Homework Assignment 6 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

Due: Friday, October 10 11:59pm CST on Canvas

Questions #1-4:

Answer the questions in this pdf.

Problem	Points
1.1	3
1.2	2.5
2.1	1
2.2	3
2.3	3.5
3.1	4
3.2	1.5
4.1	4
4.2	2.5
4.3	3
Video	2

Video Question:

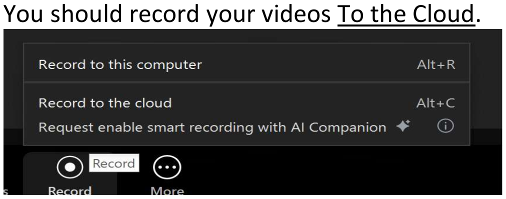
- Pretend you are a TA for this class and record a short video explaining what you did here [to a student who hasn't taken this class] for in **question 4** in this homework assignment.
- Share your screen, showing your answers.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
  - Have your camera on.
  - Show your FULL screen in Zoom (not just a particular application).
  - We should be able to hear the audio. Make sure to turn your mic on.
  - You should give a good faith attempt to answer the prompt.
  - Your video meet the minimum time requirement.
  - It should not sound like you are just reading off a script.
  - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in Canvas.

<u>Question 1</u>: Solar Panel Installation

Problem Statement

FreshBrew Coffee Company must determine how many coffee bags should be produced during each of the next 12 months.

Month	Demand
0	400
1	700
2	800
3	900
4	1,100
5	1,400
6	1,600
7	1,900
8	2,200
9	2,000
10	1,500
11	1,200

<u>Monthly Demand</u>

The demand for coffee bags over the next 12 months is shown in the table to the right. FreshBrew must meet demand on time each month.

<u>Initial Inventory</u>

At the beginning of the first month, FreshBrew has 200 coffee bags in inventory.

<u>Monthly Production</u>

At the beginning of each month, FreshBrew must decide how many coffee bags to produce. Bags roasted during a month can be used to meet demand for that same month.

- FreshBrew can produce up to 500 coffee bags per month using standard beans, at a cost of $5 per bag.
- By using premium beans, FreshBrew can produce additional bags with no upper limit, but at a higher cost of $7 per bag.

<u>Monthly Inventory</u>

At the end of each month (after production has occurred and that month's demand has been satisfied), a holding cost of $0.50 per bag is incurred for any remaining inventory.

<u>Goal</u>

Determine a production schedule that minimizes the total cost of production and inventory over the next 12 months.

1.1. Model

Formulate a mathematical program that finds an optimal solution the problem given above.

- Make sure to define your decision variables.
- You can define your own <u>input parameters</u> and <u>sets</u> to make this model more compact if you'd like. Just make sure to define any sets and input parameters that you use.

1.2. Solution

Find an optimal solution to this problem using PuLP.

Question 2: Renewable Energy Grid Flow

Problem Statement

An international relief organization is coordinating the delivery of humanitarian aid supplies to a Central Refugee Camp Distribution Hub. Supplies arrive at a Port of Entry and are shipped through regional warehouses (A, B, C, D, and E) before reaching the distribution hub.

Network Structure

Each shipping route between facilities has a maximum capacity (measured in tons/day of aid supplies) and an associated transportation cost (USD per ton), which reflects fuel, labor, and infrastructure conditions along that route.

The organization must decide how much aid to send along each route in order to meet urgent demand at the refugee camp while keeping costs as low as possible.

Shipping Route	Capacity (tons/day)	Cost ($/ton)
Port of Entry → A	3,500	18
Port of Entry → B	4,000	22
Port of Entry → C	2,500	20
A → D	3,000	15
B → D	2,000	28
B → E	2,500	24
C → E	3,000	19
D → Distribution Hub	4,500	21
E → Distribution Hub	3,500	23

Goals

- Ensure that at least 5,000 tons/day of aid reach the Central Refugee Camp Distribution Hub.
- Minimize the total transportation cost.

Stipulations

- The daily flow of aid along each route cannot exceed its given daily capacity.
- At each intermediate warehouse (A–E) the amount of aid received should be the amount shipped out. We cannot leave any aid in storage overnight.
- Only the Port of Entry provides aid supplies, and only the Central Refugee Camp consumes them.

2.1. Network Graph

Sketch the network graph along with the cost and edge capacities based on the table information above. Make sure your edges are pointed in the correct direction.

2.2. Model

Formulate a mathematical program for this problem.

- Make sure to define your decision variables.
- You can define your own input parameters and sets to make this model more compact if you'd like. Just make sure to define any sets and input parameters that you use.

2.3. Solution

Find an optimal solution to this problem using PuLP.

<u>Question 3</u>: Conference Scheduling

Problem Statement

You are responsible for scheduling talks at a one-day academic conference that features a lineup of 30 invited speakers, including 5 keynote speakers. The conference consists of 10 time slots spread across 3 parallel tracks (rooms 1, 2, 3). Each time slot on each track must be filled, and every invited speaker must present exactly once.

Time Slot	Time
1	9am
2	10am
3	11am
4	12pm
5	1pm
6	2pm
7	3pm
8	4pm
9	5pm
10	6pm

Stipulations

Your task is to assign speakers to rooms and time slots in a way that adheres to the following constraints:

1. <u>Keynote Speakers</u>: There are 5 keynote speakers, and no two can present at the same time across any rooms.
2. <u>Room Capacity</u>: Each room must host exactly one talk per time slot (no empty slots).
3. <u>Availability</u>: Speakers can only present in time slots where they are available.
4. <u>Sequencing Agreements</u>: Some keynote speakers have agreements to present immediately after another speaker, in the same room.
5. <u>Every Speaker Presents</u>: Each speaker must present exactly once.

Data

The table below lists the following for each invited speaker

a. The time slots they are <u>available</u>
b. The time slot that would <u>prefer</u> the most.
c. Whether they are a <u>keynote speaker</u> or <u>regular speaker</u>.
d. Whether or not they have a <u>sequencing agreement</u> and if so, which speaker they must present exactly after in the same room.

Goal

Ideally we would like for as many presenters as possible to have their <u>preferred</u> time slot

Speaker	Tier	Availability (Time Slots)	Sequencing Agreement	Time Slot Preference
Speaker 1	Keynote	[1, 2, 3]	After Speaker 6	2
Speaker 2	Keynote	[2, 3, 4]		3
Speaker 3	Keynote	[5, 6, 7]	After Speaker 10	6
Speaker 4	Keynote	[6, 7, 8]		7
Speaker 5	Keynote	[1, 2, 3]		3
Speaker 6	Regular	[1, 2]		1
Speaker 7	Regular	[3, 4]		4
Speaker 8	Regular	[5, 6]		5
Speaker 9	Regular	[7, 8]		7

Speaker 10	Regular	[4, 5, 6]		5
Speaker 11	Regular	[1, 2, 3]		1
Speaker 12	Regular	[2, 3, 4]		2
Speaker 13	Regular	[5, 6, 7]		5
Speaker 14	Regular	[6, 7, 8]		7
Speaker 15	Regular	[8, 9, 10]		8
Speaker 16	Regular	[1, 2, 3, 4]		2
Speaker 17	Regular	[2, 3, 4, 5]		3
Speaker 18	Regular	[6, 7, 8]		6
Speaker 19	Regular	[1, 2, 3]		1
Speaker 20	Regular	[3, 4, 5]		5
Speaker 21	Regular	[6, 7, 8]		7
Speaker 22	Regular	[9, 10]		9
Speaker 23	Regular	[1, 2]		2
Speaker 24	Regular	[2, 3, 4]		4
Speaker 25	Regular	[5, 6, 7, 8]		6
Speaker 26	Regular	[6, 7, 8, 9]		8
Speaker 27	Regular	[2, 3, 4]		2
Speaker 28	Regular	[4, 5, 6]		4
Speaker 29	Regular	[7, 8, 9]		7
Speaker 30	Regular	[9, 10]		9

3.1. Conference Schedule Model

Formulate a mathematical program that finds an optimal solution the problem given above.

- Make sure to define your decision variables.
- You can define your own input parameters and sets to make this model more compact if you'd like. Just make sure to define any sets and input parameters that you use.

3.2. Model Size

- How many decision variables does your model have?
- How many main constraints does your model have?

<u>Question 4</u>: Sudoku

4.1. Sudoku Solution Model

- <u>Task</u>
  Formulate a **<u>binary</u> integer linear program** that finds a <u>solution</u> to the following Sudoku puzzle.

- <u>Sudoku Rules Reminder</u>
  Remember that a correct completed Sudoku board must have the following rules met.
  a. Each <u>cell</u> of the puzzle must contain exactly one number.
  b. Every <u>row</u> on the board must have all 9 integers.
  c. Every <u>column</u> on the board must have all 9 integers.
  d. Every one of the 9 3-by-3 <u>square subgrids</u> shown below must have all 9 integers.

- <u>Objective function hint</u>
  Because we simply just want to find a <u>solution</u> to this puzzle below (any solution will do), we are not necessarily optimizing for anything. We simply just want to find a feasible solution to the optimization model that you formulate that meets all of the rules of a complete Sudoku board. Thus, your objective function can be anything.

4.2. Model Size

4.2.1. How many <u>decision variables</u> does your model have?
4.2.2. How many <u>main constraints</u> does your model have?

4.3. Testing Model Feasibility

4.3.1. Come up with a scenario that violates Sudoku rule (**a**) stated above. Show that this translates to an infeasible solution in your model.
4.3.2. Come up with a scenario that violates Sudoku rule (**b**) stated above. Show that this translates to an infeasible solution in your model.
4.3.3. Come up with a scenario that violates Sudoku rule (**c**) stated above. Show that this translates to an infeasible solution in your model.
4.3.4. Come up with a scenario that violates Sudoku rule (**d**) stated above. Show that this translates to an infeasible solution in your model.

5		9				4		
7		8	3		4	9		
6		1				7	3	
4	6	2	5					
3	8	5	7	2	1	6	4	9
1	9	7	4		8	2		
2			1					4
		3		4			8	7
	7			5	3			6