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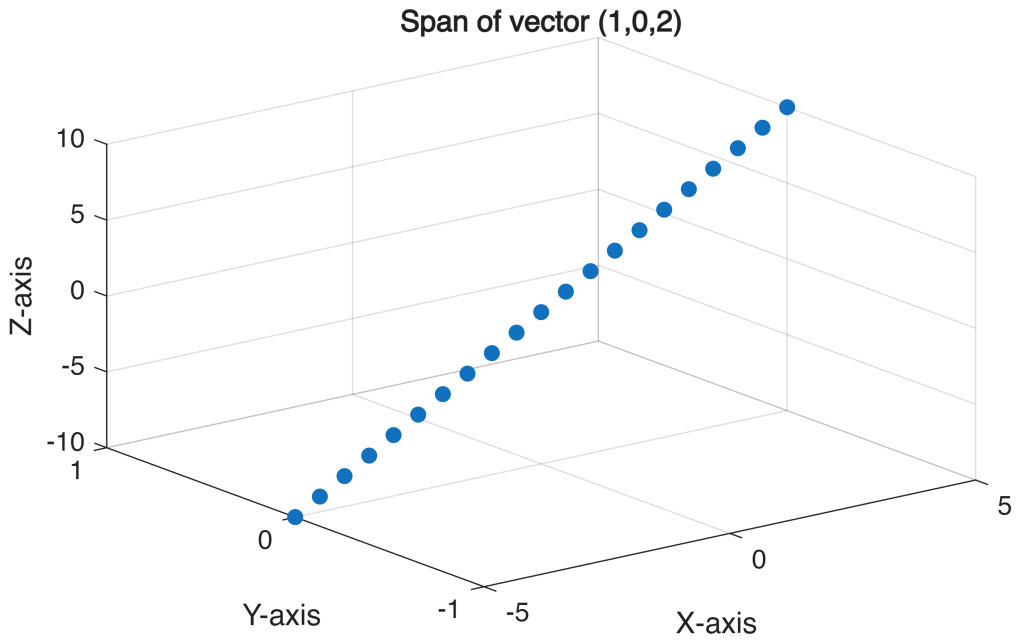
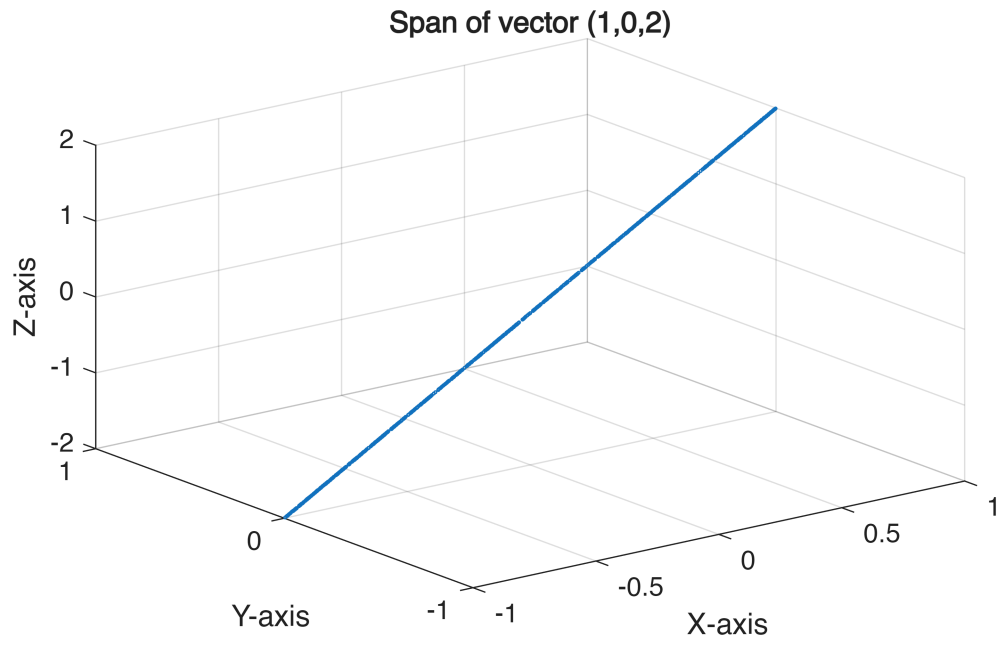
Lab Excercise 5

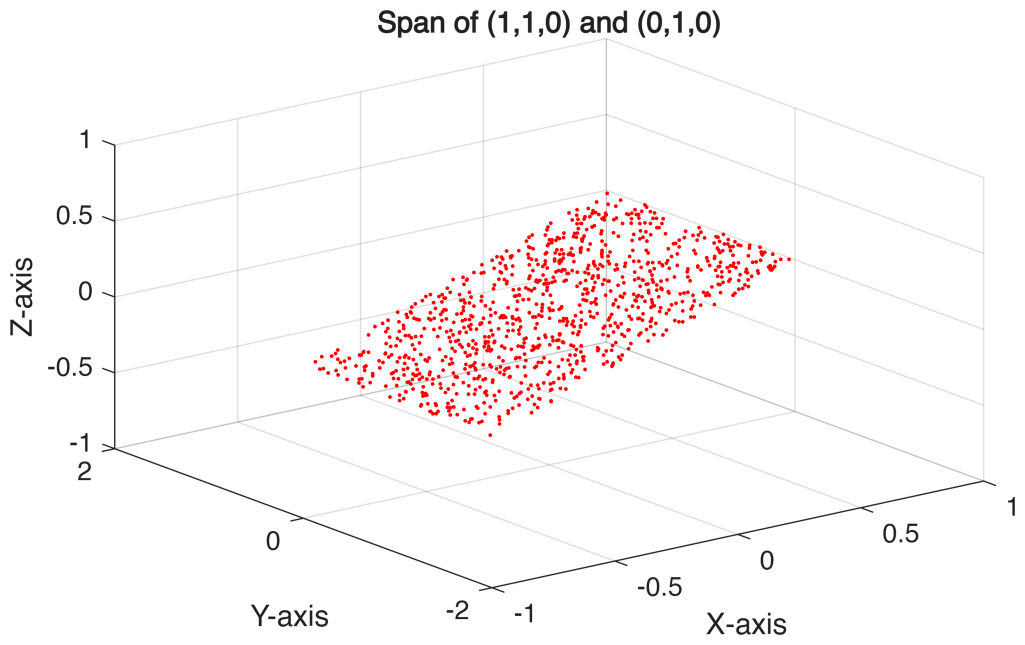
```
%qno 1
clf
v=[1 0 2];
t=-5:0.5:5;
P=t'*v;
scatter3(P(:,1), P(:,2), P(:,3), 'filled');
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
title('Span of vector (1,0,2)');
```



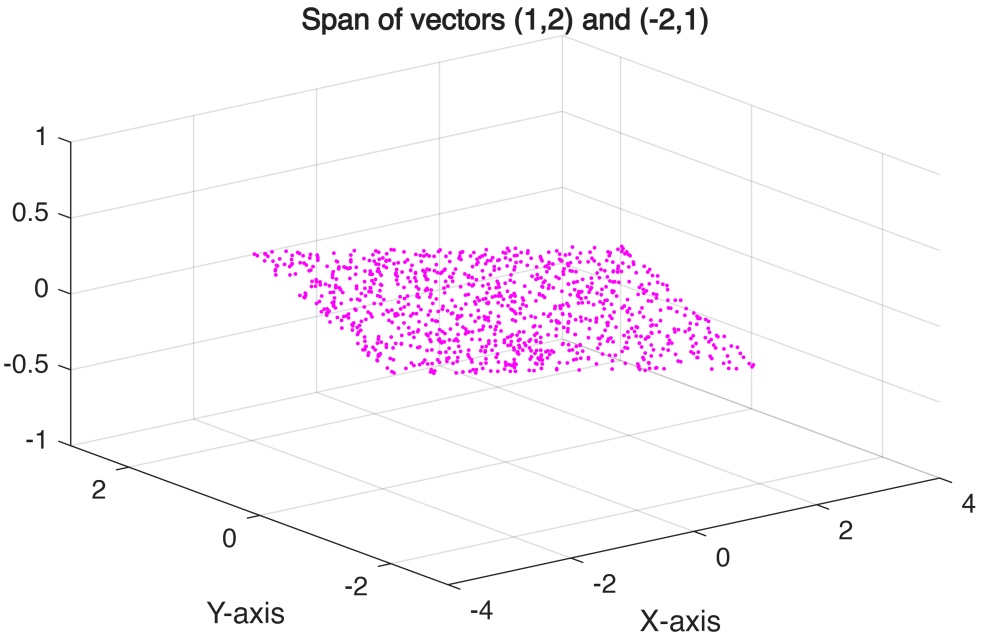

```
%another way to do it
b=[1;0;2];
pts=[];
for i=1:1000
    k1=-1+2*rand(1);
    pts=[pts,k1*b];
end
scatter3(pts(1,:),pts(2,:),pts(3,:),1);
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
title('Span of vector (1,0,2)');
```




```
%Qno 2
b1=[1;1;0];
b2=[0;1;0];
pts=[];
for i=1:1000
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
    pts=[pts,k1*b1+k2*b2];
end
scatter3(pts(1,:),pts(2,:),pts(3,:),2,'filled','r');
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
title('Span of (1,1,0) and (0,1,0)')
```




```
%Qno 2
b1=[1;2];
b2=[-2;1];
pts=[];
for i=1:1000
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
    pts=[pts,k1*b1+k2*b2];
end
scatter3(pts(1,:),pts(2,:),0,2,'filled','magenta');
xlabel('X-axis');
ylabel('Y-axis');
title('Span of vectors (1,2) and (-2,1)');
grid on
```




```
%Qno 4
clf
M=[1 0 0; 0 0 0; 0 0 3]
```

```
M = 3×3
     1     0     0
     0     0     0
     0     0     3
```

```
[RR, ic]=rref(M);
r = length(ic);
R=RR(1:r,:)
```

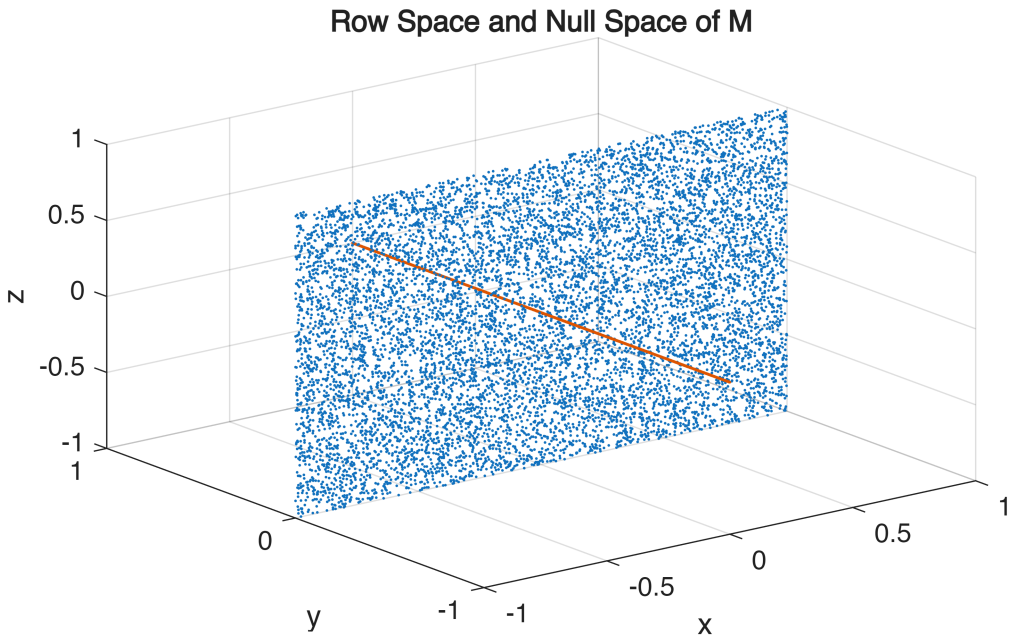
```
R = 2×3
     1     0     0
     0     0     1
```

```
RSB1=(R(1,:))';
RSB2=(R(2,:))';  % generating basis vectors of row space and writing it as column
vectors
nm=null(M)
```

```
nm = 3×1
     0
     1
     0
```

```
NSB1=(nm(:,1));  % generating basis vector of null space as column vector
RSpts=[];
NSpts=[];
for i=1:10000
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
```

```
    a1=-1+2*rand(1);
    RSpts=[RSpts,k1*RSB1+k2*RSB2];
    NSpts=[NSpts,a1*NSB1];
end
scatter3(RSpts(1,:),RSpts(2,:),RSpts(3,:),1,'filled');
hold on
scatter3(NSpts(1,:),NSpts(2,:),NSpts(3,:),1,"filled");
title('Row Space and Null Space of M');
xlabel('x');
ylabel('y');
zlabel('z');
```




```
%Qno 5
clf
a=[1,4;0,5];
b=[0,0;0,5];
c=[0,0;0,0];
[RRA,ica]=rref(a);
[RRB,icb]=rref(b);
[RRC,icc]=rref(c);
nmA=null(a);
nmB=null(b);
nmC=null(c);
disp("Nullspace of a:");disp(nmA);
```

```
Nullspace of a:
```

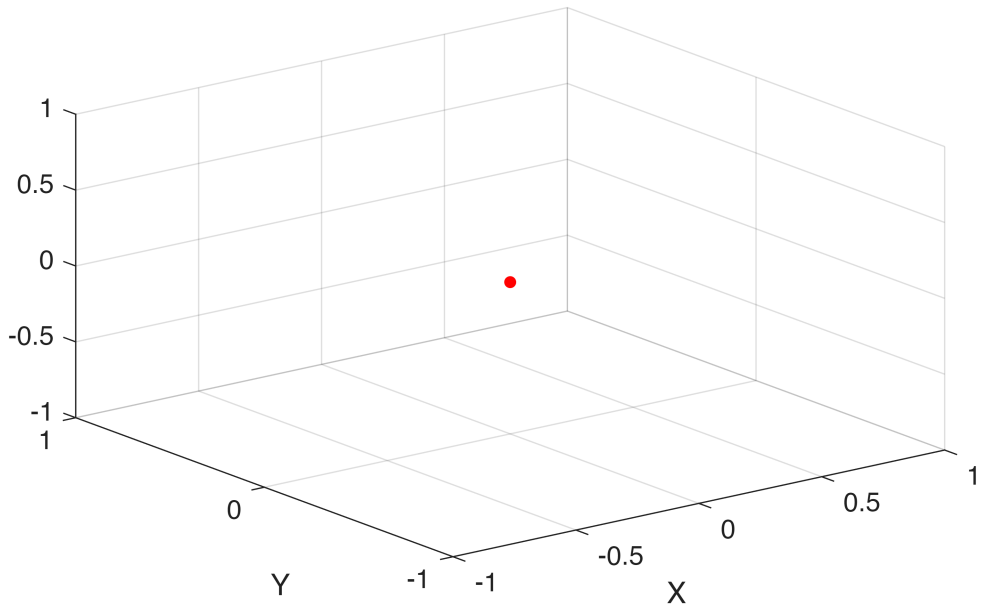
```
disp("Nullspace of b:");disp(nmB);
```

```
Nullspace of b:
     1
     0
```

```
disp("Nullspace of c:");disp(nmC);
```

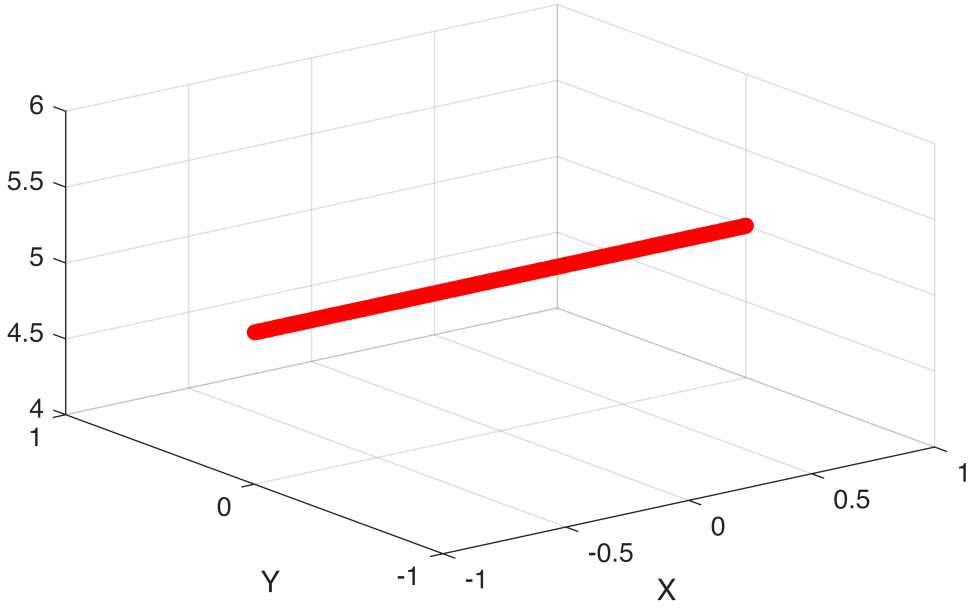
```
Nullspace of c:
     1     0
     0     1
```

```
ptsa=[];
for i=1:1000
    if isempty(nmA)
        ptsa=[ptsa,[0;0]];
    else
    ka=-1+2*rand(1);
    ptsa=[ptsa, ka*nmA(:,1)];
    end
end
scatter3(ptsa(1,:),ptsa(2,:),0,20,'filled','MarkerFaceColor','r');
grid on
xlabel("X")
ylabel("Y")
```

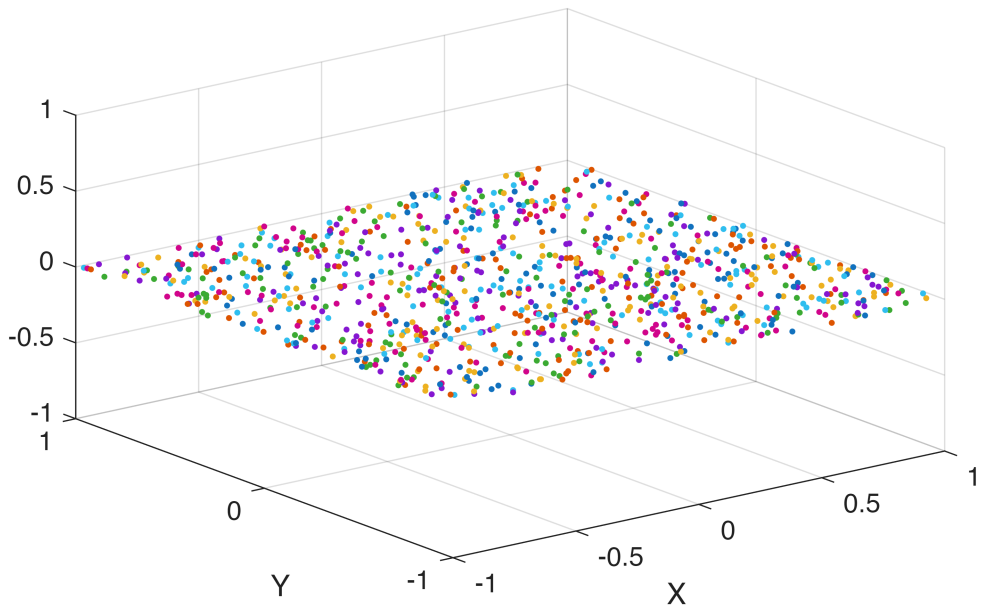



```
ptsb=[];
for i=1:1000
    if isempty(nmB)
        ptsb=[ptsb,[0;0]]
    else
        kb=-1+2*rand(1);
        ptsb=[ptsb,kb*nmB(:,1)];
    end
end
```

```
scatter3(ptsb(1,:),ptsb(2,:),5,'filled','r');
grid on
xlabel("X")
ylabel("Y")
```




```
ptsc=[];
for i=1:1000
    if isempty(nmC)
        ptsc=[ptsc,[0;0]];
    else
        kc=-1+2*rand(1);
        kc2=-1+2*rand(1);
        ptsc=[ptsc,kc*nmC(:,1)+kc2*nmC(:,2)];
    end
end
scatter3(ptsc(1,:),ptsc(2,:),0,5,'filled')
grid on
xlabel("X")
ylabel("Y")
```




```
%Qno 6
clf
A=randi([1,10],3,2)*randi([1,9],2,3)
```

```
A = 3×3
    50    36    42
    64    44    52
    47    64    65
```

```
[RRA,ica]=rref(A)
```

```
RRA = 3×3
    1.0000         0    0.2308
         0    1.0000    0.8462
         0         0         0
ica = 1×2
     1     2
```

```
Rank=length(ica)
```

```
Rank =
2
```

```
R=RRA(1:Rank,:)
```

```
R = 2×3
    1.0000         0    0.2308
         0    1.0000    0.8462
```

```
RSB1=R(1,:)'
```

```
RSB1 = 3×1
    1.0000
         0
    0.2308
```

```
RSB2=R(2,:)'
```

```
RSB2 = 3×1
         0
    1.0000
    0.8462
```

```
[RRC,icc]=rref(A')
```

```
RRC = 3×3
    1.0000         0   19.5000
         0    1.0000  -14.5000
         0         0         0
icc = 1×2
     1     2
```

```
Rankc=length(icc)
```

```
Rankc =
2
```

```
RC=RRC(1:Rankc,:)
```

```
RC = 2×3
    1.0000         0   19.5000
         0    1.0000  -14.5000
```

```
RCBS1=RC(1,:)'
```

```
RCBS1 = 3×1
    1.0000
         0
   19.5000
```

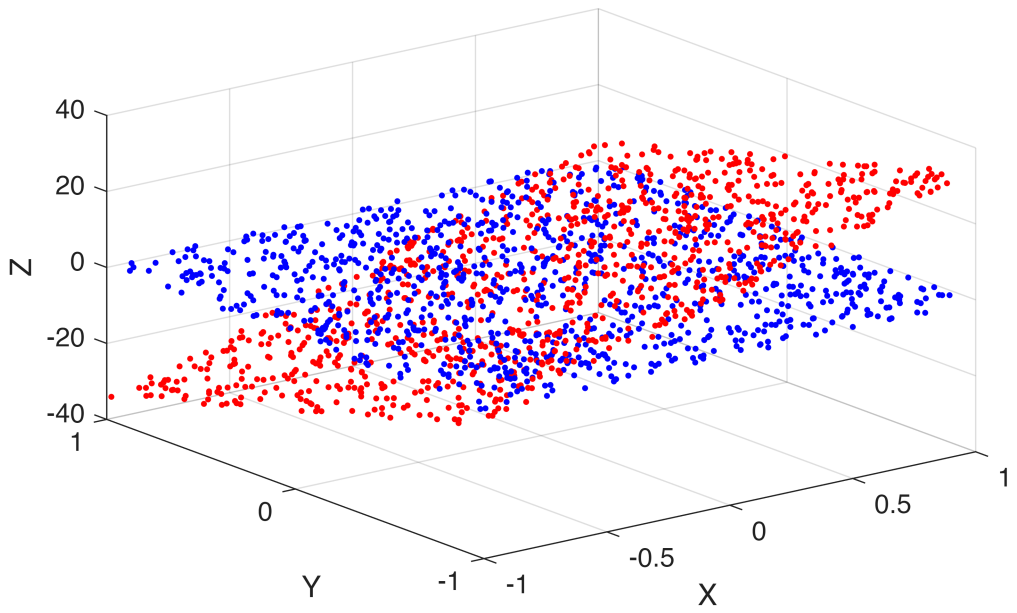
```
RCBS2=RC(2,:)'
```

```
RCBS2 = 3×1
         0
    1.0000
  -14.5000
```

```
Rspts=[];
Cspts=[];
for i=1:1000
        a1=-1+2*rand(1);
        a2=-1+2*rand(1);
        Rspts=[Rspts,a1*RSB1+a2*RSB2];
        k1=-1+2*rand(1);
        k2=-1+2*rand(1);
        Cspts=[Cspts,k1*RCBS1+k2*RCBS2];

end
scatter3(Rspts(1,:),Rspts(2,:),Rspts(3,:),5,'filled','MarkerFaceColor','b');
hold on
scatter3(Cspts(1,:),Cspts(2,:),Cspts(3,:),5,'filled','MarkerFaceColor','r');
xlabel("X")
ylabel("Y")
```

```
zlabel("Z")
hold off
```




```
%QNo 7
clf
A=randi([1,9],3,1)*randi([1,9],1,3)
```

```
A = 3×3
    54    36    18
    18    12     6
    30    20    10
```

```
[RA,ica]=rref(A')
```

```
RA = 3×3
    1.0000    0.3333    0.5556
         0         0         0
         0         0         0
ica =
1
```

```
rank=length(ica);
Co=RA(1:rank,:);
CSB1=Co(1,:)'
```

```
CSB1 = 3×1
    1.0000
    0.3333
    0.5556
```

```
na=null(A')
```

```
na = 3×2
    0.5241   -0.1449
   -0.6446   -0.7115
```

```
   -0.5566    0.6876
```

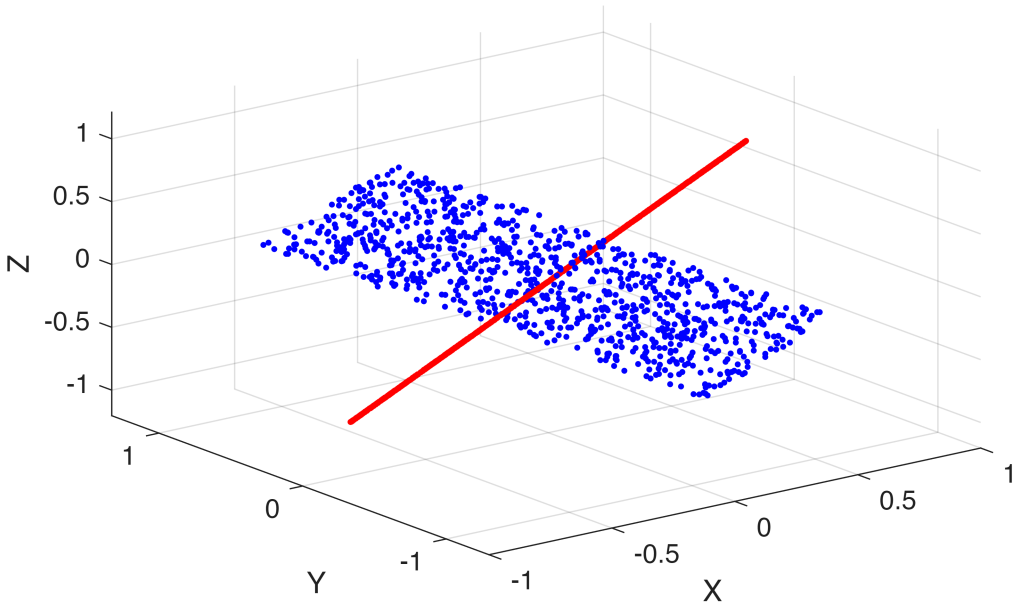
```
nsb1=na(:,1)
```

```
nsb1 = 3×1
    0.5241
   -0.6446
   -0.5566
```

```
nsb2=na(:,2)
```

```
nsb2 = 3×1
   -0.1449
   -0.7115
    0.6876
```

```
ptsc=[];
ptsn=[];
for i=1:1000
    a1=-1+2*rand(1);
    ptsc=[ptsc,a1*CSB1];
    k1=-1+2*rand(1);
    k2=-1+2*rand(1);
    ptsn=[ptsn,k1*nsb1+k2*nsb2];
end
scatter3(ptsc(1,:),ptsc(2,:),ptsc(3,:),5,'filled','MarkerFaceColor','r');
hold on
scatter3(ptsn(1,:),ptsn(2,:),ptsn(3,:),5,'filled','MarkerFaceColor','b');
grid on
hold off
xlabel("X")
ylabel("Y")
zlabel("Z")
```



```
%Qno 8
clc
clear
A=[1 3 4 7 ;2 4 6 10; 3 5 8 13; 4 6 10 16];
u=[-2;-3;1;1];
v=[5;8;11;14];
w=[1;1;2;3];
y=[1;2;0;-1];
m=[-1;1;1;-1];

% For vector u
if(rank([A,u])==rank(A))
    disp(u);disp(" u lies in column space of A")
elseif (rank([A;u'])==rank(A))
    disp(u);disp(" u lies in row space of A")
elseif norm(A*u)==0
    disp(u);disp(" u lies in null space of A")
elseif norm(u'*A)==0
    disp(u);disp(" u lies in Left Null space of A")
else
    disp("Vector u does not lie in subspace of A")
end
```

```
   -2
   -3
    1
    1

 u lies in null space of A
```

```
% For vector v
if(rank([A,v])==rank(A))
    disp(v);disp(" v lies in column space of A")
elseif (rank([A;v'])==rank(A))
    disp(v);disp(" v lies in row space of A")
elseif norm(A*v)==0
    disp(v);disp(" v lies in null space of A")
elseif norm(v'*A)==0
    disp(v);disp(" v lies in Left Null space of A")
else
    disp("Vector v does not lie in subspace of A")
end
```

```
    5
    8
   11
   14

 v lies in column space of A
```

```
% For vector w
if(rank([A,w])==rank(A))
    disp(w);disp(" w lies in column space of A")
elseif (rank([A;w'])==rank(A))
    disp(w);disp(" w lies in row space of A")
elseif norm(A*w)==0
    disp(w);disp(" w lies in null space of A")
elseif norm(w'*A)==0
    disp(w);disp(" lies in Left Null space of A")
else
    disp("Vector w does not lie in subspace of A")
end
```

```
     1
     1
     2
     3

 w lies in row space of A
```

```
% For vector y
if(rank([A,y])==rank(A))
    disp(y);disp(" y lies in column space of A")
elseif (rank([A;y'])==rank(A))
    disp(y);disp(" y lies in row space of A")
elseif norm(A*y)==0
    disp(y);disp(" y lies in null space of A")
elseif norm(y'*A)==0
    disp(y);disp(" y lies in Left Null space of A")
else
    disp("Vector y does not lie in subspace of A")
end
```

```
     1
     2
     0
    -1

 y lies in null space of A
```

```
% For vector m
if(rank([A,m])==rank(A))
    disp(m);disp(" m lies in column space of A")
elseif (rank([A;m'])==rank(A))
    disp(m);disp(" m lies in row space of A")
elseif norm(A*m)==0
    disp(m);disp(" m lies in null space of A")
elseif norm(m'*A)==0
    disp(m);disp(" m lies in Left Null space of A")
else
    disp("Vector m does not lie in subspace of A")
```

```
end
```

```
    -1
     1
     1
    -1

 m lies in Left Null space of A
```

```
%Qno 9
clc
clear
A=[1 -1 2 3; 0 2 1 4; 1 1 3 1; 2 0 5 4];
v=[5;1;-2;0];
w=[0;2;2;2];
u=[-1;2;-1;1];
m=[3;-1;7;7];

if(rank([A,v])==rank(A))
    disp(v);disp("v lies in column space of A");
elseif(rank([A;v'])==rank(A))
    disp(v);disp("v lies in row space of A");
elseif(A*v==0)
    disp(v);disp("v lies in null space of A");
elseif(v'*A==0)
    disp(v);disp("v lies in left null space of A");
else
    disp(v);disp("v doesnot lie in subspopace of A");
end
```

```
     5
     1
    -2
     0

v lies in null space of A
```

```
if(rank([A,w])==rank(A))
    disp(w);disp("w lies in column space of A");
elseif(rank([A;w'])==rank(A))
    disp(w);disp("w lies in row space of A");
elseif(A*w==0)
    disp(w);disp("w lies in null space of A");
elseif(w'*A==0)
    disp(w);disp("w lies in left null space of A");
else
    disp(w);disp("w doesnot lie in subspopace of A");
end
```

```
     0
     2
     2
```

```
     2

w lies in column space of A
```

```
if(rank([A,u])==rank(A))
    disp(u);disp("u lies in column space of A");
elseif(rank([A;u'])==rank(A))
    disp(u);disp("u lies in row space of A");
elseif(A*u==0)
    disp(u);disp("u lies in null space of A");
elseif(u'*A==0)
    disp(u);disp("u lies in left null space of A");
else
    disp(u);disp("u doesnot lie in subspopace of A");
end
```

```
    -1
     2
    -1
     1

u doesnot lie in subspopace of A
```

```
if(rank([A,m])==rank(A))
    disp(m);disp("m lies in column space of A");
elseif(rank([A;m'])==rank(A))
    disp(m);disp("m lies in row space of A");
elseif(A*m==0)
    disp(m);disp("m lies in null space of A");
elseif(m'*A==0)
    disp(m);disp("m lies in left null space of A");
else
    disp(m);disp("m doesnot lie in subspopace of A");
end
```

```
     3
    -1
     7
     7

m lies in row space of A
```

```
%Qno 10
A=randi([1,9],9,2)*randi([1,9],2,9);
rank(A)
```

```
ans =
2
```

```
%Qno 10 a
B=A+A'
```

```
B = 9×9
    76    60    68   111   108   125    82   102    34
    60    24    57   103   102    97    50    53    23
    68    57    30    90    82    55    40   104    38
   111   103    90   158   149   178   117   145    51
   108   102    82   149   140   161   110   152    52
   125    97    55   178   161   106    63   158    66
    82    50    40   117   110    63    40   116    44
   102    53   104   145   152   158   116   178    48
    34    23    38    51    52    66    44    48    14
```

```
rank(B)
```

```
ans =
4
```

```
%QNo 10 b
[V,D]=eig(B)
```

```
V = 9×9
   -0.2356   -0.0018    0.1583    0.8345    0.2403   -0.1888    0.1582    0.1079 ···
   -0.1750    0.7389   -0.1744   -0.0612   -0.3569    0.0664    0.1239    0.4340
    0.3519   -0.2181   -0.6651    0.2470   -0.4476   -0.2074    0.0029   -0.1766
   -0.2826   -0.3084   -0.4327   -0.3472    0.4694    0.0125    0.0867    0.3146
   -0.1578   -0.4385    0.3887   -0.0443   -0.5311    0.4018    0.0117    0.1081
    0.6093    0.0502    0.3866   -0.2046    0.0944   -0.4587   -0.0255    0.2253
    0.4459    0.2233   -0.0823    0.1580    0.3206    0.7081   -0.1332   -0.1610
   -0.2661    0.2487    0.0855   -0.2176    0.0132   -0.1628    0.1566   -0.7625
   -0.2106    0.0699   -0.0178    0.0468   -0.0169   -0.1306   -0.9534   -0.0051
D = 9×9
 -108.3935         0         0         0         0         0         0         0 ···
         0  -54.7825         0         0         0         0         0         0
         0         0   -0.0000         0         0         0         0         0
         0         0         0   -0.0000         0         0         0         0
         0         0         0         0   -0.0000         0         0         0
         0         0         0         0         0    0.0000         0         0
         0         0         0         0         0         0    0.0000         0
         0         0         0         0         0         0         0   54.2237
         0         0         0         0         0         0         0         0
```

```
for i=1:9
    fprintf("Eigen Vectors %d is :",i)
    disp(V(:,i))
end
```

```
Eigen Vectors 1 is :
   -0.2356
   -0.1750
    0.3519
   -0.2826
   -0.1578
    0.6093
    0.4459
   -0.2661
   -0.2106
Eigen Vectors 2 is :
   -0.0018
    0.7389
   -0.2181
   -0.3084
   -0.4385
    0.0502
```

```
    0.2233
    0.2487
    0.0699
Eigen Vectors 3 is :
    0.1583
   -0.1744
   -0.6651
   -0.4327
    0.3887
    0.3866
   -0.0823
    0.0855
   -0.0178
Eigen Vectors 4 is :
    0.8345
   -0.0612
    0.2470
   -0.3472
   -0.0443
   -0.2046
    0.1580
   -0.2176
    0.0468
Eigen Vectors 5 is :
    0.2403
   -0.3569
   -0.4476
    0.4694
   -0.5311
    0.0944
    0.3206
    0.0132
   -0.0169
Eigen Vectors 6 is :
   -0.1888
    0.0664
   -0.2074
    0.0125
    0.4018
   -0.4587
    0.7081
   -0.1628
   -0.1306
Eigen Vectors 7 is :
    0.1582
    0.1239
    0.0029
    0.0867
    0.0117
   -0.0255
   -0.1332
    0.1566
   -0.9534
Eigen Vectors 8 is :
    0.1079
    0.4340
   -0.1766
    0.3146
    0.1081
    0.2253
   -0.1610
   -0.7625
   -0.0051
Eigen Vectors 9 is :
```

```
    0.3050
    0.2318
    0.2251
    0.4362
    0.4176
    0.4052
    0.2681
    0.4243
    0.1477
```

```
for i=1:9
    fprintf("Eigenvalue %d is:",i)
    disp(D(i,i))
end
```

```
Eigenvalue 1 is:
 -108.3935
Eigenvalue 2 is:
  -54.7825
Eigenvalue 3 is:
  -1.4679e-14
Eigenvalue 4 is:
  -1.0494e-14
Eigenvalue 5 is:
  -1.0350e-15
Eigenvalue 6 is:
   9.6392e-15
Eigenvalue 7 is:
   7.9717e-14
Eigenvalue 8 is:
   54.2237
Eigenvalue 9 is:
  874.9523
```

```
Eign=eig(B);
if all(isreal(Eign))
    disp("All Eigenvalues are real numbers");
else
    disp("There are complex Eigenvalues");
end
```

```
All Eigenvalues are real numbers
```

```
fprintf('A symmetric matrix has real eigenvalues because')
```

```
A symmetric matrix has real eigenvalues because
```

```
%Qbo 10d
count=0;
for i=1:9
    if abs(Eign(i))<1e-6||abs(Eign(i))<-1e-6
        count=count+1;
    else
        continue
    end
end
fprintf("number of Zero eigen values is : %d",count);
```

```
number of Zero eigen values is : 5
```

```
fprintf("Relation: Number of zero eigenvalues = size(B,1) - rank(B)"); %number of 0
eigen values=n-r
```

```
Relation: Number of zero eigenvalues = size(B,1) - rank(B)
```

```
%Qno 10 e
check=V'*V;
if norm(check - eye(size(check)))<1e-6
    disp("Eigenvectors are orthonormal to B")
else
    disp("Eigenvectors are not orthonormal to B")
end
```

```
Eigenvectors are orthonormal to B
```

```
%Qno 10 f
[RRA,ica]=rref(B);
[RRC,icc]=rref(B');
nb=null(B)
```

```
nb = 9×5
    0.9146    0.0051    0.0558    0.0091    0.0115
   -0.1898    0.1090    0.3242   -0.1245   -0.1133
   -0.0178    0.0655    0.8304    0.2263    0.0378
   -0.2481   -0.2506   -0.1352    0.5525   -0.2964
   -0.1893    0.4571   -0.0175   -0.5929    0.0110
    0.0104   -0.4934   -0.2122   -0.2458    0.2495
    0.0631    0.6021   -0.3622    0.3263   -0.2222
   -0.1132   -0.2652   -0.0440   -0.1217   -0.0776
   -0.1135    0.1889   -0.0749    0.3078    0.8832
```

```
ranka=length(ica)
```

```
ranka =
4
```

```
rankc=length(icc)
```

```
rankc =
4
```

```
RA=RRA(1:ranka,:)
```

```
RA = 4×9
    1.0000         0         0         0    0.4090   -2.0638    0.3437    6.1975 ···
         0    1.0000         0         0   -0.2395    0.9689    0.4603   -0.8305
         0         0    1.0000         0    0.2457    1.6423    1.2939    0.4494
         0         0         0    1.0000    0.6719    1.0093   -0.5381   -3.1509
```

```
RB=RRC(1:rankc,:)
```

```
RB = 4×9
    1.0000         0         0         0    0.4090   -2.0638    0.3437    6.1975 ···
         0    1.0000         0         0   -0.2395    0.9689    0.4603   -0.8305
         0         0    1.0000         0    0.2457    1.6423    1.2939    0.4494
         0         0         0    1.0000    0.6719    1.0093   -0.5381   -3.1509
```

```
%Qno 10 g
NullityA=size(A,1)-rank(A);
NullityB=size(B,1)-rank(B);
fprintf("Nullity of A is: %d",NullityA)
```

```
Nullity of A is: 7
```

```
fprintf("Nullity of B is: %d",NullityB)
```

```
Nullity of B is: 5
```

```
%Qno 11
A=randi([1 9],10,3)*randi([1 9],3,5);
rank(A)
```

```
ans =
3
```

```
%Qno 11 a
S1=A*A'
```

```
S1 = 10×10
       61071       43574       75590       70764       42005       51783 ···
       43574       31131       53842       50276       29796       36769
       75590       53842       93767       88061       52393       64510
       70764       50276       88061       83111       49573       60918
       42005       29796       52393       49573       29690       36457
       51783       36769       64510       60918       36457       44802
       33932       24046       42395       40157       24163       29668
       29899       21311       37069       34751       20712       25526
       40174       28732       49569       46199       27323       33740
       27486       19528       34216       32276       19302       23730
```

```
S2=A'*A
```

```
S2 = 5×5
      100907       73044       92772      100881       83987
       73044       55611       69752       74670       62347
       92772       69752       87780       94284       78760
      100881       74670       94284      101871       84819
       83987       62347       78760       84819       71010
```

```
%QNo 11 b
rank(S1)
```

```
ans =
3
```

```
rank(S2)
```

```
ans =
3
```

```
%Qno 11 c
es1=eig(S1)
```

```
es1 = 10×1
```

```
10^5 ×
   -0.0000
   -0.0000
   -0.0000
   -0.0000
    0.0000
    0.0000
    0.0000
    0.0024
    0.0225
    4.1469
```

```
es2=eig(S2)
```

```
es2 = 5×1
10^5 ×
    0.0000
    0.0000
    0.0024
    0.0225
    4.1469
```

```
fprintf("What we observe is S1 and S2 have the same non zero eigenvalues \n");
```

```
What we observe is S1 and S2 have the same non zero eigenvalues
```

```
fprintf("Both S1 and S2 have the same three nonzero eigenvalues (since rank A = 3),
\n while S1 has 7 and S2 has 2 zero eigenvalues, so rank(S1)=rank(S2)=rank(A) and
their nonzero eigenvalues are identical.")
```

```
Both S1 and S2 have the same three nonzero eigenvalues (since rank A = 3),
 while S1 has 7 and S2 has 2 zero eigenvalues, so rank(S1)=rank(S2)=rank(A) and their nonzero eigenvalues are identi
```

```
%Qno 11 d
nA=size(A,1)-rank(A);
ns1=size(S1,1)-rank(S1);
ns2=size(S2,1)-rank(S2);
fprintf("Nullity of A is: %d ",nA);
```

```
Nullity of A is: 7
```

```
fprintf("Nullity of S1 is: %d ",ns1);
```

```
Nullity of S1 is: 7
```

```
fprintf("Nullity of S2 is: %d ",ns2);
```

```
Nullity of S2 is: 2
```