Homework Assignment 10 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

Questions: 1-4

Video Question:

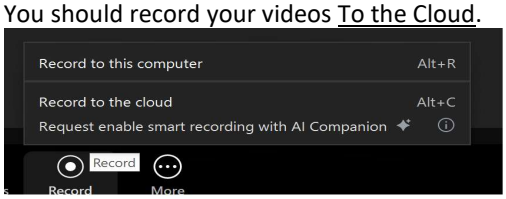
Give a 3-4 minute summary of question 4.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
  - Have your camera on.
  - Show your FULL screen in Zoom (not just a particular application).
  - We should be able to hear the audio. Make sure to turn your mic on.
  - You should give a good faith attempt to answer the prompt.
  - Your video meet the minimum time requirement.
  - It should not sound like you are just reading off a script.
  - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in Canvas.

Problem	Points
1.1	1.5
1.2	1.5
1.3	1.5
2.1	1.5
2.2	2
2.3	2
2.4	2
2.5	2
2.6	2
2.7	2
3	2
4.1	1.5
4.2	1.5
4.3	4
4.4	1
Video	2

Question #1: LPs in Standard Form

Write the following LPs in LP Standard Form.

1.1.

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 5x_2 + 2x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 + x_3 \geq 10, \\ & 4x_1 + x_2 + 2x_3 \leq 12, \\ & x_1 + 2x_2 + 3x_3 = 8, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

1.2

$$\begin{aligned} \text{Maximize} \quad & Z = 5x_1 + 3x_2 \\ \text{subject to} \quad & 2x_1 + x_2 = 8, \\ & x_1 + 3x_2 = 6, \end{aligned}$$

1.3

$$\begin{aligned} \text{Minimize} \quad & Z = 2x_1 + 4x_2 \\ \text{subject to} \quad & 3x_1 + 5x_2 = 20, \\ & 4x_1 + 2x_2 = 12, \\ & x_1, x_2 \leq 0. \end{aligned}$$

Question #2: Basic Solutions for an LP (Not Necessarily in Standard Form)

Consider the following LP below that is not in LP standard form.

$$\begin{aligned} \text{Maximize} \quad & Z = 4x_1 + 3x_2 + 5x_3 + 6x_4 \\ \text{subject to} \quad & x_1 + x_2 + 2x_3 + 2x_4 \leq 18 \quad (1), \\ & x_1 + x_2 + 2x_3 \leq 20 \quad (2), \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

2.1. What is the minimum number of constraints that need to be active for a given solution to a basic solution in the LP above?

2.2. Select a set of constraints from the LP above that would NOT yield a basic solution when active. The number of constraints that you pick should match the number of constraints that you discussed in 2.1.

2.3. Is $(0,0,10,-1)$ a basic solution? Is it a basic feasible solution? Is it an extreme point? Explain.

2.4. Is $(0,0,0,9)$ a basic solution? Is it a basic feasible solution? Is it an extreme point? Explain.

2.5. Is $(18,0,0,0)$ a basic solution? Is it a basic feasible solution? Is it an extreme point? Explain.

2.6. Are $(0,0,0,9)$ and $(18,0,0,0)$ adjacent basic feasible solutions? Explain.

2.7. If $(0,0,0,9)$ and $(18,0,0,0)$ are adjacent basic feasible solutions, give an equation for the edge of the feasible region that exists between them.

Question #3: General Basic Feasible Solution

Suppose there is an LP (not necessarily in standard form) that has M total constraints (this includes main constraints and variable type constraints) and n decision variables, what is the maximum number of basic solutions this LP can have?

Hint: Remember that $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ counts up the number of ways that you can choose a set of size b from a total set of size a.

Question #4: LP in Standard Form

Let's return to our LP from question #2.

$$\begin{aligned} \text{Maximize} \quad & Z = 4x_1 + 3x_2 + 5x_3 + 6x_4 \\ \text{subject to} \quad & x_1 + x_2 + 2x_3 + 2x_4 \leq 18 \quad (1), \\ & x_1 + x_2 + 2x_3 \leq 20 \quad (2), \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

4.1. Put this into LP standard form.

4.2. In order to construct a basic solution from this LP in standard form using the method that we discussed in class, how many variables should be set as basic? And how many variables should be set as nonbasic?

4.3. Construct two distinct basic solutions from this LP in standard form, using the method discussed in class.

For EACH of these two basic solutions also discuss:

- What are the basic variables?
- What are the nonbasic variables?
- Is the basic solution also a basic feasible solution?

4.4. If a slack variable is 0 for a solution of an LP in standard form, what does this imply about the constraint that corresponds to this slack variable in the original LP at this solution?