Assignment 1: Introduction to Systems Programming

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1 Mathematical Analysis

1.1 Algorithm 1

```
Data: Integer array A of size N
Result: Greatest Sum of Subarray
for i ← 0 to N do
    for j ← i to N do
        s ← 0
        for k ← i to j do
            s ← s + A[k]
        end
        if s > max then
            max ← s
        end
    end
end
```

Algorithm 1: Pseudocode for Basic Enumeration

1.2 Algorithm 2

```
Data: Integer array A of size N
Result: Greatest Sum of Subarray
for i ← 0 to N do
    s ← 0
    for j ← i to N do
        s ← A[j]
        if s > max then
            max ← s
        end
    end
end
```

Algorithm 2: Pseudocode for Better Enumeration

1.3 Algorithm 3

```
Data: Integer array A of size N
Result: Greatest Sum of Subarray
```

Algorithm 3: Pseudocode for Divide and Conquer

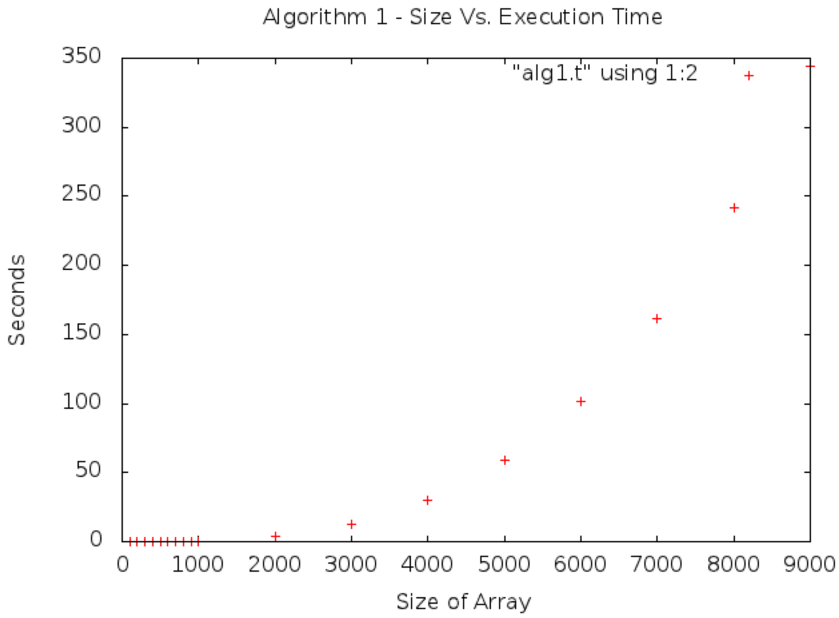
2 Theoretical Correctness

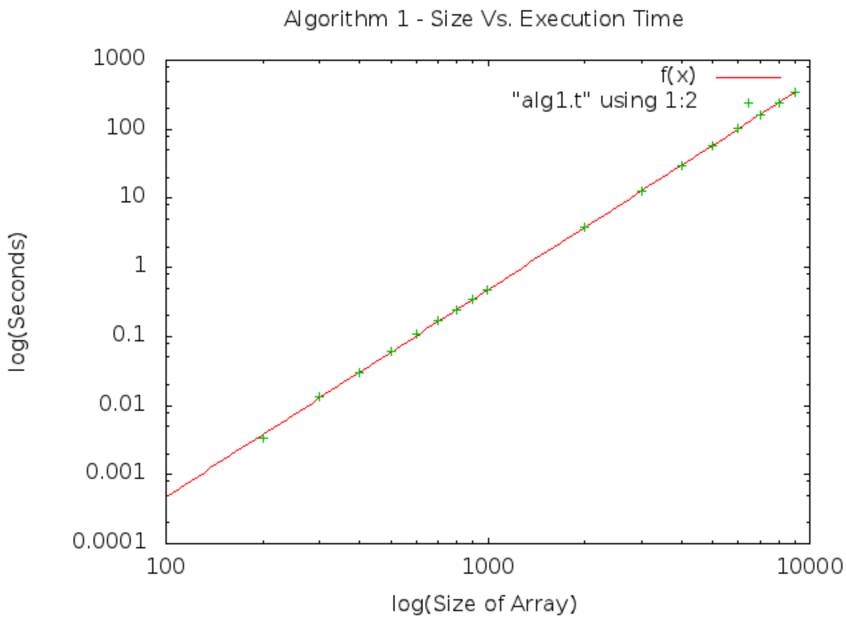
3 Testing

931678074 - 5703
930569466 - 8184
932086449 - 4949

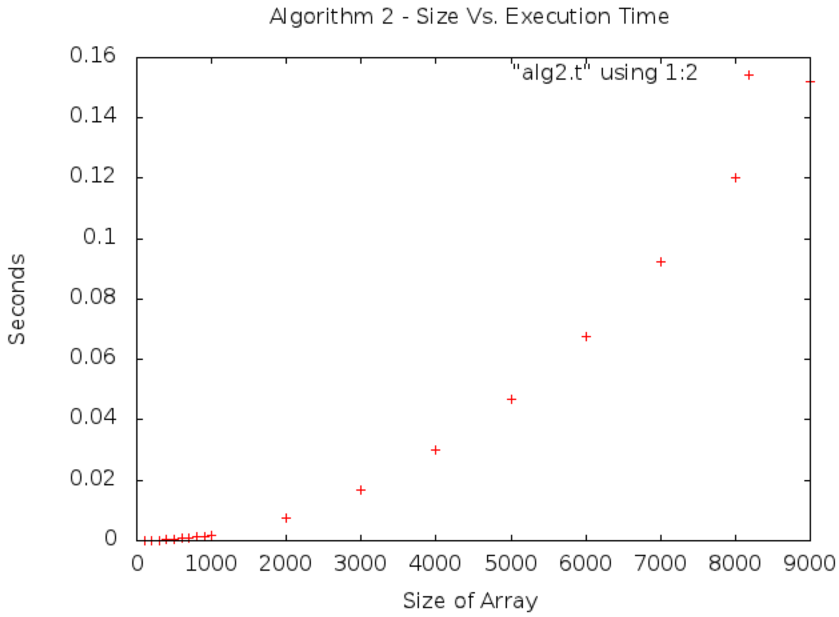
4 Experimental Analysis

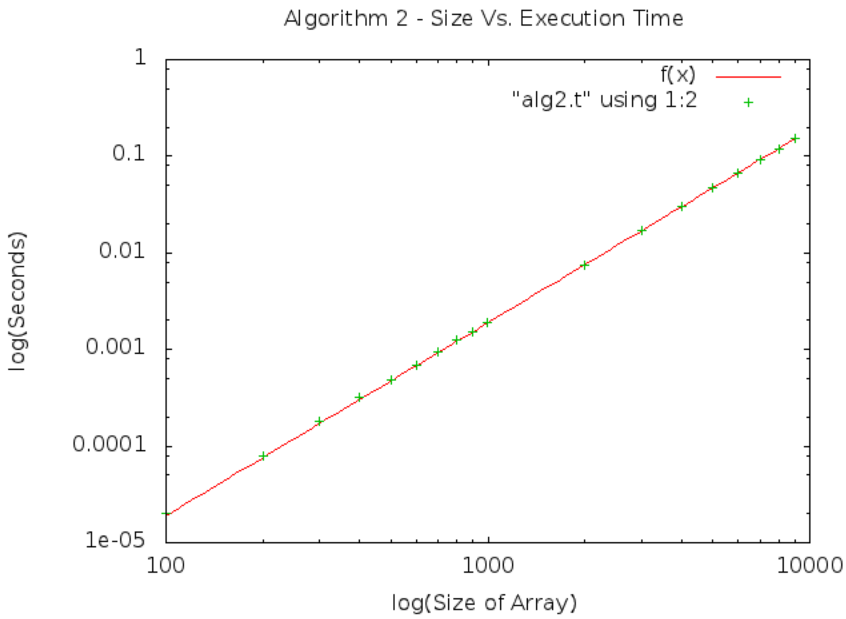
4.1 Algorithm 1



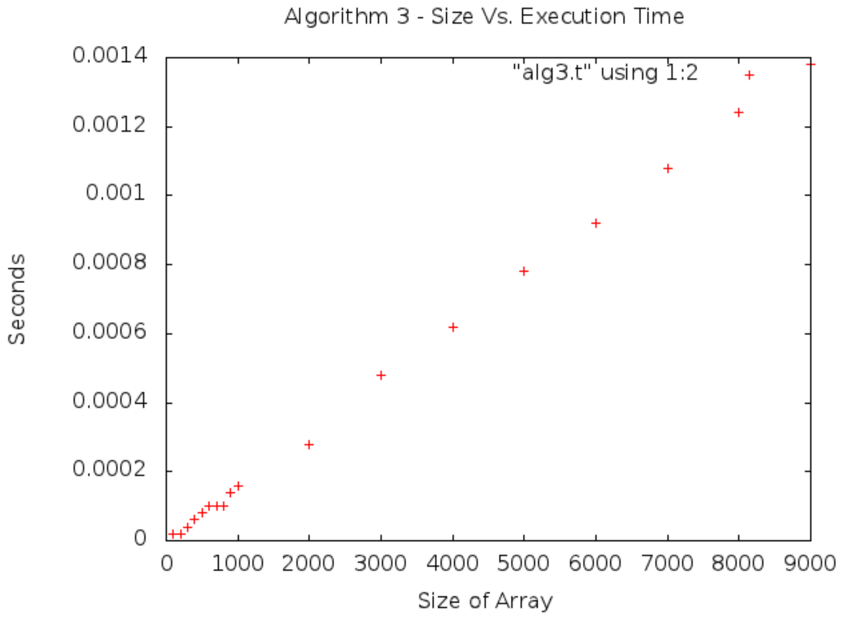


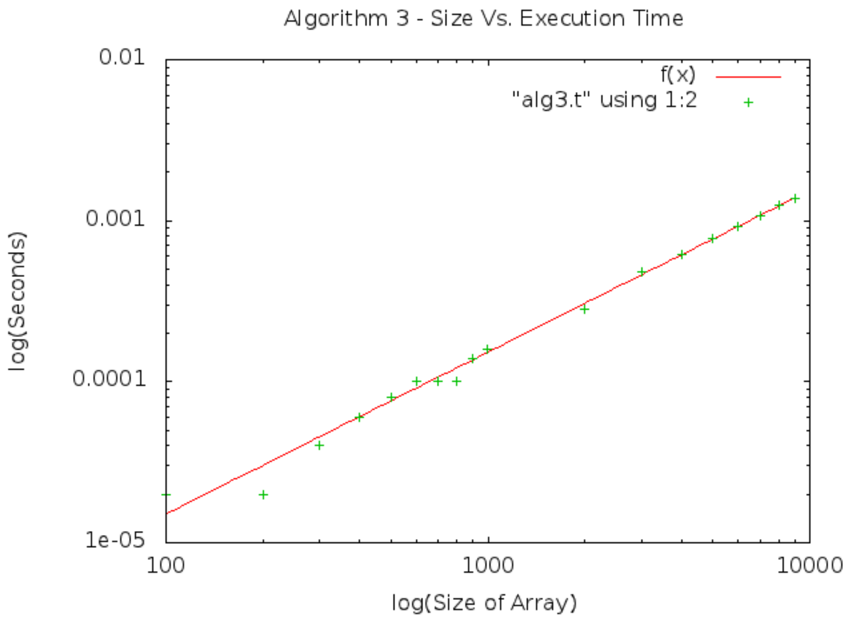
4.2 Algorithm 2





4.3 Algorithm 3





5 Extrapolation and Interpretation

5.1 Extrapolation

5.2 Interpretation

5.3 Algorithm 1

5.3.1 Extrapolation

$f(n) = 4.71599 \times 10^{-10} \times n^3$
$f(n) = 3600 \rightarrow n = 19690$

5.3.2 Interpretation

Slope = 2.99734

5.4 Algorithm 2

5.4.1 Extrapolation

$f(n) = 1.87761 \times 10^{-9} \times n^2$
$f(n) = 3600 \rightarrow n = 1384678$

5.4.2 Interpretation

Slope = 1.99602

5.5 Algorithm 3

5.5.1 Extrapolation

$f(n) = 1.74832 \times 10^{-8} \times n \times log(n)$
$f(n) = 3600 \rightarrow n = 8984428998 = 8.98 \times 10^9$

5.5.2 Interpretation

Slope = 1.00506

6 Code

6.1 Files

alg1.cpp - function for algorithm 1
alg2.cpp - function for algorithm 1
alg3.cpp - function for algorithm 1
analysis.cpp - code to run algorithm and measure times for the number of array then outputs .t file
makefile - to compile files
maxSubarray.pdf -
maxSubarray.tex - to create pdf filename
test.cpp - allows input of file, and runs algorithm on input file
analysis/
test/
timingfiles/ - holds files for creating plots
timingfiles/*.t - files that holds rum times for different array sizes
timingfiles/*.gp - code for gnuplot. 2 plots of each algorithm: 1 normal plot, and 1 log-log plot

6.2 Algorithm 1

```cpp
/*
 * Enumeration
 * Loop over each pair of indices i; j and compute the sum from k=i to j of a[k].
 * Keep the best sum you have found so far.
 */

using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    for (j = i; j < n; ++j){
      sum = 0;
      for (k = i; k <=j; ++k){
        sum += a[k];
      }
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg1.cpp

6.3 Algorithm 2

```
/*
 * Better Enumeration
 * Notice that in the previous algorithm, the same sum is computed many times.
 * In particular, notice that sum from k=i to j of a[k] can be computed from sum from k=i to j - 1 of a[k] in O(1) time, rather than starting from scratch.
 * Write a new version of the frst algorithm that takes advantage of this observation.
 */


using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    sum = 0;
    for (j = i; j < n; ++j){
      sum += a[j];
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg2.cpp

6.4 Algorithm 3

```cpp
/*
 * Divide and Conquer
 * If we split the array into two halves, we know that the maximum subarray will either be
 *       * contained entirely in the frst half,
 *       * contained entirely in the second half, or
 *       * made of a suffix of the frst half of maximum sum and a prefix of the second half of maximum sum
 * The frst two cases can be found recursively. The last case can be found in linear time.
 */

using namespace std;

void MaxSubarray_h(int a[], int n, int b[]){
  if(n <= 1){
    b[0] = a[0];
    b[1] = a[0];
    b[2] = a[0];
    b[3] = a[0];
    return;
  }
  int i = n/2;
  int *left = new int[4];
  MaxSubarray_h(a,i,left);
  int *right = new int[4];
  MaxSubarray_h(a+i,n-i,right);

  int sum = left[0] + right[0];
  int l = left[0]+right[1];
  int r = left[2]+right[0];
  int mid = left[2]+right[1];

  l = l > left[1] ? l : left[1];
  r = r > right[2] ? r : right[2];
  int m = left[3] > right[3] ? left[3] : right[3];
  m = m > mid ? m : mid;

  b[0] = sum;
  b[1] = l;
  b[2] = r;
  b[3] = m;
}


int MaxSubarray(int a[], int n){
  int *p = new int[4];
  MaxSubarray_h(a,n,p);
  int s1 = p[0] > p[1] ? p[0] : p[1];
  int s2 = p[2] > p[3] ? p[2] : p[3];
  s1 = s1 > s2 ? s1 : s2;
  return s1;
}
```

alg3.cpp