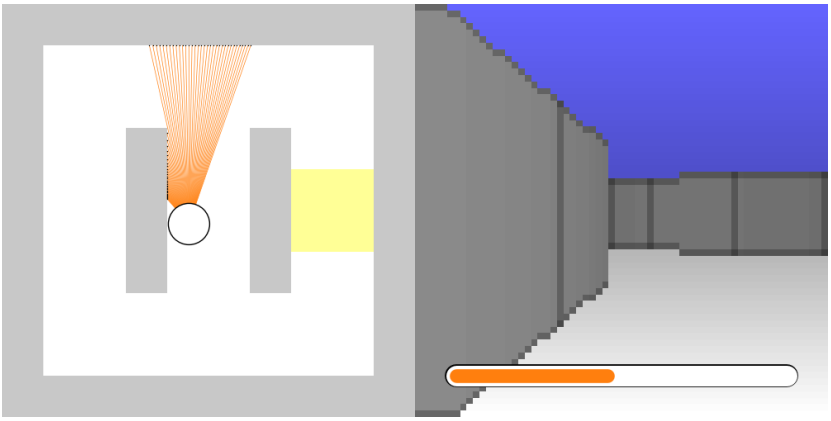
Contents

1 Programming with programmatoid and neuronoid

Let us consider the following digital experimental setup, as described in braincraft challenge presentation[1].

1.1 Simplified problem statement



Reference: The braincraft challenge[2].

Variables

Input θ_l Max average of the *left* sensors input, between 0 and 1 when very close, 30 sensor field.

θ_r Max average of the *right* sensors input, idem.

$I_{\textbf{left color sensor}}$, for each color max-average value of the *left* color sensors, see usage in the sequel.

$I_{\textbf{right color sensor}}$, idem.

ε Energy indicator, between 1 when full and 0 when dead.

Output $d\theta$ Orientation relative increment, between -1 for -5 leftward and 1 for 5 rightward.

Internal λ Orientation preference 0 if left, 1 if right, $\lambda = 0$ at start.

With respect to the original we consider only two leftward and rightward sensor, and re-normalize the input/output values.

[1] `https://github.com/rougier/braincraft/blob/master/README.md#introduction`
[2] `https://github.com/rougier/braincraft`

Computation unit: neuronoid

By "neuronoid" we name the not very biologically plausible[3] simplest biological neuron or neuron small ensemble inspired by mean-field modelisation[4] of the Hodgkin–Huxley neuronal axon model[5].

$$\tau \frac{\partial v_i}{\partial t}(t) + v_i(t) = z_i(t), \quad z_i(t) \stackrel{\text{def}}{=} h\left(\sum_j w_{ij}\, v_j(t) + w_i\right),$$
$$h(v) \stackrel{\text{def}}{=} \frac{1}{1+\exp(-4\,v)}, \qquad H(v) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if} \quad v > 0 \\ 0 & \text{if} \quad v < 0 \end{cases}$$

where v_i is the membrane potential, so that:

$$\begin{array}{rcl} v(t) & = & 1/\tau \int_0^t z(s)\, exp(-(t-s)/\tau)\, ds + v(0)\, exp(-t/\tau) \\ & = & z(0) + (v(0) - z(0))\, e^{-t/\tau}\big|_{z(t)=z(0)}\,. \\ & = & z(t)|_{\tau=0} \\ & \simeq & (1-\gamma)\, v(t-\Delta t) + \gamma\, z(t-\Delta t)) \\ & = & \sum_{s=0}^{t-1} z(s)\, \gamma\, (1-\gamma)^{t-s+1}\, z(s) + v(0)\, (1-\gamma)^t \\ & = & z(0) + (v(0) - z(0))\, (1-\gamma)^t|_{z(t)=z(0)} \\ & = & z(t)|_{\gamma=1}\,. \end{array}$$

writing also the corresponding discrete approximation using an Euler schema with $0 < \gamma < 1, 0 < \tau$:

$$\gamma \stackrel{\text{def}}{=} 1 - \exp(-1/\tau) \Leftrightarrow \tau = 1/\log(1/(1-\gamma))$$
$$\lim_{\gamma \to 0} \tau = +\infty, \lim_{\gamma \to 1} \tau = 0.$$

Here, $h(\cdot)$ is the normalized sigmoid with $h(-\infty) = 0, h(0) = 1/2, h'(0) = 1, h(+\infty) = 1$, linearly related to the hyperbolic function:

$$h(v) = \frac{1+\tanh(2\,v)}{2}.$$

It is the mollification of the Heaviside function $H(\cdot)$, as detailed below.

It is thus a very common 1st order "neuronoid" model, but with an adjustable bias (or offset) w_i.

1.2 Programmatoid and neuronoid computation

Programmatoid computation

We name "programmatoid" computation the conception of an input-output straight-line program[6] implementing test operator on numerical value expressions using the Heaviside function, considering 1 as the true value and 0 as the false value. The $H(v)$ implements the test of the positive sign of v, while for $v_i \in \{0, 1\}$:

[3] https://en.wikipedia.org/wiki/Biological_neuron_model#Relation_between_artificial_and_biological_neuron_models

[4] https://inria.hal.science/cel-01095603v1

[5] https://en.wikipedia.org/wiki/Hodgkin-Huxley_model

[6] https://en.wikipedia.org/wiki/Straight-line_program

programmatoid conjunction The $v_0 = H(v_1 + v_2 + \cdots)$ formula performs a *or* operation.

programmatoid disjunction The $v_0 = v_1\, v_2 \cdots$ formula performs a *and* operation.

programmatoid negation The $v_0 = (1 - v_1)$ formula performs a negation.

so that we can combine any boolean expression on any test of value sign, thus value comparison, and value interval inclusion, switches between two expressions, etc. This defines real semi-algebraic[7] sets of degree 1.

Using local feedback we can also design several functions detailed in the Appendix of this section, while we also can combine neuronoid and programmatoid functions to design temporal functions.

Mollification of the Heaviside function

The Heaviside function is related to a conditional expression by a simple relation:

$$H(v) = \mathit{conditional\ value} > 0\ ?\ 1 : \mathit{conditional\ value} < 0\ ?\ 0 :\ H(0),$$

where $\mathit{condition}\ ?\ \mathit{value\ if\ true} : \mathit{value\ if\ false}$ is a conditional expression.

The Heaviside function approximates sigmoid with huge slope at zero, i.e.:

$$\forall v \neq 0, H(v) = \lim_{W_\infty \to +\infty} h(W_\infty\, v),\ h'(W_\infty\, v)|_{v=0} = W_\infty$$

while the convergence is also obtained for $v = 0$ in the distribution sense with $H(0) = h(0) = 1/2$. More precisely:

$$|H(\cdot) - h(\cdot)|_{\mathcal{L}_1} = \frac{\log(2)}{2}\, \frac{1}{W_\infty}$$

and, for $0 < \epsilon_\infty \ll 1 < v_\infty$:

$$h(W_\infty\, v_\infty) = 1 - \epsilon_\infty \Leftrightarrow v_\infty = \frac{\log(1-\epsilon_\infty) - \log(\epsilon_\infty)}{4\, W_\infty} = \frac{-\log(\epsilon_\infty)}{4\, W_\infty} + O\left(\epsilon_\infty\right).$$

Numerically, the convergence is very fast, e.g. $W_\infty = 2$, for $\epsilon_\infty = 0.1\%$:

ϵ_∞	10^{-1}	10^{-2}	10^{-3}	10^{-6}
W_∞	0.55	1.15	1.73	3.46

A step further, the local variation of sigmoid writes:

$$\begin{aligned} h(v) &= h(v_0) + \left[4\, \exp(-8\, v_0) + O\left(\exp(-12\, v_0)\right)\right]\, (v - v_0) + O\left((v - v_0)^2\right) \\ &\simeq h(v_0) + 4\, \exp(-8\, v_0)\, (v - v_0). \end{aligned}$$

Numerically, values decrease very rapidly:

v_0	1	2	5	10
$4\, \exp(-8\, v_0) \simeq$	10^{-1}	10^{-3}	10^{-8}	10^{-17}

Neuronoid implementation of programmatoid computation

We call "neuronoid" computation the conception of an input-output transform based on feed-forward and recurrent combination of neuronoids, as defined previously.

[7] `https://en.wikipedia.org/wiki/Real_algebraic_geometry`

By successive combination, any programmatoid computation involving the $H(\cdot)$ function can be approximated by a neuronoid, with $\tau \simeq 0$.

A step ahead, we considering neuronoid with $\tau > 0$ it seems obvious that we can designed temporizing mechanisms, oscillators and sequence generator, sleep sort mechanism, etc. (not detailed here because not used at this stage, see appendix).

A putative controller

Navigation

Heuristic : The bot runs at constant velocity, (i) ahead by default and (ii) if passing in the preferred orientation is possible, makes a quarter turn.

Programmatoid solution :

$$\begin{aligned} d\theta &= \underbrace{\gamma\,(\theta_l - \theta_r)}_{\substack{\text{linear correction to}\\ \text{maintain direction ahead}}} + \underbrace{\frac{\pi}{2}\,(\Delta\theta_r - \Delta\theta_l)}_{\text{direction change}} \\ \Delta\theta_r &= \lambda\, H(\beta - \theta_r) \\ \Delta\theta_l &= (1-\lambda)\, H(\beta - \theta_l) \end{aligned}$$

where:

$\Delta\theta_*$ raises from 0 to 1 when the left sensor detects a passing, i.e., the fact tat the side wall is not close anymore.

γ is a feedback loop gain $0 < \gamma < 1$ to be adjusted high enough to correct the direction, small enough to avoid oscillations.

β is a threshold below which the side sensor input corresponds to no side wall but a passing.

in words: the bot navigates ahead thanks to the linear correction parameterized by γ and perform a quarter turn in the preferred direction as soon a passing is detected.

Neuronoid implementation : The mollification of this system uses 3 neuronoids and writes:

$$\begin{aligned} d\theta &= h\left(\gamma\,(\theta_l - \theta_r) + \tfrac{\pi}{2}\,(\Delta\theta_r - \Delta\theta_l)\right) \\ \Delta\theta_r &= h(W_\infty\,(\beta - \theta_r) + 2\,W_\infty\,(\lambda - 1)) \\ \Delta\theta_l &= h(W_\infty\,(\beta - \theta_l) - 2\,W_\infty\,\lambda) \end{aligned}$$

as easily verified considering the four cases $\beta \lessgtr \theta_*$ versus $\lambda \in \{0, 1\}$.

With respect to the programmatoid solution, the output value is "saturated" by the $h(\cdot)$ function while the $\Delta\theta_*$ almost binary values are approximated by the mollification of the Heaviside function. This part of the system is feed-forward thus without convergence or stability issue.

Direction choice

Heuristic : The initial direction is right, but as soon as an input contradicts this assumption, it is turned left once, and this remains.

Case 1 If the energy is to low, it means we turn in the wrong direction, so it changes.

Case 2 If there is a blue color on the left it means we must change from right to left.

Case 3 If there is a red (or yellow, etc) color somewhere, then change the turn direction if i see it again on the left.

Programmatoid solution :

$$\begin{aligned}
\lambda &= \lambda + \Delta\lambda_1 + \Delta\lambda_2 + \Delta\lambda_3 + \cdots \\
\Delta\lambda_1 &= (\lambda - 1)\, H(\alpha - \varepsilon) \\
\Delta\lambda_2 &= (\lambda - 1)\, H(\iota - I_{\text{left blue color sensor}}) \\
\Delta\lambda_3 &= (\lambda - 1)\, (\Upsilon_{\text{again red color}} + \Upsilon_{\text{again yellow color}} + \cdots) \\
&\cdots \\
\Upsilon_{\text{again this color}} &= \Upsilon_{\text{seen this color}}\, H(\iota - I_{\text{left this color sensor}}) \\
\Upsilon_{\text{seen this color}} &= \Upsilon_{\text{seen this color}} + (1 - \Upsilon_{\text{seen this color}})\, H(\iota - I_{\text{this color sensor}}) \\
&\cdots
\end{aligned}$$

where:

α is a the energy threshold, corresponding to the energy consumption during one turn.

ι is some color detection threshold.

$\Delta\lambda_1$ raises to one if the energy decreases below a threshold.

$\Delta\lambda_2$ raises to one if the blue color is seen on the left.

$\Delta\lambda_3$ raises to one if some color is seen again.

$\Upsilon_{\text{again this color}}$ raises to one a previously seen color is seen again on the left.

$\Upsilon_{\text{seen this color}}$ raises to one a color is seen for the first time.

$I_{\text{left this color sensor}}$ combines color sensor input.

$I_{\text{this color sensor}}$ combines left and right color sensor input.

Neuronoid implementation : The mollification uses 3 neuronoids for cases 1 and 2 plus 3 neuronoids by color for case 3. The derivation of the neuronoid equations is straightforward after the previous one.

Appendix: a few programmatoid and neuronoid components

Conditional expression

For two inputs $v_{i1}(t) \in \{0,1\}, v_{i0}(t) \in \{0,1\}$, an output $v_o(t) \in \{0,1\}$ and a control $v_l \in \{0,1\}$, the condition expression equation, for $0 < W_\delta \ll 1 < W_\sigma$:

$$
\begin{aligned}
v_o(t+1) &= v_l(t) == 1 \ ? \ v_{i1}(t) : v_{i0}(t) && (1)\\
&= v_l(t)\, v_{i1}(t) + (1 - v_l(t))\, v_{i0}(t) && (2)\\
&= H(v_{i1}(t) - W_\sigma\,(1 - v_l(t)) - W_\delta) + H(v_{i0}(t) - W_\sigma\, v_l(t) - W_\delta) && (3)\\
&\simeq h(W'_\infty\,(W_\omega\, v_{i1}(t) - W_\sigma\,(1 - v_l(t)) - W_\delta))\\
&+ h(W'_\infty\,(W_\omega\, v_{i0}(t) - W_\sigma\, v_l(t) - W_\delta)). && (4)
\end{aligned}
$$

To explain these design choices, let us notice that:

- The W_σ value is used at the programmatoid level to ensure that given the $v_l(t)$ binary switch value, it constraints the Heaviside function output to correspond to the desired value. Here, an expression including a product by a binary function, is replaces by a sum. The rationale is that there is no explicit multiplication between two variables at the neuronoid level. This trick allows one a straightforward neuronoid approximation.
- The W_δ value is used at the programmatoid level to avoid the ambiguous 0 value and ensure that for $v \simeq 0$ we obtain $H(v - W_\delta) = 0$.
- The W'_∞ gain is used to approximate the Heaviside function by a sigmoid, as discussed previously.
- Informally, at the neuronoid level, we mimic the programmatoid mechanisms, assuming that suitable $W'_\infty, W_\omega, W_\sigma, W_\delta$ values will reproduced the programmatoid conditional expression. It works, although the parameter adjustment is not obvious, as derived now.

Let us now verify the equivalence between these equations:

- It is obvious to verify that line (1) and (2) are equivalent.
- The fact line (2) and (3) are equivalent is verified by this truth table:

$v_l(t)$	$v_{i1}(t)$	$v_{i0}(t)$	$v_{i1}(t) - W_\sigma\,(1 - v_l(t)) - W_\delta$	$v_{i0}(t) - W_\sigma\, v_l(t) - W_\delta$	$v_o(t+1)$
1	0	0	$-W_\delta < 0$	$-W_\sigma - W_\delta < 0$	$0 + 0 = 0 = v_{i1}(t)$
1	0	1	$-W_\delta < 0$	$1 - W_\sigma - W_\delta < 0$	$0 + 0 = 0 = v_{i1}(t)$
1	1	0	$1 - W_\delta > 0$	$-W_\sigma - W_\delta < 0$	$1 + 0 = 1 = v_{i1}(t)$
1	1	1	$1 - W_\delta > 0$	$1 - W_\sigma - W_\delta < 0$	$1 + 0 = 1 = v_{i1}(t)$
0	0	0	$-W_\sigma - W_\delta < 0$	$-W_\delta < 0$	$0 + 0 = 0 = v_{i0}(t)$
0	0	1	$-W_\sigma - W_\delta < 0$	$1 - W_\delta > 0$	$0 + 1 = 1 = v_{i0}(t)$
0	1	0	$1 - W_\sigma - W_\delta < 0$	$-W_\delta > 0$	$0 + 0 = 0 = v_{i0}(t)$
0	1	1	$1 - W_\sigma - W_\delta < 0$	$1 - W_\delta > 0$	$0 + 1 = 1 = v_{i0}(t)$

- The approximation of line (3) at line (4) by continuous quantities corresponds now to:

$$v_*(t) \in \{[0, \epsilon_\infty], [1 - \epsilon_\infty, 1]\} = \{\simeq 0, \simeq 1\}, \epsilon_\infty \ll 1/2,$$

in words: values to be either below or above a threshold close to either

0 or 1. The truth table now involves intervals and has been generated using the computer algebra piece of code associated to this document[8]. Considering the following intuitive design constraints:

$$0 < W_\delta < \{W_\omega, W_\sigma\}, 0 < W_\omega < W_\sigma, 0 < \epsilon_\infty < 1$$

the computer algebra derivations show that the approximation at line (4) is valid as soon as:

$$W_\omega\,\epsilon_\infty < W_\delta, W_\sigma\,\epsilon_\infty < W_\delta, W_\delta + (W_\omega + W_\sigma)\,\epsilon_\infty < W_\omega.$$

Furthermore, let us consider the margin:

$$0 < \mu = \min(W_\delta - \epsilon_\infty\,W_\sigma, W_\omega - (W_\omega + W_\sigma)\,\epsilon_\infty - W_\delta),$$

yielding:

$$|v_o(t+1) - 1/2| > h(W'_\infty\,\mu).$$

We thus can adjust $W'_\infty = W_\infty/\mu$ in order $v_o(t+1) \in \{[0, \epsilon_\infty], [1-\epsilon_\infty, 1]\}$ for further calculation.

For instance $\{W_\delta = 1, W_\omega = 2, W_\sigma = 4, \epsilon_\infty = 1/8\}$ is a suitable solution with $\mu = 1/4$.

RS input/output gate

For an input $v_i(t) \in \{0,1\}$, an output $v_o(t) \in \{0,1\}$, and a control $v_l \in \{0,1\}$, the equation:

$$\begin{array}{rcl} v_o(t+1) & = & v_l(t) == 1\ ?\ v_o(t) : v_i(t), \\ & = & v_l(t)\,v_o(t) + (1 - v_l(t))\,v_i(t) \\ & = & H(v_o(t) - W_\sigma\,(1 - v_l(t)) - W_\delta) + H(v_i(t) - W_\sigma\,v_l(t) - W_\delta) \\ & \simeq & h(W'_\infty\,(W_\omega\,v_o(t) - W_\sigma\,(1 - v_l(t)) - W_\delta)) \\ & + & h(W'_\infty\,(W_\omega\,v_i(t) - W_\sigma\,v_l(t) - W_\delta)). \end{array}$$

implements a 1 bit memory, i.e., also called a RS gate, and reusing the previous conditional instruction parameters.

The key point is that it is now a recurrent system, which stability is obvious at the programmatoid level, but not necessarily at the neuronoid level, since we have a continous equation. More precisely:

$$v_o(t+1) = h(W'_\infty\,W_\omega\,v_o(t) - W_\beta(t)) + h_\beta(t),$$

$$\left\{ \begin{array}{rcl} W_\beta & \stackrel{\text{def}}{=} & W_\sigma\,(1 - v_l(t)) + W_\delta \\ h_\beta(t) & \stackrel{\text{def}}{=} & h(W'_\infty\,(W_\omega\,v_i(t) - W_\sigma\,v_l(t) - W_\delta)) \in [-1,1] \end{array} \right.$$

with a non contracting recurrent function, since:

$$h'(W'_\infty\,W_\omega\,v) > 1 \text{ for } |v| \leq W'_\infty\,W_\omega.$$

monostable binary value The $v_0 = v_0 + (1 - v_0)\,H(v_1)$ formula sets v_0 to 1 for ever, as soon v_1 has raised once to 1.

[8] `https://github.com/vthierry/braincraft/raw/master/data/programmatic-solution.mpl.txt.out`

and by combination RS gates, bistable mechanisms, etc. (not detailed here because not used at this stage, see appendix).

To be done.

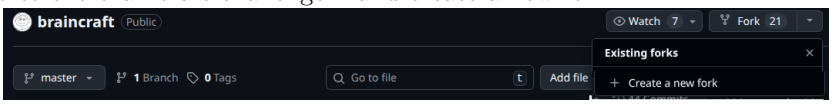
2 Using the braincraft challenge setup

Reference: The braincraft challenge[9].

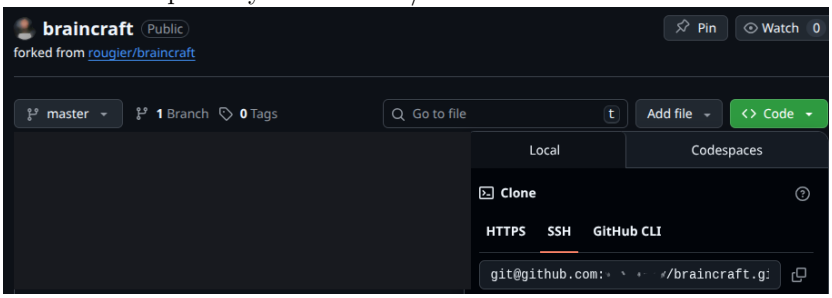
You must be familiar with basic `git` usage and basic `python` programmation.

Installation of the setup

- Connect to `https://github.com` with your login.
- Go to the braincraft challenge[10] and create a new fork:



- Download the repository in SSH read/write mode:



-In the braincraft local git directory, run `make test`
+ You may have to run `make install`, before.
+ You are advised to use a virtual environment, running `make venv`

Running at the programmatic level

The 'challenge_callback_1.py' file contains the support routines

Running at the artificial neural network level

```
* Note : vthierry veut PAS gagner la compétition is veut just vérifier des hypothèse quant l

- Warningup : duration (bot don't move before warmup period is over) just 1 to allow a 1st i

- Quelles dimensions pour Win (quelles entrées où ? le truc de dimension P), W, et Wout(lign
```

[9] `https://github.com/rougier/braincraft`
[10] `https://github.com/rougier/braincraft`

- Avis sur calcul de depth et couleur ?
 + depth: prendre le min des capteurs de gauche/droite ou vaut mieux leur moyenne pondérée ?
 + couleur: les murs ont le vert comme couleur par defaut ? prendre la valeur de couleur la

- Pour expérimenter l'approche programmatique avant de passer à l'approche connexiviste :
 - comment ''débrancher´´ le réseau et just avoir (P inputs) -> output ? sans reimplementer

- Pour expérimenter l'approche connectiviste sans apprentissage ... juste implémenter def tr