# Homework Assignment 9 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

| Problem | Points |
|---|---|
| 1 | 7 |
| 2 | 6 |
| 3 | 7 |
| 4 | 7 |
| Video | 3 |

## Questions: 1-4

## Video Question:

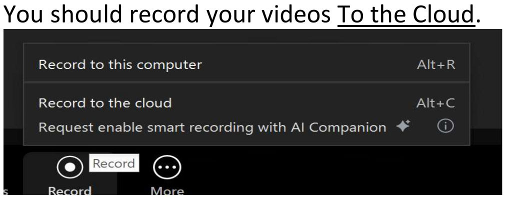
Explain your answers to all 4 questions. Your video should be about 7-10 minutes long.

**IMPORTANT Video Element of ALL Homework Assignments:**

- In order to receive points for each video submission, you need to do **ALL** of the following.
    - Have your camera on.
    - Show your FULL screen in Zoom (not just a particular application).
    - We should be able to hear the audio. Make sure to turn your mic on.
    - You should give a good faith attempt to answer the prompt.
    - Your video meet the minimum time requirement.
    - It should not sound like you are just reading off a script.
    - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

**How to Submit Videos:**

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.




- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in Canvas.

## Question #1: Transforming NLPs

The optimization model below is not a **linear program.** Come up with an equivalent formulation for the model below such that:

1. your objective function and main constraints are linear
2. none of your constraints are disjunctive (ie. “or” constraints).

*Hint: It is ok if you need to introduce binary decision variables.*

$$\begin{aligned} \text{Minimize} \quad & |x - 5| + y \\ \text{Subject to} \quad & |x - y| = 10 \\ & x, y \geq 0 \end{aligned}$$

## <u>Question #2</u>: Minimizing the Maximum Value

The optimization model below is not an **integer <u>linear</u> program,** because the objective function is not linear. Reformulate the model below into an equivalent model that *is* an integer <u>linear</u> program.

That is, you should come up with some way to get rid of the $max\{y_1, 2y_2, 3y_3\}$ function, while still *representing* that you are minimizing this value.

$$\begin{array}{ll} \text{Minimize} & max\{y_1, 2y_2, 3y_3\} \\ \text{Subject to} & y_1 + y_2 + y_3 \leq 2 \\ & y_1, y_2, y_3 \in \{0, 1\} \end{array}$$

*<u>Hint</u>: This is a very small model with only 3 possible feasible solutions to the model above. You can use this to help check your answer.*

## <u>Question #3</u>: Military Supply Bases and Frontline Outposts

Imagine a military operation where several <u>frontline outposts</u> require reliable and rapid access to <u>supply bases</u> for medical supply kits.

**<u>Decisions</u>**

1. We can build <u>supply bases</u> in 5 potential locations. The amount of medical kits that a given base location can supply is shown to the right.
2. We must also determine what percentage of an <u>outposts</u> medical kit demand is met by each <u>supply base</u>. The monthly medical kit demand for each outpost is shown to the right.

| Supply Base | Monthly # of Medical Kits the Base can Supply | Cost of Opening Supply Base ($ millions) |
|---|---|---|
| 1 | 120 | 100 |
| 2 | 100 | 120 |
| 3 | 150 | 150 |
| 4 | 80 | 90 |
| 5 | 130 | 110 |

**<u>Stipulations</u>**

1. **Supply Base Limit**<u>:</u> The military operation can only afford to establish at most 3 <u>supply bases</u>.
2. **All Outpost Demands Met**: We must ensure that 100% of each <u>outposts</u> medical supply demand is met.
3. **Supply Base Capacity:** If a <u>supply base</u> is opened, then the amount of medical kits that it supplies to the outposts cannot it's total monthly supply. Furthermore, if a <u>supply base</u> is not opened, it cannot supply any medical kits to any outposts.

| Outpost | Monthly Demand for Supplies (# of medical kits) |
|---|---|
| 1 | 50 |
| 2 | 70 |
| 3 | 60 |
| 4 | 40 |
| 5 | 90 |
| 6 | 55 |
| 7 | 80 |
| 8 | 65 |

**<u>Goals</u>**

The overall goal of this problem is minimize total costs.

1. If a <u>supply base</u> is sending *some* (non-zero amount) of supplies to a given <u>outpost</u>, then there is a fixed cost for maintaining the security of this supply route. You can see these costs (in $ millions) in the table below.

| | Outposts ($ millions) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Base** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| 1 | 10 | 15 | 12 | 20 | 25 | 18 | 22 | 30 |
| 2 | 14 | 10 | 18 | 8 | 22 | 25 | 17 | 20 |
| 3 | 20 | 18 | 15 | 10 | 30 | 12 | 28 | 26 |
| 4 | 25 | 20 | 22 | 15 | 10 | 24 | 18 | 20 |
| 5 | 16 | 12 | 25 | 18 | 14 | 22 | 19 | 30 |

2. It also costs money to open a given supply base. You can see these costs in the first table above.

Formulate this problem with a optimization model. All of your model constraints should be <u>linear</u>.

## Question #4: Traveling Salesman – No Return Trip

Suppose that a traveling salesman starts their journey from city 1 and needs to visit all cities (1,2,…,7). All cities must be visited exactly once in the journey.

- However, *in this special case,* once they visit their final city, they do not return to city 1. They remain at this final destination city.
- It is technically fine for the salesman to end their journey at *any* city.

The distances between each pair of cities (in miles) is given below.

Formulate an optimization model that finds their optimal travel path that minimizes the total amount of distance traveled.

***Simplifying*: For the sake of simplicity, assume for now that a "subtour" is ok in your formulation. However, if you want extra credit, try to come up with a way to eliminate subtours from your path in your model**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **1** | 0 | 29 | 20 | 21 | 17 | 34 | 28 |
| **2** | 29 | 0 | 15 | 29 | 28 | 40 | 34 |
| **3** | 20 | 15 | 0 | 15 | 14 | 25 | 24 |
| **4** | 21 | 29 | 15 | 0 | 22 | 36 | 27 |
| **5** | 17 | 28 | 14 | 22 | 0 | 31 | 23 |
| **6** | 34 | 40 | 25 | 36 | 31 | 0 | 30 |
| **7** | 28 | 34 | 24 | 27 | 23 | 30 | 0 |