Homework Assignment 3 – [30 points]

STAT430 Mathematical Optimization – Fall 2025

Due: Friday, September 19 11:59pm CST on Canvas

Questions #1-5

See next pages.

Video Question #6 [2 points]

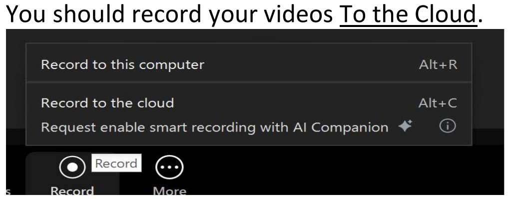
- Select the **question number (out of 3,4,5)** in this assignment that is closest to the **last digit** of your **netid**.
    - ***Ex***: *Netid: cw8**0** -> **0***
      *Select question **1**.*
- Pretend you are a TA for this class and record a 3-4 minute video explaining what you did here [to a student who hasn't taken this class] for this **particular question number** in this homework assignment.
- Share your screen, showing your answers.

IMPORTANT Video Element of ALL Homework Assignments:

- In order to receive points for each video submission, you need to do **ALL** of the following.
    - Have your camera on.
    - Show your FULL screen in Zoom (not just a particular application).
    - We should be able to hear the audio. Make sure to turn your mic on.
    - You should give a good faith attempt to answer the prompt.
    - Your video meet the minimum time requirement.
    - It should not sound like you are just reading off a script.
    - It's ok if your video recording is not the most eloquent. What's important is that you are putting together YOUR authentic thoughts on your particular understanding of the assignment and the lecture content.

How to Submit Videos:

- You should record your videos in your UIUC Zoom client.
- You should record your videos To the Cloud.



- You can find your recording link at https://illinois.zoom.us/recording/.
- Click on the corresponding video and Copy shareable link to paste the link to the video prompt in the corresponding Jupyter notebook.

1. **<u>Original Optimization Problem Model Formulation</u>**

Formulate the following optimization problem as an <u>optimization model</u> in <u>standard form.</u>

<u>How many products of each type to manufacture</u>

A company is attempting to determine how many products of each type (<u>product 1</u> and <u>product 2</u>) it should produce to maximize their **<u>expected</u>** <u>profits</u> for *this quarter*.

<u>Time of the Calculations and Product Manufacturing</u>

Let's assume that they need to find this optimal solution on **July 1**. After determining this optimal solution, they make the final decision to start producing the specified number of <u>products 1</u> and <u>products 2</u>. Assume that they cannot not make any more or less than this specified number as new information comes in.

<u>Required Manufacturing Resources and Needed as of **July 1**</u>

In order to manufacture <u>products 1 and 2</u>, different amounts of <u>resources A, B, C, and D</u> are required.

As of **July 1**, the data scientists at this company are **predicting** the following about each of the resources.

- <u>Resource A</u>
  - Producing each unit of Product 1 uses 2 units of Resource A.
  - Producing each unit of Product 2 uses 3 units of Resource A.
  - The factory has a total of 20 units of Resource A available.
- <u>Resource B</u>
  - Producing each unit of Product 1 uses 4 units of Resource B.
  - Producing each unit of Product 2 uses 1 units of Resource B.
  - The factory has a total of 20 units of Resource B available.
- <u>Resource C</u>
  - Producing each unit of Product 1 uses 1 units of Resource C.
  - Producing each unit of Product 2 uses 3 units of Resource C.
  - The factory has a total of 18 units of Resource C available.
- <u>Resource D</u>
  - Producing each unit of Product 1 uses 1 unit of Resource D.
  - Producing each unit of Product 2 <u>no units</u> of Resource D.
  - The factory has a total of 7 units of Resource D available.

<u>Manufacturing Beginning Date</u>

After deciding how many products to make on **July 1**, the products cannot begin to be produced until **August 1**.

<u>Expected Profit</u>

As of **July 1**, the company's data scientists have predicted that the ***expected*** **profit** for <u>selling</u> each type of product on **September 1** will be:

- $4 for each unit of <u>product 1</u>
- $2 for each unit of product 2

2. **Original Model**
    2.1. Plot the feasible region.
    2.2. Plot at least 2 contour curves. Make sure to label and indicate what objective function value $(ex: c)$ corresponds to each of the contour curves that you drew.
    2.3. Find the optimal solution and optimal objective function value using the **graphic solution method.**
    2.4. Find the optimal solution and optimal objective function value using the **PuLP** package in **Python.**

3. Considering Profit Fluctuations

After building and solving this original model on **July 1,** as a pragmatic data scientist, you'd like to perform sensitivity analysis to determine how robust your optimal solution is to fluctuations in the expected profit of product 1 in the event that the actual profit of selling either product 1 were to change by **September 1**, when we actually start to sell the products.

Note: *That a more "complete" analysis would explore the impact of profit fluctuations on product 2 as well.*

3.1. Expected General Model Changes

If the expected profit of either product 1 or 2 were to change, what would you expect to happen to the following components of your **original model**?

Select one of the following answers for (i, ii, iii) below. a.) definitely change, b.) definitely will not change, or c.) MIGHT change depending on the direction and size of the change.

3.1.1. the feasible region of the original model

3.1.2. whether the original model is unbounded or not

3.1.3. whether the original model is infeasible or not

3.2. **INCREASING the expected profit of product 1**

3.2.1. By how much is the expected profit p_1 of selling one unit of product 1 able to INCREASE (above \$4), before the optimal solution of the original model changes? Call this maximum new expected profit of product 1 p_1^+.
If increasing the expected profit of product 1 would NOT change the optimal solution, say so.
Show your work **by hand**.

3.2.2. What would the new optimal solution be once you increase the expected profit of product 1 slightly beyond p_1^+?

3.2.3. Verify your answers to 3.2.1 and 3.2.2 in Python, using the **PuLP** package.

3.2.4. Suppose we were to keep increasing the expected profit of product 1 even more BEYOND this p_1^+ that you found in 3.2.1. How much more would you need to increase this p_1 beyond p_1^+ before the optimal solution changes AGAIN to something new?

If increasing the expected profit of product 1 even further would NOT change the optimal solution again, say so and explain why.

Show your work **by hand**.

3.2.5. Suppose that on **September 1** the profit of selling one unit of product 1 were to have increased to $\boldsymbol{p_1^+} + \$\mathbf{1}$. But, rather than using the new optimal solution from 3.2.2, you were locked into using the old optimal solution (from #2) from your original model that you ran on **July 1**.

How much profit would you be losing by using the old optimal solution from your original model as opposed to your new optimal solution from 3.2.2? Explain.

3.3. **DECREASING the expected profit of product 1**

3.3.1. By how much is the expected profit p_1 of selling one unit of product 1 able to DECREASE (below $4), before the optimal solution of the original model changes? Call this minimum new expected profit of product 1 p_1^-.

If decreasing the expected profit of product 1 would NOT change the optimal solution, say so and explain..

Show your work **by hand**.

3.3.2. What would the new optimal solution be once you decrease the expected profit of product 1 slightly beyond p_1^-?

3.3.3. Verify your answers to 3.3.1 and 3.3.2 in Python, using the **PuLP** package.

3.3.4. Suppose we were to keep decreasing the expected profit of product 1 even more BEYOND this p_1^- that you found in 3.3.1. How much more would you need to decrease this p_1 beyond p_1^- before the optimal solution changes AGAIN to something new?

If decreasing the expected profit of product 1 even further would NOT change the optimal solution again, say so and explain why.

Show your work **by hand**.

4. <u>**Considering Available Resource A Fluctuations**</u>

Also after building and solving the original model on **July 1,** as a pragmatic data scientist, you'd like to perform sensitivity analysis to determine how robust your optimal solution is to <u>fluctuations in how much available units</u> of <u>resource **A**</u> in the event that the <u>actual amount</u> of units of <u>resource A</u> were to change by **August 1**, when we actually start to manufacture the products.

Note: *That a more "complete" analysis would explore the impact of fluctuations of availability in resource B, C, and D as well.*

4.1. **Expected General Model Changes**

If the <u>expected amount of available</u> units of either <u>resource A</u> were to change, what would you expect to happen to the following components of your **original model**?
Select one of the following answers for (i, ii, iii) below. a.) definitely change, b.) definitely will not change, or c.) MIGHT change depending on the <u>direction</u> and <u>size</u> of the change.

4.1.1. the <u>feasible region</u> of the original model

4.1.2. whether the original model is <u>unbounded</u> or not

4.1.3. whether the original model is <u>infeasible</u> or not

4.2. **INCREASING the amount of available resource A**

4.2.1. By how much is the amount $(call\ it\ A)$ of available resource A (above 20) able to INCREASE, before the optimal solution of the original model STOPS changing? Let's call A^+ this maximum amount of resource A.
If the optimal solution would NOT change even by slightly increasing A, say so and explain why.
Show your work **by hand**.

4.2.2. What would the new optimal solution (that doesn't change anymore) be once you increase the amount of resource A slightly beyond A^+?

4.2.3. Verify your answers to 4.2.1 and 4.2.2. in Python, using the **PuLP** package.

4.2.4. Suppose that on **August 1** you actually had A^+ amount of available resource A. But, given that you had already committed to manufacturing the amount of product 1 and 2 that you had determined with the optimal solution of the original model, you were not able to adapt how many products you made.

How much profit would you have lost in this case because you used the optimal solution from the original model instead of the optimal solution that you found in 4.2.2 that was able to factor in more resource A availability?

4.3. **DECREASING the amount of available resource A**

4.3.1. By how much is the amount $(call\ it\ A)$ of available resource A (above 20) able to DECREASE, before the optimal solution of the original model STOPS changing? Let's call A^{-} this minimum amount of resource A.
If the optimal solution would NOT change even by slightly decreasing A, say so and explain why.
Show your work **by hand**.

4.3.2. As we decrease A all the way down to 0, describe how the nature of the resulting optimal solutions will change. Or in other words, describe "path" that the resulting optimal solutions will take as you decrease A.

4.3.3. Verify your answers to 4.3.1 and 4.3.2. in Python, using the **PuLP** package.

4.3.4. Suppose that on **August 1** you actually had 15 amount of available resource A. But, given that you had already committed to manufacturing the amount of product 1 and 2 that you had determined with the optimal solution of the original model, you were not able to adapt how many products you made. So suppose that you had to purchase the remaining additional 5 units of resource A units that you needed. Each resource A unit that you had to purchase costed $3.

4.3.4.1. How much TOTAL profit would you have made in this scenario had you used your original optimal solution from the original model?

4.3.4.2. How much TOTAL profit would you have made in this scenario had found and used the optimal solution to a new model with $A = 15$?

5. <u>**Considering Available Resource D Fluctuations**</u>

Also after building and solving the original model on **July 1,** as a pragmatic data scientist, you'd like to perform sensitivity analysis to determine how robust your optimal solution is to <u>fluctuations in how much available units</u> of <u>resource **D**</u> in the event that the <u>actual amount</u> of units of <u>resource A</u> were to change by **August 1**, when we actually start to manufacture the products.

Note: *That a more "complete" analysis would explore the impact of fluctuations of availability in resource B, C, and D as well.*

5.1. Expected General Model Changes

If the <u>expected amount of available</u> units of either <u>resource **D**</u> were to change, what would you expect to happen to the following components of your **original model**?
Select one of the following answers for (i, ii, iii) below. a.) definitely change, b.) definitely will not change, or c.) MIGHT change depending on the <u>direction</u> and <u>size</u> of the change.

5.1.1. the <u>feasible region</u> of the original model

5.1.2. whether the original model is <u>unbounded</u> or not

5.1.3. whether the original model is <u>infeasible</u> or not

5.2. **INCREASING the amount of available resource D**

5.2.1. By how much is the amount $(call\ it\ D)$ of available resource D (above 7) able to INCREASE, before the optimal solution of the original model STARTS changing? Let's call D^{+} this maximum amount of resource D.

If the optimal solution would NEVER change by increasing D, say so and explain why.

5.2.2. Attempt to verify your answer to 5.2.1. in Python, using the **PuLP** package.

5.3. **DECREASING the amount of available resource D**

5.3.1. By how much is the amount $(call\ it\ D)$ of available resource D (below 7) able to DECREASE, before the optimal solution of the original model STARTS changing? Let's call D^{-} this minimum amount of resource D.
If the optimal solution would NEVER change by decreasing D, say so and explain why.
Show your work **by hand**.

5.3.2. As we decrease D all the way down to 0, describe how the nature of the resulting optimal solutions will change. Or in other words, describe "path" that the resulting optimal solutions will take as you decrease D.

5.3.3. Verify your answers to 5.3.1 and 5.3.2. in Python, using the **PuLP** package.