

Score: _____ / 35

PSTAT 5A / **MIDTERM EXAM 2** / Sum. Sess. A 2023

Instructor: **Ethan Marzban**

Name: _____
First, then Last

UCSB NetID: _____
NOT your Perm Number!

Circle Your Section: Olivier 12:30 - 1:20pm Mengrui 2 - 2:50pm Mengrui 3 - 3:50pm

FREE RESPONSE QUESTIONS

Instructions:

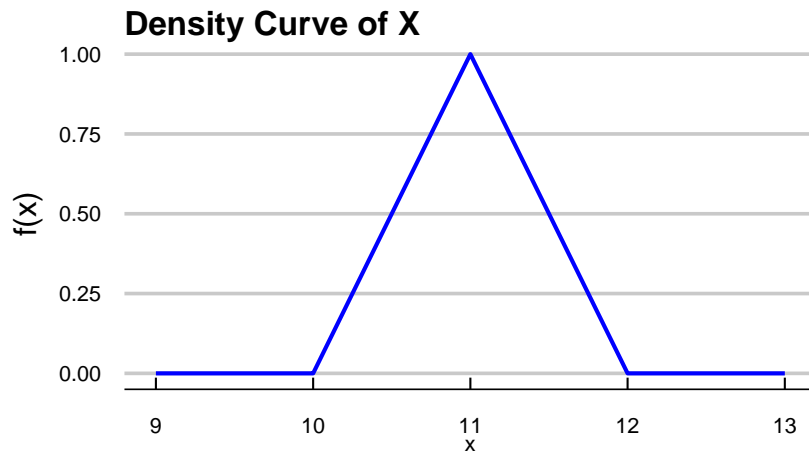
- You will have **75 minutes** to complete the entire exam
 - Do not begin working on the exam until instructed to do so.
 - During the final 10 minutes of the exam, we will ask everyone to remain seated until the exam concludes.
- This exam comes in **TWO PARTS**: this is the **FREE RESPONSE** part of the exam.
 - There is a separate booklet containing Multiple Choice questions that should have been distributed to you at the same time as this booklet.
- Write your answers directly in the space provided on this exam booklet.
 - You do not need to write anything on your scantron for this part of the exam.
- Be sure to show all of your work; correct answers with no supporting work will not receive full credit.
- The use of calculators is permitted; the use of any other aids (including notes, laptops, phones, etc.) is strictly prohibited. A list of formulae, as well as a collection of tables, is included with this exam.
- **PLEASE DO NOT DETACH ANY PAGES FROM THIS EXAM.**
- Good Luck!!!

Honor Code: In signing my name below, I certify that all work appearing on this exam is entirely my own and not copied from any external source. I further certify that I have not received any unauthorized aid while taking this exam.

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Free Response Questions

Problem 1. The length of a *GachoSteel*-brand rod is meant to be 11 feet; due to imperfections in the manufacturing process, however, the length of a randomly-selected *GachoSteel*-brand rod is actually a random variable X that has the following density curve:



(a) What is the probability that a randomly-selected *GachoSteel*-brand rod is exactly 11 meters in length? [2pts.]

(b) What is the probability that a randomly-selected *GachoSteel*-brand rod is longer than 11.5 meters? [2pts.]

- (c) A sample of 10 *GaachoSteel*-brand rods is taken with replacement, and the number of rods longer than 11.5 meters is recorded. What is the probability that this sample contains exactly 4 rods that are longer than 11.5 meters? Be sure to define any new random variables clearly and explicitly, and make sure to check any/all relevant conditions! You do **not** need to report your final answer as a decimal. [4pts.]

Problem 2. Alayah is interested in performing inference on the true average monthly rent (in thousands of dollars) of a 1-bedroom apartment in Santa Barbara. To that effect, she takes a representative sample of 100 1-bedroom apartments in Santa Barbara, and finds that these 100 apartments have a combined average monthly rent of 2.2 thousand dollars per month. From prior studies, she knows that the standard deviation of all monthly rents of 1-bedroom apartments in Santa Barbara is 0.75 thousand dollars.

- (a) Define the parameter of interest. [1pts.]

(b) Define the random variable of interest. [1pts.]

(c) What distribution should Alayah use when making inferences about the true average monthly rent of a 1-bedroom apartment in Santa Barbara? Be sure to check any/all relevant conditions. [3pts.]

(d) Construct a 97% confidence interval for the true average monthly rent of a 1-bedroom apartment in Santa Barbara. Be sure to interpret your interval in the context of the problem! [3pts.]

- (e) Would a 95% confidence interval for the true average monthly rent of a 1-bedroom apartment in Santa Barbara be wider or narrower than the interval you constructed in part (d) above? Explain briefly; you do not need to construct the interval. [2pts.]

Problem 3. In the field of Psychology, a Reaction Time Test is used to measure the time it takes a given person to respond to a specific stimulus; for example, how long it takes a person to press a button once the button has lit up. Suppose that for a particular stimulus, response times of randomly-selected individuals follow a normal distribution centered at 3 seconds with a standard deviation of 0.5 seconds. A person is selected at random, administered the stimulus, and their reaction time is recorded.

- (a) Define the random variable of interest, and call it X . [1pts.]

- (b) What is the probability that a randomly-selected person has a reaction time between 2.5 seconds and 3.7 seconds? [3pts.]

(c) Can you foresee any potential difficulties in modeling response times using a normal distribution? Specifically, think in terms of state spaces. [1pts.]

(d) What sort of plot would be best-suited for assessing whether or not a set of reaction times could plausibly have been drawn from a normal distribution? [1pts.]

Problem 4. According to the World Bank, only 54.2% of households in Ethiopia live with access to electricity. To test these claims, a sociologist takes a representative sample of 130 Ethiopian households, and observes that 67 of these households live with access to electricity. Suppose that the sociologist wants to test the World Bank's claims against a two-sided alternative, at a 5% level of significance.

(a) Define the parameter of interest, and call it p . [1pts.]

(b) Define the random variable of interest, and call it \hat{P} . [1pts.]

(c) State the null and alternative hypotheses in terms of p . [2pts.]

(d) Compute the observed value of the test statistic. [2pts.]

(e) Compute the critical value of the test. Be sure to check any/all relevant conditions first! [3pts.]

(f) Now, perform the test and interpret your conclusions in the context of the problem. [2pts.]