1-V_1\nu_2}{\nu_2+\nu_1}\right)e$

(D) $\left(\frac{V_2-V_1}{\nu_2-\nu_1}\right)e, \left(\frac{V_2\nu_1-V_1\nu_2}{\nu_2-\nu_1}\right)e$

Solution **Ans. (D)**

Equation of straight line $\frac{V-V_1}{\nu-\nu_1}=\frac{V_2-V_1}{\nu_2-\nu_1} \Rightarrow V=\left(\frac{V_2-V_1}{\nu_2-\nu_1}\right)\nu-\left(\frac{V_2\nu_1-V_1\nu_2}{\nu_2-\nu_1}\right)$

But $V=\left(\frac{h}{e}\right)\nu-\frac{\phi_0}{e} \Rightarrow h=\left(\frac{V_2-V_1}{\nu_2-\nu_1}\right)e$ and $\phi_0=\left(\frac{V_2\nu_1-V_1\nu_2}{\nu_2-\nu_1}\right)e$

Example#11

Consider a hydrogen like atom whose energy in n^{th} excited state is given by $E_n=-\frac{13.6Z^2}{n^2}$ when this excited atom makes a transition from excited state to ground state, most energetic photons have energy E_{max} = 52.2224 eV and least energetic photons have energy E_{min} = 1.224 eV.The atomic number of atom is

(A) 2 (B) 5 (C) 4 (D) None of these

Solution **Ans. (A)**

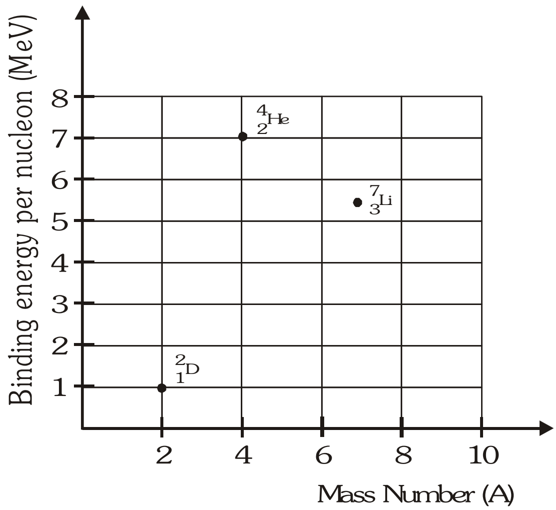
Maximum energy is liberated for transition $E_n \to E_1$ and minimum energy for $E_n \to E_{n-1}$

Hence $\frac{E_1}{n^2}-E_1=52.224eV$ and $\frac{E_1}{n^2}-\frac{E_1}{(n-1)^2}=1.224eV \Rightarrow E_1$ = –54.4eV and n=5

Now $E_1 = -\frac{13.6Z^2}{1^2}$ = –54.4 eV. Hence Z = 2

Example#12

The positions of $^2_1D, ^4_2He$ and ^7_3Li are shown on the binding energy curve as shown in figure.



The energy released in the fusion reaction. $^2_1D + ^7_3Li \to 2\,^4_2He + ^1_0n$

Solution **Ans. (B)**

Released energy = 2 4 7 – 2 1 – 7 5.4 = 16 MeV

Example#13

How many head-on elastic collisions must a neutron have with deuterium nuclei to reduce it energy from 6.561 MeV to 1 keV ?

(A) 4 (B) 8 (C) 3 (D) 5

Solution **Ans. (A)**

$$\frac{\text{Energy loss}}{\text{Initial KE}} = \frac{4m_1m_2}{(m_1+m_2)^2} = \frac{4(1)(2)}{(1+2)^2} = \frac{8}{9}$$

After 1[st] collision $\Delta E_1 = \frac{8}{9}E_0$, After 2[nd] collision $\Delta E_2 = \frac{8}{9}E_1$, After n[th] collision $\Delta E_n = \frac{8}{9}E_{n-1}$

Adding all the losses

$$\Delta E = \Delta E_1 + \Delta E_2 + ... + \Delta E_n = \frac{8}{9}(E_0 + E_1 + ... E_{n-1})\ ;\ \text{here } E_1 = E_0 - \Delta E_1 = E_0 - \frac{8}{9}E_0 = \frac{1}{9}E_0$$

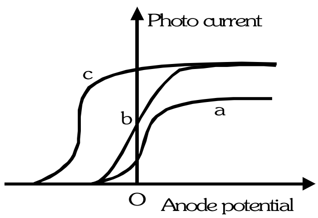
$$E_2 = E_1 - \Delta E_2 = E_1 - \frac{8}{9}E_1 = \frac{1}{9}E_1 = \left(\frac{1}{9}\right)^2 E_0 \text{ and so on}$$

$$\Rightarrow \Delta E = \frac{8}{9}\left[E_0 + \frac{1}{9}E_0 + \left(\frac{1}{9}\right)^2 E_0 + ... + \left(\frac{1}{9}\right)^{n-1} E_0\right] = \frac{8}{9}\left[\frac{1-\frac{1}{9^n}}{1-\frac{1}{9}}\right]E_0 = \left(1-\frac{1}{9^n}\right)E_0$$

$$E_0 = 6.561 \text{ MeV},\ \Delta E = (6.561 - 0.001)\text{ MeV} \Rightarrow \frac{6.561-0.001}{6.561} = 1-\frac{1}{9^n} \Rightarrow \frac{1}{6561} = \frac{1}{9^n} \Rightarrow n = 4$$

Example#14

The figure shows the variation of photo current with anode potential   for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively. Choose the **incorrect** relation.



(A) $f_a = f_b$ (B) $I_a < I_b$ (C) $f_c < f_b$ (D) $I_c > I_b$

Solution **Ans. (C)**

Here $I_B = I_C > I_a$ and $f_c > f_a = f_b$

Example#15

Statement-1: Radioactive nuclei emit β-particles (fast moving electrons)

and

Statement-2: Electrons exist inside the nucleus.

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is NOT a correct explanation for Statement–1

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

Example#16

A vessel of 831cc contains ${}_{1}^{3}H$ at 0.6 atm and 27 C. If half life of ${}_{1}^{3}H$ is 12.3 years then the activity of the gas is-

(A) 3.04 10^{13} dps (B) 582 Ci (C) 2.15 10^{13} dps (D) 823 Ci

Solution **Ans. (B,C)**

Number of moles of gas $n = \dfrac{PV}{RT} = \dfrac{(0.6 \times 10^5)(831 \times 10^{-6})}{(8.31)(300)}$ =0.02

Activity = λN = $\dfrac{(0.693)nN_A}{T_{1/2}} = \dfrac{(0.693)(0.02)(6.02 \times 10^{23})}{12.3 \times 3.15 \times 10^7}$ = 2.15 10^{13} dps = $\dfrac{2.15 \times 10^{13}}{3.7 \times 10^{10}}$ =582 Ci

Example#17

Choose the **CORRECT** statement(s)

(A) Mass of products formed is less than the original mass in nuclear fission and nuclear fusion reactions.

(B) Binding energy per nucleon increases in α-decay and β-decay.

(C) Mass number is conserved in all nuclear reactions.

(D) Atomic number is conserved in all nuclear reactions.

Solution **Ans. (ABC)**

Fusion and fission are always exothermic and α & β decay will result in more stable product.

Mass number is conserved but atomic number is not conserved.

${}_{A}^{N}X \rightarrow {}_{B}^{M}Y + {}_{C}^{0}Z$, N = M + 0 and A may not be equal to B + C

Example#18 to 20

Einstein in 1905 propounded the special theory of relativity and in 1915 proposed the general theory of relativity. The special theory deals with inertial frames of reference. The general theory of relativity deals with problems in which one frame of reference. He assumed that a fixed frame is accelerated w.r.t. another frame of reference of reference cannot be located. Postulates of special theory of relativity

- The laws of physics have the same form in all inertial systems.
- The velocity of light in empty space is a universal constant the same for all observers.

Einstein proved the following facts based on his theory of special relativity. Let v be the velocity of the speceship w.r.t. a given frame of reference. The observations are made by an observer in that reference frame.

- All clocks on the spaceship will go slow by a factor $\sqrt{1 - v^2/c^2}$
- All objects on the spaceship will have contracted in length by a factor $\sqrt{1 - v^2/c^2}$
- The mass of the spaceship increases by a factor $\sqrt{1 - v^2/c^2}$
- Mass and energy are interconvertable $E = mc^2$
- The speed of a material object can never exceed the velocity of light.
- If two objects A and B are moving with velocity u and v w.r.t. each other along the x-axis, the relative velocity of A w.r.t. B = $\dfrac{u - v}{1 - uv/c^2}$

18. One cosmic ray particle approaches the earth along its axis with a velocity of 0.9c towards the north pole and another one with a velocity of 0.5c towards the south pole. The relative speed of approach of one particle w.r.t. another is-

(A) 1.4 c (B) 0.9655 c (C) 0.8888c (D) c

19. The momentum of an electron moving with a speed 0.6 c is (Rest mass of electron is 9.1 10^{-31}kg)

(A) 1.6 10^{-22} kg ms^{-1} (B) 8 10^{-22} kg ms^{-1} (C) 5.46 10^{-31} kg ms^{-1} (D) 5.46 10^{-22} kg ms^{-1}

20. A stationary body explodes into two fragments each of rest mass 1kg that move apart at speeds of 0.6c relative to the original body. The rest mass of the original body is-

(A) 2 kg (B) 2.5 kg (C) 1.6 kg (D) 2.25 kg

Solution

18. Ans. (B)

$$\text{Relative speed} = \frac{u+v}{1+\frac{uv}{c^2}} = \frac{0.9c+0.5c}{1+\frac{(0.9c)(0.5c)}{c^2}} = \frac{1.4c}{1.45} = 0.9655c$$

19. Ans. (B)

$$p = mv = \frac{m_0 v}{\sqrt{1-v^2/c^2}} = \frac{(9.1\times10^{-31})\left(\frac{3}{5}\times3\times10^8\right)}{\sqrt{1-\left(\frac{3}{5}\right)^2}} = 2 \quad 10^{-22} \text{ kg ms}^{-1}$$

20. Ans. (B)

$$m_0c^2 = \frac{m_{01}c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} + \frac{m_{02}c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \Rightarrow m_0 = \frac{1}{0.8}+\frac{1}{0.8} = 2.5\text{kg}$$

Example#21 to 23

A mercury arc lamp provides 0.1 watt of ultra-violet radiation at a wavelength of λ = 2537 Å only. The photo tube (cathode of photo electric device) consists of potassium and has an effective area of 4 cm^2. The cathode is located at a distance of 1m from the radiation source. The work function for potassium is ϕ_0 = 2.22 eV.

21. According to classical theory, the radiation from arc lamp spreads out uniformly in space as spherical wave. What time of exposure to the radiation should be required for a potassium atom (radius 2Å) in the cathode to accumulate sufficient energy to eject a photo-electron ?

(A) 352 second (B) 176 second (C) 704 seconds (D) No time lag

22. To what saturation current does the flux of photons at the cathode corresponds if the photo conversion efficiency is 5%.

(A) 32.5 nA (B) 10.15 nA (C) 65 nA (D) 3.25 nA

23. What is the cut off potential V_0 ?

(A) 26.9 V (B) 2.69 V (C) 1.35 V (D) 5.33 V

Solution

21. Ans. (A)

UV energy flux at a distance of 1m $= \frac{0.1}{4\pi\times1^2}$

cross section (effective area) of atom = $\pi \times (2 \times 10^{-10})^2 = 4\pi \times 10^{-20}$ m^2

Energy required to eject a photoelectron from potassium = 2.2 eV $\equiv 2.2 \times 1.6 \times 10^{-19}$ J.

$$\Rightarrow \text{Exposure time} = \frac{2.2\times1.6\times10^{-19}}{\left(\frac{0.1}{4\pi\times1^2}\right)\left(4\pi\times10^{-20}\right)} = 352 \text{ seconds.}$$

22. Ans. (A)

$$\text{Flux of photon at the cathode} = \left(\frac{0.1}{4\pi\times1^2}\right)\left(\frac{1}{\text{photon energy}}\right) = 1.015\times10^{16} \text{ photons/ sec m}^2$$

23. Ans. (B)

Cut off potential $= \dfrac{(4.897 - 2.22)\text{eV}}{e} = 2.69$ volts.

Example#24

With respect to photoelectric experiment, match the entries of Column I with the entires of Column II.

	Column I		Column II
(A)	If ν(frequency) is increased keeping I (intensity) and ϕ (work function) constant	(P)	Stopping potential increases
(B)	If I is increased keeping ν and ϕ constant	(Q)	Saturation photocurrent increases
(C)	If the distance between anode and cathode increases.	(R)	Maximum KE of the photoelectrons increase
(D)	If ϕ is decreased keeping ν and I constant	(S)	Stopping potential remains the same

Solution Ans. (A)→(P,R); (B)→(Q,S); (C)→(S); (D)→(P,R)

From $eV_0 = h\nu - \phi$ and $K_{max} = h\nu - \phi$. If ν increases keeping ϕ constant, then V_0 and K_{max} both increase.

If ϕ decreases keeping ν constant, then V_0 and K_{max} increase.

If I increases more photoelectrons would be liberated, hence saturation photocurrent increases.

If separation between cathode and anode is increased, then there is no effect on ν_0, K_{max} or current.

Example#25

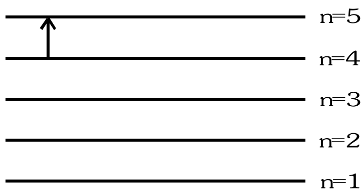
A sample of hydrogen gas is excited by means of a monochromatic radiation. In the subsequent emission spectrum, 10 different wavelengths are obtained, all of which have energies greater than or equal to the energy of the absorbed radiation. Find the initial quantum number of the state (before absorbing radiation).

Solution Ans. 4

10 emission lines $\Rightarrow$ final state n = 5

If the initial state were not n = 4, in the emission spectrum, some lines with energies less than that of absorbed radiation would have been observed.

$\therefore$ initial state n = 4.



Example#26

An electron collides with a fixed hydrogen atom in its ground state. Hydrogen atom gets excited and the colliding electron loses all its kinetic energy. Consequently the hydrogen atom may emit a photon corresponding to the largest wavelength of the Balmer series. The K.E. of colliding electron will be $\dfrac{24.2}{N}$ eV. Find the value of N.

Solution Ans. 2

Kinetic energy of electron $= 13.6\left[1 - \dfrac{1}{9}\right]\text{eV} = 12.1$ eV.

Example#27

. Neutrons in thermal equilibrium with matter at 27 C can be thought to behave like ideal gas. Assuming them to have a speed of v_{rms}, what is their De broglie wavelength λ (in nm). Fill $\left(\frac{156}{11}\right)\lambda$ in the OMR sheet. [Take m_n = $1.69 \quad 10^{-27}$ kg. k = $1.44 \quad 10^{-23}$ J/K, h = $6.60 \quad 10^{-34}$ Jsec]

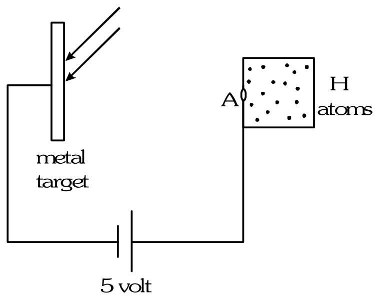
Solution **Ans. 2**

$$v_{rms} = \sqrt{\frac{3kT}{m}}; \lambda = \frac{h}{p} \Rightarrow \lambda = \frac{h}{m_n \times v_{rms}} = \frac{h}{\sqrt{3kTm_n}} \Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1.44 \times 10^{-23} \times 1.69 \times 10^{-27} \times 300}} = \frac{2.2 \times 10^{-10}}{1.2 \times 1.3}$$

$$\Rightarrow \frac{156}{11}\lambda = \frac{156 \times 2.2 \times 10^{-10}}{11 \times 1.2 \times 1.3} = \frac{220 \times 10^{-10}}{11} = \frac{22}{11} \text{ nm } = 2$$

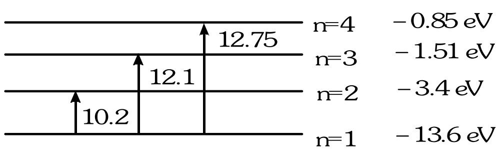
Example#28

Electromagnetic waves of wavelength 1242 Å are incident on a metal of work function 2eV. The target metal is connected to a 5 volt cell, as shown. The electrons pass through hole A into a gas of hydrogen atoms in their ground state. Find the number of spectral lines emitted when hydrogen atoms come back to their ground states after having been excited by the electrons. Assume all excitations in H-atoms from ground state only. (hc = 12420 eVÅ)



Solution **Ans. 6**

$$kE_{max} = \frac{12420\,\text{Å eV}}{1242\,\text{Å}} - 2 \text{ eV} = 8 \text{ eV}$$



The electrons are emitted with kinetic energy varying from zero to 8 eV.

When accelerated with 5 volt potential difference their energies increases by 5eV.

Hence hydrogen will get photons of energies in the range from 5 eV to 13 eV.

So maximum possible transactions are upto n = 4. Hence number of spectra lines is ${}^4C_2 = 6$