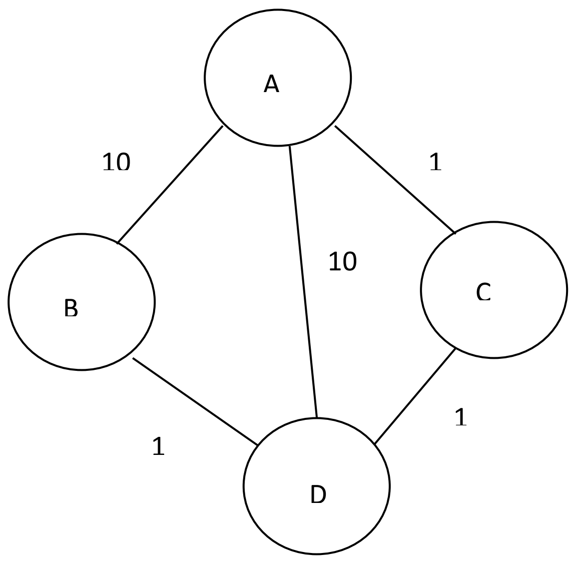
<u>Solution 1</u>

No, The algorithm does not work as it does not take the minimum distance among the neighbors which leads to incorrect solution.
For example consider the following graph,



We start from A, => dist [C] = 1, dist[B] =dist[D]=10
Then we suppose take B(because now we are not going with minimum neighbor distance)

Q : A **B** D C
We don't update anything

Q : A B **D** C
Now we take D, Again we do nothing

Q : A B D **C**
Now we take C, we update D as dist[D]=2
The algorithm ends and we have dist[B] = 10 which is not true as the dist[B] should be clearly 3 (path A->C->D->B).

Solution 2

Pseudo-code:

```
Dijkstra ( G=(V, E, w), s )
1. Let H=∅
2. For every vertex v do
        3. dist[v]=∞
4. dist[s]=0
5. Update (s)
6. For i=1 to n-1 do
        7. u=extract vertex from H of smallest cost
        8. Update(u)
9. Return dist[]
```

```
Update (u)
1. For every neighbor v of u
        2. If dist[v]>dist[u]+w(u,v) then
                3. dist[v]=dist[u]+w(u,v)
                4. If v not in H then
                5. Add v to H
```

Verbal Description:

To find single-source shortest path on positive-vertex-weighted undirected graphs from the given source we can quite simply implement the Dijkstra's Algorithm. The above algorithm is Dijkstra's Algorithm.
This algorithm in a way solves the shortest path problem in a dynamic way.
The heart here is: the min distance between any two points.
The only difference here is that we have weights directly on the vertices which does not make any changes in Algorithm but a little change in the program (depending on the implementation).

Correctness:

Say, there is a shortest path defined between S and T (S=> source and T=> Target)

And for contradiction, there is another path
from S to X X being an intermediate path.
and X to T
which is shorter.

Then this algorithm would not have chosen the path S-T instead the path S-X-T.
Thus by contradiction we know that this algorithm gives the shortest path.

Running Time:

$O(n^2)$
As the Dijkstra's Algorithm runs in $O(n^2)$.

<u>Solution 3</u>

<u>Pseudo-code:</u>

```
StartDijkstra(G=(V, E, w))
1. for i=1 to n
        2. cnt[] = Dijkstra (G, i )
        3. Print (dist[i]/cnt[i])

Dijkstra ( G=(V, E, w), s )
1. Let H=∅
2. For every vertex v do
        3. dist[v]=∞
        4. cnt[v] = 1
5. dist[s]=0.
6. ret[][] = Update (s, 1, cnt)
7. cnt[ret[0][0]] = ret[0][1];
8. For i=1 to n-1 do
        9. u=extract vertex from H of smallest cost
        10. ret[][] = Update(u, 1, cnt)
        11. cnt[ret[0][0]] = ret[0][1];
11. return cnt  .

Update (u, cnt1, cnt)
1. For every neighbor v of u
        2. If dist[v]>dist[u]+w(u,v) then
                3.  cnt1++
                4. ret[0][0] = v;
                5. ret[0][1] = cnt1;
        6. If dist[v]>dist[u]+w(u,v) then
                7. cnt[v] = cnt[u];
                8. dist[v]=dist[u]+w(u,v)
                9. If v not in H then
                10. Add v to H
11. return ret;
```

<u>Verbal Description & Proof of Correctness:</u>

We run the Dijkstra algorithm for all the pair of vertices . As the weights are non negative, Dijkstra works perfectly for all pairs.

To get the count of all multiple paths:

If there exists multiple paths of minimum distances, for any vertices (u)→(v) they could be merging on the vertice (v) or if we pick a path of minimum distances from(u)→(v) they could be merging on any intermediate vertice(s) on that minimum path.

So for the condition

If dist[v]=dist[u]+w(u,v)

we increase the count of vertice (v) as there is a different path of same length to (v).

    If dist[v]>dist[u]+w(u,v) then

dist[v]=dist[u]+w(u,v)

we assign all the paths of (u) to (v) coz if multiple paths exists to (u) they all will reach (v) through (u)

<u>Running Time:</u>
Running time will be same as , Dijkstra . that is O(n^2) for a single source , here we compute for every vertex so the running time will be O(n^3).