Amrita School of Engineering, Bengaluru-35

23MAT106
Mathematics for Intelligent Systems – 1
Lab Material 2

Rank, Random number generation, Generation of matrices with given rank (one lab session)

- rank(A) ← Finds the rank of matrix A
- rand(1) ← Generation of a random number between 0 and 1
- rand(2)
- rand(2,3) ← Generation of a random matrix(square/rectangular) with elements between 0 and 1
- randi(5) ← Generates an integer random number between 0 and 5
- randi([3,15]) ← Generates an integer random number between 3 and 15
- randi([0,5],2) ← Generates random square matrix of order 2, with elements between 0 and 5
- randi([2,6],3,4) ← Generates random 3×4 matrix with elements between 2 and 6

Generation of random integer matrices with given rank

- Results used –
  1. Rank of an m×n matrix is min(m,n)
  2. Rank(AB)≤min(Rank(A),Rank(B))

```
A=randi([0 9],3,1)*randi([0 9],1,3)
                % Generates a random 3×3 matrix of rank 1.
rank(A)
B=randi([0 9],5,2)*randi([0 9],2,7)
                % Generates a random 5×7 matrix of rank 2.
rank(B)
C=randi([0 9],6,3)*randi([0 9],3,4)
                % Generates a random 6×4 matrix of rank 3.
rank(C)
```

Generation of a random symmetric matrix

- From a random square matrix
  A=randi([0,9], 4, 4); S=A+A'
- From a random rectangular matrix

```
A=randi([0,9], 4, 2);
S1=A'*A;
        % 2x2 symmetric matrix with rank 2
S2=A*A';
        % 4x4 symmetric matrix with rank 2
```

Practice Problems:

1. Obtain a random square matrix of order 20 and find the rank of it.
2. Obtain 2 random integer square matrices A and B of order 5.
   (a) Find the rank of A
   (b) Find the rank of B
   (c) Find the rank of A+B
   (d) Find the rank of A-B
   (e) Find the rank of A*B
   (f) Find the rank of kB, by choosing k as any real number
3. Find a matrix X which when multiplied with matrix Y gives matrix Z. Matrix Y and Z can be generated as random integer matrices of order 5.
4. Just by checking the rank of the coefficient matrix of the system mention whether the solution of the given homogeneous linear equations represent a trivial solution / a non-trivial solution representing a line / a non-trivial solution representing a plane.
   (a) $x + y – z = 0$; $3x – 2y +9z = 0$; $7x -3y +17z = 0$
   (b) $2x_1+3x_2+5x_3-8x_4+x_5=0$; $-x_1+ x_2+5x_3+2x_4+3x_5=0$; $5x_1+9x_2-2x_3+x_4+3x_5=0$;
   $2x_1+3x_2-5x_3+8x_4-x_5=0$; $6x_1-3x_2-7x_3+2x_4-x_5=0$;
   (c) $x_1-3x_2+5x_3+x_4=0$; $3x_1+ x_2+2x_3+4x_4=0$; $4x_1-2x_2+7x_3+5x_4=0$; $2x_1-6x_2+10x_3+2x_4 =0$;
5. Generate a 5x5 matrix A of rank 4.
   a. Retrieve an element with row index 3, and column index 5.
      Ans: a=A(3,5)
   b. Retrieve first row from A and store in b
      Ans: b=A(1, :) or b=A(1, [1 2 3 4 5]) or b= A(1, 1:5)
      Or b= A(1, 1:end)
   c. Retrieve first and third row from A and store in C.
      Ans: C=A([1, 3], :) or C=A([1,3] , [1 2 3 4 5])
      or C= A([1,3], 1:5) or C= A([1, 3], 1:end)
   a. Note that ‘end’ is a built-in reserved word in Matlab
   d. Retrieve second column from A and store in b
      Ans: b=A(:, 2) or b=A([1 2 3 4 5], 2) or b= A(1:5, 2)
      or b= A(1:end,2)
   e. Retrieve second and fourth column from A and store in C.
      Ans: C=A(:, [2, 4]) or C=A([1 2 3 4 5],[2, 4]) or C= A(1:5,[2,4])
      or C= A(1:end, [2 4])

6. Using MATLAB generate a 9× 9 matrix A of rank 2. Obtain a symmetric matrix B=A+A’ and find rank of B.

7. Using MATLAB generate a 10×5 matrix A of rank 3.
   (a) Obtain a symmetric matrix $S1=A*A^T$ and $S2=A^T*A$
   (b) Find the rank of S1 and S2.

8. Solve the following systems of linear equations (rref command can be used in MATLAB to get the row reduced echelon form). Mention what the solution geometrically represents (a point or a line or a plane or a hyperplane).

a) $3x+3y-z=4$; $3x-8y+6z=7$; $x+y+10z=22$
b) $4x-3y+2z+5w=10$; $9x-2y-3z+6w=7$; $2x+11y+3z-6w=13$; $8x-3y+5z-w=14$
c) $x-3y+2z+5w= 3$; $2x-2y+3z+6w=11$; $2x+11y-3z-6w=40$; $5x+6y+2z+5w=54$
d) $4x-3y+2z+5w=10$; $9x-2y-3z+6w=7$; $5x+1y-5z+w=13$; $8x-6y+4z+10w=20$
e) $x+y-z=7$; $2x-2y+3z=9$; $3x+2y-5z=10$
f) $x-3y+2z+5w= 0$; $2x-2y+3z+6w=0$; $2x+11y-3z-6w=0$; $5x+6y+2z+5w=0$

g) 4x-3y+2z+5w=0; 9x-2y-3z+6w=0; 5x+1y-5z+w=0; 8x-6y+4z+10w=0
h) x+y-2z=0; 2x-3y+z=0; 3x-2y-z=0
i) x+y-5z+3w=0; 2x-3y-10z+4w=0; x-9y-5z+w=0; 4x-11y-20z+8w=0

9. Write the augmented matrix AB for the following linear system of equations, write it in row-reduced echelon form using MATLAB and solve. Mention what the solution geometrically represents.
   j) 3x+3y-z=4; 3x-8y+6z=7; x+y+10z=22
   k) 4x-3y+2z+5w=10; 9x-2y-3z+6w=7; 2x+11y+3z-6w=13; 8x-3y+5z-w=14
   l) x-3y+2z+5w= 3; 2x-2y+3z+6w=11; 2x+11y-3z-6w=40; 5x+6y+2z+5w=54
   m) 4x-3y+2z+5w=10; 9x-2y-3z+6w=7; 5x+1y-5z+w=13; 8x-6y+4z+10w=20
   n) x+y-z=7; 2x-2y+3z=9; 3x+2y-5z=10
10. Another way of solving a system AX=B using MATLAB is to use the command, X=A\B. Which of the systems in Question 14 can be solved using this method? Explain.
11. Another way of solving a system AX=B using MATLAB is using inverse, $X = A^{-1}B$. Try to find the solution of systems in Q.14 using this way.