Extending the Streaming Instability to Vertically Stratified Protoplanetary Disks

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1. Aims

The streaming instability has been proposed as a method to form planetesimals. It has been widely explored through simulations in the literature for non-vertically stratified protoplanetary disks. We attempt to extend these simulations to protoplanetary disks with vertical structure, thereby incorporating the effect of vertical gravity within a two fluid model. We discovered that the streaming instability is destroyed in these conditions, as the disk becomes turbulent, preventing the formation of the instability. Instead, we propose physical reasoning for the Kelvin-Helmholtz instability to be the dominant source of observed turbulence.

2. Introduction

Background

- Protoplanetary disks are clouds of gas and dust that form around young (Class-II) stars. These disks provide the necessary matter for planetary formation.
- Neither dust coagulation nor gravitational collapse can explain how 1μm sized dust particles can clump into planetesimals, eventually forming 100km sized planets.
- Fluid instabilities have been proposed to solve this discrepancy. We propose the Kelvin Helmholtz instability as an additional mechanism [1].

Fluid instabilities

- *The streaming instability* – Occurs when gas exerts a drag on dust, causing rapid clumping. It was first proposed by Youdin and Goodman in 2005 [1] and is characterized by streaks of high density that accrete in a wave-like motion. An example of it can be seen in *Figure 1*.
- *The Kelvin-Helmholtz instability (KHI)* – Generates turbulence when two different fluids meet at an interface and move at different velocities [2].

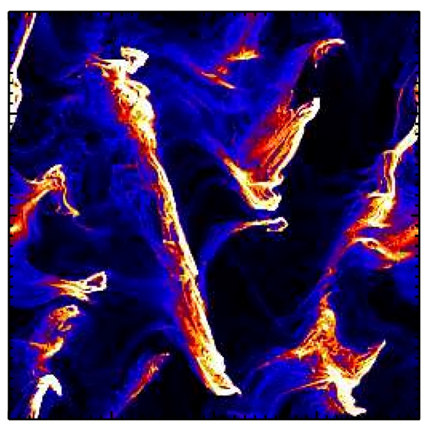

Figure 1: Expected density profile of the streaming instability [3]

3. Aims

1. To reproduce Youdin and Goodman's streaming instability of 2005 with Dedalus, using a non-vertically stratified two-fluid model to simulate its fluid mechanics.
2. Apply vertical structure to the disk in simulations to investigate whether the streaming instability can still be identified in vertically stratified disks.
3. Identify processes that may dominate in these vertically stratified disk simulations.

4. Theory & Method

- The protoplanetary disk is modeled in a two-fluid approximation where dust and gas fluids interact via a drag force.
- The time-evolution of the disk is simulated by solving the following fluid-dynamics equations (1 and 2) for a small region, central to the disk [4]. The Python-based Dedalus library does this using spectral methods. Different parameters used in these equations are defined in *Table 1*.

	Material derivative		Fluid drag	Coriolis force	Radial gravity	Vertical gravity	Pressure
(1) Dust	$\frac{\partial \vec{V}_D}{\partial t} + (\vec{V}_D \cdot \vec{\nabla})\vec{V}_D$	=	$-\frac{\vec{V}_D - \vec{V}_G}{t_{stop}}$	$+ 2\vec{V}_D \times \Omega$	$+ 2q\Omega^2 x\,\hat{\boldsymbol{x}}$	$- \Omega^2 z\hat{\boldsymbol{z}}$	
(2) Gas	$\frac{\partial \vec{V}_G}{\partial t} + (\vec{V}_G \cdot \vec{\nabla})\vec{V}_G$	=	$\epsilon\frac{\vec{V}_D - \vec{V}_G}{t_{stop}}$	$+ 2\vec{V}_G \times \Omega$	$+ 2q\Omega^2 x\,\hat{\boldsymbol{x}}$	$- \Omega^2 z\hat{\boldsymbol{z}}$	$-\frac{\nabla P}{\rho_G}$

- These equations are solved in a shearing box placed within the protoplanetary disk [4]. A 3D example of this can be seen in *Figure 2.*
- The box is defined as periodic in $\hat{\boldsymbol{x}}$ and restricted in $\hat{\boldsymbol{z}}$ with $\frac{dv_x}{dz} = \frac{dv_y}{dz} = v_z = 0$.
- The gas density profile of the disk is described in equation 3 where H is the disk scale height (half width) and ρ_0 is the maximum density of gas.

$$\rho(z) = \rho_0\, e^{-\frac{1}{2}\left(\frac{z}{H}\right)^2} \quad (3)$$

- For the non-stratified disk, we define $H \gg z$ to model a uniform density profile of gas.
- For the stratified disk we define H and z to be on the same order of magnitude. This adds vertical structure to the gas, thereby introducing density gradients.

Terms	Description
ρ_G, ρ_D	Gas and dust density
ϵ	Density ratio of dust to gas
$\vec{V}_g, \vec{V}_D$	Gas and dust velocities
q	Shear parameter
Ω	Angular velocity
t_{stop}	Particle stopping time

Table 1: Equation parameters

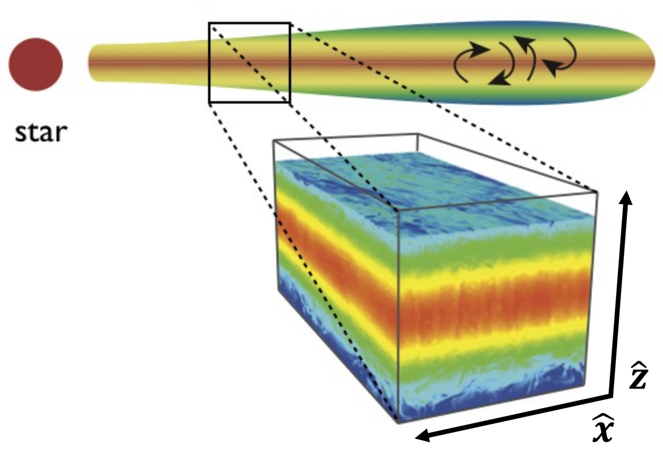

Figure 2: A 3D shearing box on a protoplanetary disk [5]

5. Results & Discussion

Non-Vertically Stratified Disks

The streaming instability was successfully reproduced in the non-stratified disk.

- This was confirmed by identifying strong similarities between our resulting radial velocity plot in *Figure 3 (b),* and Youdin and Goodman's results in *Figure 4*.
- High-density clumps were observed to travel in a wave-like motion, in *Figure 3 (a).* This is an expected property of the streaming instability, as seen in *Figure 1* [3, 6].

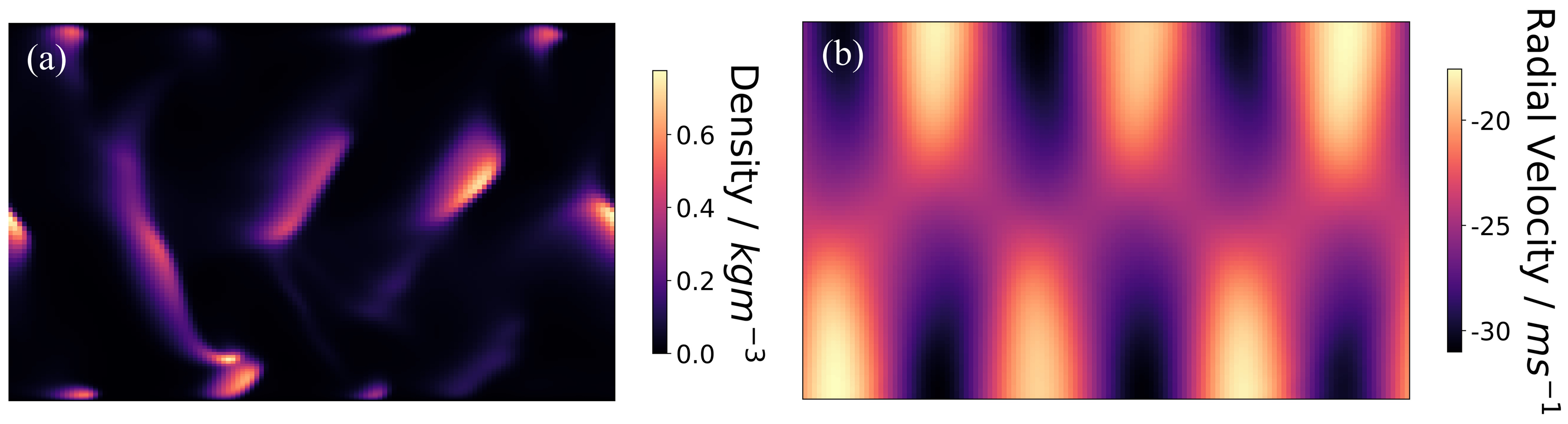

Figure 3: (a) Heatmap of the density profile of the disk in the absence of vertical stratification.
(b) Heatmap of the magnitude of the radial velocities of the dust in the same conditions and region.

Vertically Stratified Disks

The streaming instability was not recovered in any simulations of the vertically stratified disk.

- Significant turbulence, caused by larger density gradients, overpowered the subtle drag interaction required to generate the streaming instability.
- Comparing *Figure 5 (a)* to *Figure 1,* and *Figure 5* (b) to *Figure 4* shows no similarities, suggesting the streaming instability is no longer observed.

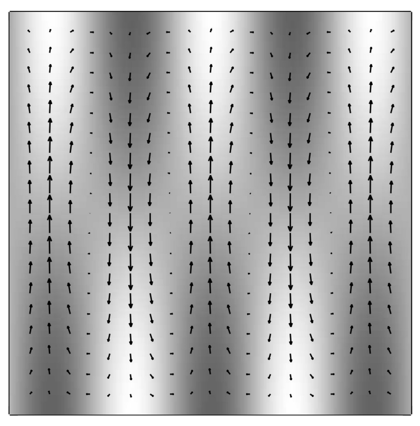

Figure 4: Expected dust velocity profile of the streaming instability [1]

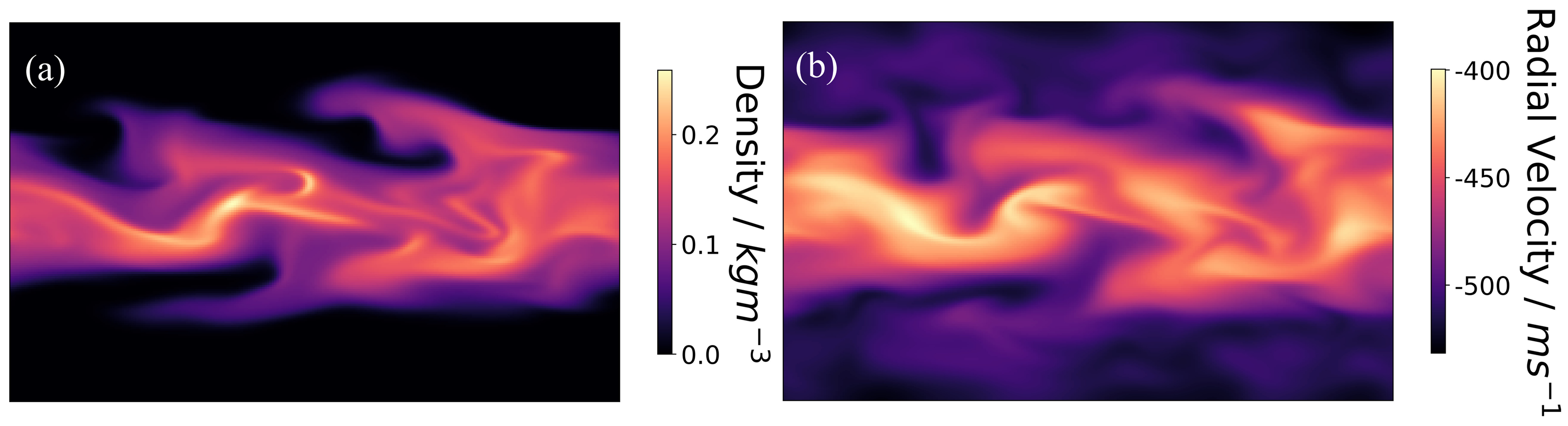

Figure 5: (a) Heatmap of the density profile of the disk in presence of vertical stratification.
(b) Heatmap of the magnitude of the velocities of the dust in the same conditions and region.

What causes the turbulence?

- We believe that we may be observing the effects of the Kelvin-Helmholtz instability (KHI) in the vertically stratified disk due to the interaction of the two fluids in our model.
- The design of our two-fluid model also meets the following requirements of the KHI:
  - The sharper density gradients of dust may generate the interface required for the KHI [2]. This would yield two interfaces in the disk, forming layers of gas above and below the central dust layer.
  - The accreting dust, from higher density zones, may generate the velocity shear required by the KHI [2].

Scan this QR code to view a simulation of each model

6. Conclusions & Further Work

- We successfully reproduced the streaming instability using Dedalus for the non-vertically stratified protoplanetary disk. This was confirmed by comparing our results to literature plots.
- In extending the model to stratified disks, the streaming instability was disrupted by fluid turbulence in the disk and therefore was never recovered.
- We propose that the Kelvin Helmholtz instability may instead occur under these conditions and be responsible for the observed turbulence in the disk.
- Further investigation would be required to prove this, but is outside the scope of the project. Higher computing power would be required to extend the simulations to three dimensions in order to investigate this behaviour more thoroughly.

7. References

[1] A. Youdin & J. Goodman (2005). 'Streaming Instabilities in Protoplanetary Disks'. *The Astrophysical Journal*. **620**, 459.
[2] C. Matsuoka. (2014). 'Kelvin-Helmholtz Instability and Roll-Up'. *Scholarpedia*. **9**, 11821.
[3] A. Johannsen & A. Youdin (2007) 'Protoplanetary Disk Turbulence Driven by the Streaming Instability: Linear Evolution and Numerical Methods'. *The Astrophysical Journal*. **662**, 613.
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