

Solution 1.

Pseudo-code:

```
SELECT-RAND (A, k) //N is length of array A
1.  x = A[i] where i = a random number from {1,...,n}
2.  rearrange A so that all elements smaller than
    x come before x, all elements larger than x
    come after x, and elements equal to x are
    next to x
3.  j1,j2 = the leftmost and the rightmost
    position of x in rearranged A
4.  if (k<j1) return SELECT-RAND(A[1...(j1-1)],k)
5.      if (j1<=k<=j2) return x
6.      if (k>j2) return SELECT-RAND(A[(j2+1)...n],k-j)
```

// X is the array with coordinates (x1,x2,x3.....xn)

1. Float Xbest = SELECT-RAND(X, n/2)
2. Calculate the sum of distance between each house and Xbest
3. Print sum.

Running Time:

O(n)

Argument:

We need $\text{dist}_{\text{best}} = \sum_{i=1}^n |x_{\text{best}} - x_i|$ to be minimum.

For $\text{dist}_{\text{best}}$ to be minimum differentiation ($\text{dist}_{\text{best}} / x_{\text{best}} = 0$ As the function can take as high value as we want by taking x_{best} very large.

Thus,

$$d \text{dist}_{\text{best}} / d x_{\text{best}} = \sum_{i=1}^n (x_{\text{best}} - x_i) / |x_{\text{best}} - x_i| = 0 \dots\dots\dots d/dx |x| = x/|x|$$

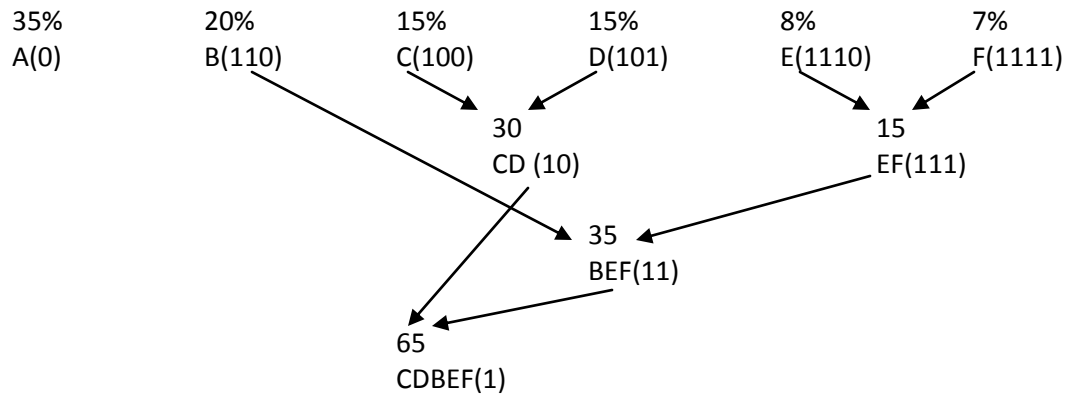
$$= (x_{\text{best}} - x_1) / |x_{\text{best}} - x_1| + (x_{\text{best}} - x_2) / |x_{\text{best}} - x_2| + (x_{\text{best}} - x_3) / |x_{\text{best}} - x_3| + \dots\dots\dots (x_{\text{best}} - x_n) / |x_{\text{best}} - x_n| = 0$$

To make it zero we need as many co-ordinates after x_{best} as many before it. Then only the summation can add up to zero.

Thus, x_{best} should be such that number of co-ordinates before and after it should be equal.

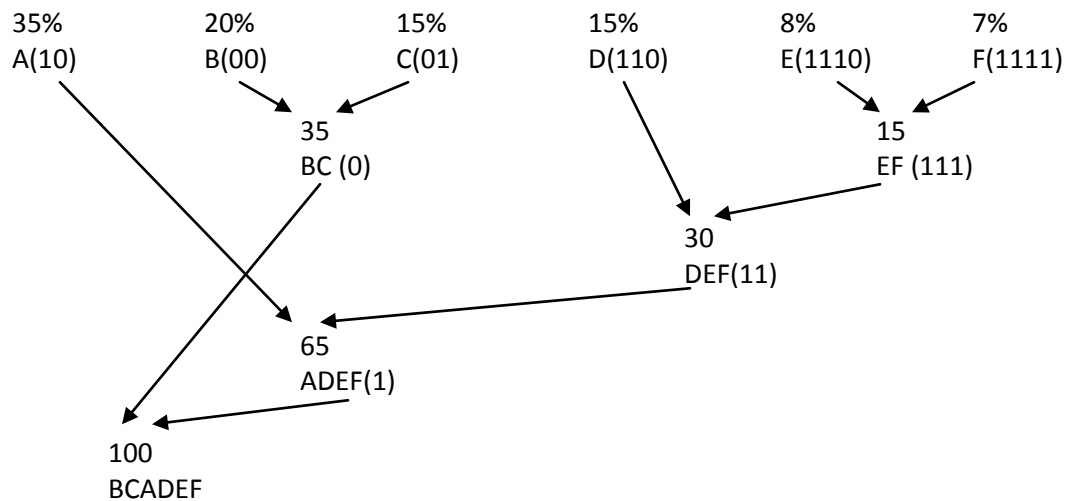
Thus, n is odd then x_{best} should be the middle element, and if n is even, then x_{best} can take any position between the two center most positions.

Solution 2.



Character	Code	Bits	Frequency
A	0	1	0.35
B	110	3	0.20
C	100	3	0.15
D	101	3	0.15
E	1110	4	0.08
F	1111	4	0.07

Expected Code-word Length = $(1 \times 0.35) + (3 \times 0.20) + (3 \times 0.15) + (3 \times 0.15) + (4 \times 0.08) + (4 \times 0.07) = 2.45$



Character	Code	Bits	Frequency
A	10	2	0.35
B	00	2	0.20
C	01	2	0.15
D	110	3	0.15
E	1110	4	0.08
F	1111	4	0.07

Expected Code-word Length = $(2 \times 0.35) + (2 \times 0.20) + (2 \times 0.15) + (3 \times 0.15) + (4 \times 0.08) + (4 \times 0.07) = 2.45$

Solution 3.

Observation about part1

The KNAP-IND-REC Algorithm only provides us with the best cost and not the set of items that should be stolen! Also it takes exponential time, thus the program "virtually" hangs when a list of only 50 items is provided to it.

On the other hand KNAPSACK-INDIVISIBLE gives us the best cost very quickly. In time: $O(n.W)$

Part 2 Pseudo-Code

KNAPSACK-INDIVISIBLE(n, c, w, W)

1. initialize $S[0][v]=0$ for every $v=0, \dots, W$
2. initialize $S[k][0]=0$ for every $k=0, \dots, n$
3. for $v=1$ to W do
4. for $k=1$ to n do
5. $S[k][v] = S[k-1][v]$
6. if $(w_k \leq v)$ AND $(S[k-1][v-w_k] + c_k > S[k][v])$ Then,
7. $S[k][v] = S[k-1][v-w_k] + c_k$
8. print $S[n][W]$ //max. cost.
9. capacityLeft = $j = W$;
10. for $i=n$ to 1 do
11. if $(j > 0$ AND $S[i][j] \neq S[i-1][j])$
12. temp[z] = i ;
13. $z++$;
14. $j = \text{capacityLeft} - w[i]$;
15. capacityLeft = j ;
16. print temp[] in reverse order. // To get the items to be stolen!

Explanation:

Till point 8 it's same as **KNAPSACK – indivisible: a dyn-prog** as in the professor Bezáková's notes.

- 1) Once we have the array with max – cost we pick the bottom right element (ie the max cost)
- 2) and check if it is same as the element above it.
 - i) If it is then we pick the element above it. And goto step 2.
 - ii) If NOT then we store the row number in the temp.
 - a. And go back to the column subtracting the weight of the item stored in temp from total weight W .
 - b. Goto step 2.