Math 5260 Ordinary Differential Equations, Assignment 1 (Chapter 4)

Due date: April 7, 2020 in class tentatively.

1. (15 points) Exercise 4.8.

2. (15 points) Exercise 4.13.

3. (**10 points**) Find $\alpha \in \mathbb{R}$ such that the planar system

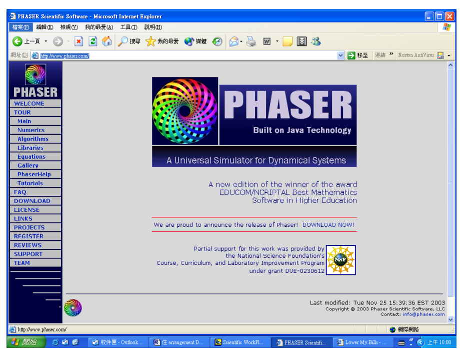
$$\begin{cases} x' = y, \\ y' = \alpha(1 - x^2 - y^2)y - x, \end{cases}$$

has an orbitally asymptotically stable periodic orbit. (Hint. Corollary 4.4.1.)

4. (**20 points**) (a) Draw the *global* phase portrait of

$$\begin{cases} \dfrac{dx_1}{dt} = 2x_2 \\ \dfrac{dx_2}{dt} = 2x_1 - 3x_1^2 + \lambda x_2(x_1^3 - x_1^2 + x_2^2). \end{cases}$$

with parameters (i) $\lambda = 0$, (ii) $\lambda = -5$, (iii) $\lambda = 5$, (ii) $\lambda = -30$, (iii) $\lambda = 30$, and any software like PHASER. Show that there exists a homoclinic orbit on the curve $x_1^3 - x_1^2 + x_2^2 = 0$.



PHASER http://www.phaser.com/

(b) Analyze the *global* phase portrait of the system with parameter (i) $\lambda = 0$, (ii) for all $\lambda < 0$, (iii) for all $\lambda > 0$ by mathematical analysis as possible.

5. (**20 points**) **Exercise B.** Consider nonlinear pendulum system

$$\begin{cases} \theta' = \omega \\ \omega' = -\sin\theta - c\omega \ (c \geq 0, \ \text{a parameter}). \end{cases}$$

Draw the *global* phase portrait of the system on (θ, ω)-plane and analyze it as possible as you can by mathematical analysis for different values $c = 0,\ 1/2,\ 1,\ 3/2,\ 2,\ 3$.