

# CS280r Spring 2017 - Final Project Report

## Ἀρμονία (Harmonia): A System for Collaborative Music Composition

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### Abstract

Increasing productivity of music composition has many positive benefits. Listeners would appreciate individually tailored music to their emotional needs and context. Composers would be facilitated by greater and more diverse cooperation yielding more innovative music. Composition agents could assist in the generation of repetitive or experimental musical forms. Therapists can use music as part of a treatment plan for autism and many other disorders. The system we propose attempts to address these myriad needs by offering two key innovations: a SharedPlan with collaborative versioning to mediate the workflow of a composition for a group of musicians, an algorithmic evaluation of a composition against the intention of the SharedPlan to provide guidance to both human and agent composers.

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### 1. Introduction

It is my design to render it manifest that no one point in its composition is referrible either to accident or intuition – that the work proceeded, step by step, to its completion with the precision and rigid consequence of a mathematical problem.

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*Edgar Allen Poe*  
*The Philosophy of Composition*

Fix references from section to section

Music composition has historically been an individual endeavor, which is counterintuitive when music is mostly played, improvised, and experienced in a group. Part of this is due to the singular nature of creative expression, but a large part is also due to a dearth of viable tools to enable collaborative efforts between composers. Most modern composition is performed using digital workstation tools, yet composers do not generally have access to the collaborative tools and evaluation processes available to an analogous entry level software engineer.

Mark: I think some word about building on collaborative ideation for the shared artifact setting is important here

Deep Neural Networks are being used for the creation of new visual[ref to Sketch paper], narrative[ref] and now musical works[wavenet? magenta ref?]. These more sophisticated tools can assist human composers both in terms of idea exploration

as well as objective evaluation against intent of the composition and expression of originality.

We propose a system for music composition that addresses collaborative music composition inspired by tools used for software teamwork, collaborative ideation, and musical structure analysis. This system leaves many components open for integration with neural networks. We will first examine related work for intentionality, collaboration and mathematical evaluation of music. We will then provide a specification for music composition workflow. This will be followed by a specification of algorithmic evaluation of the work in order to validate against intentionality. We will present three use cases for the entire system including possible failure modes. We will then conclude with a discussion of the proposal including limitations we have identified and potential future work.

Our work builds on several areas of research related to music, computer science, and creative cognition. First we discuss related work in collaborative ideation, both in general and specifically in music. Next we discuss intelligent music systems that facilitate human composition and improvisation, and related work in Music Information Retrieval (MIR). Finally we describe previous work that applies information theory to the analysis of musical structure.

## 2. Related Work

### 2.1. Shared Plans

David

decompose sets of tasks among multiple agents. Commitment among agents. Typically used in a concurrent cooperative setting where agents act on many resources and accomplish a goal. In our case there are many agents, potentially taking turns, working on one structure.

### 2.2. Collaborative Ideation

Collaborative Ideation (CI) seeks to improve the productivity of individuals and groups in generating ideas through collaboration. The people involved are interested in creating related objects (e.g., we all want to brainstorm solutions to social problems) and seek either feedback or examples of others' work to enhance their individual process. Collaboration is centered around a shared workspace, physical or virtual, that allows for communication and sharing of ideas. The ideas produced may be for individual use, or ideators may work on shared artifacts such as an essay or piece of art. The dynamics of collaboration may be real-time or not, though increasingly, today's settings are real-time and virtual. A simple CI setting is one where each ideator brainstorms solutions to a problem common to all participants, and each participant can see all other's ideas. While such approaches have been used naturally in human culture for millennia, the design of intelligent computer systems today aims to facilitate these activities to allow for increased creativity and productivity.

IdeaHound [Siangliulue 2016] addresses at-scale collaboration in this setting. Siangliulue's work claims that only a small subset of the idea pool may be relevant and inspiring to a single ideator, that it is overwhelming for each ideator to view all participant's ideas. The system creates a semantic map of all generated ideas that allows each ideator to easily view their work in the context of the entire solution space. The map is automatically generated. Each user is prompted to interact with a personal "whiteboard" where they can cluster their ideas and separate them by semantic distance, and the global map is computed from the collection of whiteboards. This approach bypasses the need for external workers to power semantic analysis of ideas. Using this map, IdeaHound recommends diverse suggestions to each ideator, eliminating the cognitive load of idea search. Our work is largely influenced by IdeaHound, but several challenges specific to collaborative music composition require new interventions. First, the generated objects are structured rather than

unordered collections of ideas. Second, ideators need to build over each other's ideas rather than only seek inspiration.

CI has surfaced in a setting closer to ours, in the space of online blogs and services designed to share visual art and music. Ideas range from small, unfinished efforts seeking directions to finished pieces seeking critique. Artists improve upon their ideas using the large-scale feedback. SoundCloud is an example of a hybrid music streaming service and CI platform. Though much of the hosted music is presented in finished form, people also post incomplete projects. Artists sometimes share "stems" to their music, which are individual sound files that feature isolated instrumental tracks, with the intention that others seeking inspiration remix their pieces into new work. A newer platform, Blend, makes the sharing of source files explicit. By default, artists share their works in progress in the format of music production software source files, which allows others to quickly pick up on their work and take it in new directions. This setting is closer to our area of application and supports building on one another's ideas. What changes when several ideators intend to create a single shared piece? With SoundCloud and Blend, one may take another's piece in a totally different direction. In our work, a collaborative composition has a goal associated with it through the duration of its existence. It is up to the composers and the system to keep a piece of music close to its shared plan.

### 2.3. Computer Facilitated Composition and Improvisation

Computer agents with the ability to facilitate and take part in music composition and improvisation are of great interest to music theorists and artificial intelligence researchers. These systems have in common a requirement to "understand" music at multiple levels, including low-level acoustic signal, mid-level theoretical constructs such as harmony and rhythm, and high-level level aspects such as mood, genre, and style. For example, music recommendation system such as Spotify seek to analyze music and extract a measure of relevance for a function such as "study music". These issues constitute the research area of Music Information Retrieval.

In systems that create music, the interest is to assist human composers, rather than replace them. Perhaps a composer has good "seeds" ideas, but the system may recommend variations of ideas, or re-orderings of ideas, to make them more conveying. Such system knowledge often comes from large-scale corpus analysis that mines patterns common to a collection of music. ChordRipple [Huang 2016] is a recent system that takes as input a progression of musical chords from a composer, and

suggests substitutions of intermediate chords that preserve the original semantics of the input while serving to replace conventional choices with more interesting ones. If the composer agrees to make one of the recommended changes, the system assists the composer in interpolating between original and substituted material before and after the initial substitution, resulting in further mixing of human and system generated music.

While our current work seeks primarily to assist teams of human composers to enrich and organize their work, we intend to design the system such that intelligent computer agent composers may be further in the loop. Last year, Google Brain launched the Magenta Project for exploring machine intelligence in music <sup>1</sup>. Magenta is an integrated environment of software tools and music-related datasets. Recently, Magenta released AI Duet <sup>2</sup>, a computer system that reacts to human improvised gestures. Improvisation is an important part of composition. Even in steps where a human is composing, it may be beneficial to have an agent for the human to go back-and-forth on ideas with, much like two friends would iteratively vary and refine their ideas. In settings where a piece is defined by a specific enough set of guidelines, such as in a therapy use case where a listener may need music at a certain tempo and with a simple beat [See section INSERT SECTION], powerful information retrieval systems make effective machine composition agents possible. Human composers may be placed at later steps of collaboration to ensure that the piece meets requirements in a humanly perceptible way.

#### 2.4. Information Theory and Music Analysis

Our system relies on the ability to model musical structure in a way that supports automated feedback for collaborating composers, where feedback is in the form of suggested rearrangements of musical ideas that help a composition reach a mutually specified structural goal. In this direction, there has been a rich body of work in automated analysis of musical structure from the 1950's to present. A prominent direction is to model musical form by way of listener perception and the expected dynamics of their attention and surprise. Understanding musical structure and its impact on listeners is a cogent goal to music theorists, cognitive scientists, and machine learning researchers. Many of these approaches have drawn on probability theory and information theory. A survey of approaches historical to contemporary can be found in the *Con Espressione*

Manifesto [Widmer 2016], which is a strong position paper on the coming decade of research directions for music information retrieval. Many of these works borrow from original principles described in ? in largely unmodified ways.

Our work builds on the the Information Dynamics Approach [Abdallah et al 2012]. Abdallah uses predictive information rate, an entropy/divergence based metric that measures how a listener's mid-piece distribution over future musical events is continually revised as new information is presented, to compute a curve that summarizes aspects of surprise in redundancy in a piece of music. Our work assumes that musical structure can be effectively summarized by this criteria. We assume that pieces from a particular genre/mood are defined by characteristic balances of surprise and redundancy over time, with peaks of information content (communicated by the composer) in genre-specific locations. We leverage this metric as the foundation of our automatic analysis system, which compares a collaborative work-so-far against the characteristic curves for the genre and mood specified by the mutual plan for the piece, and suggests edits to the composers to keep them in line with their goal.

### 3. System Design

Mark: Need to rephrase this

After reviewing the typical workflow of editing a piece of music as part of a team of composers, we describe the details of our system design. The design is centered around addressing two aspects of collaborative composition. The first includes the composition interface, the representation of music, the editing capabilities of composers, and the version control system used to track musical revisions. These all focus on issues of communication and coordination in collaboration over a shared artifact of a specified format, and with a specification of editing actions. Then we discuss the automated music analysis used in our system for giving feedback and suggestions to composers. This facilitates the collaborative composition process by helping composers assess the affect of their edits on the movement of the work in progress toward the goal, and is specific to the domain of music. We discuss how these two system functions in combination help composers by guiding aspects of revision on a shared musical artifact, and by providing an interface that makes such revisions convenient, modular, and amenable to collaboration.

#### 3.1. Workflow Overview

David

<sup>1</sup><https://magenta.tensorflow.org/welcome-to-magenta>

<sup>2</sup><https://aiexperiments.withgoogle.com/ai-duet>

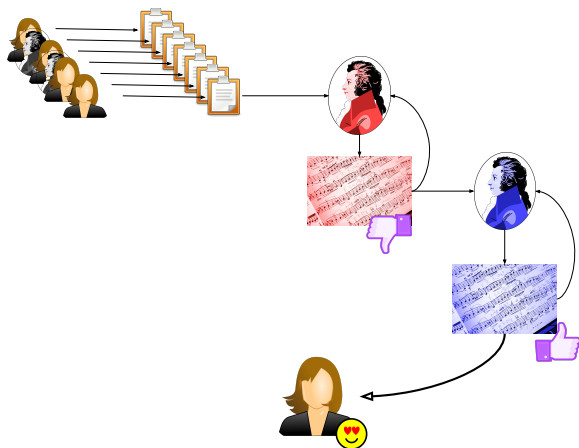


Figure 1: Overall workflow: a non-musical user or a composer define a Shared-Plan. Composer 1 (red) starts iterating and commits a work-in-progress. Composer 2 (blue) continues iterating until the original user is satisfied.

someone creates a shared plan (individual or composers)  
 information retrieval system gets characteristic metrics using  
 as much of the sharedplan info as it can  
 composers iteratively work on the piece  
 Within a particular composer/system interaction, automated  
 analysis is run on the piece as it is. suggests some actions to  
 the composer, particularly suggesting to switch two parts of a  
 piece. Composer can follow suggestions or do their own modi-  
 fications, or do nothing and finish.

### 3.2. MIDI and Edit Actions

Mark: fix above image spacing

Our system represents music in MIDI format. MIDI is a protocol for communicating discrete information about the pitch, duration, and dynamics of individual notes. A musical work is described by specifying the vertical arrangement of individual notes as chords and their horizontal arrangement over time. Additionally, there is a standard MIDI soundbank that maps integers to specific synthetic instruments. For example, one can specify that this MIDI part is meant to be played by sound 0, grand piano or sound  $x$ , xylophone. Most familiar software for music notation and music production build user interfaces on top of a basic MIDI file editor. Extensions to this protocol include MusicXML, which allows for the specification of additional parameters such as expression markings and articulation information. In our system, a simple MIDI editor is sufficient for now. Through the use of additional metadata (next section), composers are able to segment MIDI files into separate segments. For example, a piece may be subdivided into several

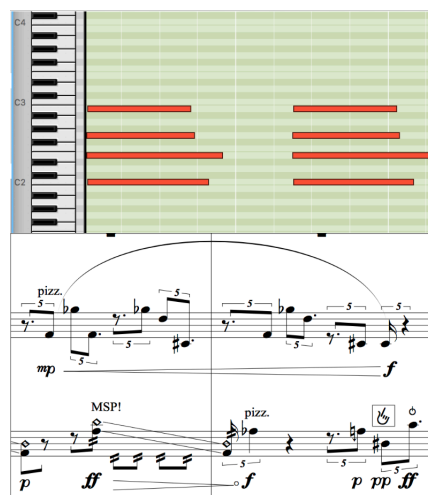


Figure 2: WHY DOES THIS LOOK LIKE THIS. Top: MIDI. Bottom: MusicXML

sections. Segmentation may represent intentions about musical form, for example one may segment part of a composition into an exposition and a development section.

Mark: clean up language about segmentation

During an interaction with the system, a composer is able to change a composition by editing the MIDI in several low-level or high-level ways. Composers can add a new block of material, edit an existing block, remove an existing block, swap the order of two blocks, merge two blocks, or split a block into two. At each iteration of editing, the system suggests an option that may bring the work in progress closer to what is specified by the SharedPlan. This is primarily in the form of "switch blocks A and B?" (See Section on Automatic Evaluation). The final product of collaboration is a MIDI file that represents the pitches, rhythms, and dynamics of musical events, as well as a collection of metadata that concisely states additional information, such as tempo and instrumentation.

This final product can be produced as an electronic piece of music by importing the MIDI into any music production software and specifying which MIDI track should be played by which sound or synthetic instrument. If desired, composers may take additional actions on that finished work that are external to the system, such as produce the piece of music with live musicians, but this is beyond our scope, which is mainly to facilitate collaboration during the composition process.

### 3.3. GIT

David

- git + intention
- version control not available in existing MIDI systems, along with tracking additional metadata
- attributing credit
- reference splice.com and blend.io

### 3.4. Shared Plan + Metadata, inter-composer communication

David

- genre and mood specification
- tempo specification optional
- don't discuss information retrieval much here
- how do composers or users express intentions concisely?
- What sharing of information occurs between users and composers, and among composers? Decoupling (be explicit)
- added metadata when composer adds a block and wants to communicate what else should be done to it
- NOTE FROM MARK: I talked to Pao about this a little. Finding out a nice way for composers to share their intentions about an added passage will help because it will distinguish between two kinds of disagreements
  1. composer agrees with another composer's intention about an edit, but disagrees with the actual implementation
  2. composer fundamentally disagrees with the intention
- should discussion of inter-composer communication be separate? Maybe parts relevant to each part of the system specification should be mentioned, but the discussion of their synthesis as an inter-composer system should be saved for discussion section?
- should failure modes for inter-composer communication go here, or elsewhere?

## 4. User Interface

David

- moo

## 5. Automated Analysis of Musical Structure

Harmonia facilitates collaborative composition in two ways. First, the interface as a whole, including the revision system and the shared metadata associated with each composition, helps with practical aspects of communication and coordination. Second, the analysis system lets composers know how close their work-in-progress is to their goal, as measured by similarity to characteristic pieces relevant to their goal. The analysis system also suggests structural edits such as swapping the order of existing material, or deleting material, that could further improve the piece. In this section, we describe the automated analysis of musical structure that is used by our system. Crucial to our system, our computational approach models the structure of a piece of music in relation to the expected trajectory of surprise and redundancy that a listener experiences. We first discuss the nature of the musical analysis used in our system, and then discuss how this method supports goal checking and suggested musical edits.

### 5.1. Entropy of Musical Events and Divergence

Let  $X$  be a discrete random variable that takes on values from the set  $\mathcal{X}$ . For example,  $X$  may represent the next chord that a listener hears in a piece of music. The event  $X = x$  indicates that the listener heard  $X$  take on a specific value  $x$ . Let  $p_X(x) = p(x)$  denote the probability that  $X$  will take on value  $x$ , *before* the listener hears the chord, as estimated by a distribution that the listener brings with them from prior musical experiences, as well as from what they have heard in the piece so far.  $-\log p(x)$  then corresponds to the *surprise* of the event, because the more the listener expects the event (higher  $p(x)$ ), the lower the surprise, where the log is taken for convenience (it is monotonic in the  $p(x)$ ). Since  $X$  represents the event that the listener is about to hear, we can represent the expected surprise of  $X$  averaged over all possible values as:

$$H(X) = - \sum_{x \in \mathcal{X}} p(x) \log p(x)$$

which corresponds to the Entropy of  $X$ ,  $H(X)$ . Intuitively, this means that the listener does not know what the next event will be (e.g. which chord will be played next), but from context (from their state of listening as represented by their current distribution over future events), they expect a certain extent of surprise from the next event in general.

Because we choose to represent musical structure in terms of the surprise dynamics of the listener, it is necessary to describe



the way in which the listener’s distribution over future events changes as they hear present events. In the running example, by hearing  $X = “Ab^{Δ7}”$  in the present, how does their distribution over future events differ from how it was before they heard that chord. It is necessary to describe this “difference” between the distributions. The Kullback-Leibler Divergence of one distribution from another captures this notion of distance. Avoiding subscripting  $X$  with timesteps, let  $X'$  be the revised distribution over the next event *after* hearing  $X = x$  in the context of the existing distribution.

$$D_{KL}(X' \parallel X) = \sum_{x \in X} p_{X'}(x) \log \frac{p_{X'}(x)}{p_X(x)}$$

This is read as “the divergence from  $X'$  of  $X$ ”, and is the average over the ratio of point-wise log probabilities between the two distributions, weighted by  $p_{X'}(x)$ . For an accessible yet informative discussion of the significance of entropy as a measure of information and KL Divergence<sup>3</sup>, see Christopher Olah’s post on Visual Information Theory<sup>4</sup>. This divergence describes the amount of revision to a listener’s distribution over the future that happens as they hear each event. Let this be called the *predictive information* of the event  $X = x$  as the listener hears it. When a surprising event occurs and causes the listener to drastically revise their distribution (i.e. this same event will be less surprising in the future), the event had high predictive information. On the other hand, if 30 strikes of the same chord have just happened, hearing a 31<sup>st</sup> articulation does not communicate much predictive information. Our system measures the predictive information *rate* (PIR) over the duration of the piece (or work-in-progress), and uses this trajectory of this rate to summarize the structure of a piece of music as it is expected to be perceived by the listener. Note that in the running example, entropy and divergence were discussed in terms of a sequence of chords heard by the listener, and the expectation over next chords in context. In even a simple piece of music, the listener tracks multiple such parameters and their interactions: evolving harmony, rhythm, timbre, and more (see section Future Work). This application of divergence to the revision of a listeners expectation over events is directly motivated by the work of Abdallah et al. We base the summarization of musical structure in our system on their work.

<sup>3</sup>Another interpretation of entropy is the average number of bits required to send a message from a distribution  $p$  under an optimal variable-length coding scheme. The KL Divergence of  $q$  from  $p$  is the increase in the average bits per message when one communicates items from  $p$  using a code optimized for  $q$ . This is the difference between the cross entropy  $H(p, q)$  and entropy  $H(p)$ .

<sup>4</sup><http://colah.github.io/posts/2015-09-Visual-Information>

## 5.2. Current Design: Analyze, Suggest, and Edit

Harmonia uses predictive information rate to calculate the proximity of a musical work in progress to a characteristic piece from the category specified by the SharedPlan. Using these measures, the system gives feedback to a composer with respect to the composer’s editing decisions, and provides suggestions that may bring a piece closer to the goal. It is this analysis and suggestion loop, along with interaction with the SharedPlan metadata, that characterize the main experience for an individual composer. This experience is further enriched by the fact that each time the composer enters the feedback loop, the piece and aspects of SharedPlan may have changed by other composers.

We assume that music information retrieval systems exist to facilitate calculating PIR on sample pieces of music queried by genre, mood, and other metadata keywords (HipHop, Chill, Slow, Study)[footnote to youtube link]. Because we currently analyze MIDI representation of music, this retrieval is done on a corpus of MIDI and score representations of music rather than audio recordings. Some examples of existing query systems include BLAH BLAH and BLAH. We calculate the PIR for the the most popular  $\beta\%$  of pieces matching a specified query, and average the PIR curves to create a “characteristic curve” that represents the typical structure of a piece of music fitting the metadata criteria.

Mark: Is the method for calculating rate clear?

Mark: HUGE PROBLEM #1 HOW IS THIS AVERAGE CREATED?

Comparing the characteristic curve with the PIR curve of the work in progress, our system can estimate some notion of distance from the musical goal specified in the SharedPlan. Let the difference be denoted as  $\Delta$ . This comparison supports several important features of our system. First, not considering any edit suggestions made by our system, a composer may simply see whether their latest edit brings the piece of music closer to (lower  $\Delta$ ) or further from (higher  $\Delta$ ) the SharedPlan.

Mark: HUGE PROBLEM #2 HOW IS THIS DIFFERENCE CALCULATED?

Composers may prefer to go with edits that decrease  $\Delta$ , or may choose to stick with their edit even if it increases  $\Delta$ . Reasons for going with a “worsening” action include choosing to lay down material that further edits will re-contextualize, whether by the same composer or by others. In this case, it is important for a composer to commit their intention for the new edit (as concise text) into the SharedPlan metadata. Future

work involves specifying this format more explicitly, so that an automated agent may be able to act in response to this intention. It is also the case that unstructured, non-concise description of intentions by one composer may be difficult or overwhelming for another composer to deal with.

Also using these PIR scores, our system may give edit suggestions. As specified in [Section Midi and Edit Actions], a composer may edit a piece by adding a new block of material (of any length), edit an existing block, remove a block, swap two blocks, merge two blocks into one, or split one block into two. Excluding adding or editing blocks because this requires intelligent automated composition, and excluding splitting and joining blocks because this does not actually change any musical material or its ordering, the system may recommend repeating or deleting any existing block, or swapping any pair of existing blocks. Because for any reasonably-lengthed work in progress piece there is a tractably enumerable set of such choices, the system can just try each choice of deleting, repeating, and swapping, and suggest to the user the choice the minimizes  $\Delta$ . This choice may be the “make no change” choice at a given iteration, because the best thing to do may be to add more material before considering such actions.

## 6. Use Cases

### 6.1. Individual User, Individual Composer

David

Our first use case considers the following scenario: a listener who may be a non-musician would like a new piece of music, perhaps to use for a function such as study music. We consider the case that the listener specifies a new project defined by a mood and genre. At this point, multiple composers could collaborate on the music specification, but we first consider the case that a single composer iterates over the piece with assistance from our system until the requester is happy.

### 6.2. Multiple Composers

Mark: include failure modes

Our second use case considers the case where multiple composers create a music specification together, and then collaboratively compose music that stays on track with the original specification.

### 6.3. Therapist with Agent - Human Composition Team

David

Our third case considers the situation where a music therapist would like music to use with their patients. These pieces may have a more highly-refined specification than music for casual listening. The specification may follow a treatment plan and may require a specific tempo or special therapeutic timbres (sound qualities).

high volume necessity  
given the detailed specification, an artificially intelligent agent may do a large amount of work, which is then checked by a human composer

## 7. Evaluation Methodology

Mark

## 8. Discussion

David + Mark

### 8.1. Enhancing or Stifling Creativity

Notes: evaluation is optional. Can be ignored by committer.

### 8.2. Limitations

David

Collaboration is offline, not real-time  
Mark

Are there any generalizations of the automated analysis? It's a special case of semantic analysis that is specific to temporal structure. What would it be like to work on a story? The focus should be on considering things that do not rely on external crowdsourced work (point made in IdeaHound).

Mark

Current music representation is discrete MIDI, not audio. Limits for vocals, ocean sounds MIDI not Music XML. Sure, MIDI blocks could be used as placeholders for later-inserted, real-audio sounds, but it would be nice to avoid the need for future post-production work. Would be nice for composers to collaborate and have a finished piece at the end of the collaboration.

Mark + David

Presume that reliable corpus-based genre and mood classification solutions exist, particularly information retrieval procedures

## 9. Future work

David + Mark

- MusicXML

Mark: MusicXML instead of MIDI. Makes automatic evaluation more difficult

- Improved agent composition

David: 1. maybe re-mention here how could work when human agents communicate their intentions concisely and formally. 2. reinforcement learning

- Intelligent ad hoc composition

David: what would this be?

- Facilitator of scalable music composition

David: How is this different? What what change? is it the “scalable” aspect?

- improved evaluator

Mark: Improved evaluator. Multiple mutual information for interacting parameters of music.

## 10. Conclusion

David + Mark

Mark: Probably important to mention again how this is an extension to things like SoundCloud and Blend

Two Novel Contributions:

- Collaborative music composition system Intentionality, SharedPlan and Agents
- Algorithmic evaluation of composition against intention

## 11. References

David + Mark: clean this up, add proper citations in text

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