Mathematical modelling of Power Transformer based on State Space vectors in MATLAB Simulink

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Abstract

A methodology for the mathematical modelling of a power transformer has been proposed. The transformer has been modelled using its symbolic notations and the differential equations in MATLAB Simulink. A20 MVA, 132/ 11 KV, 50 Hz single and three-phase distribution grid transformers have been developed and simulated in Simulink. To enhance the understanding and reduce the complexity of the model alpha-beta state space vector representation has been used for the three-phase transformer in this paper. A model is created that can be effectively used for the analysis of different phenomenon in power transformers like internal winding faults, insulation failures, incipient faults etc. In this study, it is possible to get a closer understanding regarding the mathematical modeling and its significance in analyzing any electrical equipment in the electrical power system.

Key words: Power Transformer, Mathematical Modeling, State Space Vector, MATLAB, Simulink.

Introduction

The power transformer is an important and costliest equipment of electrical power system (EPS). To maintain the normal operation of the power transformer, its monitoring and fault analysis is very important. For the fault analysis of any electrical equipment, the very first step is to clarify the concept of the nature and influence of the fault upon the equipment. It is very dangerous, risky and expensive to induce such faults upon the practical electrical equipment for research.Therefore, the simulation model is the most suitable approach for research purpose and can portray an accurate estimation of the real-time behaviour of the equipment by simulating such models.

The approach of modelling and simulation is now getting quite

popular, catching the attention of the industry and now widely adopted for estimating the real-lifebehaviour of anequipment [1].One of the first study reported for the analysis of transformer winding-faults based on inductance models has been suggested in [2]. In [3,4], the reluctance of core, magnetic flux, permeances and MMF sources have been used to develop the magnetic equivalent circuit of the transformer. The equations of the magnetic equivalent circuit were arranged in state-space vector form in [5] to calculate differentparameters of transformer i.e. current, flux etc. In [6], a similar approach for the measurement of inrush currents in the transformer has been used, whereas the sameapproach is used to model three-phase, three legstransformer using DC bias in [7]. In recent years, finite-element models (FEM) are widely used for the analysis of electrical equipment as it portrays the behaviour of the machines more accurately by providing a Computer Aided Design (CAD) interface. One of the earliest studies for FEM based transformer modelling is reported in [8]. This technique has also been reported in [9] for modelling the non-linearity in transformer core, in [10] for the fringing effect of flux in airgaps, in [11-13] for computing the stray losses and for eddy-current losses in [14-16]. Moreover, in [17] the arching and ageing circuits along with 2D FEM have been used to analyse the incipient faults in the transformer. Each of the modelling technique discussed in the literature has its own advantage but the most common drawback in all these models is the requirement of large memory, lengthy simulation timings,and complex design.

Mathematical modellingis the most basic approach tomodelling which provides an opportunity for analysing the comprehensive dynamic model of the whole system from the scratch level and understand its basics. Hence one can develop new control techniques and analyze the behaviour of the system under a range of conditions. the mathematical model proposed in this paper overcomes all those short-comings of the previous models and can be effectively used to portray the real-time behaviour of a power transformer. Moreover, this model can be used for the analysis of different transformer faults like incipient faults, winding faults, insulation failures,and internal short circuit faults. This study will contribute to understanding the procedure of mathematical modelling and its importance in analyzing the real-life operations of the power transformer.

Methodology

Using the concept of thesimpleelectromagnetic circuit and ideal transformer (ITF), symbolic and reciprocal models are developed for single phase and three phase transformers. Moreover, a Simulink model for20 MVA, 132/ 11 KV, 50 Hz single and three-phase power transformers have been developed.

Simple electro-magnetic circuitmodelling

Ideal inductance

An inductor having an annular magnetic is consideredas shown in figure 1. The circuit will have a reluctanceR_m, given by [18]

$$R_m = \frac{1}{\mu}\frac{l_m}{A_m} \qquad (1)$$

Where μ is the core permeability, A_m is the cross-sectional area and $l_m$showsthe average flux path length. The inductance L is given by

$$L = \frac{n^2 \mu A_m}{l_m} = \frac{n^2}{R_m} \qquad (2)$$

Where n is the number of turns. Now

$$\psi = L\, i \qquad (3)$$

Where ψ is the flux linking the circuit. By Faraday's law

$$u = \frac{d\psi}{dt} \qquad (4)$$

From equation (3) and (4), we get

$$u = L\frac{di}{dt} \qquad (5)$$

or

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(t)dt \qquad (6)$$

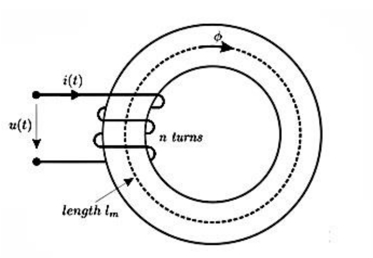

Figure 1: Annular Inductor

At $t = 0$;

$$i(t) = \frac{1}{L}\int_{0}^{t} u(t)\,dt + i(0) \qquad (7)$$

By further solving the integral

$$\Delta i = \frac{\Delta\psi}{L} \qquad (8)$$

$$\Delta\psi = \int_{0}^{t0} u(t)dt \qquad (9)$$

Where $\Delta\psi = \psi(t) - \psi(0)$, shows the flux-linkage variation with voltage. The symbolic and reciprocal models for an ideal inductor is shown in figure 2.

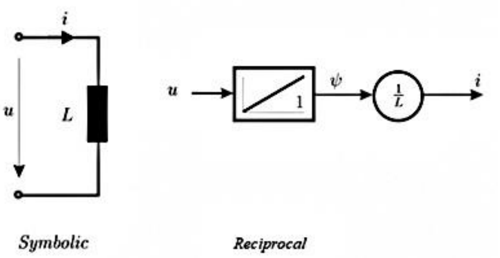

Figure 2: Ideal inductor symbolic and reciprocal models

The simulink model based on the symbolic and reciprocal model is developed and simulated in MATLAB simulink as shown in figure 3.

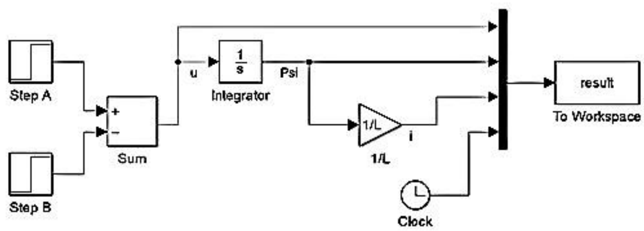

Figure 3: Simulink model of anideal inductor

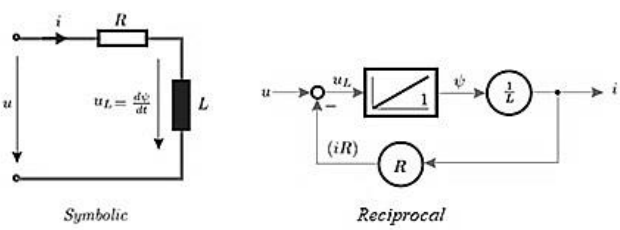

Figure 4: Ideal inductance with resistance

Ideal inductance with resistance

Ideally, the behaviour of an inductor is linear but in the realcase, there must be some wire resistance which will dissipate the energy stored in the inductor. The wire resistance can be modelled as a series resistor with an ideal inductor as shown in figure 4.

The terminal equation for the model is given as

$$u = iR + \frac{d\psi}{dt} \qquad (10)$$

Where R is the resistance of the inductor. From the reciprocal model, it is clearly shown that the voltage drop across the resistor will decrease the total inductor voltage u_L. The respective simulink model in figure 5, has been developed for the symbolic and reciprocal models in simulink MATLAB.

An inductor with magnetic saturation

There is a limit of magnetic flux density in every magnetic material. At the saturation point, with further increase in the magnetic flux will drop the permeability of the magnetic material equal to the permeability of air i.e. $\mu \rightarrow \mu_0$.

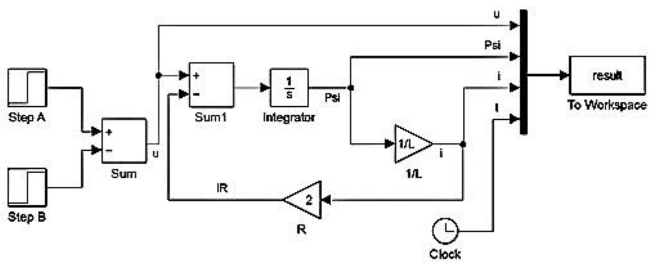

Figure 5: Simulink model for aninductor with resistance

The relation between the flux and current has been shown in figure 6. Ideally there is a linear relationship but in reality, it is a non-linear relationship.To add this non-linearity effect, a non-linear function block is used in the reciprocal model shown in figure 7. In which the function $i(\psi)$ has flux-linkageψ as an input and current i as an output. Equation (10) will remain unchanged only the gain module $1/_L$ is replaced by a non-linear module by introducing a saturation effect.

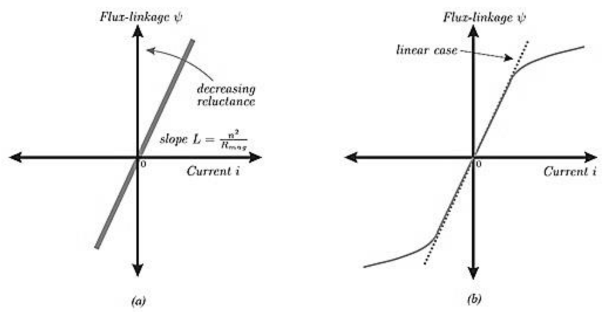

Figure 6: (a) Ideal relationship and (b) Real relationship

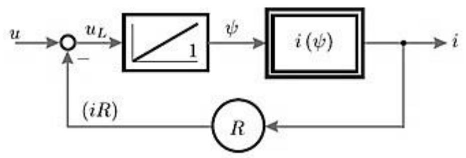

Figure 7: Reciprocal model for Inductor with saturation

The simulink model developed for the saturation effect is shown in figure 8. In which $\psi = \tanh(i)$ is used as a non-linear function in the form of a look-up table module. Furthermore, a sine wave module is used for the voltage signal having a value of $u = \hat{u}\cos\omega t$.

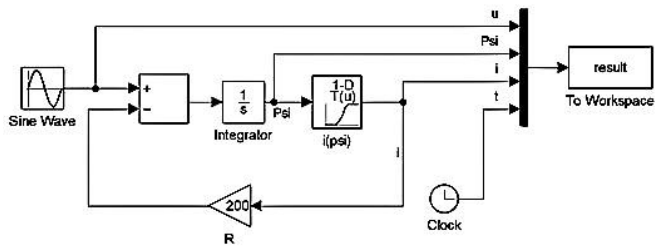

Figure 8: Simulink model for an inductor with saturation

The concept of the ideal transformer

The structureshown in figure 9, represents a transformer having an outer tube as a primary winding made up of magnetic material of infinite permeability and an inner rod as secondary winding with n_1 and n_2 turn windings respectively. Where i_1 is the primary current, i_2 is the secondary current. Moreover, ϕ_m is the mutual flux linking both the primary and secondary windings. For the ideal transformer, no leakage flux has been considered [18].

From equation (1), as $\mu = \infty$, hence $R_m = 0$, therefore $u_{core} = 0$ as $u_{core} = \phi_m R_m$. Where u_{core} is the potential inside the core. The MMF of the two inductors shown in figure 9 is shown as

$$MMF_{coil\ 1} = +n_1 i_1 \qquad (11)$$

$$MMF_{coil\ 2} = -n_2 i_2 \qquad (12)$$

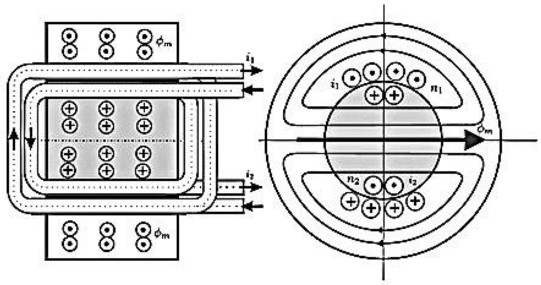

Figure 9: Ideal Transformer

The negative sign is due to the direction of currents. Hence the $MMF_{total} = 0$ as $u_{core} = 0$ i.e.

$$n_1 i_1 - n_2 i_2 = 0 \qquad (13)$$

Or,

$$i_1 = \frac{n_2}{n_1} i_2 \qquad (14)$$

Which is required current relation of ideal transformer and $^{n_2}/_{n_1}$ represents the winding ratio. Now by considering a voltage source at the primary side, hence $\phi_m = \frac{\psi_1}{n_1}$, and $\psi_2 = n_2 \phi_m$. Therefore, the flux-linkage relation for an ideal transformer will be

$$\psi_2 = \frac{n_2}{n_1} \psi_1 \qquad (15)$$

And the corresponding voltages will be given as

$$u_1 = \frac{d\psi_1}{dt} \qquad (16)$$

$$u_2 = \frac{d\psi_2}{dt} \qquad (17)$$

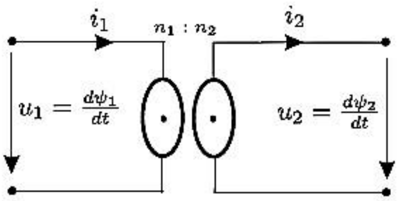

Figure 10: Symbolic model for the ideal transformer

The symbolic model for theideal transformer in the form of current directions is shown in figure 10. The corresponding reciprocal model of an ideal transformer is shown in figure 11.

Two versions of the ideal transformer are presented in figure 11, because sometimes we may need to change the direction of the current for the simplicity of the simulation circuit.

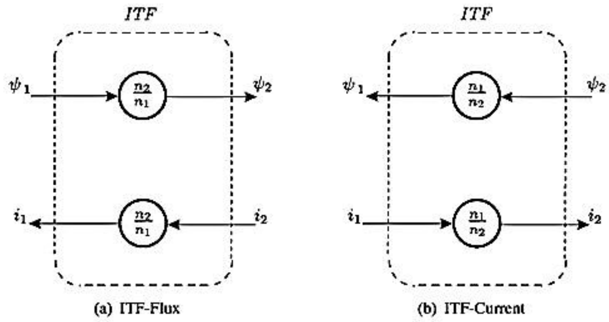

Figure 11: Reciprocal models of ideal transformer

Single-phase transformer

Making the concept of an ideal transformer as the basis, the mathematical model of single-phase transformer is proposed, considering all the non-linearity in practical transformer i.e. magnetizing resistance R_M and inductance L_M of the core, resistance R_1 and R_2 of the primary and secondary winding respectively and the leakage inductance L_σ of the windings. Airgaps are introduced in the primary and secondary windings to increase the total reluctance of the transformer. In this study the voltage is given at the primary side of the transformer, hence the primary current will have a component i_M called 'magnetizing current' given by

$$i_M = \frac{\psi_1}{L_M} \qquad (18)$$

Where $L_M = \frac{n_1^2}{\mathcal{R}_M}$ and $\mathcal{R}_M$ is the magnetic reluctance. This component i_M will cause an $MMF = n_1 i_M$. Now the total MMF becomes

$$n_1 i_1 - n_2 i_2 = n_1 i_M$$

Hence,

$$i_1 = i_M + \frac{n_2}{n_1} i_2$$

$$i_1 = i_M + i'_2 \qquad (19)$$

Where $i'_2 = \frac{n_2}{n_1} i_2$, which is the primary referred secondary current. The symbolic model for single-phase transformer is shown in figure 12. While the corresponding reciprocal model is shown in figure 13 respectively.

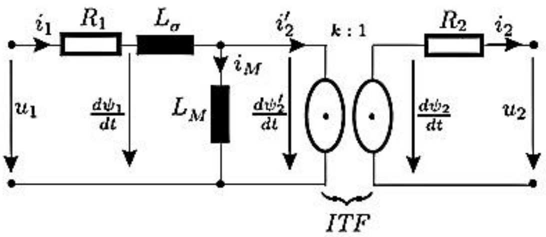

Figure 12: Symbolic diagram for single-phase transformer

The complete set of derived equations for single-phase transformer are given as

$$u_1 - R_1 i_1 = \frac{d\psi_1}{dt} \qquad (20)$$

$$u_2 - R_2 i_2 = \frac{d\psi_2}{dt} \qquad (21)$$

$$\psi_1 = i_1 L_\sigma + \psi'_2 \qquad (22)$$

$$\psi_2 = i_M L_M \qquad (23)$$

$$i_M = i_1 - i'_2 \qquad (24)$$

$$\psi'_2 = k\psi_2 \qquad (25)$$

$$i_2 = k i'_2 \qquad (26)$$

Where $k = {}^{n_1}/_{n_2}$ is the winding ratio and $L_\sigma = L_{\sigma 1} + L'_{\sigma 2}$ is the total leakage inductance of both the primary and secondary windings.

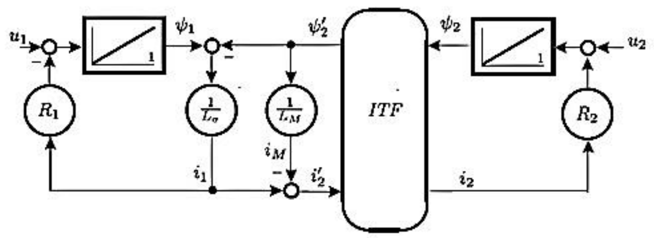

Figure 13: A Reciprocal model of single-phase transformer

Finally, $20\ MVA, 132/11\ KV, 50Hz$ single-phase transformer has been developed in simulink MATLABas shown in figure 14. For the model presented in this paper, the name-plate readings of $20\ MVA, 132/11\ KV, 50\ Hz$ transformer has been considered as shown in table 1 [19]. The first step in modeling the transformer is to calculate the circuit parameter of the transformer. For the transformer parameters calculation, the equations of 'open-circuit test' and 'short-circuit test' are being used. OCT was used to determine the magnetizing parameters R_M and L_M, while SCT was used to determine the winding resistances R_1, R_2 and leakage inductance $L_\sigma$by solving their respective equations in MATLAB. The equations used for OCT are given as

$$\cos\varphi_o = \frac{P_o}{U_o I_o} \qquad (27)$$

$$I_w = I_o \cos\varphi_o \qquad (28)$$

$$I_m = I_o \sin\varphi_o \qquad (29)$$

$$R_M = \frac{U_o}{I_w} \qquad (30)$$

$$L_M = \frac{U_o}{I_m} \qquad (31)$$

Where $\cos\varphi_o$ is a no-load or OC power factor, P_o is the no-load power or iron losses of the transformer, U_o is the no-load voltage i.e. rated voltage of LV side,I_o is the no-load currentmostly taken as $2 - 10\%$ of the full rated current of the LV winding [20], I_w is current throughthe magnetizing resistance R_M andI_m is the current through the magnetizing inductance L_M

Table 1: Name-plate readings of the Power transformer

Nameplate readings		*Rated values*
Nominal Power	$P_{nominal}$	20 MVA
Frequency	f	50 Hz
Primary voltage	$V_{1_{RMS}}$	132 kV
Secondary voltage	$V_{2_{RMS}}$	11 kV
No load losses	$P_{iron\ loss}$	14 kW
Full load losses	$P_{copper\ loss}$	70 kW
Rated current (HV)	$i_{1_{RMS}}$	87.5 A
Rated current (LV)	$i_{2_{RMS}}$	1050 A

The equations for SCT are as follow

$$R_p = \frac{P_{SC}}{I_{SC}^2} \qquad (32)$$

$$R_1 = \frac{R_p}{2} \qquad (33)$$

$$R_2 = R_1 k^2 \qquad (34)$$

$$Z_p = \frac{U_{SC}}{I_{SC}} \qquad (35)$$

$$X_\sigma = \sqrt{Z_p^2 - R_p^2} \qquad (36)$$

$$L_\sigma = \frac{X_\sigma}{\omega} \qquad (37)$$

Where $k = {}^{U_1}/_{U_2}$, is the winding ratio, R_p is the total primary side resistance, P_{SC} is the full-load power or the copper losses of the transformer, I_{SC} is full-load current i.e. full rated current of the HV winding, U_{SC} is the full-load voltage mostly taken as $5 - 10\%$ of the rated voltage of the HV winding voltage [20], Z_p is the total primary side impedance and X_σ is the total leakage reactance of the windings, while ω is the frequency of the transformer.

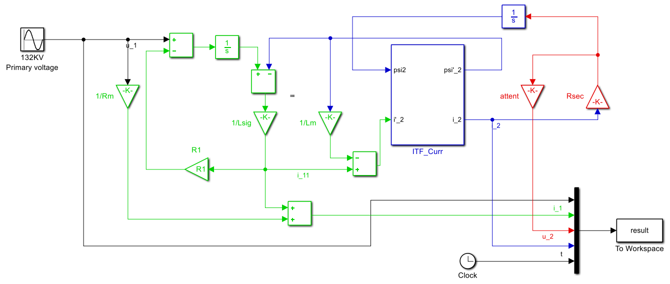

Figure 14: Simulink model for single-phase transformer

Three-phase transformer

Three-phase transformer model has been proposed and developed based on the model of single-phase transformer. For the convenient and understanding of the reader, an αβ-space vector representation of a three-phase system has been used [18]. The model for the three-phase transformer is the same as single-phase transformer except for the inclusion of three-phase to αβ-space vector configuration as shown in figure 15.there is now a pair of 'alpha' and 'beta' windings, orthogonal to each other on the primary and secondary side of the transformer.The 'effective' primary and secondary turns are represented by n_1 and n_2 respectively and $i_{1\alpha}, i_{1\beta}$ and $i_{2\alpha}, i_{2\beta}$ are the primary and secondary windings currents respectively.

$\alpha\beta$ – state space vectors

Introduction of the three-phase system increases the complexity of the model; hence state-space vector conversion is the best possible solution [18]. It is the representation of athree-phasevariables x_a, x_b, x_c to space vector form $\vec{x}_{abc} = x_\alpha + jx_\beta$. The general equation for state space vector is

$$\vec{x}_{abc} = C\{x_a + x_b e^{j\gamma} + x_c e^{j2\gamma}\} \qquad (38)$$

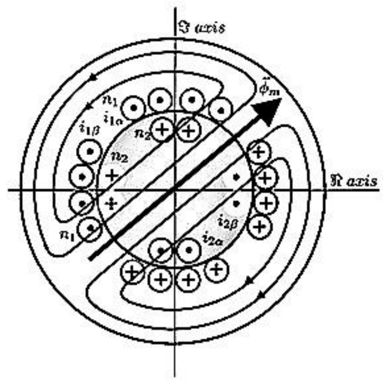

Figure 15: Three-phase transformer

The conversion matrix used for the conversion from $3\varphi \rightarrow \alpha\beta$ vector is given as

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} C & -\frac{C}{2} & -\frac{C}{2} \\ 0 & \frac{C\sqrt{3}}{2} & -\frac{C\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \qquad (39)$$

Where $C = \sqrt{\frac{2}{3}}$represents power invariant and $C = \frac{2}{3}$represents amplitude invariant state-space vectors. While the conversion matrix used for the conversion from $\alpha\beta \rightarrow 3\varphi$ vector is given as

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \frac{2}{3C} & 0 \\ -\frac{1}{3C} & \frac{1}{C\sqrt{3}} \\ -\frac{1}{3C} & -\frac{1}{C\sqrt{3}} \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_0 \qquad (40)$$

Where $x_0 = \frac{x_a + x_b + x_c}{3}$, is the zero sequence input.

Three-phase transformer based on $\alpha\beta$ – state space vectors

Vector notation has been used in the models developed for single-transformer for the modelling of a three-phase transformer. Hence, the same transformer parameters used for single-phase transformer are used for the $\alpha\beta$-system. Moreover, $Y(star) \rightarrow \Delta(delta)$ conversion has been implemented from primary to secondary winding. The symbolic model for the three-phase transformer is shown in figure 16. In which primary is start configured and the secondary is delta configured and R_L represents a three-phase delta connected symmetrical load i.e. each phase has a load resistance of R_L.

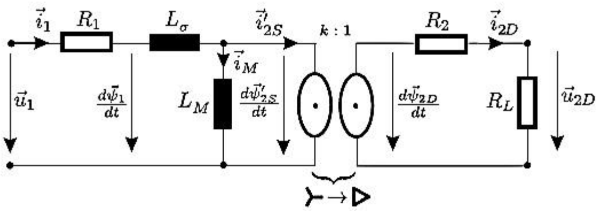

Figure 16: Symbolic model for the three-phase transformer

The equations derived for the three-phase transformer in vector form are

$$\vec{u}_1 = \vec{\imath}_1 R_1 + \frac{d\vec{\psi}_1}{dt} \quad (41)$$

$$\vec{\psi}_1 = \vec{\imath}_1 L_\sigma + \vec{\psi}'_{2S} \quad (42)$$

$$\vec{\psi}'_{2S} = \vec{\imath}_M L_M \quad (43)$$

$$\frac{d\vec{\psi}_{2D}}{dt} = \vec{u}_{2D} + \vec{\imath}_{2D} R_2 \quad (44)$$

$$\vec{\imath}_M = \vec{\imath}_1 - \vec{\imath}'_{2S} \quad (45)$$

$$\vec{\psi}'_{2S} = k\vec{\psi}_{2D} \quad (46)$$

$$\vec{\imath}_{2D} = k\vec{\imath}'_{2S} \quad (47)$$

$$\vec{u}_{2D} = \vec{\imath}_{2D} R_L \quad (48)$$

Where $S \rightarrow star$ and $D \rightarrow delta$ configuration. A three-phase sinusoidal grid has been connected to the primary side of the transformer, having ω as an angular frequency, i.e.$\vec{u}_1 = \hat{u}_1 e^{j\omega t}$in state-space vector form. Based on the equation (41-48) the reciprocal model is shown in figure 17.

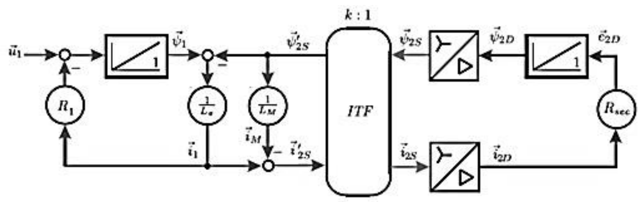

Figure 17: Reciprocal model of three-phase transformer

Furthermore, based on the reciprocal model, the simulink model for $20\,MVA, 132/11\,KV, 50Hz$ three-phase transformer has been developed in simulink MATLAB as shown in figure 18. The same OCT, SCT parameter equations and the name-plate readings mention in table 1, have been used for the parameters calculation of three-phase transformer.

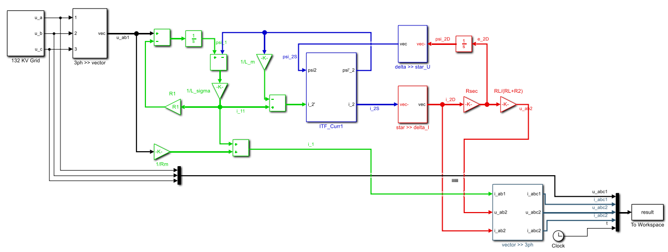

Figure 18: Simulink model of three-phase transformer

Where the $3\varphi \rightarrow \alpha\beta$ and $\alpha\beta \rightarrow 3\varphi$ modules are modeled with the help of equation (39) and (40). Moreover, the equations used for modeling the $Y \rightarrow \Delta$module is given as

$$\begin{bmatrix} i_{D\alpha} \\ i_{D\beta} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_{S\alpha} \\ i_{S\beta} \end{bmatrix} \quad (49)$$

Which shows that $\vec{\imath}_{star}$ must be scaled by $\frac{1}{\sqrt{3}}$ and shifted by $\theta = 30^\circ$ to get $\vec{\imath}_{delta}$. And the equation used for modeling the $\Delta \rightarrow Y$ module is given as

$$\begin{bmatrix} u_{S\alpha} \\ u_{S\beta} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_{D\alpha} \\ u_{D\beta} \end{bmatrix} \quad (50)$$

Which shows that $\vec{u}_{delta}$must be scaled by $\frac{1}{\sqrt{3}}$and rotated by $\theta = 30^\circ$ to get $\vec{u}_{star}$.A voltage vector $\vec{e}_{2D} = \frac{d\vec{\psi}_{2D}}{dt}$ has been usedto find $\vec{\psi}_{2D}$ at the secondary side as shown in figur18. Where $\vec{e}_{2D} = R_{sec}\vec{\imath}_{2D}$ and $R_{sec} = R_2 + R_L$.

Simulation Results

The simulink models developed in simulink MATLAB were simulated and the results were obtained.

Results of electro-magnetic circuit

ideal inductor

A step voltage of 1V has been generated by using two-step function modules as shown in figure 3. The results have been shown in figure 19, for taking $L = 0.9$ H.The inductor model simulated, showsa constant flux-linkage and current in the results even if the voltage pulse provided to it is zero. Hence showing the ideal behavior of the inductor.

Inductor with resistance

A resistance of 2Ω in the simulink model shown in figure 5 and the resultswere obtained by simulating the model as shown in figure 20. The decay in the flux-linkage and current of the inductor has been clearly shown in the simulation results. Hence showing the real behaviour of the inductor.

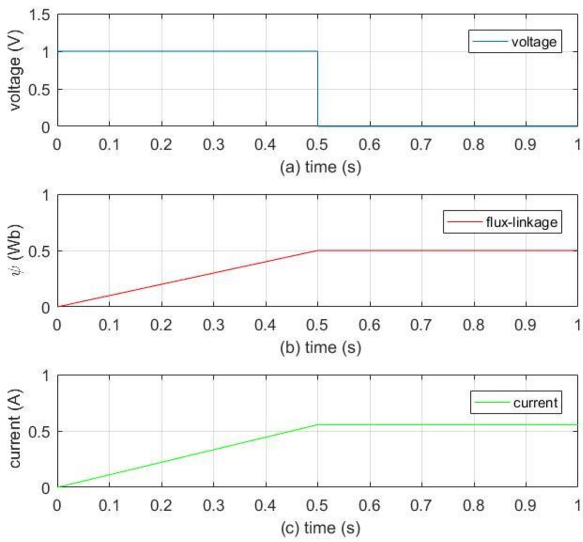

Figure 19: Simulation results of an ideal inductor

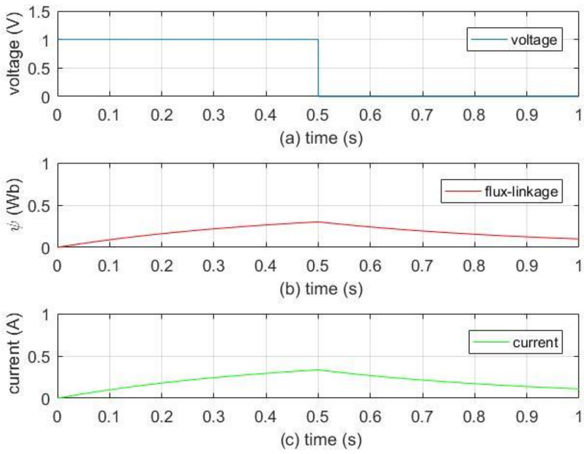

Figure 20: Simulation results of the inductor with resistance

Inductor with saturation

A sine-wave module is used for the signal generation having $\hat{u} = 240\sqrt{2}$ V, $\omega = 100\pi$ rad/sec and phase of $\pi/2$ for the cosine voltage signal as shown in figure 8. Inductor resistance of 200Ω and the table data of tanh ([−5: 0.01: 5]) and breakpoint [−5: 0.01: 5] for the look-up table has been considered. The results obtained by simulating the model are shown in figure 21. To validate the correctness of the simulink model, phasor analysis has been done through MATLAB code. In phasor analysis the input voltage function $u = \hat{u}\cos\omega t$ may also be written as [18]

$$u(t) = \Re\left\{\underbrace{\hat{u}}_{\underline{u}} e^{j(\omega t)}\right\} \tag{51}$$

Where in this case $\underline{u} = \hat{u} = 240\sqrt{2}$.The simulation results show the unifying of both the simulink and phasor analysis while there is a great transient difference between the simulink and the results calculated by phasor analysis.

Results for single-phase transformer

calculated for the transformer are given in table 4.

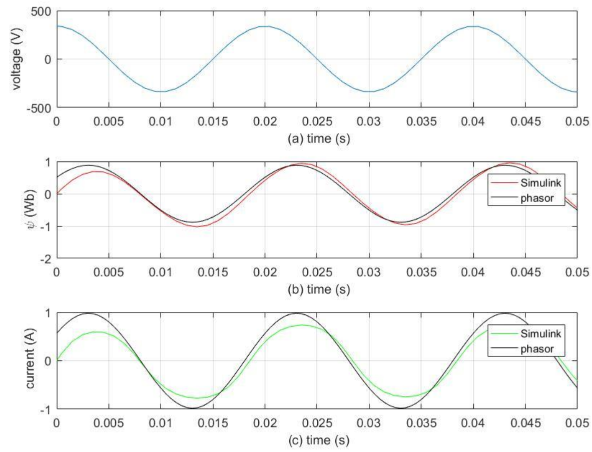

Figure 21: Simulation results for an inductor with saturation

Table 2: OCT/ No-load test data

Open-circuit data		*values*
OC voltage	U_o	11 KV
OCcurrent	I_o	21 A
OCpower	P_o	14 kW

Table 3: SCT/ Full-load test data

Short circuit data		*values*
SC voltage	V_{SC}	6600 V
SC current	I_{SC}	87.5 A
SC power	P_{SC}	70 kW

Simulink model for $20\ MVA, 132/11\ KV, 50\ Hz$ single phase transformer has been developed as shown in figure 14. The parameters of the transformer were calculated using OCT, SCT and the name-plate readings in table 1 of a grid transformer. To calculate R_M and L_M no-load or OCT were implemented on the LV side of the transformer i.e. 11 KV, in MATLAB. The data used for OCT is given in table 2.

Whereas, to calculate L_σ, R_1 and R_2 full-load or SCT were implemented on the HV side of the transformer i.e. 132 KV, in MATLAB. The data used for SCT is given in table 3. Finally, the parameter

Table 4: Transformer parameters

Parameters		*Values*
Winding ratio	k	0.083
Primary resistance	R_1	4.57 Ω
Secondary resistance	R_2	0.03 Ω
Shunt resistance	R_M	2.82 kΩ
Magnetizing inductance	L_M	2824 H
Leakage inductance	L_σ	238.3 mH

After simulating the model, the resultant primary and secondary voltage and current waveforms were acquired as shown in figure 22. And the RMS values of the primary and secondary voltages and currentsare given in table 5, which were quite similar to the name-plate readings of a $20\ MVA, 132/11\ KV, 50\ Hz$ grid transformer.

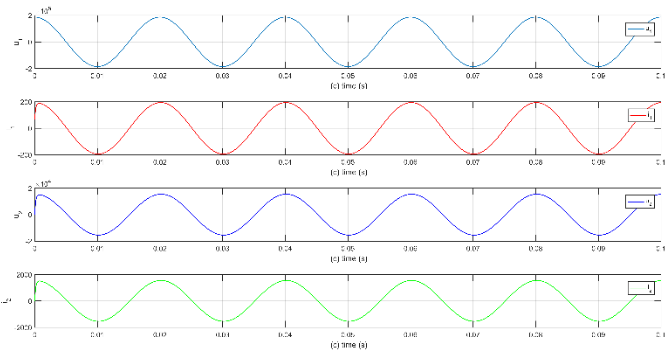

Figure 22: Simulation results of single-phase transformer

Table 5: Input/output Values of single-phase transformer model

Simulation results		*Values*
Primary voltage	U_{1_RMS}	132 kV
Secondary voltage	U_{2_RMS}	10.9 kV
Primary current	I_{1_RMS}	138 A
Secondary current	I_{2_RMS}	1099 A

Results for a three-phase transformer

The simulink model developed for 20 MVA, 132/11 KV, 50 Hz $Y \rightarrow \Delta$three-phase transformer shown in figure 18 is simulated and the results have been obtained. A three phase source of 132 KV has been used which is converted to $\alpha\beta$-state space vector. The same transformer parameters calculated for the single-phase transformer in table 4, are used for the simulation of a three-phase transformer. After simulating the model, the resultant primary and secondary voltage and current waveforms were acquired as shown in figure 23. And the RMS values of the primary and secondary voltages and currents are given in table 6, which were quite similar to the name-plate readings of a $20\ MVA, 132/11\ KV, 50\ Hz$ grid transformer.

Conclusions

A real-time 132 KV grid power transformer has been successfully modelled and simulated. Based on the concept of an ideal transformer, a mathematical model of $20\ MVA, 132/11\ KV, 50\ Hz$ power transformer is successfully developed, implemented and tested in MATLAB Simulink.Transformer parameters have been accurately calculated through short-circuit and open-circuit tests with the help of the name-plate readings of the power transformer.The real-time behavior of a grid transformer has been accurately portrayed by the mathematical model presented in this research work. Furthermore, the model proposed and implemented in this study is a cutting-edge for multiple data sets and applicable to different transformer types. The approach proposed in this study can further be applied to model the insulation of the transformer and could be used for the incipient fault analysis of the power transformers.

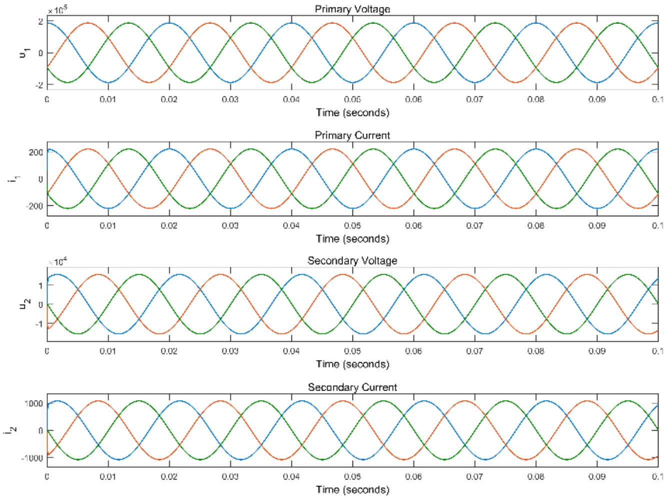

Figure 23: Simulation results for a three-phase transformer

Table 6: Input/output values of the three-phase transformer

Simulation results		*Values*
Primary phase voltage	U_{1_RMS}	132 kV
Secondary phase voltage	U_{2_RMS}	10.9 kV
Primary current	I_{1_RMS}	156 A
Secondary current	I_{2_RMS}	761 A

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References

[1] M. S. Noah and A. A. Shaltout, "Mathematical Modeling of Power Transformers," pp. 1–9.

[2] Bastard, P., Bertrand, P., & Meunier, M. (1994). A transformer model for winding fault studies. *IEEE Transactions on Power Delivery*, *9*(2), 690-699.

[3] Oliveira, L. M., & Cardoso, A. M. (2010). A permeance-based transformer model and its application to winding inter-turn arcing fault studies. *IEEE Transactions on Power Delivery*, *25*(3), 1589-1598.

[4] Chen, X. S., & Neudorfer, P. (1992, July). Digital model for transient studies of a three-phase five-legged transformer. In *IEE Proceedings C (Generation, Transmission and Distribution)* (Vol. 139, No. 4, pp. 351-358). IET Digital Library.

[5] Elrefaie, H. B., & Megahed, A. I. (2002). Modeling transformer internal faults using Matlab. In *Electrotechnical Conference, 2002. MELECON 2002. 11th Mediterranean* (pp. 226-230). IEEE.

[6] Yacamini, R., & Bronzeado, H. (1994). Transformer inrush calculations using a coupled electromagnetic model. *IEE Proceedings-Science, Measurement and Technology*, *141*(6), 491-498.

[7] Fuchs, E. F., You, Y., & Roesler, D. J. (1999). Modeling and simulation, and their validation of three-phase transformers with three legs under DC bias. *IEEE Transactions on Power Delivery*, *14*(2), 443-449.

[8] Allcock, R., McClelland, R., Holland, S. A., & Roué, A. (1995, January). Transformer design and analysis using finite element methods. In *Computation in Electrostatics, IEE Colloquium on* (pp. 8-1). IET.

[9] Mohammed, O. A., Liu, Z., Liu, S., & Abed, N. Y. (2006). Finite-element-based nonlinear physical model of iron-core transformers for dynamic simulations. *IEEE Transactions on Magnetics*, *42*(4), 1027-1030.

[10] Hoke, A. F., & Sullivan, C. R. (2002). An improved two-dimensional numerical modeling method for E-core transformers. In *Applied Power Electronics Conference and Exposition, 2002. APEC 2002. Seventeenth Annual IEEE* (Vol. 1, pp. 151-157). IEEE.

[11] Adalja, C. C., & Jain, M. L. (2008, December). Analysis of stray losses in power transformers by 3-D magnetic field simulation. In *Fifteenth National Power Systems Conference (NPSC), IIT Bombay* (pp. 498-503).

[12] Susnjic, L., Haznadar, Z., & Valkovic, Z. (2006). Stray losses computation in power transformer. In *Electromagnetic Field Computation, 2006 12th Biennial IEEE Conference on* (pp. 490-490). IEEE.

[13] Kralj, L., & Miljavec, D. (2010, September). Stray losses in power transformer tank walls and construction parts. In *Electrical Machines (ICEM), 2010 XIX International Conference on* (pp. 1-4). IEEE.

[14] Da Luz, M. V. F., Leite, J. V., Benabou, A., & Sadowski, N. (2010). Three-phase transformer modeling using a vector hysteresis model and including the eddy current and the anomalous losses. *IEEE Transactions on Magnetics*, *46*(8), 3201-3204.

[15] Fawzi, T. H., & Elkhalek, A. A. (2005, June). New approaches for the applications of FEM in the routine design of power transformers using PC's. In *Electricity Distribution, 2005. CIRED 2005. 18th International Conference and Exhibition on* (pp. 1-5). IET.

[16] Mayuri, R., Sinnou, N. R., & Ilango, K. (2010, December). Eddy current loss modeling in transformer iron losses operated by PWM inverter. In *Power Electronics, Drives and Energy Systems (PEDES) & 2010 Power India, 2010 Joint International Conference on* (pp. 1-5). IEEE.

[17] H. Wang and K. L. Butler, "Modeling transformers with internal incipient faults," *IEEE Trans. Power Deliv.*, vol. 17, no. 2, pp. 500–509, 2002.

[18] A. Veltman, D. W. J. Pulle and R. W. De Doncker, *Power Systems-Fundamentals of Electrical Drives*, 2007.

[19] Chief Engineer/MM, HVPNL, Panchkula (April-2017), "Haryana Vidyut Prasaran Nigam Limited Technical Specification for Power Transformers" Specification no. HGD/S-67/DGMS-171.

[20] Theraja, B. L. Technology Electrical. Volume II. AC & DC Machnies. *New Delhi: Nirja Construction & Development Co*, 1994.