<u>Homework Assignment 5 – [30 points]</u>

STAT430 Mathematical Optimization – Fall 2025

<u>Due</u>: Friday, October 3 11:59pm CST on Canvas

<u>Questions #1-3</u>:

Answer the questions in this pdf.

Problem	Points
1.1	2.5
1.2	1.5
1.3	1.5
1.4	1.5
1.5	1.5
1.6	1.5
1.7	3
1.8	2
1.9	2.5
2.1	2.5
2.2	3
3.1	2.5
3.2	2.5
Video Question	2

<u>Video Question</u>:

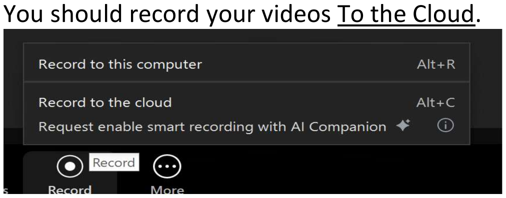
- Select the **question number** in this assignment that is closest to the **last digit** of your **netid**. ***(If you selected question 1, just explain 1.8.)***
    - ***<u>Ex</u>**: <u>Netid</u>: cw8**0** -> **0***
      *Select question **1**.*
- Pretend you are a TA for this class and record a 3-4 minute video explaining what you did here [to a student who hasn't taken this class] for this **particular question number** in this homework assignment.
- Share your screen, showing your answers.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
    - Have your camera on.
    - Show your FULL screen in Zoom (not just a particular application).
    - We should be able to hear the audio. Make sure to turn your mic on.
    - You should give a good faith attempt to answer the prompt.
    - Your video meet the minimum time requirement.
    - It should not sound like you are just reading off a script.
    - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos <u>To the Cloud</u>.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and <u>Copy shareable link</u> to paste the link to the video prompt in Canvas.

Question 1: Minimum Cost Nurse and Assistant Hospital Scheduling

Problem Statement

Nurse/Assistant Staffing

A hospital runs outpatient clinics Monday–Friday, with a morning shift (8am–12pm) and an afternoon shift (12pm–4pm) each day. The hospital manager is trying to decide how many nurses and assistants he needs to hire in order to be fully staffed.

Staff Work Constraints

- Nurses must work exactly 8 hours/week.
- Assistants must work exactly 8 hours/week.

Shifts

Each day of the week has a morning shift and an afternoon shift. Each shift is 4 hours. If assigned to a shift, all staff must work the full 4 hours of that shift.

	Day	**Shift**	**Minimum Staff Needed**
0	Monday	Morning	5
1	Monday	Afternoon	7
2	Tuesday	Morning	6
3	Tuesday	Afternoon	8
4	Wednesday	Morning	6
5	Wednesday	Afternoon	9
6	Thursday	Morning	7
7	Thursday	Afternoon	10
8	Friday	Morning	5
9	Friday	Afternoon	6

Needed Staff

Based on patient load data, the hospital has determined the minimum number of total staff (nurses and assistants) needed in each shift. This is given in the table above.

At least one nurse/shift

However, each shift must have at least one nurse staffed.

Course Staff Costs

The hospital pays $6,000 per nurse and $4,000 per assistant per month.

The hospital wants to determine how many nurses and assistants to hire to satisfy the staffing requirements at minimum cost.

1.1. Model: Formulate an optimization model for this problem. Make sure to define your decision variables.

<u>Hint</u>: Using summation and set notation can make writing up this model less tedious. If you choose to define input parameters (rather than writing out the actual numbers), make sure you define them.

*<u>Important Note</u>: Technically, your decision variables should be forced to be INTEGER. However, let's first formulate this model not explicitly stipulating this integer constraint. In 1.6 we'll try to find the optimal solution to this "**relaxed optimization model**" and see if the optimal solution values end up being integer even though we weren't explicitly forcing them to be integer.*

1.2. Decision Variables: How many total decision variables does your model above have? *<u>Hint</u>: Remember that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ Represents the total number of ways to choose k objects out of a set of n objects.*

1.3. Infeasible Scenario 1:

a) Come up with a "scenario" (ie. example) of a potential schedule that fails to meet ONE of your schedule stipulations stated in the problem above and try to translate this "scenario" into a solution to your model.

b) Is this solution a feasible solution in your model above?

c) If not, give at least one constraint or decision variable definition it violates?

1.4. Infeasible Scenario 2:

a) Come up with a "scenario" (ie. example) of a potential schedule that fails to meet ANOTHER ONE of your schedule stipulations stated in the problem above and try to translate this "scenario" into a solution to your model. *(It should be different type of stipulation than in 1.3).*

b) Is this solution a feasible solution in your model above?

c) If not, give at least one constraint or decision variable definition it violates?

1.5. Infeasible Scenario 3:

d) Come up with a "scenario" (ie. example) of a potential schedule that fails to meet ANOTHER ONE of your schedule stipulations stated in the problem above and try to translate this "scenario" into a solution to your model. *(It should be different type of stipulation than in 1.3 and 1.4).*

e) Is this solution a feasible solution in your model above?

f) If not, give at least one constraint or decision variable definition it violates?

1.6. Feasible Scenario:

a) Come up with a "scenario" (ie. example) of a potential schedule that MEETS all of the desired schedule stipulations stated in the problem above and translate this "scenario" into a solution to your model.

b) Is this solution a feasible solution in your model above?

c) Does this solution measure everything that you want it to measure in your constraints and objective function?

1.7. Optimal Solution and Objective Function Value:

a) Find the optimal solution and optimal objective function value to your model from 1.1 above using PuLP.

b) Were all of your optimal solution values integer?

c) If not all of your optimal solution values were integer, come up with the best way to round your fractional optimal solution values (up or down) such that:

- your resulting rounded solution is feasible and
- yields the lowest possible cost that you can achieve by rounding these solutions up or down.

1.8. Integer Constraints

a) Reformulate your model again in PuLP, but this time stipulate that your decision variables (all of them) must be 'Integer'.

b) Find the optimal solution and minimum cost for this new model with integer-enforced decision variables.

c) Compare the optimal objective function value of your model in 1.8 to the objective function value of your best rounded solution from 1.7. Was your best rounded solution from 1.7 optimal?

d) Do you think that your model (which explicitly forces the decision variables to be integer) has multiple optimal solutions?

1.9. Searching Amongst the Multiple Optimal Solutions

a) If you found that your integer-constrained model from 1.8 had multiple optimal solutions, did any of these optimal solutions yield one in which NO nurses and assistants had both of their shifts on the exact same day (ie. working back-to-back shifts)?

b) We would like our nurses/assistants schedule to be made at the lowest possible cost. However, if there are multiple optimal solutions (ie. solutions that yield the same lowest possible cost), ideally it would be great if the number of nurses and assistants that worked back-to-back shifts on the same day were minimized. Formulate a new optimization model that now:

- enforces our minimum cost that we found in 1.8 as a constraint,
- uses a new objective function to minimize the total number of nurses/assistants that work back-to-back shifts on the same day,
- and enforces the integer decision variable constraint.

c) Find the optimal solution and optimal objective function value to this new model in PuLP. How many nurses and assistants are required to work back-to-back shifts with this new schedule?

Question 2: Minimum Cost Nurse and Assistant Hospital Scheduling – *with Assistant Mentors*

New Problem Statement

Suppose that the hospital manager from question 1 would like to try adding an *additional set up stipulations* when building their nurse and assistant schedule. **(Ignore the additional stipulations from 1.9)**.

Assistant Mentors vs. New Assistants

Suppose that there are assistant mentors (ie. assistants that have worked at the hospital for over a year) and new assistants (that the hospital manager would choose to hire as needed). Ideally, during each shift, the manager would like for every new assistant to be mentored by their own personal assistant mentor. Or in other words, there should be AT LEAST as many assistant mentors as there are new assistants during each shift.

Limited Assistant Mentors

However, there are only 4 total assistant mentors that are available.

In addition, all 4 assistant mentors *should be staffed with 2 shifts.*

2.1. New Model: Formulate the new model, factoring in these new stipulations.

2.2. Model Feasibility: Use PuLP to determine if this new model feasible (**force the decision variables to be integer).**

Question 3: Hospital Medicine Supply Allocation

A healthcare system manages three hospitals (Hospital X, Y, and Z) that require steady supplies of medicine to operate. The medicines can be sourced from three suppliers: domestic manufacturer, regional distributor, and international supplier. The goal is to minimize the total cost of purchasing the medicine while ensuring that each hospital's medicine demands are met and that the storage space usage does not exceed predefined limits for each hospital.

- **Medicine requirements:**
  - Hospital X needs 2,000 units of medicine,
  - Hospital Y needs 3,000 units, and
  - Hospital Z needs 2,500 units.
- **Medicine source costs:**
  - Domestic: $6/unit at X, $7/unit at Y, $8/unit at Z.
  - Regional distributor: $5/unit at X, $6/unit at Y, $7/unit at Z.
  - International supplier: $4/unit at X, $5/unit at Y, $6/unit at Z.
- **Supply availability:**
  - Domestic: 5,000 units available.
  - Regional distributor: 6,000 units available.
  - International supplier: 4,500 units available.
- **Storage impacts (cubic cm per unit of medicine):**
  - Domestic medicine requires 50 cm^3 per unit.
  - Regional distributor medicine requires 100 cm^3 per unit.
  - International supplier medicine requires 200 cm^3 per unit.
- **Storage capacity limits:**
  - Hospital X can store up to 150,000 cm^3,
  - Hospital Y can store up to 200,000 cm^3,
  - Hospital Z can store up to 180,000 cm^3.

The healthcare system needs to decide how much medicine from each source each hospital should use while ensuring that all demand requirements are met, the total cost is minimized, and storage capacity limits are not exceeded.

3.1. Model Formulation

Formulate this problem with an optimization model. Make sure to define your decision variables.

3.2. Solve

Find an optimal solution to this model in PuLP as well as the minimum total cost.