Moving-average smoothing

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Features of economic series: Trend, cycle, residual

- It is common to think of economic series as consisting of a long-term trend, a seasonal cycle, and a residual.
- A popular way to visualize and understand a data series is to try to decompose it into these components.
  - To be clear, there are many potential ways to do this.
  - They should all be interpreted carefully, as visualization tools, not necessarily as revealing any reliable statistical patterns.
- The most common decomposition is based on moving averages. We will overview this approach today.

Centered moving averages

A **centered moving average** of order m is as follows:

If m is odd, then let $k = \frac{m-1}{2}$, and

$$CMA_t \quad = \quad \frac{1}{m} \times (y_{t-k} + \ldots + y_{t+k})$$

If m is even, then let $k = \frac{m}{2}$, and

$$CMA_t \quad = \quad \frac{1}{m} \times \left(\frac{1}{2} y_{t-k} + y_{t-k+1} + \ldots + y_{t+k-1} + \frac{1}{2} y_{t+k} \right)$$

Centered moving averages are a common way to visualize the "trend" component of a data series. A common choice of m is the order of seasonality in the data, for example $m = 12$ for monthly data.

Seasonal component

After calculating the series of CMAs, a common approach to modeling the seasonal component of a time series is as follows:

- Calculate a "detrended" series as $y_{t,\text{detrend}} = y_t - CMA_t$.
- For each stage l within a cycle, calculate a seasonal average $\bar{y}_l$ as the average of $y_{t,\text{detrend}}$ across all the l observations.
- For example, if the data is monthly, then $\bar{y}_l$ is the average of $y_{t,\text{detrend}}$ for a specific month across all years in the sample.
- The seasonal component for each stage l is its seasonal average, minus the average of all the seasonal averages.

Finally, the residual is y_t minus the trend and seasonal components.

See example on Canvas.

Variations on the moving-average smoothing

The simple approach described above is very common in practice, but you will also see many variations on it. Two examples are:

- Trailing averages: When we want to build *forecasts*, we don't have the future values and must average only over past values. This often leads us to also use weighted averages (next point).
- Weighted averages: In a two-sided moving average, it is common to weight the central observation by the most. In a trailing average, it is common to weight the *most recent* observation by the most, to put the emphasis on the newest data.

Next week we will cover exponential smoothing, a popular approach that uses a trailing average with a specific weighting scheme.