Homework Assignment 2 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

Due: Friday, September 12 11:59pm CST on Canvas

Questions #1-5

Question #1-#5 below involve finding the optimal solution to Models A-H using two techniques:

- graphical solution method AND
- Python.

For the graphical solution method, you should do this BY HAND *(this will help you prepare for the exam in which you will have to do it by hand).*

You can use the attached Jupyter notebook file as a template for coding Models A-H in Python.

Video Question #6

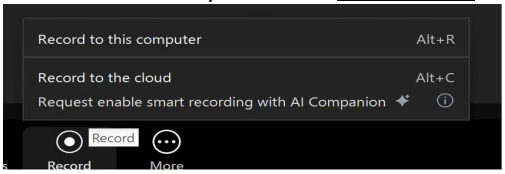
- Select the **question number** in this assignment that is closest to the **last digit** of your **netid**.
  - ***Ex**: Netid: cw8**0** -> **0***
    *Select question **1**.*
- Pretend you are a TA for this class and record a 3-4 minute video explaining what you did here [to a student who hasn't taken this class] for this **particular question number** in this homework assignment.
- Share your screen, showing your answers.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
  - Have your camera on.
  - Show your FULL screen in Zoom (not just a particular application).
  - We should be able to hear the audio. Make sure to turn your mic on.
  - You should give a good faith attempt to answer the prompt.
  - Your video meet the minimum time requirement.
  - It should not sound like you are just reading off a script.
  - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in the corresponding Jupyter notebook.

Question #1

The feasible region for the two optimization models to the right is the same.

1. Plot the feasible region.
2. Plot at least 2 contour curves. Make sure to label and indicate what objective function value (ex: c) corresponds to each of the contour curves that you drew.
3. **Model A**
   a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
   b. Use Python to either:
      i. find the optimal solution(if one exists) or
      ii. to indicate that one does not exist (and WHY it does not exist).

      Make sure you verify that the solver ran successfully.
   c. Do your Python and graphical solution optimal solutions match?

Model A

$$\begin{aligned} \text{Maximize} \quad & z = -3x_1 + 4x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \leq 25, \\ & 4x_1 - x_2 \geq 15, \\ & x_1 + 2x_2 \leq 15, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Model B

$$\begin{aligned} \text{Minimize} \quad & z = -3x_1 + 4x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \leq 25, \\ & 4x_1 - x_2 \geq 15, \\ & x_1 + 2x_2 \leq 15, \\ & x_1, x_2 \geq 0 \end{aligned}$$

4. **Model B**
   a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
   b. Use Python to either:
      i. find the optimal solution(if one exists) or
      ii. to indicate that one does not exist (and WHY it does not exist).

      Make sure you verify that the solver ran successfully.
   c. Do your Python and graphical solution optimal solutions match?

Question #2

Model C

$$\begin{aligned} \text{Maximize} \quad & z = -3x_1 + 0.75x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \leq 25, \\ & 4x_1 - x_2 \geq 15, \\ & x_1 + 2x_2 \leq 15, \\ & x_1, x_2 \geq 0 \end{aligned}$$

1. Plot the feasible region **(it's the same as question 1).**
2. Plot at least 2 contour curves. Make sure to label and indicate what objective function value $(ex: c)$ corresponds to each of the contour curves that you drew.
3. **Model C**
   a. Use the graphical solution method to find **ALL** optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
   b. Use Python to either:
      i. find **an** optimal solution(if one exists) or
      ii. to indicate that one does not exist (and WHY it does not exist).

   Make sure you verify that the solver ran successfully.
   c. Do your Python and graphical solution optimal solutions match?

Question #3

1. Plot the feasible region **(it's the same as questions 1-2)**
2. Come up with an objective function such that the optimal solution**s** of this new **Model D** is all points along the line segment between $(8.33,0)$ and $(7,4)$.
3. Plot at least 2 contour curves. Make sure to label and indicate what objective function value $(ex: c)$ corresponds to each of the contour curves that you drew.
4. Find the optimal objective function value.
5. Use Python to either:
    a. find **an** optimal solution(if one exists) or
    b. to indicate that one does not exist (and WHY it does not exist).
6. Do your Python and graphical solution **optimal objective function values** match?

Model D

$$\begin{aligned} \text{Maximize} \quad & z = \\ \text{subject to} \quad & 2x_1 + x_2 \leq 20, \\ & 4x_1 - x_2 \geq 15, \\ & x_1 + 2x_2 \leq 15, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Question #4

The feasible region for the two optimization models to the right are the same **as each other.**

Model E

$$\begin{aligned} \text{Maximize} \quad & (x_1 - 1)^2 + (x_2 - 2)^2 \\ \text{subject to} \quad & 3x_1 + 2x_2 \leq 12, \\ & 4x_1 - x_2 \leq 10, \\ & x_1 + x_2 \leq 7, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Model F

$$\begin{aligned} \text{Minimize} \quad & (x_1 - 1)^2 + (x_2 - 2)^2 \\ \text{subject to} \quad & 3x_1 + 2x_2 \leq 12, \\ & 4x_1 - x_2 \leq 10, \\ & x_1 + x_2 \leq 7, \\ & x_1, x_2 \geq 0 \end{aligned}$$

1. Plot the feasible region.
2. Plot at least 2 contour curves. Make sure to label and indicate what objective function value $(ex: c)$ corresponds to each of the contour curves that you drew.
3. **Model E**
    a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
    b. Use Python to either:
        i. find the optimal solution(if one exists) or
        ii. to indicate that one does not exist (and WHY it does not exist).
    c. Do your Python and graphical solution optimal solutions match?

4. **Model F**
    a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
    b. Use Python to either:
        i. find the optimal solution(if one exists) or
        ii. to indicate that one does not exist (and WHY it does not exist).
    c. Do your Python and graphical solution optimal solutions match?

Question #5

The feasible region for the two optimization models to the right is the same.

Model G

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 4x_2 \\ \text{subject to} \quad & 3x_1 - x_2 \geq 2, \\ & -x_1 + x_2 \geq 1, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Model H

$$\begin{aligned} \text{Minimize} \quad & z = 2x_1 + 4x_2 \\ \text{subject to} \quad & 3x_1 - x_2 \geq 2, \\ & -x_1 + x_2 \geq 1, \\ & x_1, x_2 \geq 0 \end{aligned}$$

1. Plot the feasible region.
2. Plot at least 2 contour curves. Make sure to label and indicate what objective function value $(ex: c)$ corresponds to each of the contour curves that you drew.
3. **Model G**
    a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
    b. Use Python to either:
        i. find the optimal solution(if one exists) or
        ii. to indicate that one does not exist (and WHY it does not exist).
    c. Do your Python and graphical solution optimal solutions match?
4. **Model H**
    a. Use the graphical solution method to find the optimal solution(s) to the model as well as the optimal objective function value. If an optimal solution does not exist for this model, say so and say WHY one does not exist.
    b. Use Python to either:
        i. find the optimal solution(if one exists) or
        ii. to indicate that one does not exist (and WHY it does not exist).
    c. Do your Python and graphical solution optimal solutions match?