Assignment 1: Introduction to Systems Programming

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1 Mathematical Analysis

1.1 Algorithm 1

Data: Integer array A of size N
Result: Greatest Sum of Subarray
for $i \leftarrow 0$ **to** N **do**
    for $j \leftarrow i$ **to** N **do**
        $s \leftarrow 0$;
        for $k \leftarrow i$ **to** j **do**
            $s \leftarrow s + A[k]$;
        end
        if $s > max$ **then**
            $max \leftarrow s$;
        end
    end
end

Algorithm 1: Pseudocode for Basic Enumeration

1.2 Algorithm 2

Data: Integer array A of size N
Result: Greatest Sum of Subarray
for $i \leftarrow 0$ **to** N **do**
    $s \leftarrow 0$;
    for $j \leftarrow i$ **to** N **do**
        $s \leftarrow A[j]$;
        if $s > max$ **then**
            $max \leftarrow s$;
        end
    end
end

Algorithm 2: Pseudocode for Better Enumeration

1.3 Algorithm 3

Data: Integer array A of size N
Result: Greatest Sum of Subarray
MaxSubarray$A$ $sums \leftarrow MaxSubarray_recursion(A)$;
return $max(sums)$;
MaxSubarray$_recursion$A**if**$A.size() \leq 1$**then**
  $sums.all \leftarrow A[0]$;
  $sums.left \leftarrow A[0]$;
  $sums.right \leftarrow A[0]$;
  $sums.overall \leftarrow A[0]$;
end
$left_sums \leftarrow MaxSubarray_recursion(A.left_branch)$;
$right_sums \leftarrow MaxSubarray_recursion(A.right_branch)$;
$a = left_sums.all + right_sums.all$;
$l = max(left_sums.left, left_sums.all + right_sums.left)$;
$r = max(right_sums.right, left_sums.right + right_sums.all)$;
$m = left_sums.right + right_sums.left$;
$overall = max(a, l, r, m)$;
$sums.all = a$;
$sums.left = l$;
$sums.right = r$;
$sums.overall = overall$;
return sums

Algorithm 3: Pseudocode for Divide and Conquer

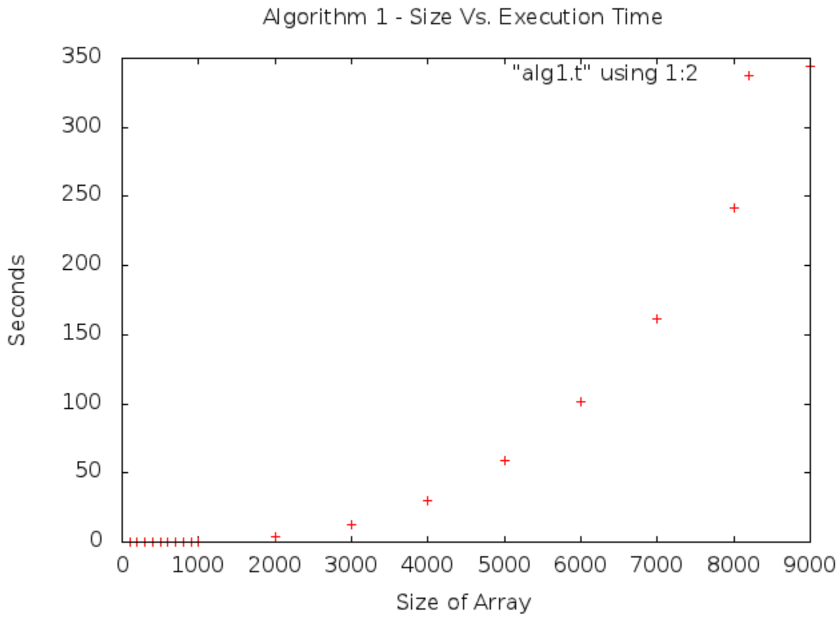
2 Theoretical Correctness

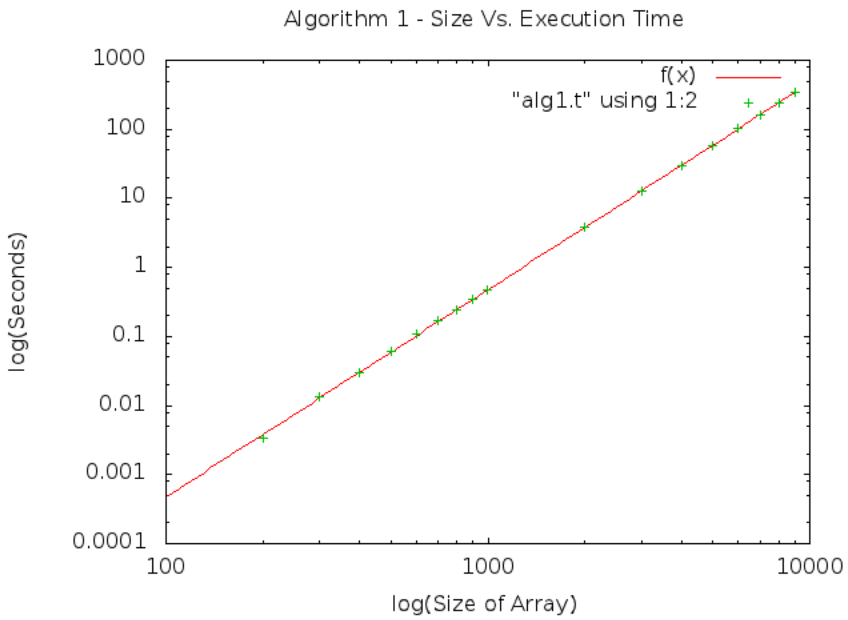
3 Testing

931678074 - 5703
930569466 - 8184
932086449 - 4949

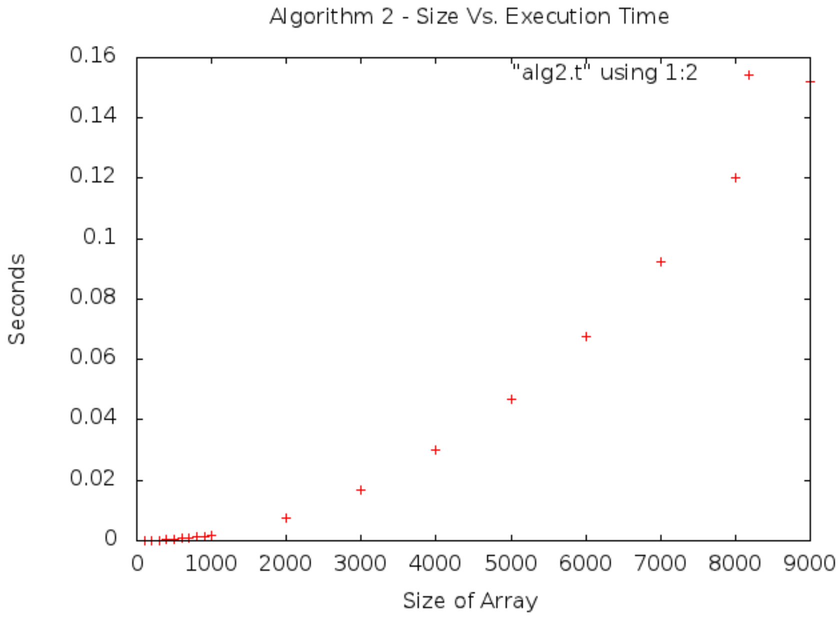
4 Experimental Analysis

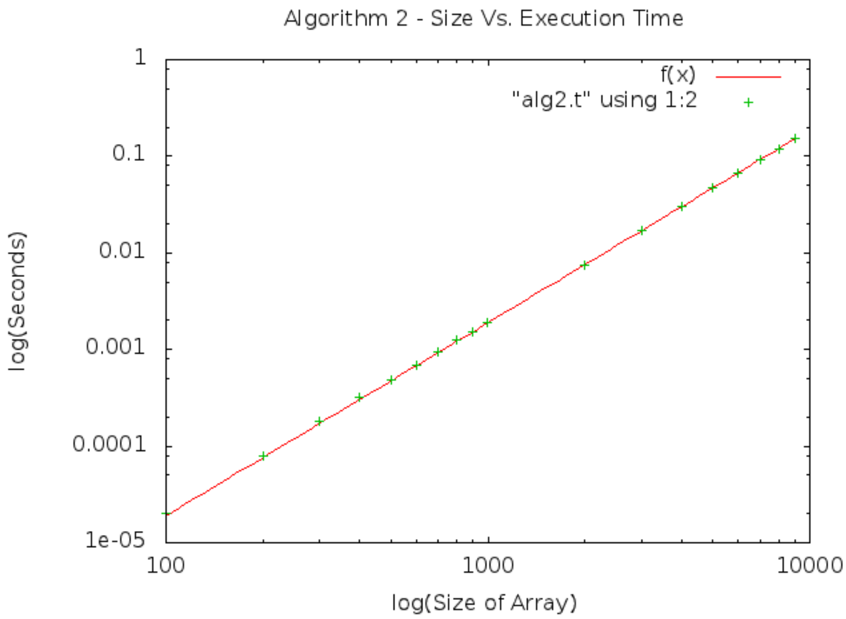
4.1 Algorithm 1



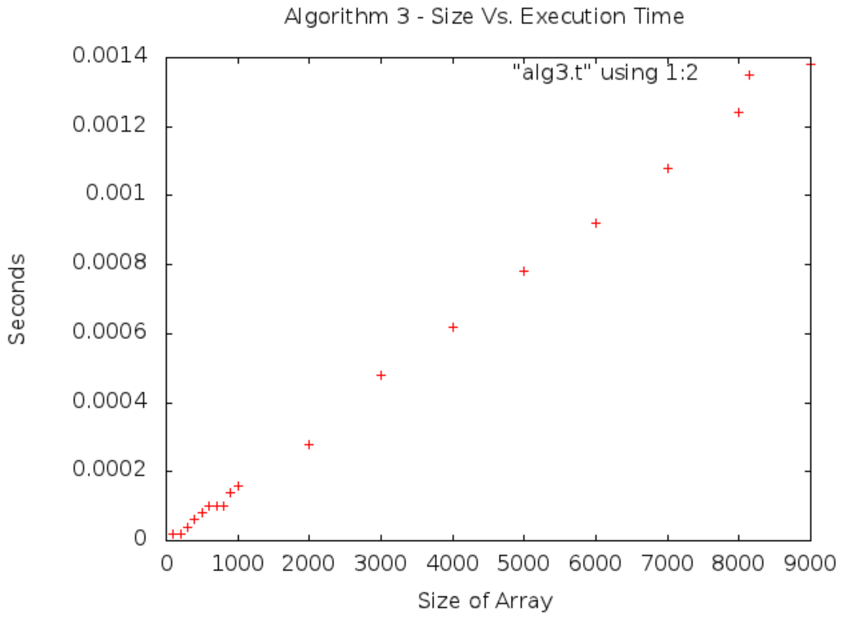


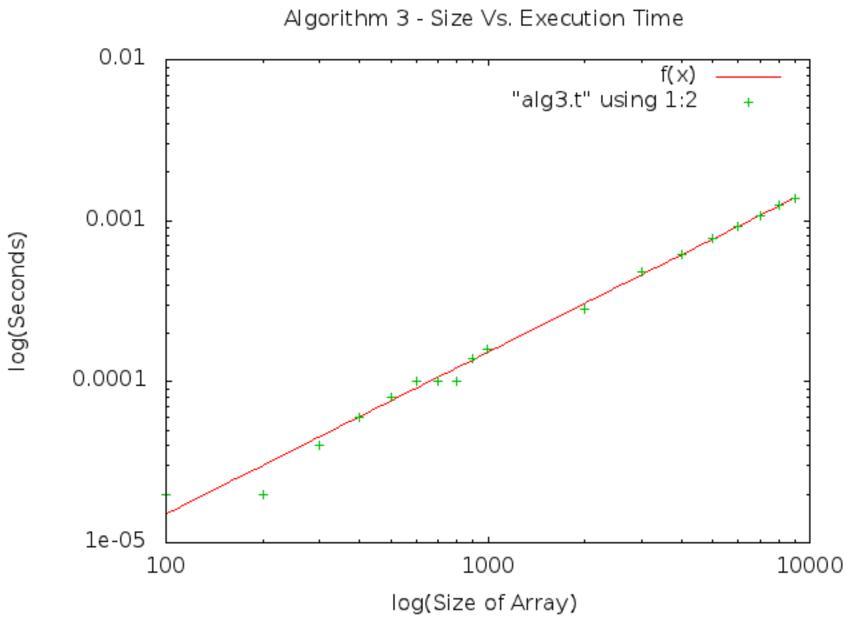
4.2 Algorithm 2





4.3 Algorithm 3





5 Extrapolation and Interpretation

5.1 Extrapolation

5.2 Interpretation

5.3 Algorithm 1

5.3.1 Extrapolation

$f(n) = 4.71599 \times 10^{-10} \times n^3$
$f(n) = 3600 \rightarrow n = 19690$

5.3.2 Interpretation

Slope = 2.99734

5.4 Algorithm 2

5.4.1 Extrapolation

$f(n) = 1.87761 \times 10^{-9} \times n^2$
$f(n) = 3600 \rightarrow n = 1384678$

5.4.2 Interpretation

Slope = 1.99602

5.5 Algorithm 3

5.5.1 Extrapolation

$f(n) = 1.74832 \times 10^{-8} \times n \times log(n)$
$f(n) = 3600 \rightarrow n = 8984428998 = 8.98 \times 10^9$

5.5.2 Interpretation

Slope = 1.00506

6 Code

6.1 Files

alg1.cpp - function for algorithm 1
alg2.cpp - function for algorithm 1
alg3.cpp - function for algorithm 1
analysis.cpp - code to run algorithm and measure times for the number of array then outputs .t file
makefile - to compile files
maxSubarray.pdf -
maxSubarray.tex - to create pdf filename
test.cpp - allows input of file, and runs algorithm on input file
analysis/
test/
timingfiles/ - holds files for creating plots
timingfiles/*.t - files that holds rum times for different array sizes
timingfiles/*.gp - code for gnuplot. 2 plots of each algorithm: 1 normal plot, and 1 log-log plot

6.2 Algorithm 1

```cpp
/*
 * Enumeration
 * Loop over each pair of indices i; j and compute the sum from k=i to j of a[k].
 * Keep the best sum you have found so far.
 */

using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    for (j = i; j < n; ++j){
      sum = 0;
      for (k = i; k <=j; ++k){
        sum += a[k];
      }
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg1.cpp

6.3 Algorithm 2

```cpp
/*
 * Better Enumeration
 * Notice that in the previous algorithm, the same sum is computed many times.
 * In particular, notice that sum from k=i to j of a[k] can be computed from sum from k=
    i to j - 1 of a[k] in O(1) time, rather than starting from scratch.
 * Write a new version of the frst algorithm that takes advantage of this observation.
 */


using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    sum = 0;
    for (j = i; j < n; ++j){
      sum += a[j];
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg2.cpp

6.4 Algorithm 3

```cpp
/*
 * Divide and Conquer
 * If we split the array into two halves, we know that the maximum subarray will either be
 *      * contained entirely in the frst half,
 *      * contained entirely in the second half, or
 *      * made of a suffix of the frst half of maximum sum and a prefix of the second half of maximum sum
 * The frst two cases can be found recursively. The last case can be found in linear time.
 */

#define ALL     0
#define LEFT    1
#define RIGHT   2
#define OVERALL 3

using namespace std;

void MaxSubarray_h(int array[], int size, int sums[]){
  // Base case.
  if(size <= 1){
    sums[ALL]     = array[0];   // Sum of entire array
    sums[LEFT]    = array[0];   // Largest sum from left end of array
    sums[RIGHT]   = array[0];   // Largest sum from right end of array
    sums[OVERALL] = array[0];   // Largest sum found so far
    return;
  }
  int i = size/2;   // Index of middle element

  // Recurse.
  int *left  = new int[4];
  int *right = new int[4];
  MaxSubarray_h(array,i, left);
  MaxSubarray_h(array+i, size-i, right);

  // Calculate various possible maximum sums.
  int a = left[ALL]   + right[ALL];     // Sum of everything
  int l = left[ALL]   + right[LEFT];    // Possible max sum from the left
  int r = left[RIGHT] + right[ALL];     // Possible max sum from the right
  int m = left[RIGHT] + right[LEFT];    // Possible max sum straddling both branches

  // Check for and find new maximums.
  l = l > left[LEFT] ? l : left[LEFT];     // Is the new left sum larger?
  r = r > right[RIGHT] ? r : right[RIGHT];    // Is the new right sum larger?
  int overall = left[OVERALL] > right[OVERALL] ? left[OVERALL] : right[OVERALL];
  overall = overall > m ? overall : m;

  // Final answers!
  sums[0] = a;
  sums[1] = l;
  sums[2] = r;
  sums[3] = overall;
}


int MaxSubarray(int a[], int n){
  int *p = new int[4];
  MaxSubarray_h(a,n,p);
  int s1 = p[0] > p[1] ? p[0] : p[1];
  int s2 = p[2] > p[3] ? p[2] : p[3];
  s1 = s1 > s2 ? s1 : s2;
  return s1;
}
```

alg3.cpp