FMNN01/NUMA11: Numerical Linear Algebra
Numerisk Analys, Matematikcentrum

Exercise 6

The purpose of these exercises is to train more on various orthogonalization techniques.
Hand-in your results electronically *latest* Oct. 16, 2013, 24:00h.

This lab has 3 task.

Task 1

Implement the QR method with Rayleigh shifts and deflations (Alg. 28.2). Test it on several symmetric matrices and compare the result with `eig`. Test it even on a symmetric orthogonal matrix. Apply it on many random symmetric matrices and make a statement about the average number of iterations you need to get the eigenvalues with a relative error $\leq 10^{-8}$.
Note, a matrix given by

```
A=rand(n,n)
A=0.5*(A+A.T)
```

is a random symmetric matrix.

Task 2

Solve Exercise 25.1 in the book. Please note the hint given there. Explain how the statement $\text{rank}(A - \lambda I) \geq m - 1 \quad \forall \lambda \in \mathbb{C}$ is related to the statement you should prove.

Task 3

Study the description of the bisection algorithm on pp. 227-229 of the course book. Implement the method and test it on matrices which are in tridiangular form. You might take a general symmetric matrix and transform it to Hessenberg form with the python command `hessenberg` or the matlab command `hessenb`.