Assignment 1: Introduction to Systems Programming

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October 16, 2012

1 Mathematical Analysis

1.1 Algorithm 1

Data: Integer array A of size N
Result: Greatest Sum of Subarray
for $i \leftarrow 0$ **to** N **do**
  for $j \leftarrow i$ **to** N **do**
    $s \leftarrow 0$;
    for $k \leftarrow i$ **to** j **do**
      $s \leftarrow s + A[k]$;
    end
    if $s > max$ **then**
      $max \leftarrow s$;
    end
  end
end

Algorithm 1: Pseudocode for Basic Enumeration

This Algorithm has 3 for-loops so it is O(n^3).

1.2 Algorithm 2

Data: Integer array A of size N
Result: Greatest Sum of Subarray
for $i \leftarrow 0$ **to** N **do**
  $s \leftarrow 0$;
  for $j \leftarrow i$ **to** N **do**
    $s \leftarrow A[j]$;
    if $s > max$ **then**
      $max \leftarrow s$;
    end
  end
end

Algorithm 2: Pseudocode for Better Enumeration

This Algorithm has 2 for-loops so it is O(n^2).

1.3 Algorithm 3

Data: Integer array A of size N
Result: Greatest Sum of Subarray

```
def MaxSubarray:
    sums = MaxSubarray_recursive(A)
    return max(sums)
```

Algorithm 3.1: Starting function pseudocode for Divide and Conquer

Data: Integer array A of size N
Result: Integer array of size 4

```
def MaxSubarray_recursive:
    if A.size <= 1:
        sums.all = A[0]
        sums.left = A[0]
        sums.right = A[0]
        sums.overall = A[0]

    left_sums = MaxSubarray_recursive(A, left_branch)
    right_sums = MaxSubarray_recursive(A, right_branch)

    sums.all = left_sums.all + right_sums.all
    sums.left = max(left_sums.left, left_sums.all + right_sums.left)
    sums.right = max(right_sums.right, left_sums.right + right_sums.all)
    m = left_sums.right + right_sums.left
    sums.overall = max(sums.all, sums.left, sums.right, m)

    return sums
```

Algorithm 3.2: Recursive function pseudocode for Divide and Conquer

This algorithm is recursive and decreases by half every step. Each lower step has double the number of calls. Thus, this algorithm is O($n \log n$).

2 Theoretical Correctness

The sum of all the elements in array denoted as Sum_All
The largest sum starting from the left denoted as Sum_Left
The largest sum starting from the right denoted as Sum_Right
The overall max sum denoted as Sum_Overall

Base case: Let $n = 1$. Then Sum_All = A[0], Sum_Left = A[0], Sum_Right = A[0], Sum_Overall = A[0]

Inductive hypothesis:
Left = MaxSubarray(A[0:$\frac{n}{2}$-1])
Right = MaxSubarray(A[$\frac{n}{2}$:n])
Sum_All = Left.Sum_All + Right.Sum_All
Sum_Left = Left.Sum_Left or Left.Sum_All + Right.Sum_Left
Sum_Right = Left.Sum_Right + Left.Sum_All or Right.Sum_Right
Sum_Overall = Sum_Left or Sum_Right or Left.Sum_Overall or Right.Sum_Overall or Left.Sum_Right + Right.Sum_Left

Proof:
Case 1: Contained entirely in the first half
This will be returned from the recursive call on the Left.

Case 2: Contained entirely in the second half
This will be returned from the recursive call on the Right.

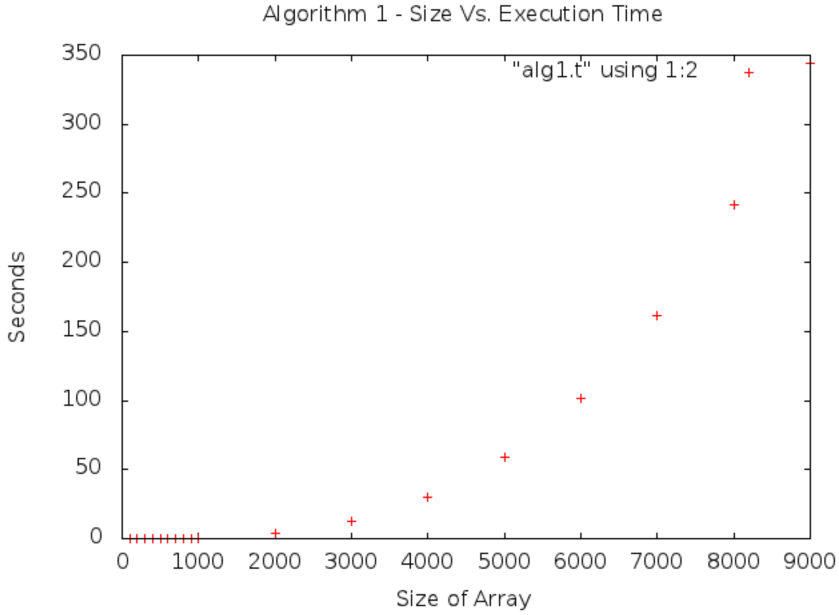
Case 3: Made of a suffix of the first half of maximun sum and the prefix of the second half of the maximum This will be found using Left.Sum_Right + Right.Sum_Left

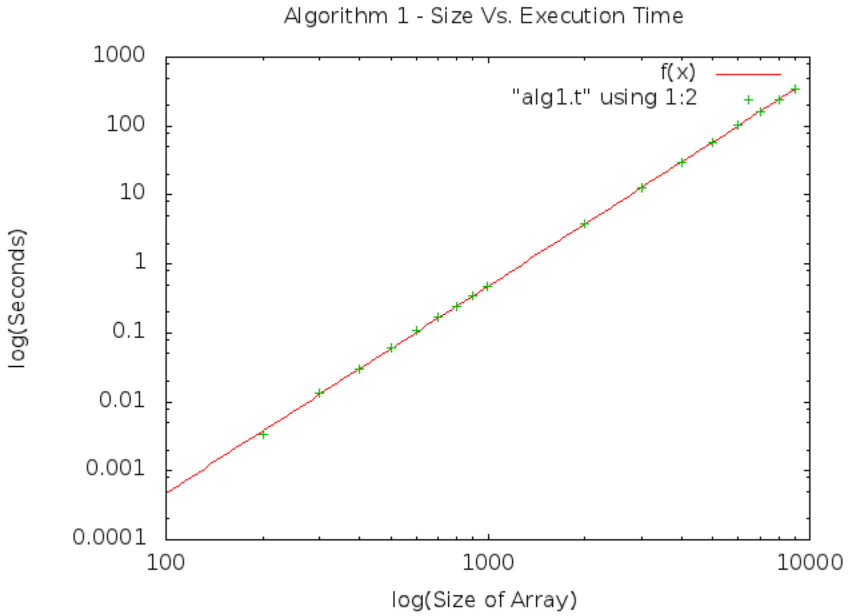
3 Testing

Student ID	Answer
931678074	5703
930569466	8184
932086449	4949

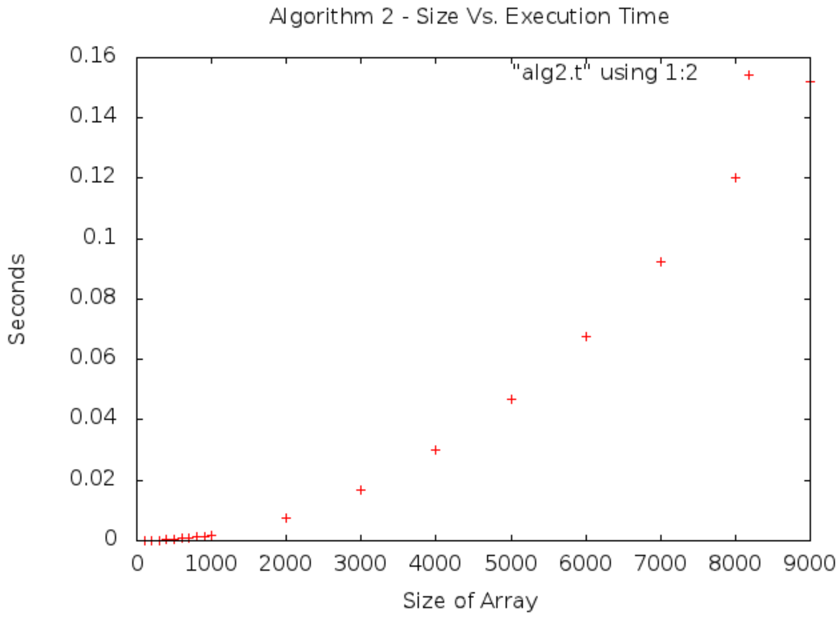
4 Experimental Analysis

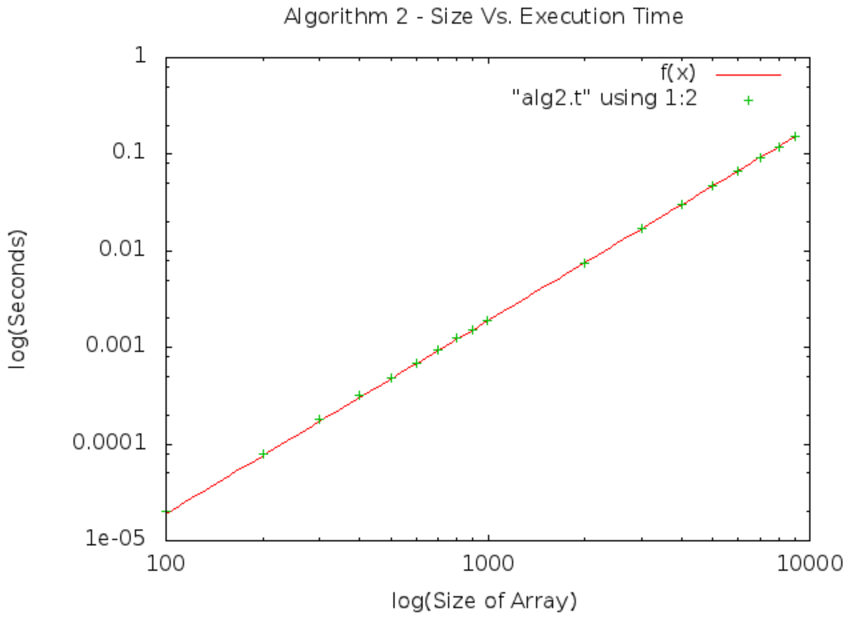
4.1 Algorithm 1



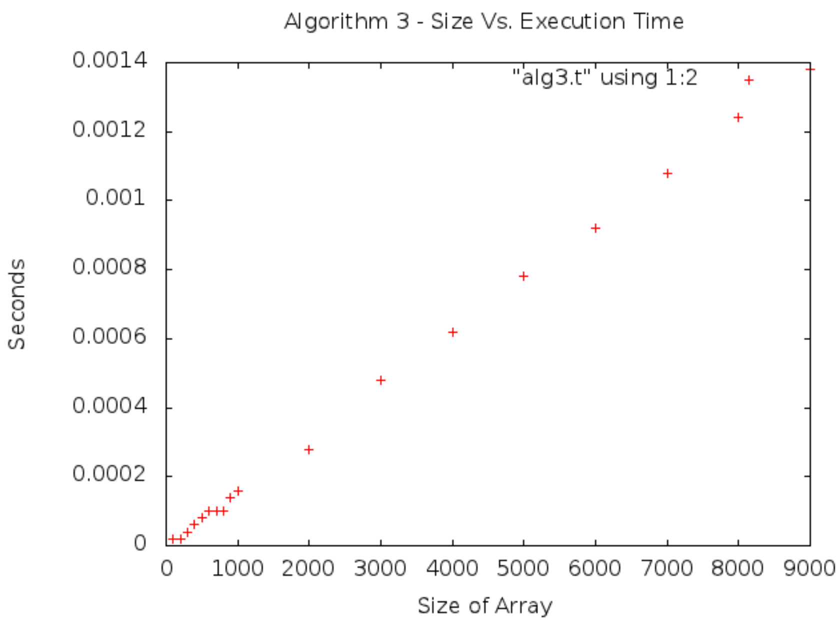


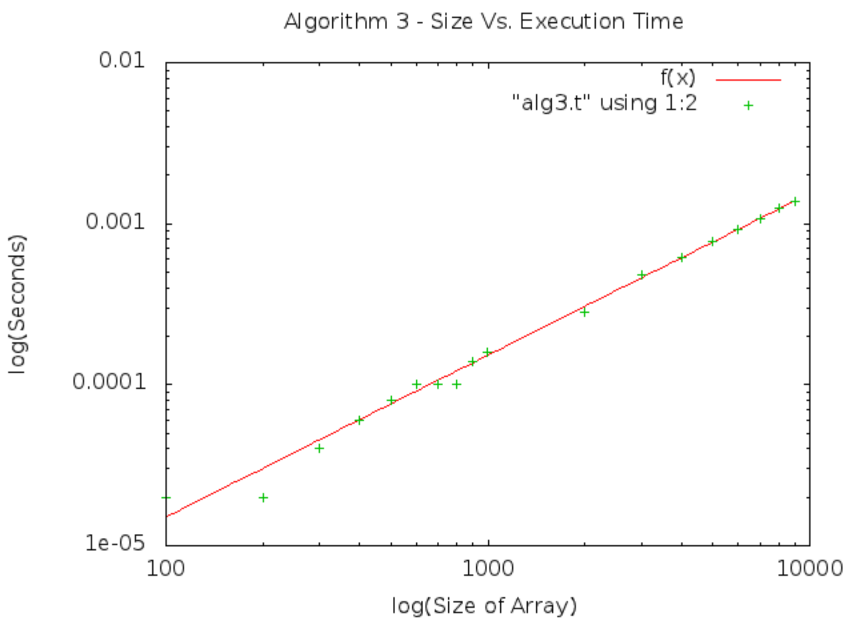
4.2 Algorithm 2





4.3 Algorithm 3





5 Extrapolation and Interpretation

5.1 Extrapolation

The functions were calculated using gnuplot's fit function.

5.2 Interpretation

The functions were calculated using gnuplot's fit function and fitting the data to $f(n) = 10^{m\log_{10} n + c}$ The slopes for each algorithm is a little lower than the actual power because of the creation overhead of the function has a larger affect on arrays with small sizes. This will cause the left side to be higher and therefore decreases slope.

5.3 Algorithm 1

5.3.1 Extrapolation

$f(n) = 4.71599 \times 10^{-10} \times n^3$
$f(n) = 3600 \rightarrow n = \boxed{19690}$

5.3.2 Interpretation

Slope = $\boxed{2.99734}$

5.4 Algorithm 2

5.4.1 Extrapolation

$f(n) = 1.87761 \times 10^{-9} \times n^2$
$f(n) = 3600 \rightarrow n = \boxed{1384678}$

5.4.2 Interpretation

Slope = $\boxed{1.99602}$

5.5 Algorithm 3

5.5.1 Extrapolation

$f(n) = 1.74832 \times 10^{-8} \times n \times log(n)$
$f(n) = 3600 \rightarrow n = 8984428998 = \boxed{8.98 \times 10^9}$

5.5.2 Interpretation

Slope = $\boxed{1.00506}$

6 Code

6.1 Files

alg1.cpp - Function for algorithm 1
alg2.cpp - Function for algorithm 1
alg3.cpp - Function for algorithm 1
analysis.cpp - Code to run algorithm and measure times for the number of array then outputs .t file
makefile - To compile files
maxSubarray.pdf - This writeup
maxSubarray.tex - TEXfile for PDF
test.cpp - Allows input of file and runs algorithm on input file
analysis/ - Contains compiled executables for running analysis
test/ - Contains compiled executables for running tests on code, and test array files
timingfiles/ - Contains files for creating plots
timingfiles/*.t - Log of runtimes for different array sizes
timingfiles/*.gp - Code for gnuplot. 2 plots of each algorithm: 1 normal plot, and 1 log-log plot

6.2 Algorithm 1

```
/*
 * Enumeration
 * Loop over each pair of indices i; j and compute the sum from k=i to j of a[k].
 * Keep the best sum you have found so far.
 */

using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    for (j = i; j < n; ++j){
      sum = 0;
      for (k = i; k <=j; ++k){
        sum += a[k];
      }
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg1.cpp

6.3 Algorithm 2

```
/*
 * Better Enumeration
 * Notice that in the previous algorithm, the same sum is computed many times.
 * In particular, notice that sum from k=i to j of a[k] can be computed from sum from k=
     i to j - 1 of a[k] in O(1) time, rather than starting from scratch.
 * Write a new version of the frst algorithm that takes advantage of this observation.
 */


using namespace std;


int MaxSubarray(int a[], int n){

  int i,j,k;
  int max = a[0];
  int sum;
  for(i = 0; i < n; ++i ){
    sum = 0;
    for (j = i; j < n; ++j){
      sum += a[j];
      if(max < sum){
        max = sum;
      }
    }
  }
  return max;
}
```

alg2.cpp

6.4 Algorithm 3

```cpp
/*
 * Divide and Conquer
 * If we split the array into two halves, we know that the maximum subarray will either be
 *      * contained entirely in the frst half,
 *      * contained entirely in the second half, or
 *      * made of a suffix of the frst half of maximum sum and a prefix of the second half of maximum sum
 * The frst two cases can be found recursively. The last case can be found in linear time.
 */

#define ALL     0
#define LEFT    1
#define RIGHT   2
#define OVERALL 3

using namespace std;

void MaxSubarray_h(int array[], int size, int sums[]){
  // Base case.
  if(size <= 1){
    sums[ALL]     = array[0];   // Sum of entire array
    sums[LEFT]    = array[0];   // Largest sum from left end of array
    sums[RIGHT]   = array[0];   // Largest sum from right end of array
    sums[OVERALL] = array[0];   // Largest sum found so far
    return;
  }
  int i = size/2;   // Index of middle element

  // Recurse.
  int *left  = new int[4];
  int *right = new int[4];
  MaxSubarray_h(array,i, left);
  MaxSubarray_h(array+i, size-i, right);

  // Calculate various possible maximum sums.
  int a = left[ALL]   + right[ALL];    // Sum of everything
  int l = left[ALL]   + right[LEFT];   // Possible max sum from the left
  int r = left[RIGHT] + right[ALL];    // Possible max sum from the right
  int m = left[RIGHT] + right[LEFT];   // Possible max sum straddling both branches

  // Check for and find new maximums.
  l = l > left[LEFT] ? l : left[LEFT];    // Is the new left sum larger?
  r = r > right[RIGHT] ? r : right[RIGHT];    // Is the new right sum larger?
  int overall = left[OVERALL] > right[OVERALL] ? left[OVERALL] : right[OVERALL];
  overall = overall > m ? overall : m;

  // Final answers!
  sums[0] = a;
  sums[1] = l;
  sums[2] = r;
  sums[3] = overall;
}


int MaxSubarray(int a[], int n){
  int *p = new int[4];
  MaxSubarray_h(a,n,p);
  int s1 = p[0] > p[1] ? p[0] : p[1];
  int s2 = p[2] > p[3] ? p[2] : p[3];
  s1 = s1 > s2 ? s1 : s2;
  return s1;
}
```

alg3.cpp