Name : Krish Singh

Roll: AID25072

Branch: B.Tech AI & DS 2nd sem

1. Pseudoinverse of an m×n matrix, A Every matrix Am×n has a pseudoinverse A+ or pinv(A) • If m = n = rank of A, then A is invertible and A+ = A-1 • If rank of A = n, (all columns of A are independent), then A+ = (ATA)-1AT = Left Inverse of A, and A+A = In. • If rank of A = m, (all rows of A are independent), then A+ = AT (AAT)-1 = Right Inverse of A, and AA+ = Im. • Pseudoinverse A+ of any matrix A can be obtained in MATLAB using the command 'pinv(A)'.

a. Generate a 9×9 random integer matrix and verify that inv(A) = pinv(A).

```
A=randi([0,9],9,9);
ainv=inv(A);
painv=pinv(A);
if norm(ainv-painv)<1e-10
    disp("Inverse of A is equal to pseudoinverse of A");
else
    disp("Inverse of A is not equal to pseudoinverse of A");
end
```

```
Inverse of A is equal to pseudoinverse of A
```

b. Generate a 5×4 random integer matrix and verify that pinv(A) is same as the left inverse of A.

```
A=randi([0,9],5,4);
apinv=pinv(A);
lainv=inv(A'*A)*A';
if norm(apinv-lainv)<1e-10
    disp("Pseudoinverse of A is equal to left inverse of A");
else
    disp("Pseudoinverse of A is not equal to left inverse of A");
end
```

```
Pseudoinverse of A is equal to left inverse of A
```

c. Generate a 3×7 random integer matrix and verify that pinv(A) is same as the right inverse of A.

```
A=randi([0,9],3,7);
apinv=pinv(A);
arinv=A'*inv(A*A');
if norm(apinv-arinv)<1e-10
    disp("Pseudonverse of A is equal to Right inverse");
else
    disp("Pseudoinverse of A is not equal to Right inverse");
end
```

```
Pseudonverse of A is equal to Right inverse
```

*2. Projection of a vector v on a subspace S, which is a column space of matrix A(with all independent columns) is p = A(ATA)-1ATv. If some columns in A are dependent (i.e., if Rank(Am×n) < n), then the projection of a vector v on a subspace S, which is a column space of matrix A is p = A*pinv(A)*v, where pinv(A) is the pseudo inverse of A.*

a. Find the projection of �=(−3 −3 8 9) on the space spanned by (3 1 0 1),(1 2 1 1),(−1 0 2 −1).

```
v=[-3;-3;8;9];
A=[3 1 -1;
    1 2 0;
    0 1 2;
    1 1 -1];
if norm(rank(A)-size(A,2))<1e-10
    p=A*inv(A'*A)*A'*v;
    disp(p);
else
    p=A*pinv(A)*v;
    disp(p);
end
```

```
   -2.0000
    3.0000
    4.0000
   -0.0000
```

b. Find the projection of �=(2 1 3) on the space spanned by (1 1 0),(1 2 1).

```
u=[2;1;3];
A=[1 1;1 2;0 1];
if norm(rank(A)-size(A,2))
    p=A*inv(A'*A)*A'*u;
    disp(p);
else
    p=A*pinv(A)*u;
    disp(p);
end
```

```
    0.6667
    2.3333
    1.6667
```

c. Find the projection of �=(2 1 3) on the column space of =(1 1 2 4; 1 2 3 6; 0 1 1 2).

```
u=[2;1;3];
A=[1 1 2 4;
1 2 3 6;
0 1 1 2 ];
```

```
if norm(rank(A)-size(A,2))<1e-10
    p=A*inv(A'*A)*A'*u;
    disp(p);
else
    p=A*pinv(A)*u;
    disp(p)
end
```

```
0.6667
2.3333
1.6667
```

Consider the matrices given below: A, B, C, D and E

```
A=[1 4 5 6 9;
    3 -2 1 4 -1;
    -1 0 -1 -2 -1;
    2 3 5 7 8];

B=[3 4;
    1 8];

C=[0 1 0;
    0 0 1;
    0 4 2];

D=[1 2 0 1;
    0 1 1 0;
    2 4 0 2];

E=[0 1 4 1;
    0 0 8 1;
    0 1 4 0;
    1 0 8 0;
    0 1 1 0;
    1 0 1 0];
```

a. which one of the matrices have inverse? Find if it exists.

```
if norm(size(A,1)-size(A,2))<1e-10 && det(A)~=0
    disp("Matrix A has inverse");
else
    disp("Matrix A does not have a inverse");
end
```

```
Matrix A does not have a inverse
```

```
if norm(size(B,1)-size(B,2))<1e-6 && det(B)~=0
    disp("Matrix B has inverse");
```

```
else
    disp("Matrix B does not have inverse");
end
```

```
Matrix B has inverse
```

```
if norm(size(C,1)-size(C,2))<1e-10 && det(C)~=0
    disp("Matrix C has inverse");
else
    disp("Matrix C does not have inverse");
end
```

```
Matrix C does not have inverse
```

```
if norm(size(D,1)-size(D,2))<1e-10 && det(D)~=0
    disp("Matrix D has a inverse");
else
    disp("Matrix D does not have a inverse");
end
```

```
Matrix D does not have a inverse
```

```
if norm(size(E,1)-size(E,2))<1e-10 && det(E)~=0
    disp("Matrix E has a inverse");
else
    disp("Matrix e does not have a inverse");
end
```

```
Matrix e does not have a inverse
```

3. Least square solution of a linear system, AX=B is:

x̂= (ATA)-1ATb, if all the column vectors in A are independent (i.e. if ATA is invertible).

*x̂= pinv(A)*b, if the column vectors in A are not all independent.*

• Find the least square solution for the following systems:

a. x + y = 10; 2x + y = 10; 3x + y = 16; 4x + y = 27; 5x + y = 23.

```
A=[1 1;
    2 1;
    3 1;
    4 1;
    5 1];
B=[10;10;16;27;23];
if rank(A)==size(A,2)
    x=(A'*A)\(A'*B);
    disp(x);
```

```
else
    x=pinv(A)*B;
    disp(x);
end
```

```
    4.3000
    4.3000
```

b. 2x – 3y + z = 1; x + 3y – 4z = -4; 3x – 2y – z = -1; 5x – y – 4z = -4.

```
A=[2 -3 1;
    1 3 -4;
    3 -2 -1;
    5 -1 -4];
B=[1;-4;-1;-4];
if rank(A)==size(A,2)
    x=(A'*A)\(A'*B);
    disp(x);
else
    x=pinv(A)*B;
    disp(x);
end
```

```
   -0.3333
   -0.3333
    0.6667
```

c. 2x + y = 20; 2x - 3y = 4; 3x + y = 28; 4x + y = 36;

```
A=[2 1;
    2 -3;
    3 1;
    4 1];
B=[20;4;28;36];
if rank(A)==size(A,2)
    x=(A'*A)\(A'*B);
    disp(x);
else
    x=pinv(A)*B;
    disp(x);
end
```

```
     8
     4
```

3. Fit a parabola (quadratic function) with (-2, 0), (-1,0), (0,1), (1, 0), and (2, 0).

```
x=[-2 -1 0 1 2];
y=[0 0 1 0 0];
quad=polyfit(x,y,2);
```

```
disp("y= "+quad(1)+"x² "+quad(2)+"x "+quad(3));
```

```
y= -0.14286x² 1.7053e-17x 0.48571
```

4. A middle-aged man was stretched on a rack to lengths L = 5, 6, and 7 feet under applied forces of F = 1, 2, and 4 tons. Assuming Hookes law L = a + bF , find his normal length a by leastsquares.

```
F=[1;2;4];
L=[5;6;7];
A=[ones(size(F)) F];
x=(A'*A)\(A'*L);
a=x(1);
b=x(2);
disp("Normal length a = "+a);
```

```
Normal length a = 4.5
```

```
disp("Proportanality constant b = "+b);
```

```
Proportanality constant b = 0.64286
```

5. Find the best straight-line t (least squares) to the measurements: b = 4 at t = -2, b = 3 at t = -1, b = 1 at t = 0 and b = 0 at t = 2. Then find the projection of b = (4; 3; 1; 0) onto the column space of A =[1 –2; 1 –1 ;1 0; 1 2]

```
A=[1 -2; 1 -1; 1 0; 1 2];
t=[-2;-1;0;2];
b=[4;3;1;0];
A=[ones(size(t)) t];
x=(A'*A)\(A'*b);
a=x(1);
beta=x(2);
disp("Best-fit line: b=a+beta*t");
```

```
Best-fit line: b=a+beta*t
```

```
disp("a = "+a+" and "+"beta = "+beta);
```

```
a = 1.7429 and beta = -1.0286
```

```
bproj=A*x;
disp("projection of b onto col(A):"); disp(bproj);
```

```
projection of b onto col(A):
    3.8000
    2.7714
    1.7429
   -0.3143
```

6. Fit a best straight line passing through (-3, 4),(-2,2),(-1,0),(0,0),(1,-2),(2,-2) and (3,-3).

```
x=[-3;-2;-1;0;1;2;3];
y=[4;2;0;0;-2;-2;-3];
```

```
A=[ones(size(x)) x];
p=(A'*A)\(A'*y);
a=p(1);
b=p(2);
disp("Best-fit straight line: y=a+b*x"); disp("a = "+a+" and "+" b = "+b);
```

```
Best-fit straight line: y=a+b*x
a = -0.14286 and  b = -1.1071
```

7. Consider the following data:

a. Find the equation of a regression line for the following data using excel.

```
[B,text]=xlsread('Excel\excelsheet_1.xlsx');
x=B(:,1);
y=B(:,2);
n=length(x);
ATA=[sum(x.^2),sum(x);sum(x),n]
```

```
ATA = 2×2
      111150         650
         650          27
```

```
ATb=[sum(x.*y);sum(y)]
```

```
ATb = 2×1
10^5 ×
    5.3044
    0.0310
```

```
m_and_c = inv(ATA)*ATb
```

```
m_and_c = 2×1
    4.7729
   -0.1026
```

b. Enter the data in an excel sheet, and then find the regression line using MATLAB. Also plot the regression line and the data points in the same figure.

• Refer to the class notes for this.

```
clf;
[B,text]=xlsread('Excel\excelsheet_1.xlsx');
x=B(:,1);
y=B(:,2);
n=length(x);
A=[x,ones(n,1)]
```

```
A = 27×2
     0     1
     1     1
     2     1
     3     1
```

```
    4     1
    5     1
    6     1
    7     1
    8     1
    9     1
   10     1
   11     1
   12     1
   13     1
   14     1
    ⋮
```

```
ATA=[sum(x.^2),sum(x);sum(x),n]
```

```
ATA = 2×2
      111150          650
         650           27
```

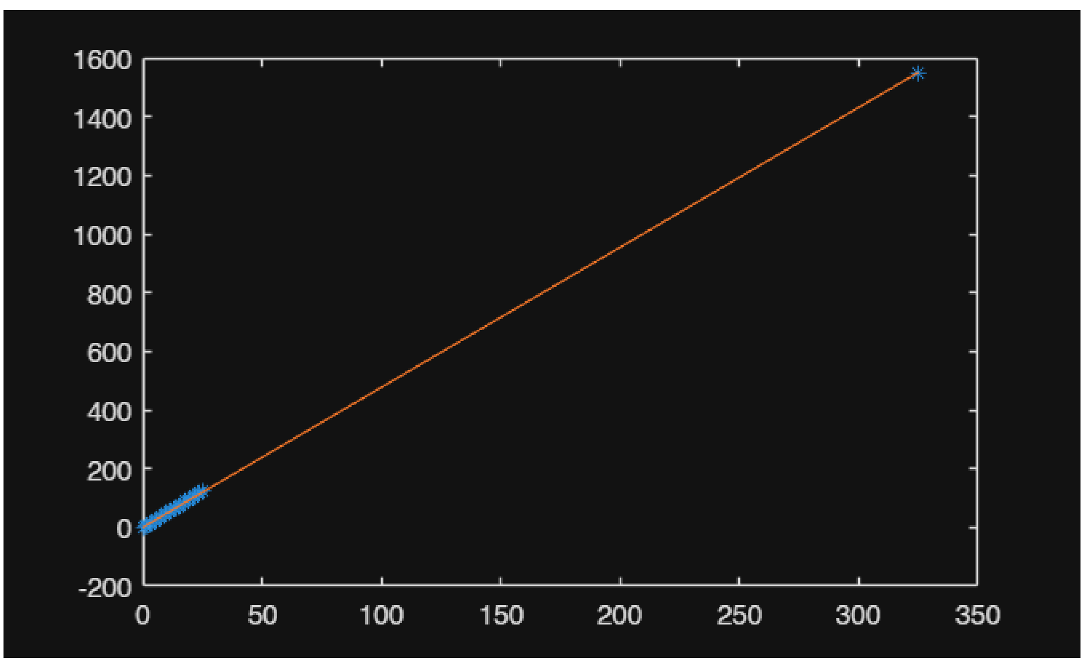
```
ATb=[sum(x.*y);sum(y)]
```

```
ATb = 2×1
10^5 ×
    5.3044
    0.0310
```

```
m_and_c = inv(ATA)*ATb
```

```
m_and_c = 2×1
    4.7729
   -0.1026
```

```
y_projected=A*inv(ATA)*ATb;
plot(x,y,'*')
hold on
plot(x,y_projected)
```



8. a. Generate a data for regression with a fixed m=9, c=5 for x = -25,-24, -23,…,24,25 and introducing a noise with standard deviation 3.

```
clf;
m=9;
c=5;
x=-25:0.1:25
```

```
x = 1×501
  -25.0000  -24.9000  -24.8000  -24.7000  -24.6000  -24.5000  -24.4000  -24.3000 ⋯
```

```
n=length(x)
```

```
n =
501
```

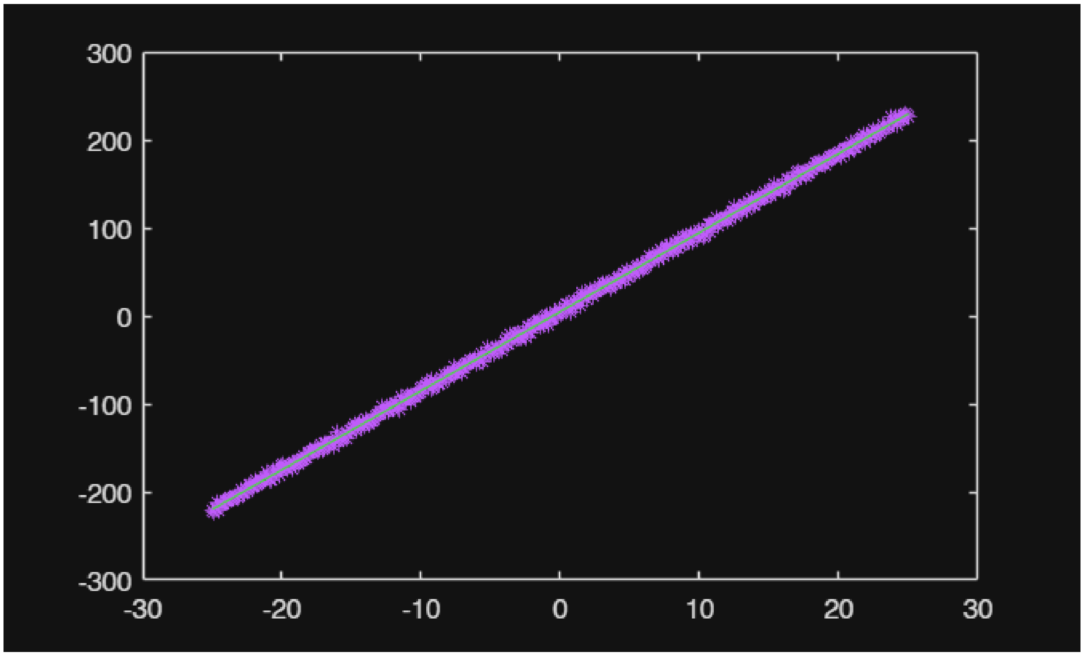
```
y=m*x+c;
std=3;
noise=std*randn(1,n)
```

```
noise = 1×501
   -0.1394   -2.9003   -1.4137    5.2057   -2.3326   -5.0514    0.5311   -0.3734 ⋯
```

```
ynoise=y+noise;
```

b. Using the data generated above, re-estimate the values of m and c. Also find the regression line and plot it in MATLAB.

```
plot(x,ynoise,'*')
hold on
A=[x' ones(n,1)];
ynoise=ynoise';
m_and_c=pinv(A)*ynoise;
y_projected=A*pinv(A)*ynoise;
plot(x,y_projected);
```




```
m_and_c
```

```
m_and_c = 2×1
    8.9903
    5.0157
```