Homework Assignment 1 – [30 points]

STAT430 Mathematical Optimization for Data Science - Fall 2025

Due: Friday, September 5 11:59pm CST on Canvas

Questions #1-3

For each of the three optimization problems given below do the following.

This must be done BY HAND.

- a. [0.5 pts] Define the decision variables.
- b. [0.25 pts] Define the objective function (in words).
- c. [0.5 pts] Define the constraint(s) (in words.)
- d. [3 pts] Formulate a mathematical optimization model for the problem, putting it in standard form. This means, that you need to *mathematically* represent your objective function and constraints.
- e. [0.25 pts] List the input parameters in this model (as well as the numbers that correspond to them).
- f. [2.5 pts] Finally, sketch the feasible region.

Questions #4-5: Both worth [2.5 pt] (4), [2 pt] (5)

Extensions of question #3. See the prompts below.

Survey Question #6 [2.5 pts]

Fill out the "Getting to Know You Survey" in Canvas quizzes.

Video Question #7 [2 pts]

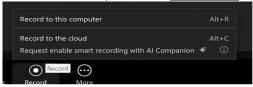
- Pretend you are a TA for this class and record a 3-4 minute video explaining what you did here [to a student who hasn't taken this class] in this homework assignment.
- Share your screen, showing your answers and explain them one by one.
- For each problem, pick an input parameter in the problem statement and explain how your model would change if this input parameter number was different.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do ALL of the following.
 - o Have your camera on.
 - Show your FULL screen in Zoom (not just a particular application).
 - We should be able to hear the audio. Make sure to turn your mic on.
 - You should give a good faith attempt to answer the prompt.
 - o Your video meet the minimum time requirement.
 - o It should not sound like you are just reading off a script.
 - It's ok if your video recording is not the most eloquent. What's important
 is that you are putting together YOUR authentic thoughts on your
 particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and <u>Copy shareable link</u> to paste the link to the video prompt in the corresponding Jupyter notebook.

Event Planning Problem

Small vs. Large Catering Events

A catering company needs to allocate its staff resources between scheduling small and larg banquet events for the upcoming week.

Staff Hour Limits

- Kitchen staff can provide up to 150 total hours of work.
- Service staff can provide up to 180 total hours of work.

Small Events

- It takes 4 kitchen hours to prepare a small event.
- It takes 3 service hours to run a small event.
- The company earns \$2,200 for each small event.

Large Events

- It takes 9 kitchen hours to prepare a large event.
- It takes 12 service hours to run a large event.
- The company earns \$5,400 for each large event.

How many small and large events should the company schedule to maximize its revenue?

Eco-Friendly Cushioning Mix Problem

Recycled Paper Pellets vs. Mushroom Foam

A packaging company is designing an eco-friendly cushioning material using two inputs: recycled paper pellets and mushroom foam.

Product Requirements

The product must meet the following requirements:

- The finished pad must weigh exactly 200 grams in total.
- The product's environmental impact score must be no more than 20 points.

Recycled Paper Pellets

- Each unit of recycled paper pellets weighs 5 grams.
- Each unit contributes 0.6 impact points.
- Each unit costs \$0.05.

Mushroom Foam

- Each unit of mushroom foam weighs 8 grams.
- Each unit contributes 0.4 impact points.
- Each unit costs \$0.12.

How many units of each material should the company use to minimize total cost while meeting the weight and impact requirements?

Equipment Allocation Problem

Laptop Loaners: Campus North vs. Campus South

A university IT department must allocate a limited pool of loaner laptops between two campuses: North and South.

Campus Requirements

- Campus North requires at least 150 laptops.
- Campus South requires at least 180 laptops.

Inventory Capacity

• The department has at most 360 laptops available to distribute this week.

Costs

- Preparing and transporting a laptop to Campus North costs \$18 per unit.
- Preparing and transporting a laptop to Campus South costs \$22 per unit.

How should the department allocate laptops to minimize total cost while meeting each campus's requirement and staying within the available inventory?

Suppose that a university administrator wanted to test the impact of adding additional constraints to the **question #3** optimization problem.

Suppose first they were considering stipulating that the ratio of the number of <u>Campus South</u> laptops over the number of <u>Campus North</u> laptops <u>cannot</u> exceed 2.

a. How would you formulate this additional constraint using your decision variables from question 3 above?

b. Would your feasible region change by adding this new constraint? If so how?

Suppose that a university administrator wanted to test the impact of adding additional constraints to the **question #3** optimization problem.

Suppose now they were considering *instead* stipulating that the ratio of the number of <u>Campus South</u> laptops over the number of <u>Campus North</u> laptops <u>should</u> exceed 2.

a. How would you formulate this additional constraint using your decision variables from question 3 above?

b. Would your feasible region change by adding this new constraint? If so how?