Claus Führer, 2014-09-25

LUNDS UNIVERSITET

FMNN01/NUMA11: Numerical Linear Algebra

Numerisk Analys, Matematikcentrum

Exercise 4

The purpose of these exercises is to train more on various orthogonalization techniques.
Hand-in your results electronically *latest* Oct. 01, 2014, 24:00h.

This lab has 4 task.

Task 1

Perform the MATLAB/Python experiments in the course book Exercise 12.3

Task 2

Solve Exercise 13.3 in the course book.

Task 3

The condition number of a matrix gives a sharp estimate of the sensitivity of x with respect to perturpations of b when solving $Ax = b$, this means there exists a right hand side b and a perturbation δb such that

$$\frac{\|\delta x\|_2}{\|x\|_2} = \kappa_2(A)\|\frac{|\delta b|_2}{\|b\|_2}$$

(Note the equal sign!). Give a vector pair $(b, \delta b)$ for which this equality holds. Hint, express these vectors in terms of left singular vectors.

Task 4

Hilbert matrices are notoriously ill conditioned. Verify your result from Task 3 by solving a linear system with a 50×50 Hilbert matrix and a worst case b and δb. Hilbert matrices and their exact inverses can be constructed in MATLAB by hilb and invhilb and in Python by the commands scipy . linalg . hilbert and scipy . linalg . invhilbert .[1]

[1] You need in Python to have Scipy version 0.10 or higher installed.