

Homework Assignment 8 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

Questions: #1-4

Video Question:

- Pretend you are a TA for this class and go over your answers to all questions. Your video should be 8-10 minutes long. Convince us that the model that you formulated does in fact represent this problem.
- Share your screen, showing your answers.

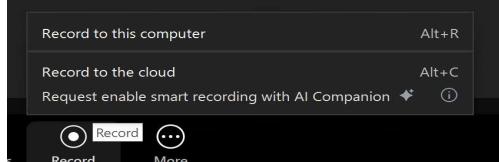
Problem	Points
1.1	5
2.1	5
2.2	2
3.1	5
3.2	2
4.1	5
4.2	2
Video Question	4

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
 - Have your camera on.
 - Show your FULL screen in Zoom (not just a particular application).
 - We should be able to hear the audio. Make sure to turn your mic on.
 - You should give a good faith attempt to answer the prompt.
 - Your video meet the minimum time requirement.
 - It should not sound like you are just reading off a script.
 - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at <https://illinois.zoom.us/recording/>.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in Canvas.

Question #1: Tech Startup Accelerator

A venture capital firm is organizing a tech startup accelerator program and has a budget of \$300,000 to invest in promising startups. There are 5 *potential* startups that they *could* invest in. They don't need to invest in all of them.

Assigning a Startup to Mentors

The program requires assigning mentors to selected startups.

- All startups should be assigned to at most one mentor. (*You can assume that if a startup was not assigned to a mentor, they were not selected*).
- Furthermore, each mentor can only be assigned to at most one selected startup. *Not all mentors need to be assigned to startups*.

Startup Expected ROI

The goal is to maximize the expected return on investment (ROI) from the selected startups. Each potential startup has an expected ROI shown to the right.

Funding Requirements

However, each startup has a specific funding requirement (shown to the right). The total firm funding must not exceed the budget of \$300,000.

Startup ID	Funding Required (\$k)	Expected ROI (\$k)	Required Expertise
S1	100	250	AI
S2	150	300	FinTech
S3	80	180	Healthcare
S4	120	220	Green Energy
S5	130	260	AI

Expertise Mentor Matching

Ideally, each startup would be paired with a mentor who has expertise in their particular area. However, this is not a hard requirement.

If a startup is matched with a mentor that does not have the required expertise, this leads to an expected ROI decline of \$100 for the start up.

Goal

The firm would like to determine which startups to fund and what mentorship pairings to make that maximizes the total expected ROI.

Mentor Team ID	Available Expertise	Ideal Startups to Support
M1	AI	S1, S5
M2	FinTech	S2
M3	Healthcare	S3
M4	Green Energy	S4
M5	AI, FinTech	S1, S2, S5

Questions:

1.1. Formulate this model. Make sure to define your decision variables (and any sets/input parameters that you might define).

Question #2: Emergency Services Network Optimization

A city is planning to optimize its emergency response network by placing service centers across various key locations. The goal is to ensure that all neighborhoods in the city are covered by at least one service center while maintaining a strategic network to minimize response time.

Required Neighborhood Coverage

- There are 5 potential locations an emergency service can be placed.
However, the city only has the budget to open at most 4 service centers.
- Each of the emergency service locations can cover just a subset of the 7 total neighborhoods (shown to the right).
- Each neighborhood *must* be covered by at least one service center.

Location ID	Can Cover Neighborhoods
L1	N1, N2
L2	N2, N3, N5
L3	N3, N4, N6
L4	N4, N5, N7
L5	N6, N7

Minimize Total *Opened* Service Center Distances

In the case of a single service center becoming overwhelmed, the sum of the distances between service centers that were opened should be minimized. The distance (miles) between each potential service center location is shown to the right.

From/To	L1	L2	L3	L4	L5
L1	0	3	7	5	8
L2	3	0	4	6	9
L3	7	4	0	2	5
L4	5	6	2	0	4
L5	8	9	5	4	0

Questions:

2.1. Formulate this model. Make sure to define your decision variables (and any sets/input parameters that you might define).

2.2. If you used and defined two different *types* of decision variables here, PROVE that the relationship that you WANTED to hold between these two types of decision variables (at least in your optimal solution) is ACTUALLY the case, based on the constraints in your model.

Question #3: Student Group Cohesion

A professor would like to create 5 groups of 27 students on the first day of class that have a lot of things in common with each other.

Maximizing Group Similarity

She conducts a survey before the first day of class and then calculates a score s_{ij} between each pair of students, counting up how many things that they have in common. Ideally the total number of things that each pair of students *in the same group* should be as large as possible.

Number of Groups

There should be 5 groups.

Group Sizes

Each group should have at least 3 students and at most 7 students.

Student Group Assignment

Each student should be assigned to exactly one group.

Questions:

3.1. Formulate this model. Make sure to define your decision variables (and any sets/input parameters that you might define).

3.2. If you used and defined two different *types* of decision variables here, PROVE that the relationship that you WANTED to hold between these two types of decision variables (at least in your optimal solution) is ACTUALLY the case, based on the constraints in your model.

Question #4: Workforce Scheduling for Emergency Response Coverage

A city's emergency management department needs to assign its available emergency response teams to various response zones in a given day.

Team Expertise

Each team has specialized skills and can cover certain zones based on their expertise.

Total Zone Coverage

Each zone must be covered by at least one team who has expertise in the area that is needed for that zone.

Team Availability

In order to ensure team efficacy, teams can only be assigned to cover at most 2 different zones.

Minimize Total Teams

Ideally the city would like to minimize the total amount of teams that are hired to work that day.

Questions:

4.1. Formulate this model. Make sure to define your decision variables (and any sets/input parameters that you might define).

4.2. If you used and defined two different *types* of decision variables here, PROVE that the relationship that you WANTED to hold between these two types of decision variables (at least in your optimal solution) is ACTUALLY the case, based on the constraints in your model.

Team ID	Expertise	Can Cover Zones
T1	Firefighting	Z1, Z2, Z4
T2	Medical Response	Z1, Z3, Z5
T3	Search and Rescue	Z2, Z3, Z6
T4	Chemical Hazard Management	Z4, Z5
T5	General Disaster Response	Z1, Z2, Z3, Z6

