Institute for Theoretical Physics | Heidelberg University
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Theoretical Statistical Physics (MKTP1) | Winter term 2020/21

Assignment 9

Handout 22.01.2021 – Return 29.01.2021 – Discussion 04./05.02.2021

Exercise 9.1 [8 points]: Quantum corrections to the classical limit

The classical limit of quantum fluids arises as the linear term in a Taylor expansion in fugacity $z = e^{\beta\mu}$ around $z = 0$ (corresponding to $\mu = -\infty$). This also corresponds to an expansion in density $\rho = N/V$ (the classical gas is a dilute gas, because then wavefunctions do not overlap). The quadratic terms in these expansions correspond to the first quantum corrections to the classical limit. As always, all quantities of interest can be calculated from the grandcanonical potential, which for the ideal quantum fluids reads

$$\Psi(T, V, \mu) = \mp g_s k_B T \frac{V}{h^3} \int d\boldsymbol{p} \ln \left[1 \pm z e^{(-\beta \frac{p^2}{2m})}\right] \tag{1}$$

with the different signs corresponding to fermions and bosons, respectively. From thermodynamics, we also know $\Psi = -pV$.

1. Expand the integrand in the formula for the grandcanonical potential Ψ to second order in z and perform the two integrals. *(2 points)*
2. Calculate the mean particle number $N = -\partial_\mu \Psi$ in the same order. Invert this relation to get $u = \rho\lambda^3/g_s$ as a function of z. *(2.5 points)*
3. Combine your results for Ψ and u to obtain the first two terms for pressure p in an expansion in ρ to second order. *(2.5 points)*
4. Discuss your results for the classical limit and the first quantum correction. Where does degeneracy g_s show up ? What do the differences in sign mean ? *(1 point)*

Exercise 9.2 [7 points]: Two-dimensional ideal Bose fluid: to condense or not to condense?

Consider a *two*-dimensional, ideal Bose fluid.

1. What is the expression for the mean particle number for a 2D system? *(1 point)*
2. First consider particles with energy $\epsilon(\mathbf{p}) = \frac{p^2}{2m}$. Evaluate the mean particle number and investigate whether there is a Bose-Einstein condensation (BEC) at finite temperature. *(2.5 points)*
3. Now consider massless bosons with energy $\epsilon(\mathbf{p}) = cp$. Again evaluate the mean particle number and investigate whether there is a BEC at finite temperature. *(2.5 points)*
4. In case you find a BEC, give the respective expression for the critical temperature. *(1 point)*