

## IE523: Financial Computing

Fall, 2016

### Programming Assignment 3: K Peg Version of the Tower of Hanoi Problem via Recursion

Due Date: 15 September, 2016

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In Lesson 1 you were introduced to the *Frame-Stewart solution* for the 4 Peg version of the Tower Hanoi Problem. For this programming assignment, I want you to write a piece of C++ code that solves the *K Peg Version of the Tower of Hanoi Problem* that uses recursion.

That is, you have pegs  $\{1, 2, \dots, K\}$  (cf. figure 1). Peg 1 has  $n$ -many disks stacked in such a way that any smaller diameter disk sits above all disks that are of larger diameter in peg 1. The objective is exactly the same as before – we want to find a way of transferring the stack of disks from peg 1 to peg  $K$  such that at no point in the process a larger disk sits on top of a smaller disk on a peg.

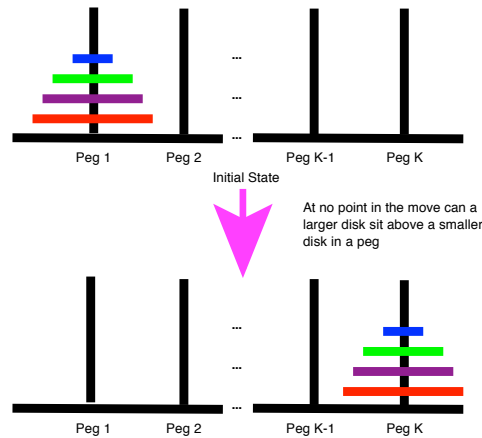


Figure 1: The *K Peg Version of the Tower of Hanoi Problem*.

This can be solved by a generalization of the Frame-Stewart Recursive Solution used in the previous programming assignment. That is, we want a recursive function  $move(n, 1, K, \{2, 3, \dots, K-1\})$ , defined as

1. Let  $p$  be some number that is less than  $n$ . Move  $p$  disks from the top of peg 1 to the  $i$ -th peg, where  $i \in \{2, 3, \dots, K-1\}$  using  $move(p, 1, i, \{2, 3, \dots, K\} - \{i\})$ .
2. Move the  $n-p$  disks from peg 1 to peg  $K$  using  $move(n-p, 1, K, \{2, 3, \dots, K-1\} - \{i\})$ . That is, this move does not involve/touch the  $i$ -th peg.
3. Move the  $p$  disks from peg  $i$  to peg  $K$  using  $move(p, i, K, \{1, 2, \dots, K-1\} - \{i\})$ .

Here is what your program should do

1. Take as input,  $K$ , the number of pegs and the number of disks,  $n$ , of decreasing size, in peg 1.
2. Write a recursive routine that moves the  $n$  disks from peg 1 to peg  $K$  such that at no point during the moves, a larger disk sits on top of a smaller one in any peg using the generalized *Frame-Stewart Solution* described above. You have some leeway in deciding what  $p$  should be. I have found that

$$p = \begin{cases} \lfloor \frac{n}{2} \rfloor & \text{if } K - 2 > 1 \text{ (i.e. \#free pegs} > 1) \\ n - 1 & \text{otherwise.} \end{cases}$$

work quite well.

3. Your code should output (1) each individual move, and (2) the total number of moves in your solution.
4. Your implementation should have a function that checks if each move generated by your implementation is legal/valid. That is, for any value of  $K$  and  $n$ , you should check if no larger disk is placed on top a smaller disk.

You could start with generalizing the 3-peg solution that I wrote (cf. `3peganoi.cpp` and `hanoi.h` on Compass). I am looking for an output that looks like what is shown in figure 2. I have given you hints on Compass.

```
Debug — bash — 80x29
Ramavarapus-MacBook-Air:Debug sreenivas$ ./K\ Peg\ Tower\ of\ Hanoi\ \(\00\
Number of pegs? 4
Number of disks? 6
-----
State of Peg 1 (Top to Bottom): 1 2 3 4 5 6
Number of Steps = 0
-----
Move disk 1 from Peg 1 to Peg 4 (Legal)
Move disk 2 from Peg 1 to Peg 2 (Legal)
Move disk 3 from Peg 1 to Peg 3 (Legal)
Move disk 2 from Peg 2 to Peg 3 (Legal)
Move disk 1 from Peg 4 to Peg 3 (Legal)
Move disk 4 from Peg 1 to Peg 4 (Legal)
Move disk 5 from Peg 1 to Peg 2 (Legal)
Move disk 4 from Peg 4 to Peg 2 (Legal)
Move disk 6 from Peg 1 to Peg 4 (Legal)
Move disk 4 from Peg 2 to Peg 1 (Legal)
Move disk 5 from Peg 2 to Peg 4 (Legal)
Move disk 4 from Peg 1 to Peg 4 (Legal)
Move disk 1 from Peg 3 to Peg 1 (Legal)
Move disk 2 from Peg 3 to Peg 2 (Legal)
Move disk 3 from Peg 3 to Peg 4 (Legal)
Move disk 2 from Peg 2 to Peg 4 (Legal)
Move disk 1 from Peg 1 to Peg 4 (Legal)
-----
State of Peg 4 (Top to Bottom): 1 2 3 4 5 6
Number of Steps = 17
-----
Ramavarapus-MacBook-Air:Debug sreenivas$ █
```

Figure 2: Sample Output.