Appendix 1: C-Band Estimation Via R

First, the libraries needed for this project need to be loaded.

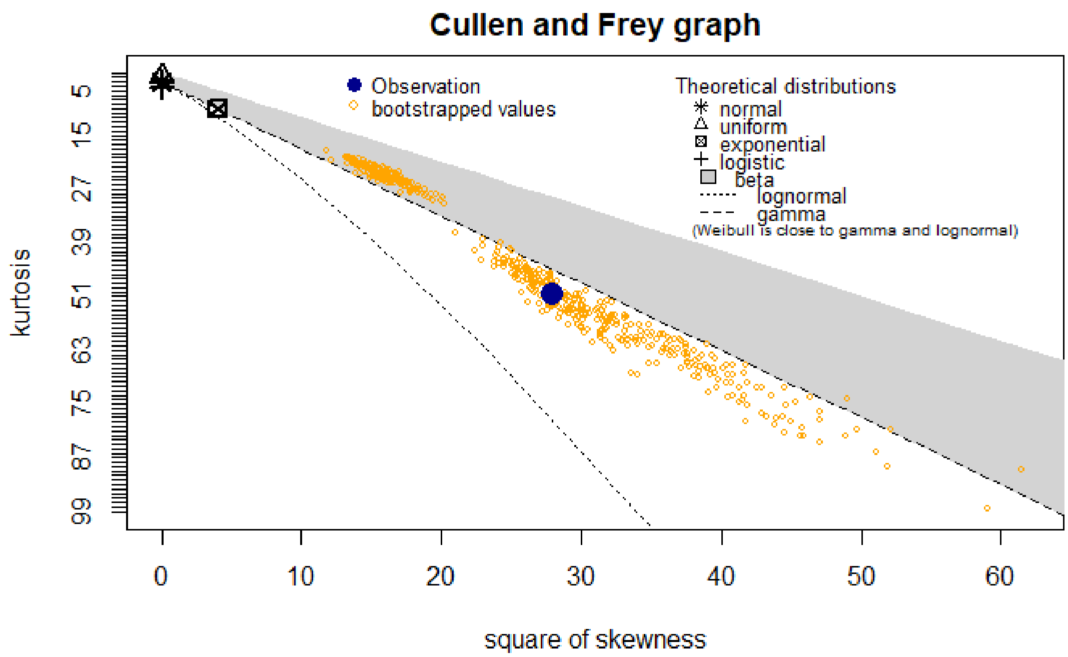
```
library(tidyr)
library(MASS)
library(dplyr)
library(fitdistrplus)
library(rstan)
library(car)
library(readxl)
```

Next, let's take a look at the previous auctions in the C-Band range, from 1 GHz up to about 5 GHz to see what the distributions look like.

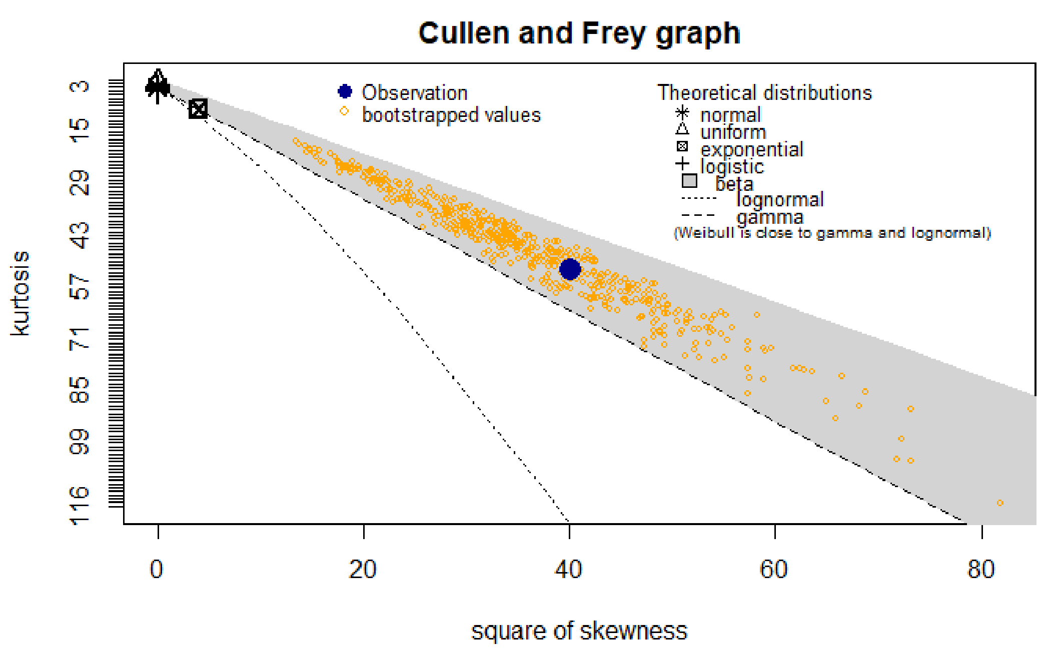
```
auction_data <- read.csv("Auction_Joins.csv", header=T) %>%
  mutate(Cost_MHz_pop = as.numeric(as.character(Cost_MHz_pop))) %>%
  filter(Cost_MHz_pop > 0)

auction_data41 <- auction_data %>% filter(AUCTION_NUMBER == "41")
auction_data53 <- auction_data %>% filter(AUCTION_NUMBER == "53")
auction_data58 <- auction_data %>% filter(AUCTION_NUMBER == "58")
auction_data66 <- auction_data %>% filter(AUCTION_NUMBER == "66")
auction_data69 <- auction_data %>% filter(AUCTION_NUMBER == "69")
auction_data71 <- auction_data %>% filter(AUCTION_NUMBER == "71")
```

All of the data has been normalized to June 2019 prices using CPI indices. How do these data fit?

```
descdist(auction_data$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.000226538   max:  6.55685
median:  0.09479346
mean:  0.2335032
estimated sd:  0.4023915
estimated skewness:  5.286192
estimated kurtosis:  50.42109
```

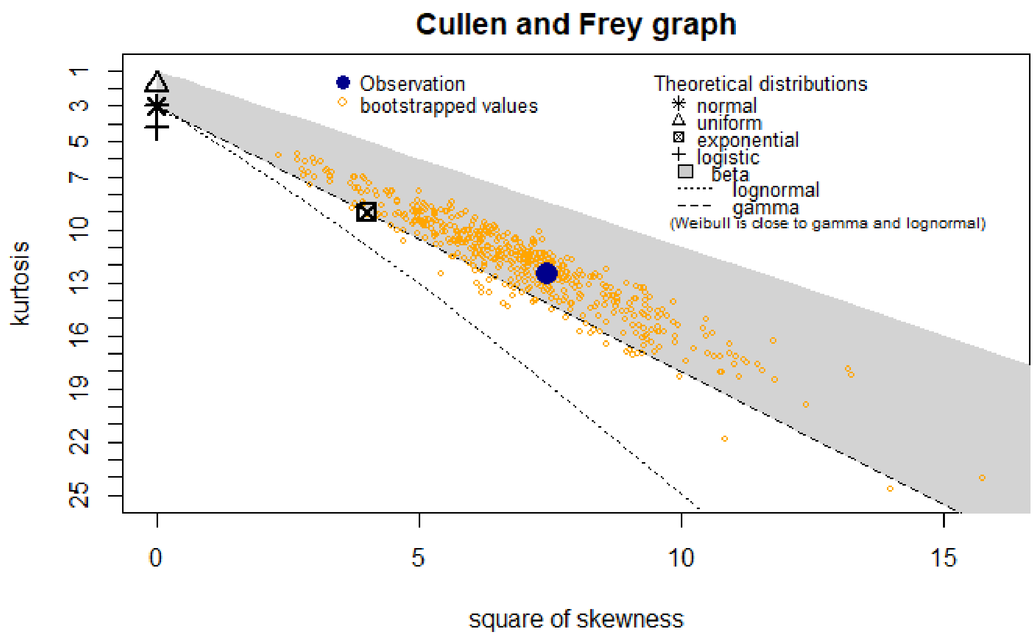



```
descdist(auction_data41$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.02054818   max:  0.5112019
median:  0.02167761
mean:  0.03591208
estimated sd:  0.04909714
estimated skewness:  6.335113
estimated kurtosis:  52.24581
```

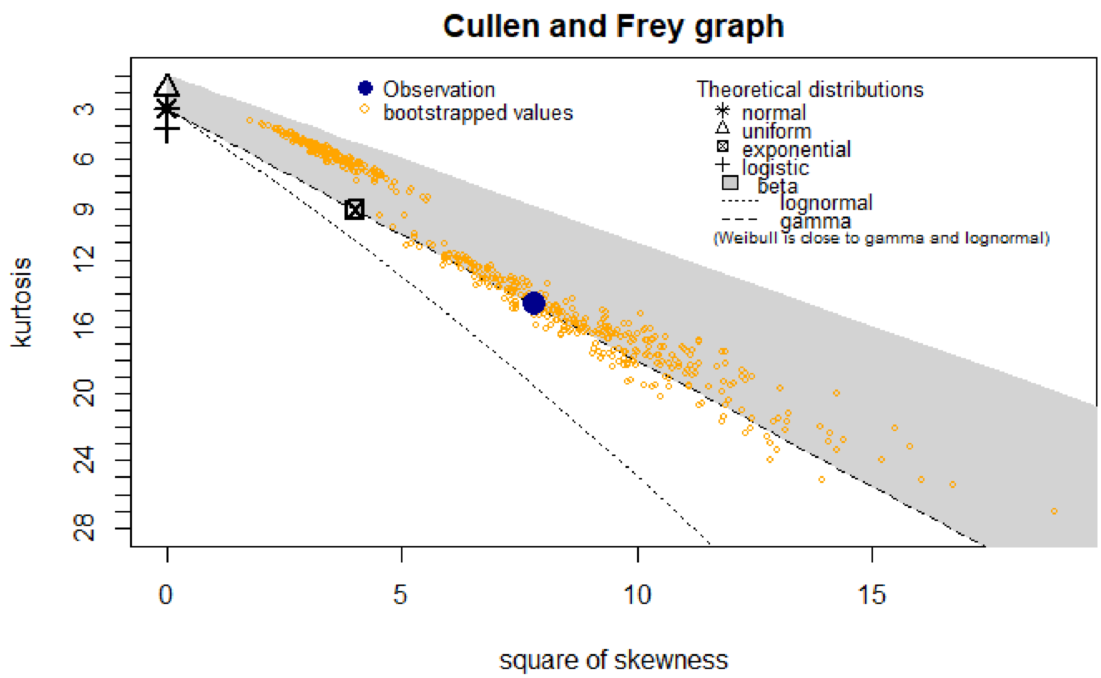



```
descdist(auction_data53$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
```

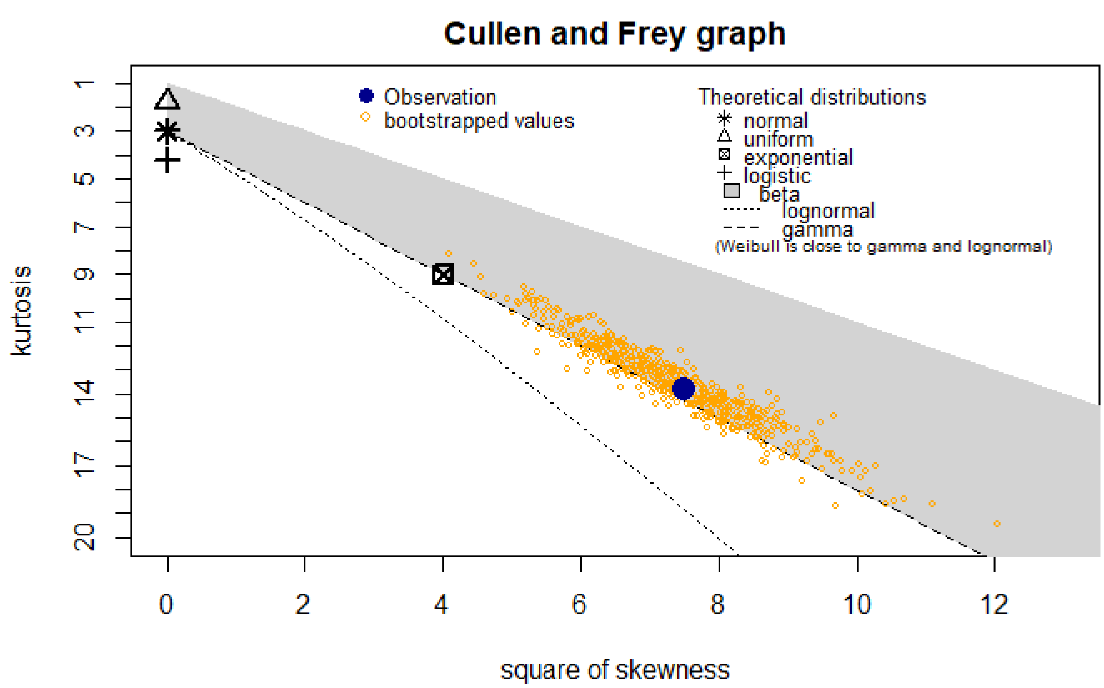
```
------
min:  0.000226538   max:  0.009314287
median:  0.000647125
mean:  0.001276631
estimated sd:  0.001524143
estimated skewness:  2.726179
estimated kurtosis:  12.48186
```



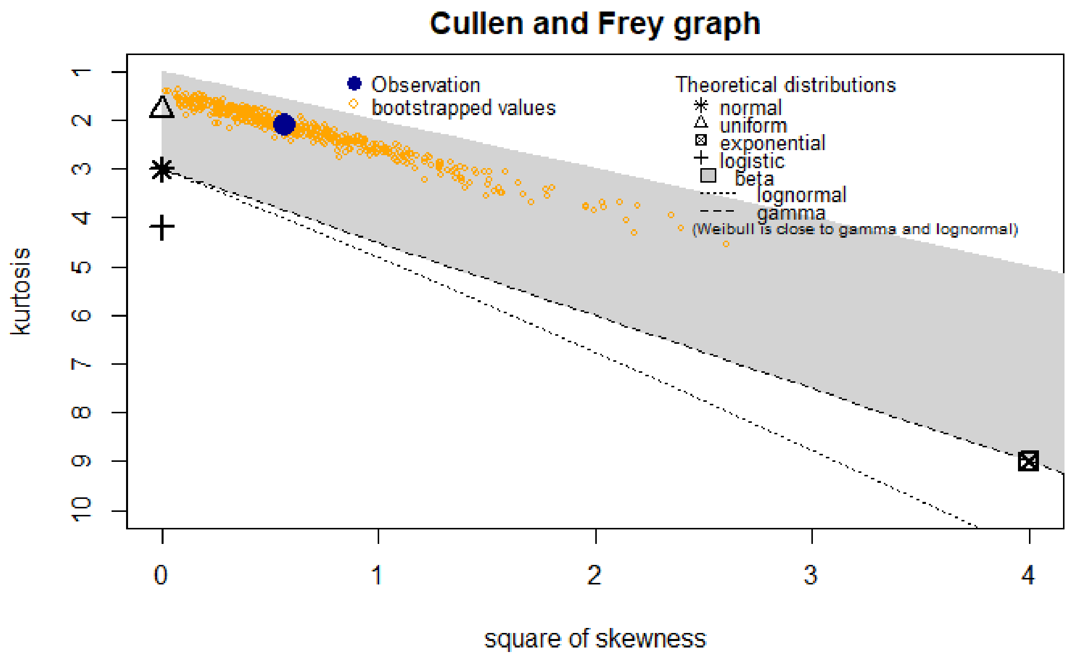

```
descdist(auction_data58$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.1488333   max:  6.55685
median:  0.3986387
mean:  0.7460484
estimated sd:  0.8318109
estimated skewness:  2.794087
estimated kurtosis:  14.60854
```



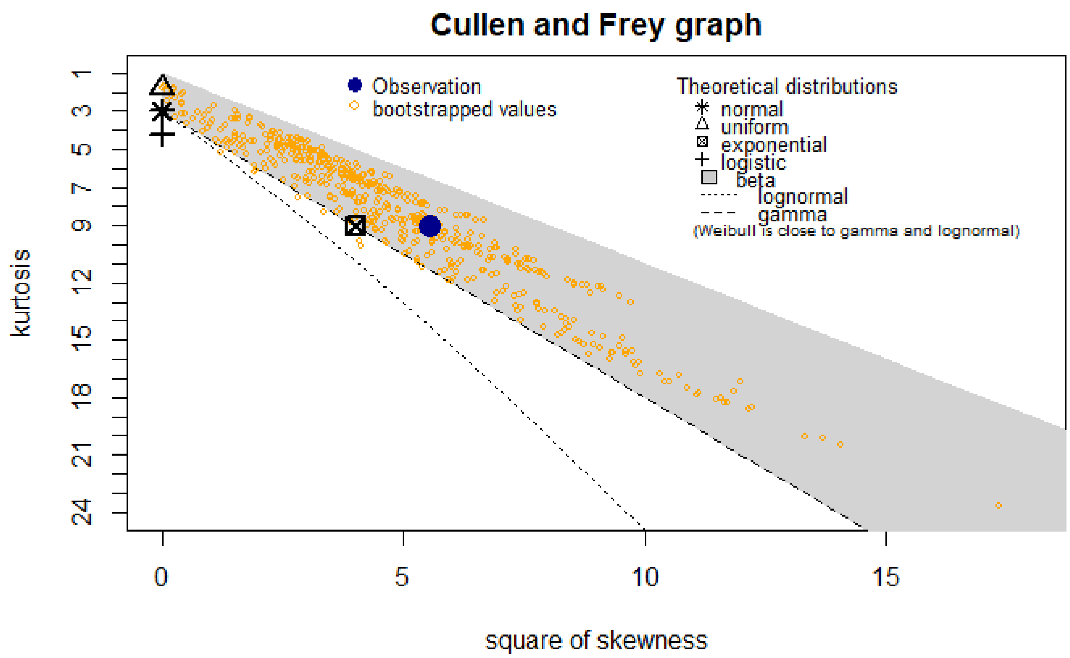

```
descdist(auction_data66$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.02759133   max:  2.003781
median:  0.1488242
mean:  0.2330977
estimated sd:  0.2574933
estimated skewness:  2.736954
estimated kurtosis:  13.8367
```



```
descdist(auction_data69$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.03063546   max:  0.2322091
median:  0.07403464
mean:  0.1032592
estimated sd:  0.05933441
estimated skewness:  0.754167
estimated kurtosis:  2.115828
```




```
descdist(auction_data71$Cost_MHz_pop, discrete=FALSE, boot=500)
summary statistics
------
min:  0.04651388   max:  1.655888
median:  0.3041648
mean:  0.3461359
estimated sd:  0.3694892
estimated skewness:  2.356229
estimated kurtosis:  9.020637
```



So, it seems the best distribution here is the Gamma distribution. So, let's fit the data we have on the C-Band auctions around the world and fit them to a Gamma distribution.

```
c_band_auctions <- read_excel("5G_spectrum_database.xlsx", sheet = 2)
fp <- fitdist(c_band_auctions$fee_per_mhz_pop_june2019, "gamma")
NaNs producedNaNs produced
summary(fp)
Fitting of the distribution ' gamma ' by maximum likelihood
Parameters :
Loglikelihood:  15.60839   AIC:  -27.21678   BIC:  -24.70059
Correlation matrix:
          shape      rate
shape 1.0000000 0.7866944
rate  0.7866944 1.0000000
```

In other words, $\alpha = 1.040088$, $\beta = 5.150726$. Since the mean is α/β (1.040088/5.150726), the calculation comes to 0.201. Next let's compute the 95% confidence interval (CI) through the quantile function in R.

```
quantile(fp, probs=c(0.05, 0.95))
Estimated quantiles for each specified probability (non-censored data)
```

This gives us a mean of 0.201 with a 95% CI of (0.01140201, 0.596599).

Appendix 2: A Cost-Benefit Analysis of C-Band

From Appendix 1, the median sales price was estimated at $0.201 per MHz per population (MHz pop), with an upper band of $0.597 per MHz pop and a lower band of $0.011 per MHz pop. Thus, there are three estimates for a price: a low, mean, and high. From here, the price per MHz per population is multiplied by 320 million Americans and is again multiplied by the amount of MHz that is projected to come to market. The potential revenue from nine potential scenarios for spectrum sales are listed below:

	C-Band Low Estimate	C-Band Mean Estimate	C-Band High Estimate
100 MHz	$364.9 million	$6.4 billion	$19.1 billion
180 MHz	$656.8 million	$11.6 billion	$34.3 billion
370 MHz	$1.3 billion	$23.9 billion	$70.6 billion

Still, changing spectrum allocations aren't common, and for good reason: It is a costly endeavor. In the 2016 Incentive Auction where broadcasters voluntarily gave up their station or switched frequencies, the total transition costs came to nearly $2.1 billion, or around $2.4 million for each of the 900-plus stations.

The C-Band is currently used by satellite companies to distribute broadcast radio and TV programming to more than 100 million U.S. households and radio listeners. Moving current C-Band emitters would require them to install a new prototype band-pass filters at a cost of around $855 million. Intelsat and SES might also need to launch a new satellite, costing around $200 million, to retain customers. Thus, the net low-end cost could be $1.055 billion.

Assuming this cost to transition the space, a simple cost-benefit analysis yields the following scenarios:

	C-Band Low Prices	C-Band Mean Prices	C-Band High Prices
100 MHz	($690.1 million)	$5.4 billion	$18 billion
180 MHz	($398.2 million)	$10.6 billion	$33.3 billion
370 MHz	$290 million	$22.9 billion	$69.6 billion

Of course, the $1.055 billion might be a low-end estimate. In a filing, economist Coleman Bazelon calculated that the costs to transition could sum to $19.7 billion. Under that cost structure, the scenarios would be reduced to the values below.

	C-Band Low Estimate	C-Band Mean Estimate	C-Band High Estimate
100 MHz	($19.3 billion)	($13.2 billion)	($608.9 million)
180 MHz	($19.0 billion)	($8.1 billion)	$14.7 billion
370 MHz	($18.4 billion)	$4.2 billion	$50.9 billion

Appendix 3: Yearly Investment Assumptions

Similar to Bazelon, this net present value calculation assumes that the spectrum sale with net cash flows that break even at the end of the 5-year mark. To simplify the calculation, the initial investment, often modeled as I_0 is set at 1 where

$$NPV = \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} - I_0$$

In this case, with a discount of 0.08, the returns for each year can be expressed:

- 1 = (x/(1+0.08)^1) + (x/(1+0.08)^2) + (x/(1+0.08)^3) + (x/(1+0.08)^4) + (x/(1+0.08)^5)
- x=0.250456

Assuming a one-year delay with a discount of 0.08:

- 1 = (0/(1+0.08)^1) + (x/(1+0.08)^2) + (x/(1+0.08)^3) + (x/(1+0.08)^4) + (x/(1+0.08)^5) + (x/(1+0.08)^6)
- x=0.270493
- The percent change from a baseline of 0.250456 equates to 8 percent.

Assuming a two-year delay with a discount of 0.08:

- 1 = (0/(1+0.08)^1) + (0/(1+0.08)^2) + (x/(1+0.08)^3) + (x/(1+0.08)^4) + (x/(1+0.08)^5) + (x/(1+0.08)^6) + (x/(1+0.08)^7)
- x=0.292132
- The percent change from a baseline of 0.250456 equates to 16.6 percent

If however, the discount were 0.12, the returns for each year can be expressed:

- 1 = (x/(1+0.12)^1) + (x/(1+0.12)^2) + (x/(1+0.12)^3) + (x/(1+0.12)^4) + (x/(1+0.12)^5)
- x=0.27741

Assuming a one-year delay with a discount of 0.12:

- 1 = (0/(1+0.12)^1) + (x/(1+0.12)^2) + (x/(1+0.12)^3) + (x/(1+0.12)^4) + (x/(1+0.12)^5) + (x/(1+0.12)^6)
- x=0.310699
- The percent change from a baseline of 0.27741 equates to 12 percent.

Assuming a two-year delay with a discount of 0.12:

- 1 = (0/(1+0.12)^1) + (0/(1+0.12)^2) + (x/(1+0.12)^3) + (x/(1+0.12)^4) + (x/(1+0.12)^5) + (x/(1+0.12)^6) + (x/(1+0.12)^7)
- x=0.347983
- The percent change from a baseline of 0.27741 equates to 25.4 percent