SWX1 Results Analysis C

Python Imports

```
import numpy as np
import pandas as pd
from prettypandas import PrettyPandas
import patsy
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
import statsmodels.formula.api
from pyomo.environ import *

from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures

from IPython.display import display, Markdown, HTML

%matplotlib inline
PlotWidth = 6

import warnings
warnings.filterwarnings('ignore')
```

```
# helper functions for this notebook

# use SVG for matplotlib-based figures
%matplotlib inline
%config InlineBackend.figure_format = 'svg'

def coded_to_actual(coded_data, actual_lows, actual_highs):
    """Converts a pandas DataFrame from coded units to actuals."""
    actual_data = coded_data.copy()
    for col in actual_data.columns:
        if not (col in actual_highs and col in actual_lows):
            continue
        try:
            # convert continuous variables to their actual value
            actual_data[col] *= 0.5 * (float(actual_highs[col]) - float(actual_lows[co
l]))
            # don't need to cast to float here, if either are not a float exception wi
ll have been thrown
            actual_data[col] += 0.5 * (actual_highs[col] + actual_lows[col])
        except ValueError:
            # assume 2 level categorical
```

```
            actual_data[col] = actual_data[col].map({-1: actual_lows[col], 1: actual_h
ighs[col]})
    return actual_data

def get_tick_labels(key, lows, highs, units):
    """Returns a list of low/high labels with units (e.g. [8mm, 10mm])"""
    return [str(lows[key]) + units[key], str(highs[key]) + units[key]]

def backward_regression(X, y,
                           threshold_out,
                           verbose=True):
    included=list(X.columns)
    while True:
        changed=False
        model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included]))).fit()
        # use all coefs except intercept
        pvalues = model.pvalues.iloc[1:]
        worst_pval = pvalues.max() # null if pvalues is empty
        if worst_pval > threshold_out:
            changed=True
            worst_feature = pvalues.idxmax()
            included.remove(worst_feature)
            if verbose:
                print('Drop {:30} with p-value {:.6}'.format(worst_feature, worst_pval
))
        if not changed:
            break

    return included

def build_model(X,values,verbose=True):
    X = [sub.replace('alh', 'model.X1') for sub in X]
    X = [sub.replace('aps', 'model.X2') for sub in X]
    X = [sub.replace('aid', 'model.X3') for sub in X]
    X = [sub.replace('arw', 'model.X4') for sub in X]
    X = [sub.replace('awt', 'model.X5') for sub in X]
    X = [sub.replace(':', '*') for sub in X]
    model = str(values[0])
    i=1
    for v in X:
        model  += " + " + str(values[i]) + " * " + v
        i += 1
    if verbose:
        print(model)
    return model
```

Process CSV Files

```
# importing the pandas library
import pandas as pd

# reading the csv file using read_csv
# storing the data frame in variable called df
df_cost = pd.read_csv('https://raw.githubusercontent.com/wilsongis/3DP_Experiments/main/Data/swx1_cost_rework.txt', sep='\t')
df_time = pd.read_csv('https://raw.githubusercontent.com/wilsongis/3DP_Experiments/main/Data/swx1_time_rework.txt', sep='\t')

# creating a list of column names by
# calling the .columns
list_of_columns_cost = list(df_cost.columns)
list_of_columns_time = list(df_time.columns)

# displaying the list of column names
print('List of Cost column names : ',
      list_of_columns_cost)
print('List of Time column names : ',
      list_of_columns_time)
```

```
List of Cost column names :  ['trial', 'lh', 'ps', 'id', 'rw', 'wt', 'alh', 'aps', 'aid', 'arw', 'awt', 'rep', 'cost']
List of Time column names :  ['trial', 'lh', 'ps', 'id', 'rw', 'wt', 'alh', 'aps', 'aid', 'arw', 'awt', 'rep', 'time']
```

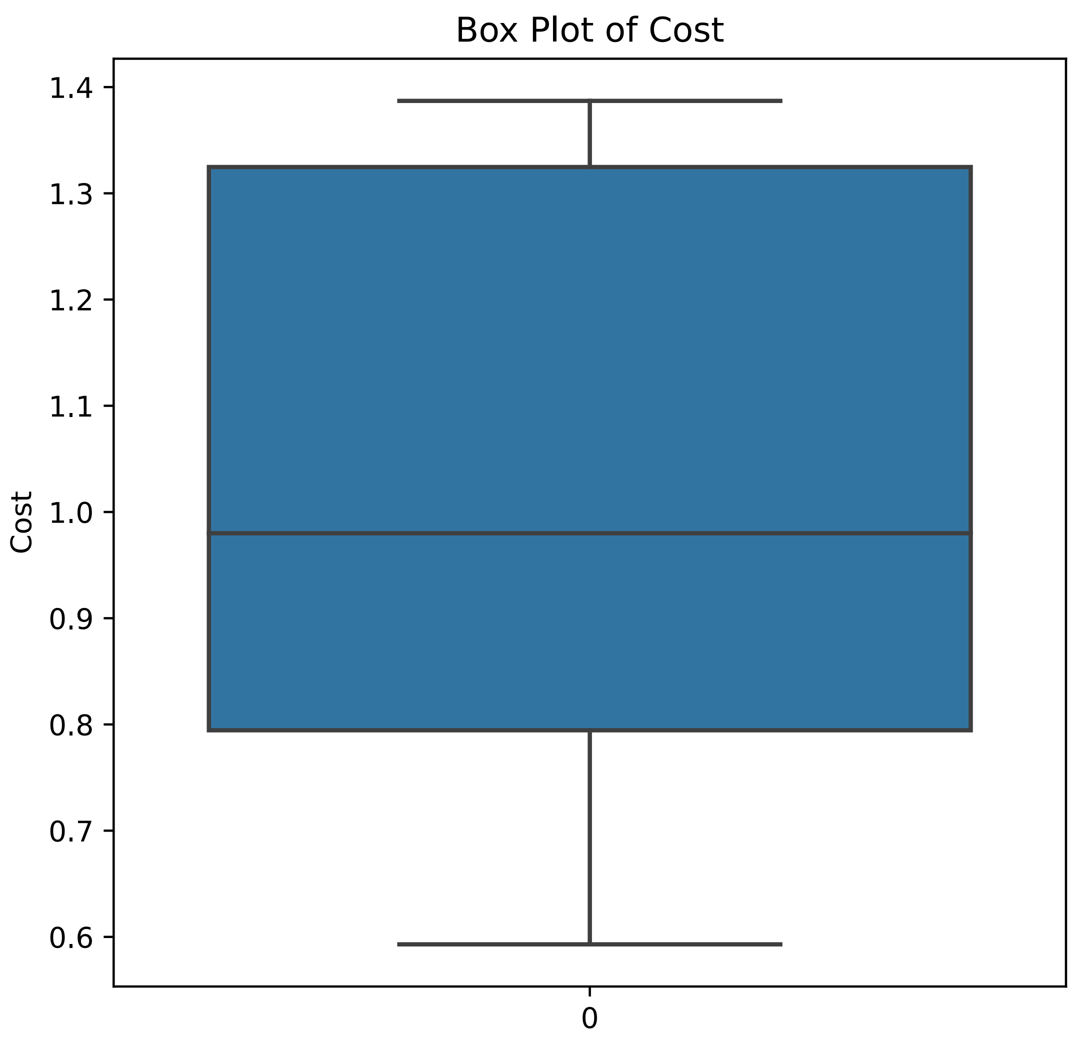
```
display((Markdown("### Statistics for Cost")))
df_cost.cost.describe()
```

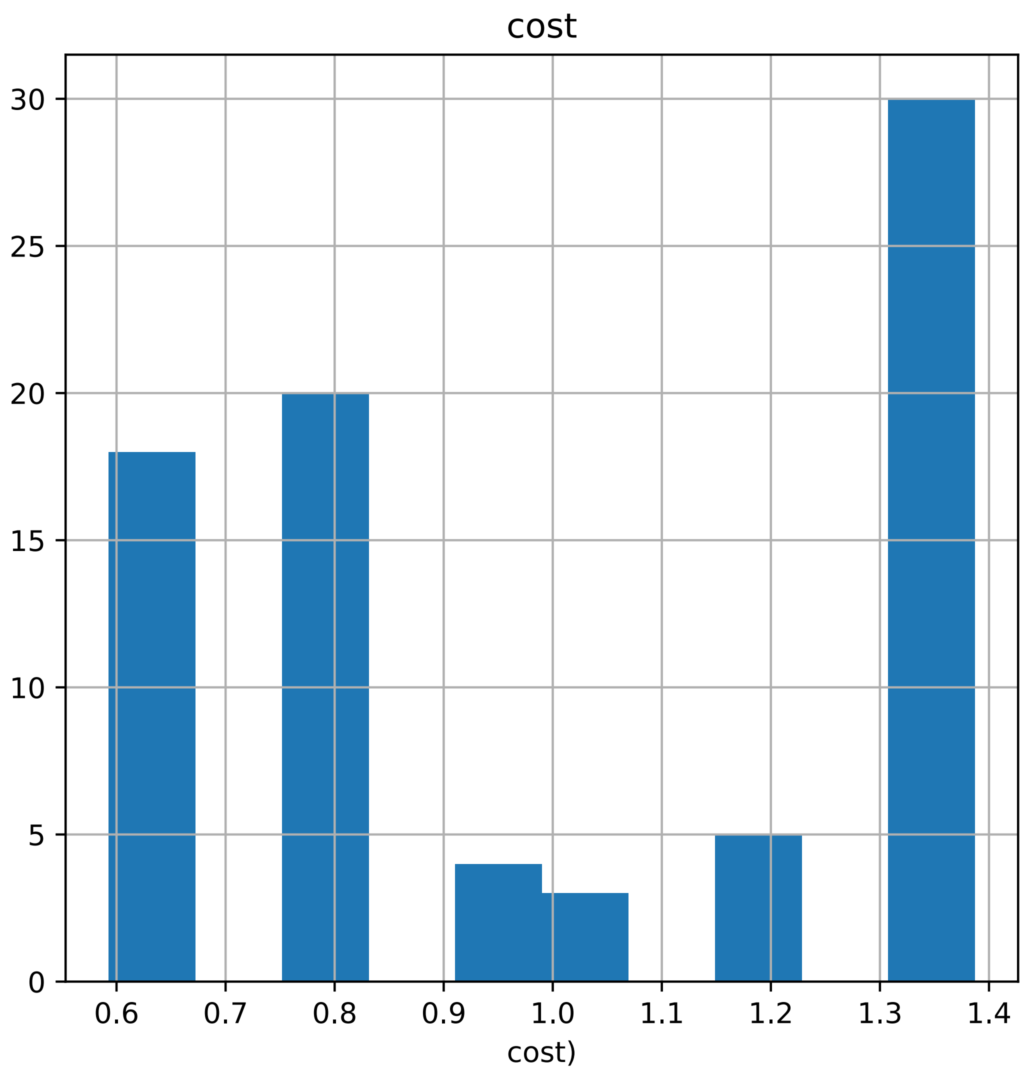
Statistics for Cost

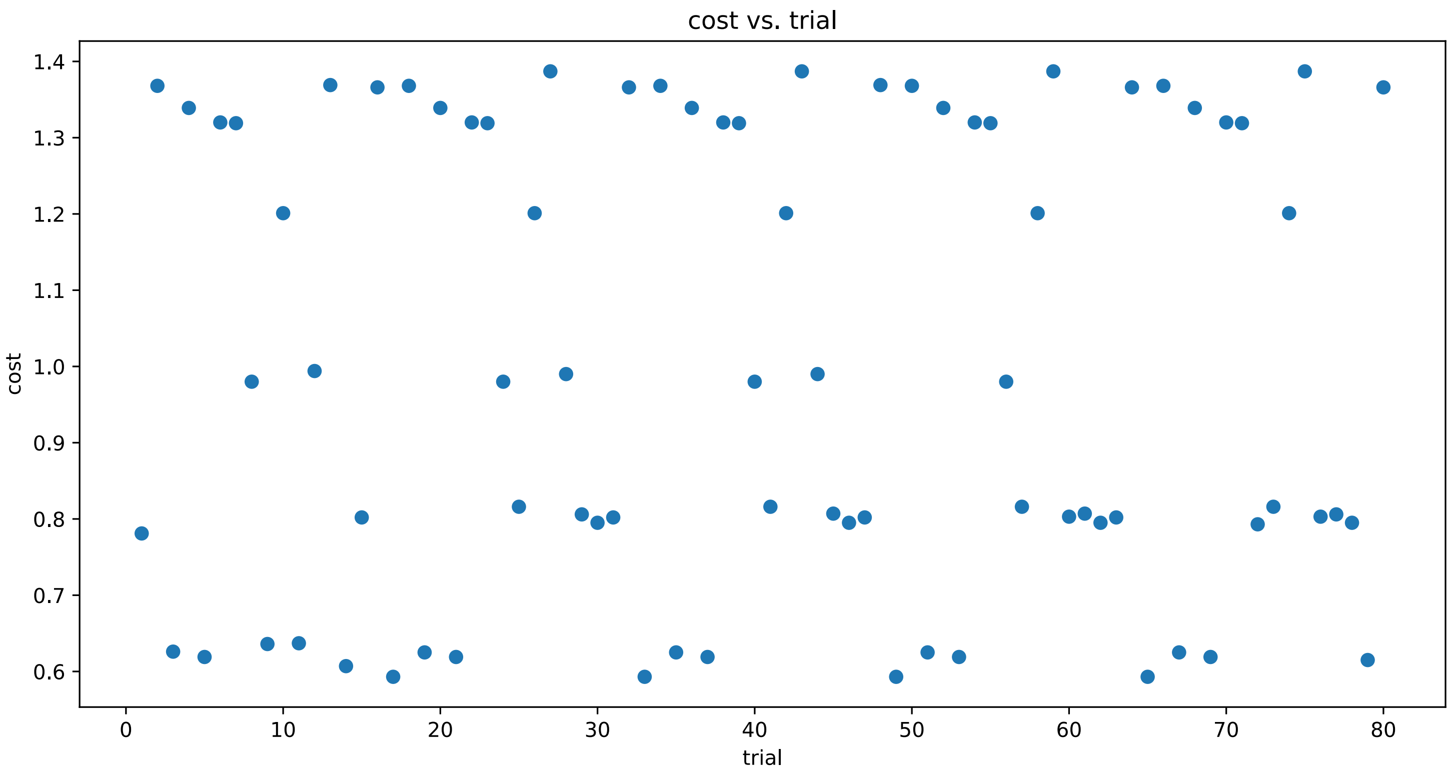
```
count    80.000000
mean      1.006563
std       0.304323
min       0.593000
25%       0.794500
50%       0.980000
75%       1.324750
max       1.387000
Name: cost, dtype: float64
```

```
plt.figure(figsize=(PlotWidth, PlotWidth))
sns.boxplot(data=df_cost['cost'])
plt.title('Box Plot of Cost')
plt.ylabel('Cost')
plt.show()
```

```
plt.figure(figsize=(PlotWidth, PlotWidth))
df_cost['cost'].hist()
plt.title('cost')
plt.xlabel('cost)')
plt.show()

plt.figure(figsize=(PlotWidth*2, PlotWidth))
plt.scatter(df_cost['trial'], df_cost['cost'])
plt.title('cost vs. trial')
plt.xlabel('trial')
plt.ylabel('cost')
plt.show()
```




cost
30
25
20
15
10
5
0
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
cost)




```
display((Markdown("### Statistics for Time")))
df_time.time.describe()
```

Statistics for Time

```
count      80.000000
mean    11719.350000
std      2959.361294
min      7767.000000
25%      9101.000000
50%     11418.500000
75%     14610.500000
max     18254.000000
Name: time, dtype: float64
```

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.boxplot(data=df_time['time'])
plt.title('Box Plot of Time')
plt.ylabel('Time')
plt.show()

plt.figure(figsize=(PlotWidth, PlotWidth))
df_time['time'].hist()
plt.title('time')

plt.xlabel(‘time)’)
plt.show()

plt.figure(figsize=(PlotWidth*2, PlotWidth))
plt.scatter(df_time[‘trial’], df_time[‘time’])
plt.title(‘time vs. trial’)
plt.xlabel(‘trial’)
plt.ylabel(‘time’)
plt.show()

Cost Analysis

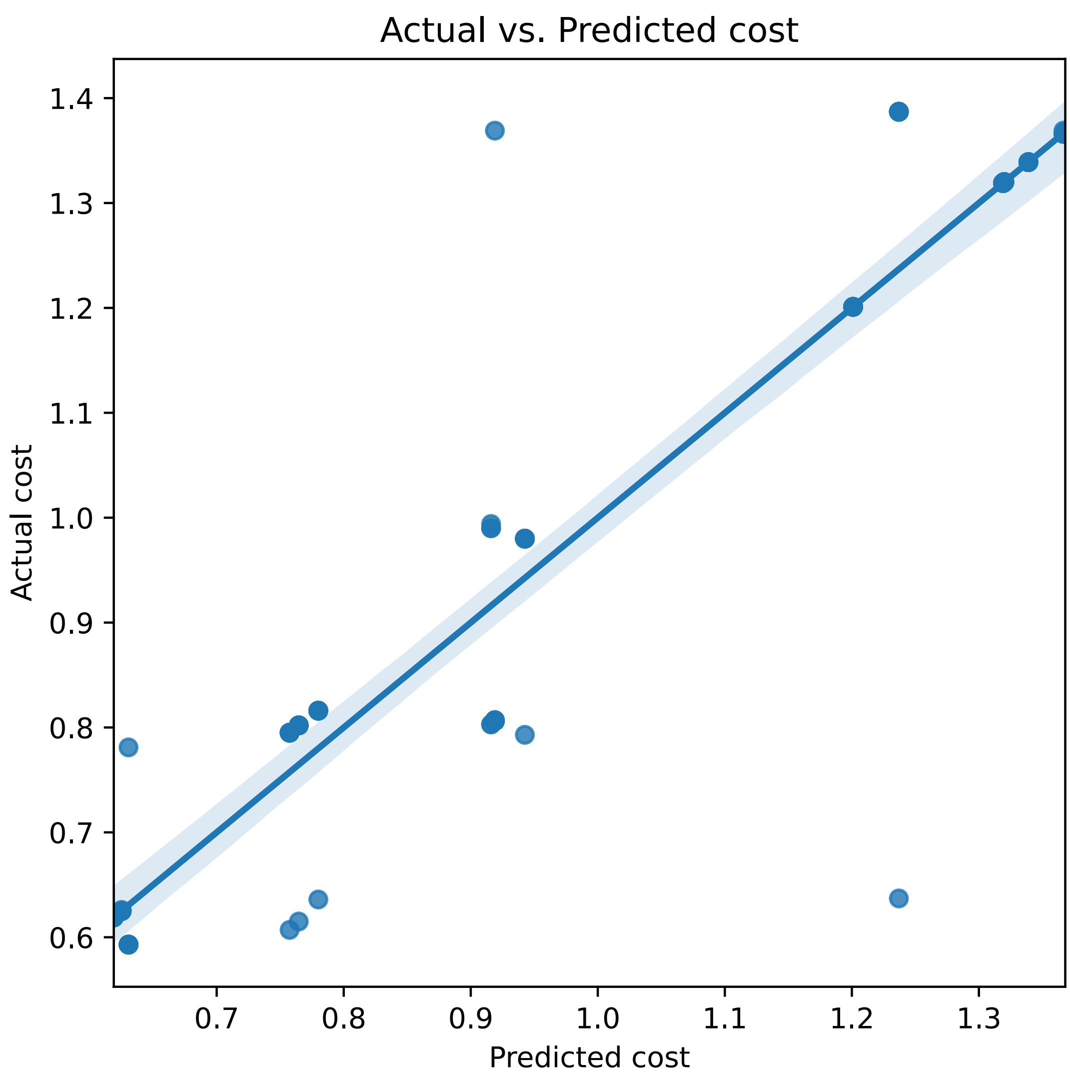
```
f = 'cost ~ (alh+aps+aid+arw+awt)**2'
y, X = patsy.dmatrices(f, df_cost, return_type='dataframe')
print(y[:5])
print(X[:5])
```

```
    cost
0  0.781
1  1.368
2  0.626
3  1.339
4  0.619
   Intercept   alh   aps   aid  arw  awt  alh:aps  alh:aid  alh:arw  alh:awt  \
0        1.0  0.16  60.0  0.25  0.4  0.8     9.60    0.040    0.064    0.128
1        1.0  0.28  60.0  0.25  0.4  1.2    16.80    0.070    0.112    0.336
2        1.0  0.16  72.0  0.25  0.4  1.2    11.52    0.040    0.064    0.192
3        1.0  0.28  72.0  0.25  0.4  0.8    20.16    0.070    0.112    0.224
4        1.0  0.16  60.0  0.15  0.4  1.2     9.60    0.024    0.064    0.192

   aps:aid  aps:arw  aps:awt  aid:arw  aid:awt  arw:awt
0     15.0     24.0     48.0     0.10     0.20     0.32
1     15.0     24.0     72.0     0.10     0.30     0.48
2     18.0     28.8     86.4     0.10     0.30     0.48
3     18.0     28.8     57.6     0.10     0.20     0.32
4      9.0     24.0     72.0     0.06     0.18     0.48
```

```

## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
```

```
plt.xlabel('Predicted cost')
plt.ylabel('Actual cost')
plt.title('Actual vs. Predicted cost')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.879
Model:                            OLS   Adj. R-squared:                  0.851
Method:                 Least Squares   F-statistic:                     31.04
Date:                Sat, 31 Jul 2021   Prob (F-statistic):           1.26e-23
Time:                        18:50:16   Log-Likelihood:                 66.692
No. Observations:                  80   AIC:                            -101.4
Df Residuals:                      64   BIC:                            -63.27
Df Model:                          15
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -6.7997      1.267     -5.365      0.000      -9.332      -4.268
alh           14.9792      2.872      5.215      0.000       9.241      20.717
aps            0.1432      0.018      8.147      0.000       0.108       0.178
aid           -3.8274      3.421     -1.119      0.267     -10.662       3.007
arw            0.1830      0.873      0.210      0.835      -1.561       1.927
awt            3.1783      0.832      3.819      0.000       1.516       4.841
alh:aps       -0.1874      0.037     -5.134      0.000      -0.260      -0.114
alh:aid       16.3792      4.380      3.739      0.000       7.629      25.130
alh:arw       -6.4344      1.095     -5.876      0.000      -8.622      -4.247
alh:awt        0.3865      1.095      0.353      0.725      -1.801       2.574
aps:aid       -0.1333      0.044     -3.043      0.003      -0.221      -0.046
aps:arw        0.0176      0.011      1.610      0.112      -0.004       0.040
aps:awt       -0.0764      0.011     -6.977      0.000      -0.098      -0.055
aid:arw        3.5262      1.314      2.683      0.009       0.901       6.151
aid:awt        7.0162      1.314      5.339      0.000       4.391       9.641
arw:awt       -0.7059      0.329     -2.149      0.035      -1.362      -0.050
==============================================================================
Omnibus:                       51.730   Durbin-Watson:                   2.230
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              778.361
Skew:                          -1.374   Prob(JB):                    9.57e-170
Kurtosis:                      18.032   Cond. No.                     3.75e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
[2] The condition number is large, 3.75e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```



Reduced Cost Model

```
cost_included = backward_regression(X,y,.05)
cost_included.pop(0)
print(cost_included)
```

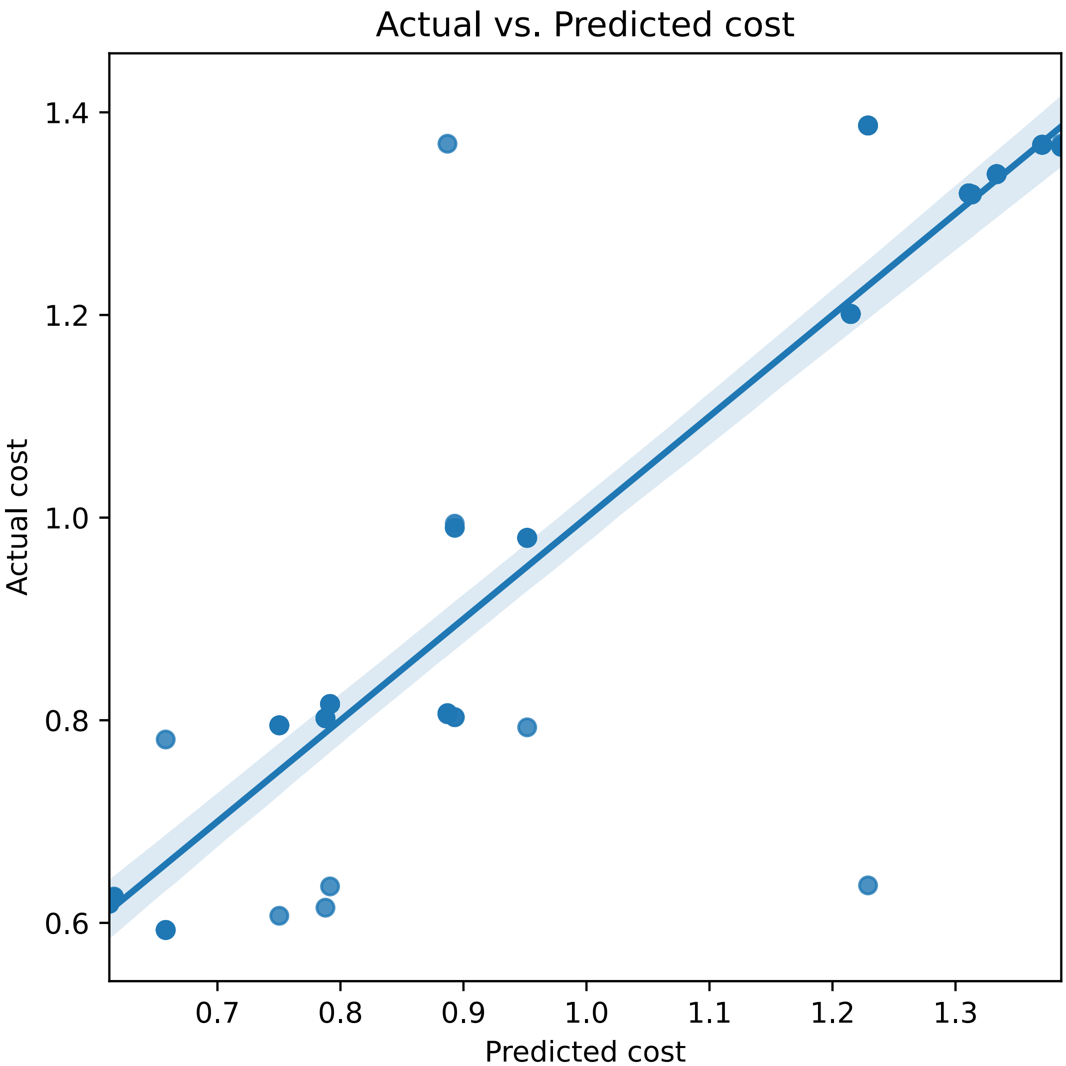
```
Drop arw                                with p-value 0.834584
Drop alh:awt                            with p-value 0.723342
Drop aid                                with p-value 0.253413
['alh', 'aps', 'awt', 'alh:aps', 'alh:aid', 'alh:arw', 'aps:aid', 'aps:arw', 'aps:awt'
, 'aid:arw', 'aid:awt', 'arw:awt']
```

```
y = df_cost['cost']
#y = df_cost['time']
X = X[cost_included]

## An intercept is not added by default, so we need to add that here
```

```python
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted cost')
plt.ylabel('Actual cost')
plt.title('Actual vs. Predicted cost')
plt.show()
```

```
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.876
Model:                            OLS   Adj. R-squared:                  0.854
Method:                 Least Squares   F-statistic:                     39.59
Date:                Sat, 31 Jul 2021   Prob (F-statistic):           1.08e-25
Time:                        18:50:18   Log-Likelihood:                 65.790
No. Observations:                  80   AIC:                            -105.6
Df Residuals:                      67   BIC:                            -74.61
Df Model:                          12                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -7.5128      0.913     -8.232      0.000      -9.334      -5.691
alh           15.5943      2.608      5.979      0.000      10.388      20.800
aps            0.1501      0.015     10.015      0.000       0.120       0.180
awt            3.3569      0.778      4.314      0.000       1.804       4.910
alh:aps       -0.1874      0.036     -5.194      0.000      -0.259      -0.115
alh:aid       14.9741      4.153      3.605      0.001       6.684      23.264
alh:arw       -6.3471      1.040     -6.102      0.000      -8.423      -4.271
aps:aid       -0.1754      0.023     -7.625      0.000      -0.221      -0.130
aps:arw        0.0203      0.006      3.356      0.001       0.008       0.032
aps:awt       -0.0764      0.011     -7.059      0.000      -0.098      -0.055
aid:arw        3.2956      1.208      2.728      0.008       0.884       5.707
aid:awt        6.4414      1.199      5.374      0.000       4.049       8.834
arw:awt       -0.6702      0.301     -2.229      0.029      -1.270      -0.070
==============================================================================
Omnibus:                       44.267   Durbin-Watson:                   2.202
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              749.960
Skew:                          -0.987   Prob(JB):                    1.41e-163
Kurtosis:                      17.869   Cond. No.                     3.44e+04
==============================================================================


Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
```

```
[2] The condition number is large, 3.44e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```




```
cost_eq = build_model(cost_included,results.params,False)
print("Cost = " + cost_eq)
```

```
Cost = -7.512756311165274 + 15.594282277667375 * model.X1 + 0.1501013849966114 * model
.X2 + 3.3568541135900944 * model.X5 + -0.18739583333333725 * model.X1*model.X2 + 14.974
06461978116 * model.X1*model.X3 + -6.3471031138125085 * model.X1*model.X4 + -0.1754447
2807322514 * model.X2*model.X3 + 0.020253573252292224 * model.X2*model.X4 + -0.07640624
999999765 * model.X2*model.X5 + 3.2956081485919615 * model.X3*model.X4 + 6.44143552627
4299 * model.X3*model.X5 + -0.6702353647414601 * model.X4*model.X5
```

Time Analysis

```
f = 'time ~ (alh+aps+aid+arw+awt)**2'
y, X = patsy.dmatrices(f, df_time, return_type='dataframe')
```

```
print(y[:5])
print(X[:5])
```

```
      time
0  16946.0
1   9136.0
2  12906.0
3   9831.0
4  14617.0
   Intercept   alh   aps   aid  arw  awt  alh:aps  alh:aid  alh:arw  alh:awt  \
0        1.0  0.16  60.0  0.25  0.4  0.8     9.60    0.040    0.064    0.128
1        1.0  0.28  60.0  0.25  0.4  1.2    16.80    0.070    0.112    0.336
2        1.0  0.16  72.0  0.25  0.4  1.2    11.52    0.040    0.064    0.192
3        1.0  0.28  72.0  0.25  0.4  0.8    20.16    0.070    0.112    0.224
4        1.0  0.16  60.0  0.15  0.4  1.2     9.60    0.024    0.064    0.192

   aps:aid  aps:arw  aps:awt  aid:arw  aid:awt  arw:awt
0     15.0     24.0     48.0     0.10     0.20     0.32
1     15.0     24.0     72.0     0.10     0.30     0.48
2     18.0     28.8     86.4     0.10     0.30     0.48
3     18.0     28.8     57.6     0.10     0.20     0.32
4      9.0     24.0     72.0     0.06     0.18     0.48
```

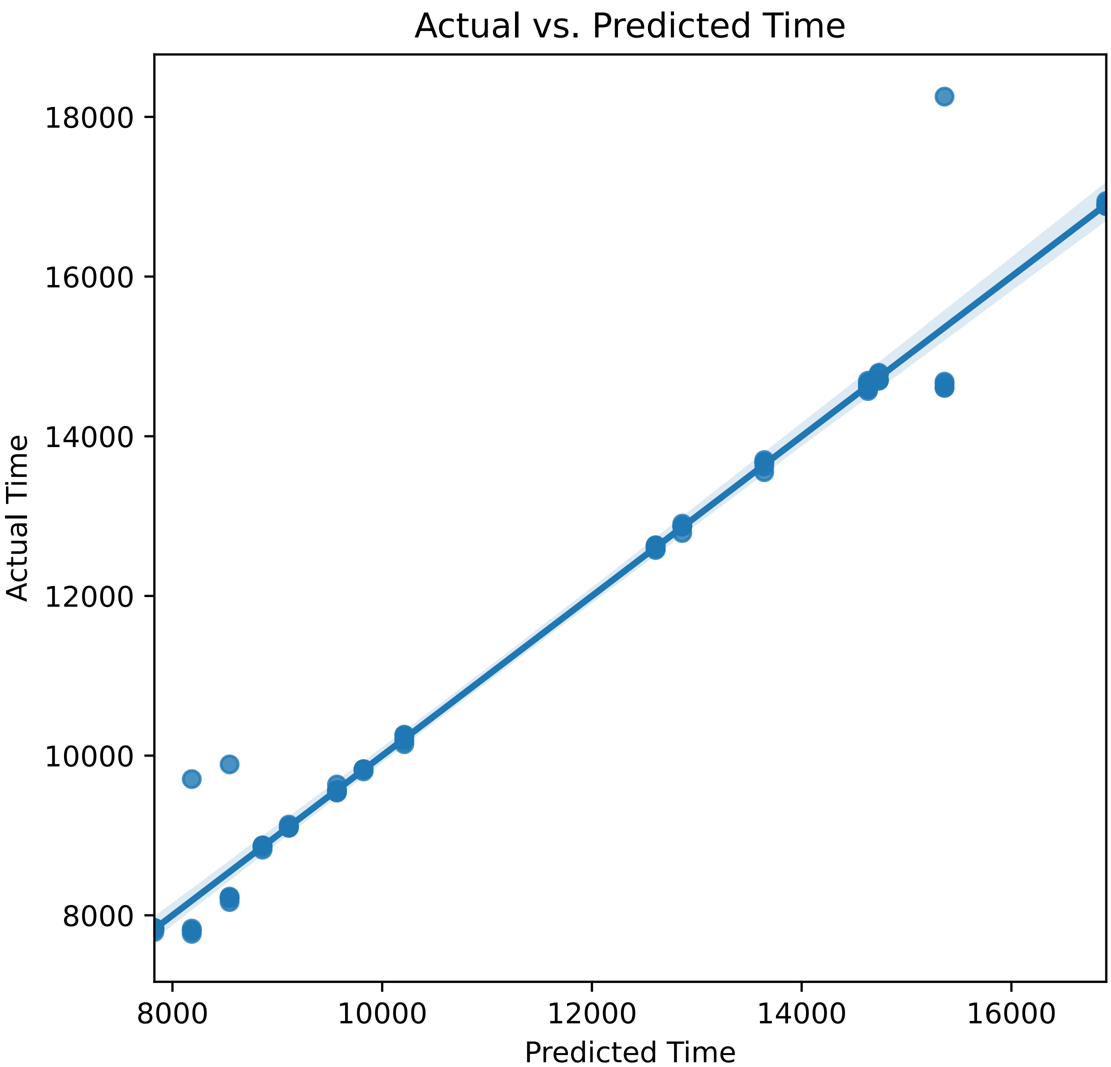
```
## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted Time')
plt.ylabel('Actual Time')
plt.title('Actual vs. Predicted Time')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   time   R-squared:                       0.977
Model:                            OLS   Adj. R-squared:                  0.972
Method:                 Least Squares   F-statistic:                     183.9
Date:                Sat, 31 Jul 2021   Prob (F-statistic):           1.37e-46
Time:                        18:50:20   Log-Likelihood:                -600.98
No. Observations:                  80   AIC:                             1234.
Df Residuals:                      64   BIC:                             1272.
Df Model:                          15
Covariance Type:            nonrobust
==============================================================================
```

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	3.908e+04	5339.602	7.319	0.000	2.84e+04	4.97e+04
alh	-9.723e+04	1.21e+04	-8.035	0.000	-1.21e+05	-7.31e+04
aps	-139.2986	74.068	-1.881	0.065	-287.266	8.669
aid	2.967e+04	1.44e+04	2.058	0.044	875.294	5.85e+04
arw	-1.258e+04	3676.724	-3.423	0.001	-1.99e+04	-5239.900
awt	-2183.1667	3506.060	-0.623	0.536	-9187.327	4820.994
alh:aps	784.8611	153.775	5.104	0.000	477.661	1092.061
alh:aid	-1.917e+04	1.85e+04	-1.039	0.303	-5.6e+04	1.77e+04
alh:arw	4518.7500	4613.237	0.980	0.331	-4697.251	1.37e+04
alh:awt	1497.9167	4613.237	0.325	0.746	-7718.084	1.07e+04
aps:aid	-334.9167	184.529	-1.815	0.074	-703.557	33.723
aps:arw	8.6875	46.132	0.188	0.851	-83.473	100.848
aps:awt	-88.8958	46.132	-1.927	0.058	-181.056	3.264
aid:arw	3845.0000	5535.884	0.695	0.490	-7214.201	1.49e+04
aid:awt	-1395.0000	5535.884	-0.252	0.802	-1.25e+04	9664.201
arw:awt	8843.7500	1383.971	6.390	0.000	6078.950	1.16e+04

Omnibus:	103.490	Durbin-Watson:	1.989
Prob(Omnibus):	0.000	Jarque-Bera (JB):	1972.446
Skew:	4.063	Prob(JB):	0.00
Kurtosis:	25.928	Cond. No.	3.75e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci fied.
[2] The condition number is large, 3.75e+04. This might indicate that there are strong multicollinearity or other numerical problems.



Time Reduced Model

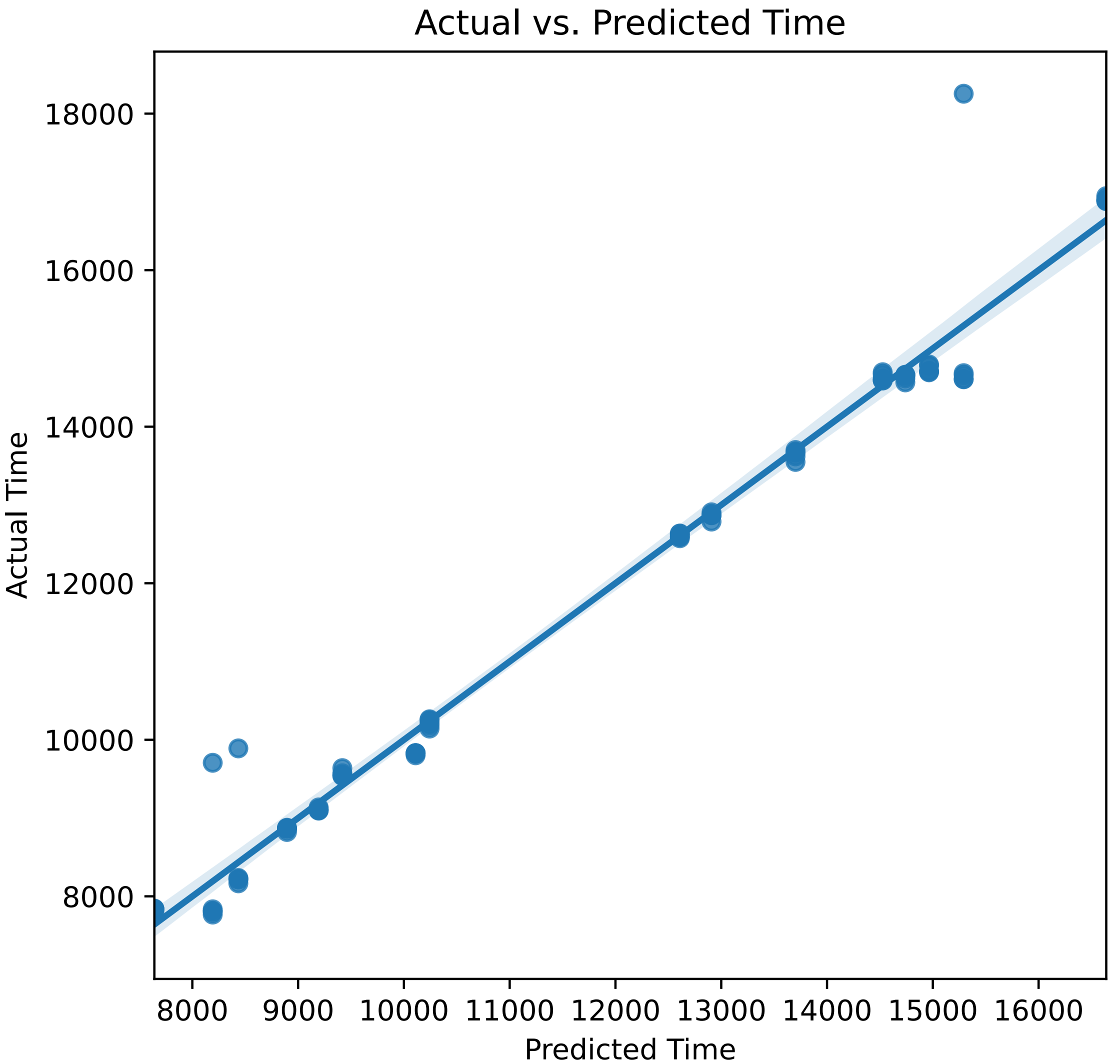
```
time_included = backward_regression(X,y,.05)
time_included.pop(0)
print(time_included)
```

```
Drop aps:arw                        with p-value 0.851224
Drop aid:awt                        with p-value 0.800387
Drop alh:awt                        with p-value 0.742837
Drop awt                            with p-value 0.494038
Drop aid:arw                        with p-value 0.478725
Drop alh:arw                        with p-value 0.316918
Drop alh:aid                        with p-value 0.288586
Drop aps:aid                        with p-value 0.0661897
['alh', 'aps', 'aid', 'arw', 'alh:aps', 'aps:awt', 'arw:awt']
```

```
y = df_time['time']
```

```
X = X[time_included]

## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted Time')
plt.ylabel('Actual Time')
plt.title('Actual vs. Predicted Time')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   time   R-squared:                       0.975
Model:                            OLS   Adj. R-squared:                  0.973
Method:                 Least Squares   F-statistic:                     401.6
Date:                Sat, 31 Jul 2021   Prob (F-statistic):           5.04e-55
Time:                        18:50:21   Log-Likelihood:                -604.83
No. Observations:                  80   AIC:                             1226.
Df Residuals:                      72   BIC:                             1245.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4.076e+04   2315.381     17.604      0.000    3.61e+04    4.54e+04
alh        -9.686e+04    1.01e+04    -9.606      0.000   -1.17e+05   -7.68e+04
aps         -171.2235     36.913     -4.639      0.000    -244.808     -97.639
aid         4256.0000   1095.376      3.885      0.000    2072.408    6439.592
arw            -1e+04   1349.427     -7.414      0.000   -1.27e+04   -7314.273
alh:aps      784.8611    152.136      5.159      0.000     481.584    1088.138
aps:awt     -118.7417     12.610     -9.416      0.000    -143.879     -93.604
arw:awt     8599.5563   1321.349      6.508      0.000    5965.496    1.12e+04
==============================================================================
Omnibus:                       98.951   Durbin-Watson:                   1.924
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1599.055
Skew:                           3.871   Prob(JB):                         0.00
Kurtosis:                      23.488   Cond. No.                     1.80e+04
==============================================================================


Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
[2] The condition number is large, 1.8e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```




```
time_eq = build_model(time_included,results.params,False)
print("Time = " + time_eq)
```

```
Time = 40760.716666663415 + -96855.41666666564 * model.X1 + -171.22354749789565 * mode
l.X2 + 4256.00000000009 * model.X3 + -10004.306297709902 * model.X4 + 784.861111111115
9 * model.X1*model.X2 + -118.74173027989448 * model.X2*model.X5 + 8599.556297709905 *
model.X4*model.X5
```

Equations

```
display(Markdown("Cost = "))
print(cost_eq)

print("------")

display(Markdown("Time = "))
```

```
print(time_eq)
```

Cost =

```
-7.512756311165274 + 15.594282277667375 * model.X1 + 0.1501013849966114 * model.X2 + 3
.3568541135900944 * model.X5 + -0.1873958333333725 * model.X1*model.X2 + 14.9740646197
8116 * model.X1*model.X3 + -6.3471031138125085 * model.X1*model.X4 + -0.17544472807322
514 * model.X2*model.X3 + 0.02025357325229224 * model.X2*model.X4 + -0.076406249999997
65 * model.X2*model.X5 + 3.2956081485919615 * model.X3*model.X4 + 6.441435526274299 *
model.X3*model.X5 + -0.6702353647414601 * model.X4*model.X5
------
```

Time =

```
40760.716666663415 + -96855.41666666564 * model.X1 + -171.22354749789565 * model.X2 +
4256.00000000009 * model.X3 + -10004.306297709902 * model.X4 + 784.8611111111159 * mod
el.X1*model.X2 + -118.74173027989448 * model.X2*model.X5 + 8599.556297709905 * model.X
4*model.X5
```

Optimization

```
model = ConcreteModel()

model.X1 = Var(within=NonNegativeReals)
model.X2 = Var(within=NonNegativeReals)
model.X3 = Var(within=NonNegativeReals)
model.X4 = Var(within=NonNegativeReals)
model.X5 = Var(within=NonNegativeReals)

model.C1 = Constraint(expr = model.X1 <= .28)
model.C2 = Constraint(expr = model.X2 <= 72)
model.C3 = Constraint(expr = model.X3 <= .25)
model.C4 = Constraint(expr = model.X4 <= .8)
model.C5 = Constraint(expr = model.X5 <= 1.2)

model.C6 = Constraint(expr = model.X1 >= .16)
model.C7 = Constraint(expr = model.X2 >= 60)
model.C8 = Constraint(expr = model.X3 >= .15)
model.C9 = Constraint(expr = model.X4 >= .4)
model.C10 = Constraint(expr = model.X5 >= .8)

model.f1 = Var()
model.f2 = Var()
model.C_f1 = Constraint(expr = model.f1 == (-7.512756311165274 + 15.594282277667375 *
model.X1 + 0.1501013849966114 * model.X2 + 3.3568541135900944 * model.X5 + -0.18739583
33333725 * model.X1*model.X2 + 14.97406461978116 * model.X1*model.X3 + -6.347103113812
```

```
5085 * model.X1*model.X4 + -0.17544472807322514 * model.X2*model.X3 + 0.02025357325229
224 * model.X2*model.X4 + -0.07640624999999765 * model.X2*model.X5 + 3.295608148591961
5 * model.X3*model.X4 + 6.441435526274299 * model.X3*model.X5 + -0.6702353647414601 *
model.X4*model.X5))
model.C_f2 = Constraint(expr = model.f2 == (40760.716666663415 + -96855.41666666564 *
model.X1 + -171.22354749789565 * model.X2 + 4256.00000000009 * model.X3 + -10004.30629
7709902 * model.X4 + 784.8611111111159 * model.X1*model.X2 + -118.74173027989448 * mod
el.X2*model.X5 + 8599.556297709905 * model.X4*model.X5))
model.O_f1 = Objective(expr = model.f1, sense=minimize)
model.O_f2 = Objective(expr = model.f2, sense=minimize)

# max f1 separately
# install glpk solver:  sudo apt-get install glpk-utils
model.O_f2.deactivate()
solver = SolverFactory('ipopt')  #'cplex', 'ipopt'
solver.solve(model)

print('( X1 , X2, X3, X4, X5 ) = ( ' + str(value(model.X1)) + ' , ' + str(value(model.
X2)) + ' , ' + str(value(model.X3)) + ' , ' + str(value(model.X4)) + ' , ' + str(value
(model.X5)) + ' )')
print('f1 = ' + str(value(model.f1)))
print('f2 = ' + str(value(model.f2)))
f2_min = value(model.f2)

# max f2 separately
model.O_f2.activate()
model.O_f1.deactivate()
solver = SolverFactory('ipopt')  #'cplex', 'ipopt'
solver.solve(model)

print('( X1 , X2, X3, X4, X5 ) = ( ' + str(value(model.X1)) + ' , ' + str(value(model.
X2)) + ' , ' + str(value(model.X3)) + ' , ' + str(value(model.X4)) + ' , ' + str(value
(model.X5)) + ' )')
print('f1 = ' + str(value(model.f1)))
print('f2 = ' + str(value(model.f2)))
f2_max = value(model.f2)

# apply augmented $\epsilon$-Constraint
# max                   f1 + delta*s
# constraint    f2 - s = e
model.O_f1.activate()
model.O_f2.deactivate()

model.del_component(model.O_f1)
model.del_component(model.O_f2)

model.e = Param(initialize=0, mutable=True)
model.delta = Param(initialize=0.00001)
model.slack = Var(within=NonNegativeReals)
model.O_f1 = Objective(expr = model.f1 + model.delta * model.slack, sense=minimize)
```

```python
model.C_e = Constraint(expr = model.f2 - model.slack == model.e)

n = 100
step = int((f2_max - f2_min) / n)
steps = list(range(int(f2_min),int(f2_max),step)) + [f2_max]

x1_l, x2_l, x3_l, x4_l, x5_l = [], [], [], [], []
f1_l, f2_l = [], []
for i in steps:
    model.e = i
    solver.solve(model)
    x1_l.append(value(model.X1))
    x2_l.append(value(model.X2))
    x3_l.append(value(model.X3))
    x4_l.append(value(model.X4))
    x5_l.append(value(model.X5))
    f1_l.append(value(model.f1))
    f2_l.append(value(model.f2))
    # print(i, value(model.X1), value(model.X2), value(model.f1), value(model.slack),
value(model.f2))
```

```
( X1 , X2, X3, X4, X5 ) = ( 0.15999999183403207 , 59.999999533653316 , 0.14999999109654624 , 0.7999999847099541 , 1.2000000088828169 )
f1 = 0.5680095811532789
f2 = 14866.22869285591
( X1 , X2, X3, X4, X5 ) = ( 0.2800000099999376 , 72.00000071997307 , 0.14999999000059022 , 0.39999999000739667 , 1.2000000119995191 )
f1 = 0.9517842950707984
f2 = 7641.0830096684
```

```python

```