Homework Assignment 4 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

<u>Due</u>: Friday, September 26 11:59pm CST on Canvas

Problem	Points
1.1	1.5
1.2	1.5
1.3	1.5
1.4	1.5
2.1	1.5
2.2	1.5
2.3	1.5
2.4	1.5
3.1	1.5
3.2	1.5
3.3	1.5
3.4	1.5
4.1	2.5
4.2	2
4.3	1.5
5.1	1.5
5.2	1.5
5.3	1.5
Video	1.5

Questions #1-5: 28.5 points

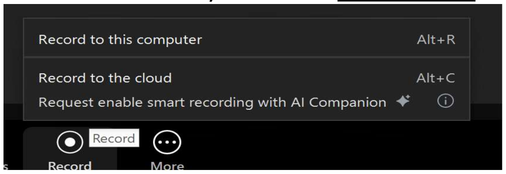
See below.

Video Question #6: 1.5 points

- Select the **question number** in this assignment that is closest to the **last digit** of your **netid**.
    - ***Ex***: *Netid: cw8**0** -> **0***
      *Select question **1**.*
- Pretend you are a TA for this class and record a 3-4 minute video explaining what you did here [to a student who hasn’t taken this class] for this **particular question number** in this homework assignment.
- Share your screen, showing your answers.

IMPORTANT Video Element of ALL Homework Assignments:
- In order to receive points for each video submission, you need to do **ALL** of the following.
    - Have your camera on.
    - Show your FULL screen in Zoom (not just a particular application).
    - We should be able to hear the audio. Make sure to turn your mic on.
    - You should give a good faith attempt to answer the prompt.
    - Your video meet the minimum time requirement.
    - It should not sound like you are just reading off a script.
    - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:
- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in Canvas.

Question 1: Potential Optimal Solution Locations of *General* Mathematical Programs

Suppose the feasible region to a given mathematical program is shown to the right.

$$x_1 + x_2 \leq 20$$
$$x_1, x_2 \geq 0$$

1.1. List all of the extreme points of this feasible region.

1.2. Come up with an objective function for this feasible region in which there is only ONE optimal solution at $\boldsymbol{x}^* = (5,6)$**.** Make sure to also do the following.

- Indicate whether you are maximizing or minimizing this objective function.
- Sketch the feasible region and at least two contour curves **by hand** that shows that this is the optimal solution.
- Show any other work that you need to in order prove that this is the optimal solution.

1.3. Come up with an objective function for this feasible region in which there are EXACTLY TWO optimal solutions at $\boldsymbol{x}^* = (20,0)$ and $\boldsymbol{x}^* = (0,20)$.
Make sure to also do the following.

- Indicate whether you are maximizing or minimizing this objective function.
- Sketch the feasible region and at least two contour curves **by hand** that shows that this is the optimal solution.
- Show any other work that you need to in order prove that this is the optimal solution.

1.4. Come up with an objective function for this feasible region in which there is EXACTLY ONE optimal solutions at $\boldsymbol{x}^* = (10,10)$.
Make sure to also do the following.

- Indicate whether you are maximizing or minimizing this objective function.
- Sketch the feasible region and at least two contour curves **by hand** that shows that this is the optimal solution.
- Show any other work that you need to in order prove that this is the optimal solution.

Question #2: Linear Program Optimal Solution Locations

$$
\begin{array}{ll}
\text{Maximize} & c_x x + c_y y \\
\text{subject to} & 2x + 3y \leq 12 \\
& y \leq 3 \\
& x \leq 8 \\
& x, y \geq 0
\end{array}
$$

For the questions below come up with input parameter values for c_x and c_y in which the resulting LP has optimal solution(s) at the following locations. If this is not possible, say so. *(No explanation required if not possible, but may help with partial credit if you are wrong).* ***Any sketching must be done by hand.***

2.1. There is just one optimal solution at $(1,3)$.

2.2. All feasible solutions in this model are optimal.

2.3. There are infinitely many optimal solutions. ONE of them is $(1,3)$. But $(0,0)$ is NOT an optimal solution.

2.4. All solutions on the line segment between $(0,3)$ and $(6,0)$ are optimal.

Question 3: 3-d Linear Program

The 3-d feasible region for the LP shown to the right is sketched below.

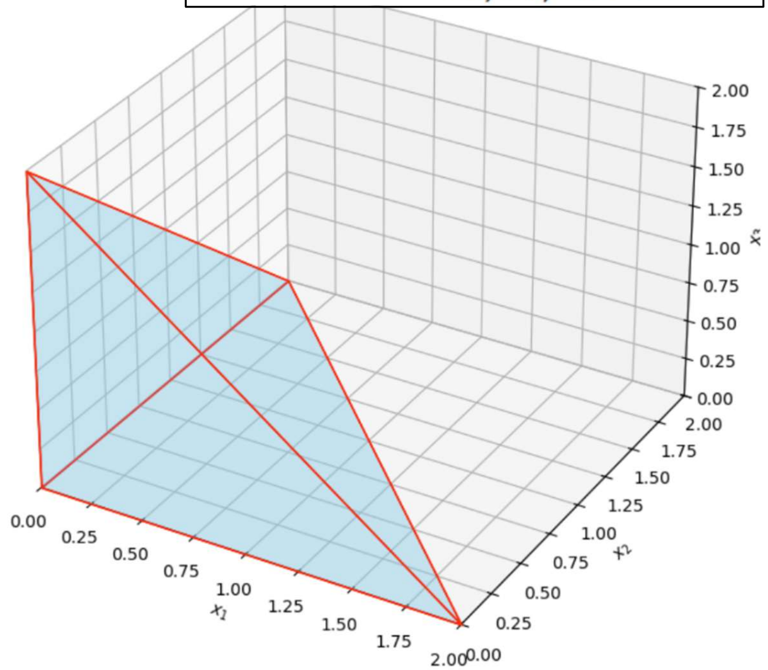
$$
\begin{array}{ll}
\text{Maximize} & 3x_1 + 4x_2 + 5x_3 \\
\text{subject to} & x_1 + x_2 + x_3 \leq 2 \\
 & x_1, x_2, x_3 \geq 0
\end{array}
$$

3.1. Find all of the extreme points of this LP.

3.2. How many constraints were active (ie. binding) at each of these extreme points?

3.3. Find an optimal solution to this LP (by hand). **You can assume that this LP *has* at least one optimal solution.**

3.4. Find an optimal solution to this LP using PuLP in Python.



<u>Question 4</u>: 4-d Linear Program

In order for a <u>solution</u> in an <u>LP with **4** decision variables</u> to be an <u>extreme point</u>, it must:

- be <u>feasible</u>,
- have at <u>least **4** constraints</u> that are <u>active</u>, and
- when solving this system of **4+** active equality constraints, there must be a <u>unique solution</u>.

$$\begin{array}{ll} \text{Maximize} & 20x_1 + x_2 + 10x_3 + x_4 \\ \text{subject to} & 4x_1 + 3x_2 + 2x_3 + x_4 \leq 1 \quad (A) \\ & x_1 \leq 100 \quad (B) \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

4.1. Find all the extreme points of this LP. Show your work.

4.2. Find <u>THREE optimal solutions</u> to this LP by hand. **You can assume that this LP *has* at least one optimal solution.**

4.3. Find at least one optimal solution to this LP with PuLP in Python.

<u>Question 5</u>: Competition Preparation Optimization

A high school robotics team is preparing for a competition. The coach wants to design an optimal weekly training schedule that balances different types of practice sessions. The team can engage in nine different activities: Coding, CAD Design, Electrical Work, Mechanical Assembly, Strategy Meetings, Driving Practice, Troubleshooting, Team Presentations, and Documentation.

Each activity takes a certain amount of time and contributes to the team's skill development in four areas: <u>Technical Skills</u>, <u>Teamwork</u>, <u>Problem-Solving</u>, and <u>Communication</u>.

Activity	Time per Session (hours)	Technical Skills	Teamwork	Problem-Solving	Communication
Coding (1)	3	4	0	1	0
CAD Design (2)	2.5	3	0	2	0
Electrical Work (3)	2	2	1	0	0
Mechanical Assembly (4)	1.5	2	2	1	0
Strategy Meetings (5)	1	0	3	0	2
Driving Practice (6)	2.5	1	2	2	0
Troubleshooting (7)	3	2	0	3	0
Team Presentations (8)	1.5	0	1	0	2
Documentation (9)	1	0	2	1	3

The coach would like to ensure that the team is getting at least each of the following:

- <u>55 Technical Skill points</u>
- <u>65 Teamwork points</u>
- <u>15 Problem-Solving points</u>
- <u>25 Communication points</u>

They would like to minimize the total amount of training hours needed to meet these requirements.

5.1. Formulate this optimization problem. Use the actual numbers from above in your formulation. Make sure to define your decision variables.

5.2. Use summation notation to make this formulation more compact. Make sure to indicate what any input parameter values that you might have used are.

5.3. Formulate this model using matrix notation. Make sure to indicate what any matrices are that you might have used.

5.4. Find an <u>optimal solution</u> to this problem using PuLP in Python. What is the amount of time the team will spend on this optimal set of activities? You can use the skills.csv. **Assume for now that it is ok to complete a "fractional amount" of each type of activity.**