CR6 Results Analysis A

Python Imports

```
import numpy as np
import pandas as pd
from prettypandas import PrettyPandas
import patsy
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
import statsmodels.formula.api
from pyomo.environ import *

from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures

from IPython.display import display, Markdown, HTML

%matplotlib inline
PlotWidth = 6

import warnings
warnings.filterwarnings('ignore')
```

```
# helper functions for this notebook

# use SVG for matplotlib-based figures
%matplotlib inline
%config InlineBackend.figure_format = 'svg'

def coded_to_actual(coded_data, actual_lows, actual_highs):
    """Converts a pandas DataFrame from coded units to actuals."""
    actual_data = coded_data.copy()
    for col in actual_data.columns:
        if not (col in actual_highs and col in actual_lows):
            continue
        try:
            # convert continuous variables to their actual value
            actual_data[col] *= 0.5 * (float(actual_highs[col]) - float(actual_lows[co
l]))
            # don't need to cast to float here, if either are not a float exception wi
ll have been thrown
            actual_data[col] += 0.5 * (actual_highs[col] + actual_lows[col])
        except ValueError:
            # assume 2 level categorical
```

```
            actual_data[col] = actual_data[col].map({-1: actual_lows[col], 1: actual_h
ighs[col]})
    return actual_data

def get_tick_labels(key, lows, highs, units):
    """Returns a list of low/high labels with units (e.g. [8mm, 10mm])"""
    return [str(lows[key]) + units[key], str(highs[key]) + units[key]]

def backward_regression(X, y,
                           threshold_out,
                           verbose=True):
    included=list(X.columns)
    while True:
        changed=False
        model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included]))).fit()
        # use all coefs except intercept
        pvalues = model.pvalues.iloc[1:]
        worst_pval = pvalues.max() # null if pvalues is empty
        if worst_pval > threshold_out:
            changed=True
            worst_feature = pvalues.idxmax()
            included.remove(worst_feature)
            if verbose:
                print('Drop {:30} with p-value {:.6}'.format(worst_feature, worst_pval
))
        if not changed:
            break

    return included

def build_model(X,values,verbose=True):
    X = [sub.replace('alh', 'model.X1') for sub in X]
    X = [sub.replace('aps', 'model.X2') for sub in X]
    X = [sub.replace('aid', 'model.X3') for sub in X]
    X = [sub.replace('arw', 'model.X4') for sub in X]
    X = [sub.replace('awt', 'model.X5') for sub in X]
    X = [sub.replace(':', '*') for sub in X]
    model = str(values[0])
    i=1
    for v in X:
        model  += " + " + str(values[i]) + " * " + v
        i += 1
    if verbose:
        print(model)
    return model
```

Process CSV Files

```
# importing the pandas library
import pandas as pd

# reading the csv file using read_csv
# storing the data frame in variable called df
df_cost = pd.read_csv('https://raw.githubusercontent.com/wilsongis/3DP_Experiments/main/Data/cr6_cost_raw.txt', sep='\t')
df_time = pd.read_csv('https://raw.githubusercontent.com/wilsongis/3DP_Experiments/main/Data/cr6_time_raw.txt', sep='\t')

# creating a list of column names by
# calling the .columns
list_of_columns_cost = list(df_cost.columns)
list_of_columns_time = list(df_time.columns)

# displaying the list of column names
print('List of Cost column names : ',
      list_of_columns_cost)
print('List of Time column names : ',
      list_of_columns_time)
```

```
List of Cost column names :  ['trial', 'lh', 'ps', 'id', 'rw', 'wt', 'alh', 'aps', 'aid', 'arw', 'awt', 'rep', 'cost']
List of Time column names :  ['trial', 'lh', 'ps', 'id', 'rw', 'wt', 'alh', 'aps', 'aid', 'arw', 'awt', 'rep', 'time']
```

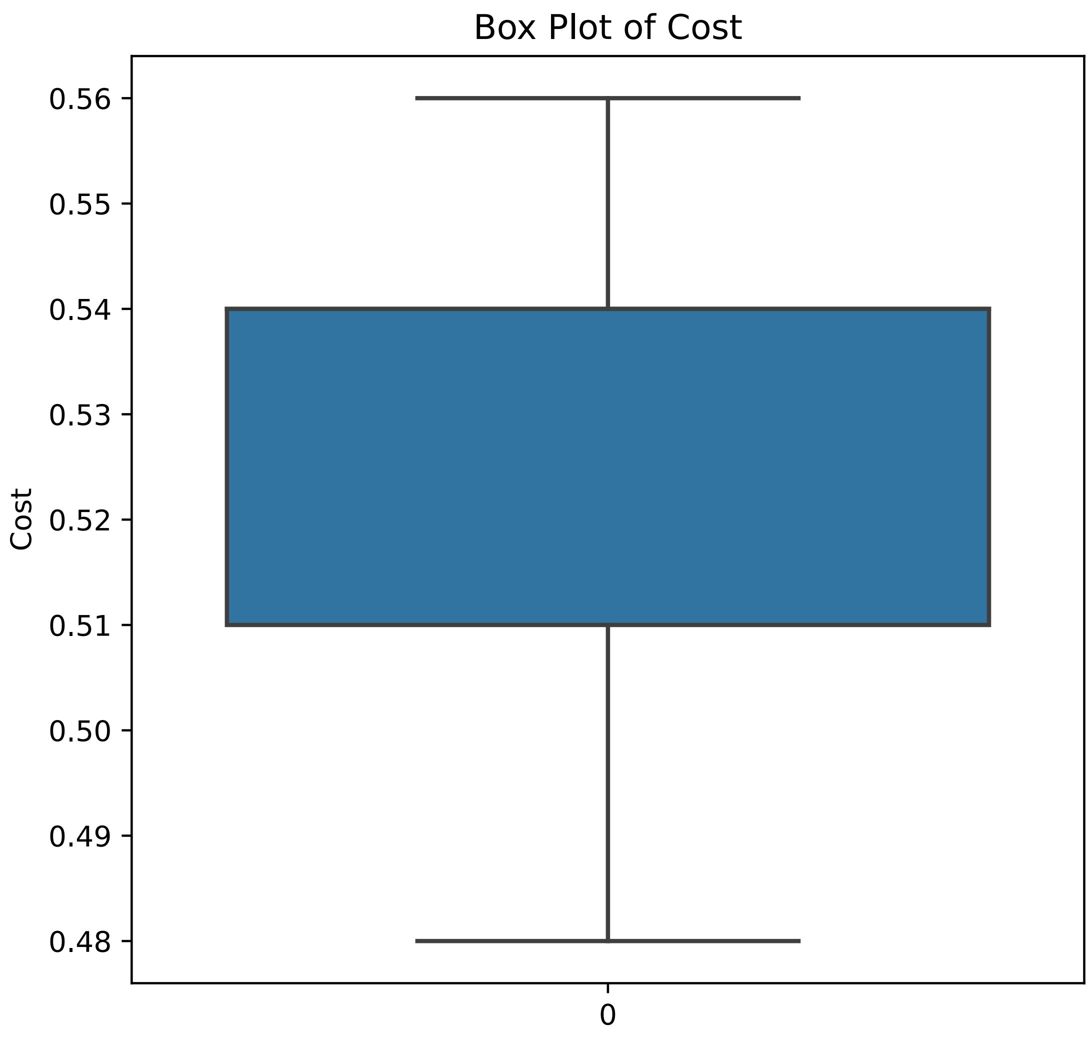
```
display((Markdown("### Statistics for Cost")))
df_cost.cost.describe()
```

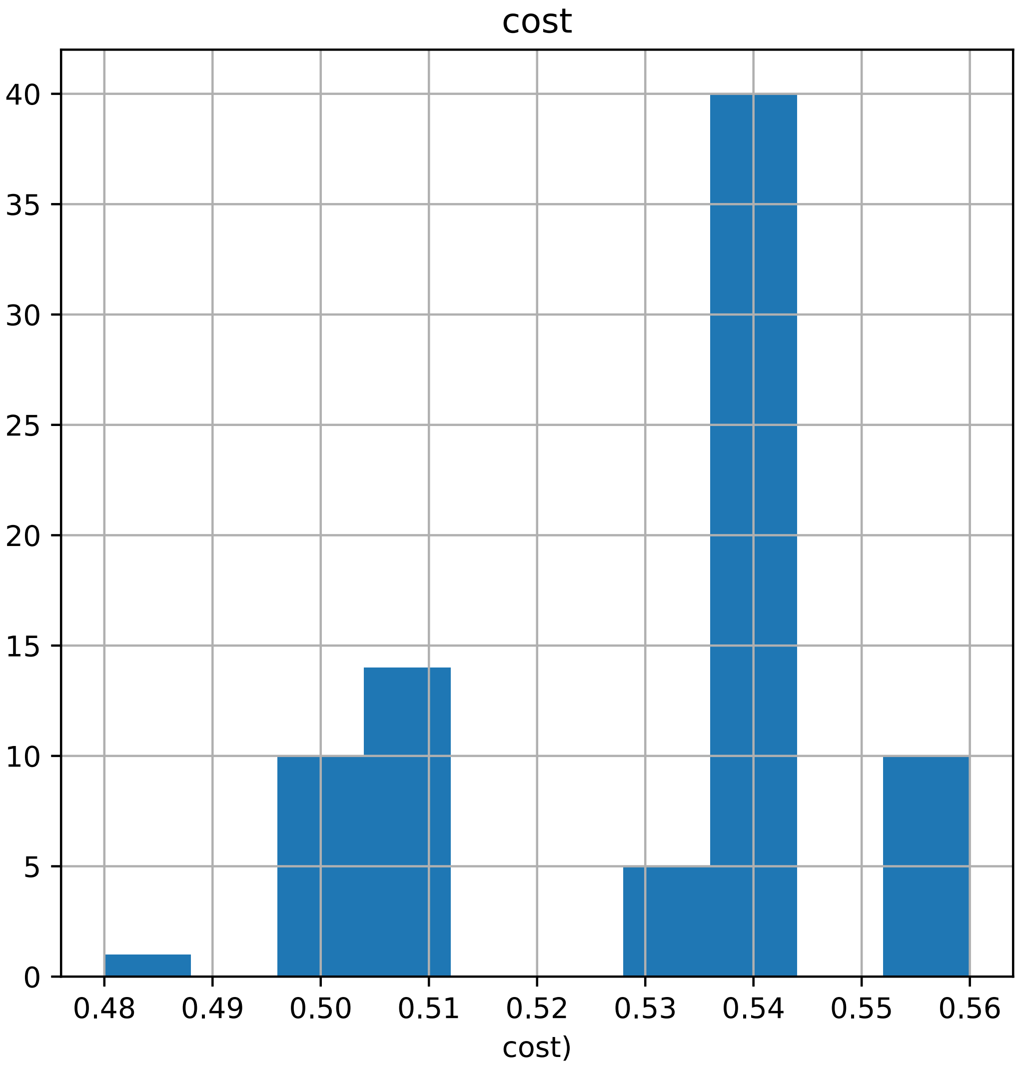
Statistics for Cost

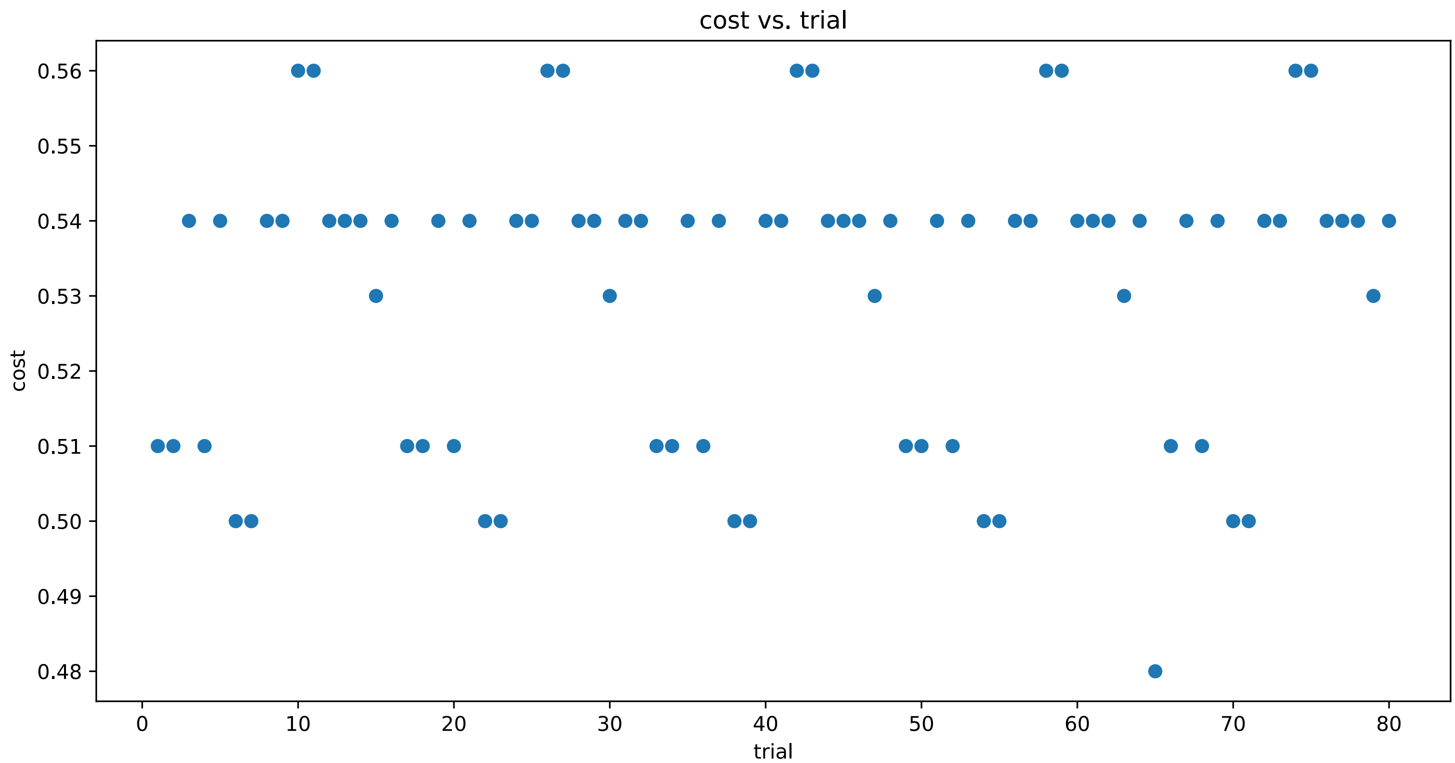
```
count    80.000000
mean      0.530875
std       0.019500
min       0.480000
25%       0.510000
50%       0.540000
75%       0.540000
max       0.560000
Name: cost, dtype: float64
```

```
plt.figure(figsize=(PlotWidth, PlotWidth))
sns.boxplot(data=df_cost['cost'])
plt.title('Box Plot of Cost')
plt.ylabel('Cost')
plt.show()
```

```
plt.figure(figsize=(PlotWidth, PlotWidth))
df_cost['cost'].hist()
plt.title('cost')
plt.xlabel('cost)')
plt.show()

plt.figure(figsize=(PlotWidth*2, PlotWidth))
plt.scatter(df_cost['trial'], df_cost['cost'])
plt.title('cost vs. trial')
plt.xlabel('trial')
plt.ylabel('cost')
plt.show()
```




cost
40
35
30
25
20
15
10
5
0
0.48
0.49
0.50
0.51
0.52
0.53
0.54
0.55
0.56
cost)




```
display((Markdown("### Statistics for Time")))
df_time.time.describe()
```

Statistics for Time

```
count       80.00000
mean     12681.40000
std       3360.13591
min       8480.00000
25%       9464.25000
50%      12989.00000
75%      15460.75000
max      18098.00000
Name: time, dtype: float64
```

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.boxplot(data=df_time['time'])
plt.title('Box Plot of Time')
plt.ylabel('Time')
plt.show()

plt.figure(figsize=(PlotWidth, PlotWidth))
df_time['time'].hist()
plt.title('time')

plt.xlabel('time)')
plt.show()

plt.figure(figsize=(PlotWidth*2, PlotWidth))
plt.scatter(df_time['trial'], df_time['time'])
plt.title('time vs. trial')
plt.xlabel('trial')
plt.ylabel('time')
plt.show()

Cost Analysis

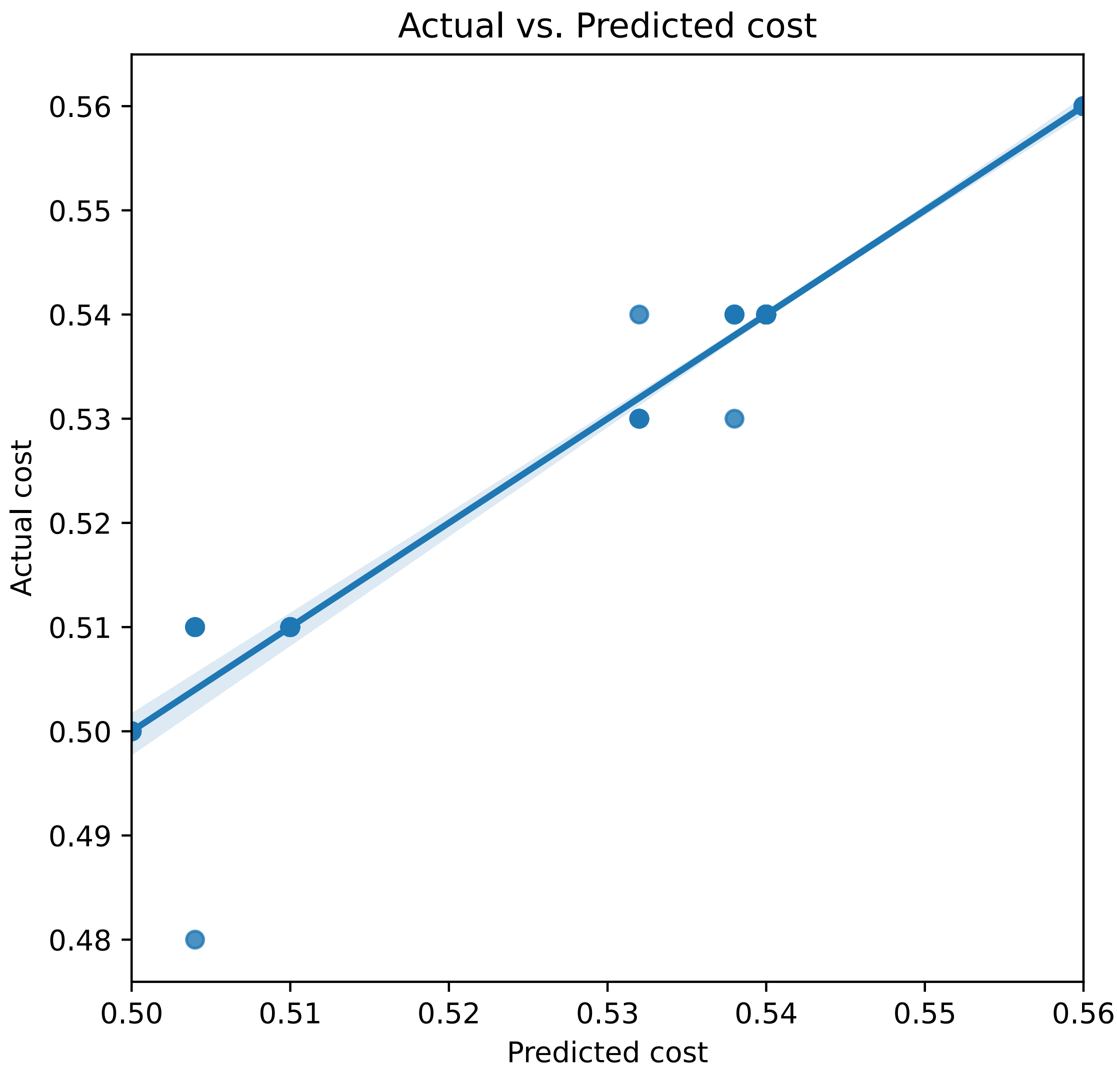
```
f = 'cost ~ (alh+aps+aid+arw+awt)**2'
y, X = patsy.dmatrices(f, df_cost, return_type='dataframe')
print(y[:5])
print(X[:5])
```

```
   cost
0  0.51
1  0.51
2  0.54
3  0.51
4  0.54
   Intercept   alh   aps   aid  arw  awt  alh:aps  alh:aid  alh:arw  alh:awt  \
0        1.0  0.16  50.0  0.25  0.4  0.8      8.0    0.040    0.064    0.128
1        1.0  0.28  50.0  0.25  0.4  1.2     14.0    0.070    0.112    0.336
2        1.0  0.16  60.0  0.25  0.4  1.2      9.6    0.040    0.064    0.192
3        1.0  0.28  60.0  0.25  0.4  0.8     16.8    0.070    0.112    0.224
4        1.0  0.16  50.0  0.15  0.4  1.2      8.0    0.024    0.064    0.192

   aps:aid  aps:arw  aps:awt  aid:arw  aid:awt  arw:awt
0     12.5     20.0     40.0     0.10     0.20     0.32
1     12.5     20.0     60.0     0.10     0.30     0.48
2     15.0     24.0     72.0     0.10     0.30     0.48
3     15.0     24.0     48.0     0.10     0.20     0.32
4      7.5     20.0     60.0     0.06     0.18     0.48
```

```
## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
```

```
plt.xlabel('Predicted cost')
plt.ylabel('Actual cost')
plt.title('Actual vs. Predicted cost')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.971
Model:                            OLS   Adj. R-squared:                  0.964
Method:                 Least Squares   F-statistic:                     141.4
Date:                Wed, 28 Jul 2021   Prob (F-statistic):           4.75e-43
Time:                        19:27:10   Log-Likelihood:                 343.19
No. Observations:                  80   AIC:                            -654.4
Df Residuals:                      64   BIC:                            -616.3
Df Model:                          15
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.3328      0.040      8.322      0.000       0.253       0.413
alh            0.0083      0.091      0.092      0.927      -0.173       0.189
aps           -0.0019      0.001     -2.880      0.005      -0.003      -0.001
aid           -0.1825      0.108     -1.691      0.096      -0.398       0.033
arw            0.3513      0.028     12.757      0.000       0.296       0.406
awt            0.2413      0.026      9.188      0.000       0.189       0.294
alh:aps        0.0029      0.001      2.111      0.039       0.000       0.006
alh:aid       -0.6250      0.138     -4.523      0.000      -0.901      -0.349
alh:arw        0.1563      0.035      4.523      0.000       0.087       0.225
alh:awt       -0.1563      0.035     -4.523      0.000      -0.225      -0.087
aps:aid        0.0105      0.002      6.332      0.000       0.007       0.014
aps:arw       -0.0026      0.000     -6.332      0.000      -0.003      -0.002
aps:awt        0.0011      0.000      2.714      0.009       0.000       0.002
aid:arw        0.4125      0.041      9.950      0.000       0.330       0.495
aid:awt       -0.4625      0.041    -11.156      0.000      -0.545      -0.380
arw:awt       -0.2594      0.010    -25.025      0.000      -0.280      -0.239
==============================================================================
Omnibus:                      114.302   Durbin-Watson:                   2.141
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3820.537
Skew:                          -4.440   Prob(JB):                         0.00
Kurtosis:                      35.669   Cond. No.                     3.13e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
[2] The condition number is large, 3.13e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```



Reduced Cost Model

```
cost_included = backward_regression(X,y,.05)
cost_included.pop(0)
print(cost_included)
```

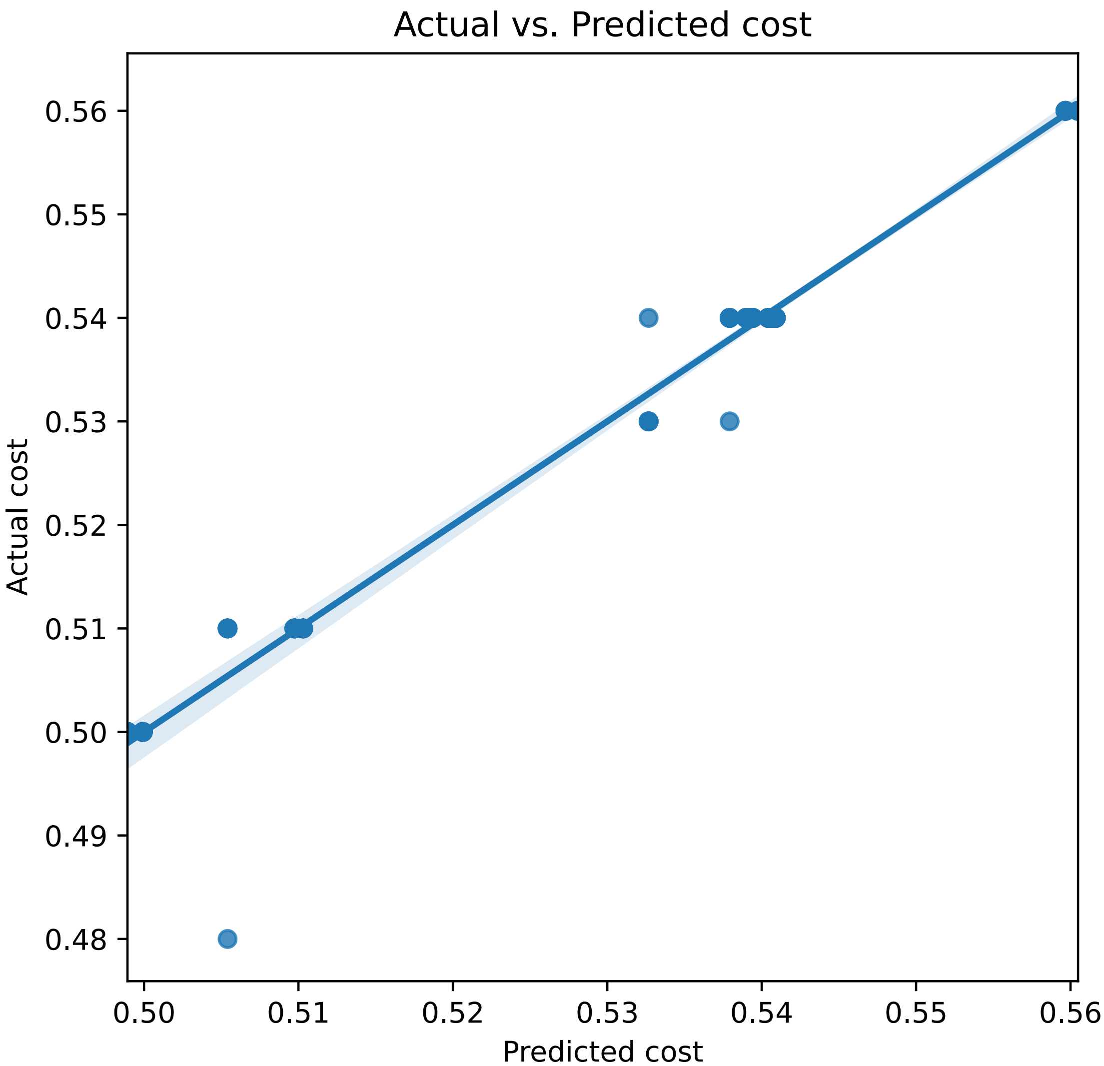
```
Drop alh                                    with p-value 0.927017
Drop aid                                    with p-value 0.0905131
['aps', 'arw', 'awt', 'alh:aps', 'alh:aid', 'alh:arw', 'alh:awt', 'aps:aid', 'aps:arw'
, 'aps:awt', 'aid:arw', 'aid:awt', 'arw:awt']
```

```
y = df_cost['cost']
#y = df_cost['time']
X = X[cost_included]

## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
```

```
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted cost')
plt.ylabel('Actual cost')
plt.title('Actual vs. Predicted cost')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   cost   R-squared:                       0.969
Model:                            OLS   Adj. R-squared:                  0.963
Method:                 Least Squares   F-statistic:                     160.7
Date:                Wed, 28 Jul 2021   Prob (F-statistic):           1.22e-44
Time:                        19:27:12   Log-Likelihood:                 341.41
No. Observations:                  80   AIC:                            -654.8
Df Residuals:                      66   BIC:                            -621.5
Df Model:                          13
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3006      0.029     10.531      0.000       0.244       0.358
aps           -0.0015      0.001     -2.669      0.010      -0.003      -0.000
arw            0.3541      0.028     12.824      0.000       0.299       0.409
awt            0.2460      0.026      9.423      0.000       0.194       0.298
alh:aps        0.0032      0.001      4.253      0.000       0.002       0.005
alh:aid       -0.6815      0.128     -5.340      0.000      -0.936      -0.427
alh:arw        0.1581      0.034      4.671      0.000       0.091       0.226
alh:awt       -0.1531      0.032     -4.764      0.000      -0.217      -0.089
aps:aid        0.0081      0.001      9.160      0.000       0.006       0.010
aps:arw       -0.0026      0.000     -6.288      0.000      -0.003      -0.002
aps:awt        0.0011      0.000      2.695      0.009       0.000       0.002
aid:arw        0.3961      0.041      9.754      0.000       0.315       0.477
aid:awt       -0.4898      0.039    -12.715      0.000      -0.567      -0.413
arw:awt       -0.2594      0.010    -24.854      0.000      -0.280      -0.239
==============================================================================
Omnibus:                      125.804   Durbin-Watson:                   2.098
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4980.386
Skew:                          -5.178   Prob(JB):                         0.00
Kurtosis:                      40.241   Cond. No.                     2.66e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
```

```
[2] The condition number is large, 2.66e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```




```
cost_eq = build_model(cost_included,results.params,False)
print("Cost = " + cost_eq)
```

```
Cost = 0.30061301677651686 + -0.0014979526099359666 * model.X2 + 0.3541048685686145 *
model.X4 + 0.24600811428103486 * model.X5 + 0.003193676289093103 * model.X1*model.X2 +
 -0.681545835829839 * model.X1*model.X3 + 0.15813870197108404 * model.X1*model.X4 + -0
.15310216338152616 * model.X1*model.X5 + 0.008101719131677326 * model.X2*model.X3 + -0
.0026250000000000544 * model.X2*model.X4 + 0.0011250000000000322 * model.X2*model.X5 +
 0.3961480849887108 * model.X3*model.X4 + -0.4897531916854896 * model.X3*model.X5 + -0
.2593750000000006 * model.X4*model.X5
```

Time Analysis

```
f = 'time ~ (alh+aps+aid+arw+awt)**2'
```

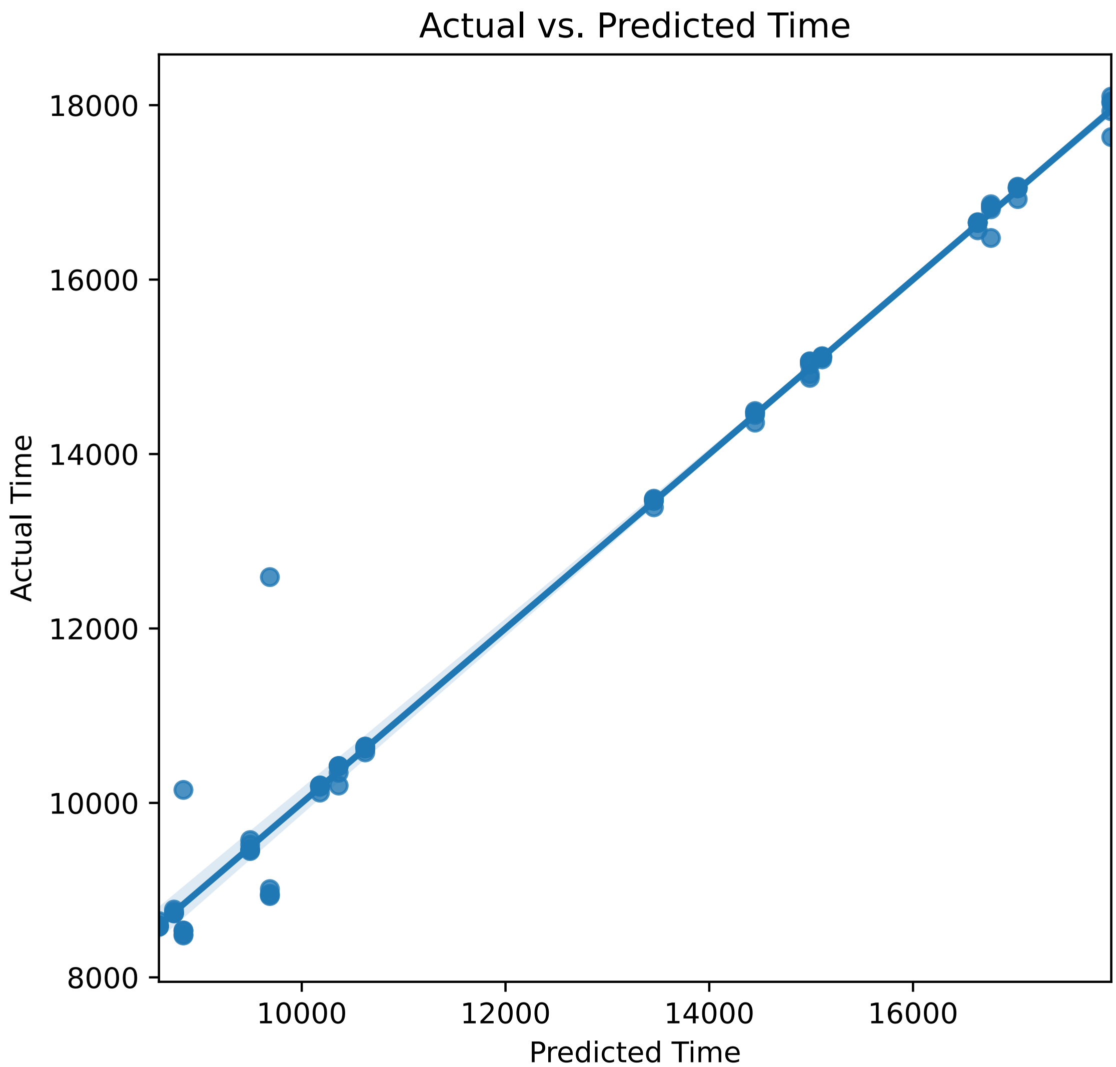
```
y, X = patsy.dmatrices(f, df_time, return_type='dataframe')
print(y[:5])
print(X[:5])
```

```
      time
0  18098.0
1   8741.0
2  14493.0
3  10191.0
4  14914.0
   Intercept   alh   aps   aid  arw  awt  alh:aps  alh:aid  alh:arw  alh:awt  \
0        1.0  0.16  50.0  0.25  0.4  0.8      8.0    0.040    0.064    0.128
1        1.0  0.28  50.0  0.25  0.4  1.2     14.0    0.070    0.112    0.336
2        1.0  0.16  60.0  0.25  0.4  1.2      9.6    0.040    0.064    0.192
3        1.0  0.28  60.0  0.25  0.4  0.8     16.8    0.070    0.112    0.224
4        1.0  0.16  50.0  0.15  0.4  1.2      8.0    0.024    0.064    0.192

   aps:aid  aps:arw  aps:awt  aid:arw  aid:awt  arw:awt
0     12.5     20.0     40.0     0.10     0.20     0.32
1     12.5     20.0     60.0     0.10     0.30     0.48
2     15.0     24.0     72.0     0.10     0.30     0.48
3     15.0     24.0     48.0     0.10     0.20     0.32
4      7.5     20.0     60.0     0.06     0.18     0.48
```

```
## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted Time')
plt.ylabel('Actual Time')
plt.title('Actual vs. Predicted Time')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   time   R-squared:                       0.985
Model:                            OLS   Adj. R-squared:                  0.982
Method:                 Least Squares   F-statistic:                     287.3
Date:                Wed, 28 Jul 2021   Prob (F-statistic):           1.17e-52
Time:                        19:27:13   Log-Likelihood:                -593.61
No. Observations:                  80   AIC:                             1219.
Df Residuals:                      64   BIC:                             1257.
Df Model:                          15
Covariance Type:            nonrobust
```

```
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   3.886e+04   4869.801      7.980      0.000    2.91e+04    4.86e+04
alh        -9.373e+04    1.1e+04     -8.494      0.000   -1.16e+05   -7.17e+04
aps         -101.9583     81.061     -1.258      0.213    -263.896      59.980
aid         6178.6667   1.31e+04      0.470      0.640   -2.01e+04    3.24e+04
arw         2096.5000   3353.230      0.625      0.534   -4602.347    8795.347
awt        -8507.9167   3197.582     -2.661      0.010   -1.49e+04   -2120.012
alh:aps      302.8333    168.294      1.799      0.077     -33.372     639.039
alh:aid    -4.994e+04   1.68e+04     -2.968      0.004   -8.36e+04   -1.63e+04
alh:arw     1.009e+04   4207.345      2.399      0.019    1688.613    1.85e+04
alh:awt     2.909e+04   4207.345      6.913      0.000    2.07e+04    3.75e+04
aps:aid      301.9000    201.953      1.495      0.140    -101.547     705.347
aps:arw     -137.1250     50.488     -2.716      0.008    -237.987     -36.263
aps:awt      -18.6750     50.488     -0.370      0.713    -119.537      82.187
aid:arw     5830.0000   5048.814      1.155      0.252   -4256.165    1.59e+04
aid:awt    -1.288e+04   5048.814     -2.552      0.013    -2.3e+04   -2798.835
arw:awt     1626.2500   1262.203      1.288      0.202    -895.291    4147.791
==============================================================================
Omnibus:                      117.964   Durbin-Watson:                   1.973
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3766.256
Skew:                           4.734   Prob(JB):                         0.00
Kurtosis:                      35.252   Cond. No.                     3.13e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
[2] The condition number is large, 3.13e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```



Time Reduced Model

```
time_included = backward_regression(X,y,.05)
time_included.pop(0)
print(time_included)
```

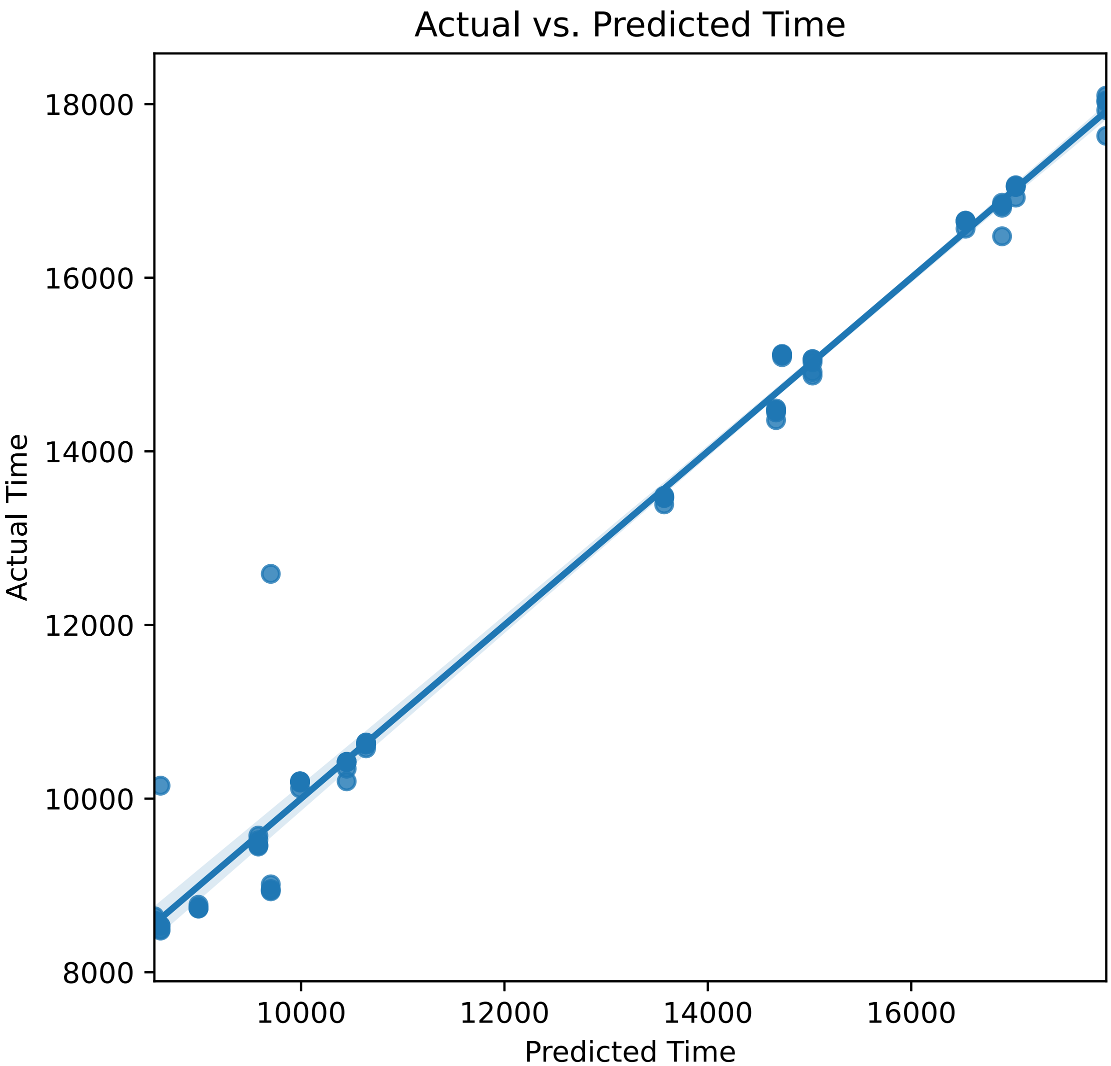
```
Drop aps:awt                             with p-value 0.712687
Drop aid                                 with p-value 0.63764
Drop arw                                 with p-value 0.549529
Drop aid:arw                             with p-value 0.12172
Drop alh:aps                             with p-value 0.0744513
Drop arw:awt                             with p-value 0.0725949
['alh', 'aps', 'awt', 'alh:aid', 'alh:arw', 'alh:awt', 'aps:aid', 'aps:arw', 'aid:awt'
]
```

```
y = df_time['time']
X = X[time_included]
```

```python
## An intercept is not added by default, so we need to add that here
X = sm.add_constant(X)
results = sm.OLS(y, X).fit()
results.summary()

print(results.summary())

plt.figure(figsize=(PlotWidth, PlotWidth))
sns.regplot(x=results.predict(X), y=y)
plt.xlabel('Predicted Time')
plt.ylabel('Actual Time')
plt.title('Actual vs. Predicted Time')
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   time   R-squared:                       0.983
Model:                            OLS   Adj. R-squared:                  0.981
Method:                 Least Squares   F-statistic:                     453.8
Date:                Wed, 28 Jul 2021   Prob (F-statistic):           2.19e-58
Time:                        19:27:14   Log-Likelihood:                -599.26
No. Observations:                  80   AIC:                             1219.
Df Residuals:                      70   BIC:                             1242.
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       3.942e+04   1154.479     34.142      0.000    3.71e+04    4.17e+04
alh        -7.914e+04   6029.887    -13.124      0.000   -9.12e+04   -6.71e+04
aps         -128.1914     25.154     -5.096      0.000    -178.360     -78.023
awt        -8860.8498   1369.537     -6.470      0.000   -1.16e+04   -6129.395
alh:aid    -4.626e+04    1.65e+04    -2.798      0.007   -7.92e+04   -1.33e+04
alh:arw      1.23e+04   4097.512      3.002      0.004    4127.272    2.05e+04
alh:awt     2.909e+04   4317.386      6.737      0.000    2.05e+04    3.77e+04
aps:aid      434.5856    102.753      4.229      0.000     229.652     639.519
aps:arw      -57.7179     16.919     -3.411      0.001     -91.462     -23.974
aid:awt    -1.138e+04   4760.148     -2.390      0.020   -2.09e+04   -1883.394
==============================================================================
Omnibus:                      103.336   Durbin-Watson:                   1.908
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2137.913
Skew:                           3.999   Prob(JB):                         0.00
Kurtosis:                      27.030   Cond. No.                     2.14e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
fied.
[2] The condition number is large, 2.14e+04. This might indicate that there are
```

```
strong multicollinearity or other numerical problems.
```




```
time_eq = build_model(time_included,results.params,False)
print("Time = " + time_eq)
```

```
Time = 39415.87499999855 + -79138.51046393832 * model.X1 + -128.19137670175314 * model.X2 + -8860.849838411767 * model.X5 + -46255.95567867106 * model.X1*model.X3 + 12299.502666120085 * model.X1*model.X4 + 29085.416666666522 * model.X1*model.X5 + 434.58559556786804 * model.X2*model.X3 + -57.717904019688284 * model.X2*model.X4 + -11377.20914127423 * model.X3*model.X5
```

Equations

```
display(Markdown("Cost = "))
print(cost_eq)
```

```
print("------")

display(Markdown("Time = "))
print(time_eq)
```

Cost =

```
0.30061301677651686 + -0.0014979526099359666 * model.X2 + 0.3541048685686145 * model.X
4 + 0.24600811428103486 * model.X5 + 0.003193676289093103 * model.X1*model.X2 + -0.681
545835829839 * model.X1*model.X3 + 0.15813870197108404 * model.X1*model.X4 + -0.153102
16338152616 * model.X1*model.X5 + 0.008101719131677326 * model.X2*model.X3 + -0.002625
000000000544 * model.X2*model.X4 + 0.0011250000000000322 * model.X2*model.X5 + 0.3961
480849887108 * model.X3*model.X4 + -0.4897531916854896 * model.X3*model.X5 + -0.259375
0000000006 * model.X4*model.X5
------
```

Time =

```
39415.87499999855 + -79138.51046393832 * model.X1 + -128.19137670175314 * model.X2 + -
8860.849838411767 * model.X5 + -46255.95567867106 * model.X1*model.X3 + 12299.50266612
0085 * model.X1*model.X4 + 29085.416666666522 * model.X1*model.X5 + 434.58559556786804
 * model.X2*model.X3 + -57.717904019688284 * model.X2*model.X4 + -11377.20914127423 *
model.X3*model.X5
```

Optimization

```
model = ConcreteModel()

model.X1 = Var(within=NonNegativeReals)
model.X2 = Var(within=NonNegativeReals)
model.X3 = Var(within=NonNegativeReals)
model.X4 = Var(within=NonNegativeReals)
model.X5 = Var(within=NonNegativeReals)

model.C1 = Constraint(expr = model.X1 <= .28)
model.C2 = Constraint(expr = model.X2 <= 60)
model.C3 = Constraint(expr = model.X3 <= .25)
model.C4 = Constraint(expr = model.X4 <= .8)
model.C5 = Constraint(expr = model.X5 <= 1.2)

model.C6 = Constraint(expr = model.X1 >= .16)
model.C7 = Constraint(expr = model.X2 >= 50)
model.C8 = Constraint(expr = model.X3 >= .15)
model.C9 = Constraint(expr = model.X4 >= .4)
model.C10 = Constraint(expr = model.X5 >= .8)
```

```
model.f1 = Var()
model.f2 = Var()
model.C_f1 = Constraint(expr = model.f1 == (0.30061301677651686 + -0.00149795260993596
66 * model.X2 + 0.3541048685686145 * model.X4 + 0.24600811428103486 * model.X5 + 0.003
193676289093103 * model.X1*model.X2 + -0.681545835829839 * model.X1*model.X3 + 0.15813
870197108404 * model.X1*model.X4 + -0.15310216338152616 * model.X1*model.X5 + 0.008101
719131677326 * model.X2*model.X3 + -0.0026250000000000544 * model.X2*model.X4 + 0.0011
250000000000322 * model.X2*model.X5 + 0.3961480849887108 * model.X3*model.X4 + -0.4897
531916854896 * model.X3*model.X5 + -0.2593750000000006 * model.X4*model.X5))
model.C_f2 = Constraint(expr = model.f2 == (39415.87499999855 + -79138.51046393832 * m
odel.X1 + -128.19137670175314 * model.X2 + -8860.849838411767 * model.X5 + -46255.9556
7867106 * model.X1*model.X3 + 12299.502666120085 * model.X1*model.X4 + 29085.416666666
522 * model.X1*model.X5 + 434.58559556786804 * model.X2*model.X3 + -57.717904019688284
 * model.X2*model.X4 + -11377.20914127423 * model.X3*model.X5))
model.O_f1 = Objective(expr = model.f1, sense=minimize)
model.O_f2 = Objective(expr = model.f2, sense=minimize)

# max f1 separately
# install glpk solver:  sudo apt-get install glpk-utils
model.O_f2.deactivate()
solver = SolverFactory('ipopt')  #'cplex', 'ipopt'
solver.solve(model)

print('( X1 , X2, X3, X4, X5 ) = ( ' + str(value(model.X1)) + ' , ' + str(value(model.
X2)) + ' , ' + str(value(model.X3)) + ' , ' + str(value(model.X4)) + ' , ' + str(value
(model.X5)) + ' )')
print('f1 = ' + str(value(model.f1)))
print('f2 = ' + str(value(model.f2)))
f2_min = value(model.f2)

# max f2 separately
model.O_f2.activate()
model.O_f1.deactivate()
solver = SolverFactory('ipopt')  #'cplex', 'ipopt'
solver.solve(model)

print('( X1 , X2, X3, X4, X5 ) = ( ' + str(value(model.X1)) + ' , ' + str(value(model.
X2)) + ' , ' + str(value(model.X3)) + ' , ' + str(value(model.X4)) + ' , ' + str(value
(model.X5)) + ' )')
print('f1 = ' + str(value(model.f1)))
print('f2 = ' + str(value(model.f2)))
f2_max = value(model.f2)

# apply augmented $\epsilon$-Constraint
# max                   f1 + delta*s
# constraint    f2 - s = e
model.O_f1.activate()
model.O_f2.deactivate()

model.del_component(model.O_f1)
```

```
model.del_component(model.O_f2)

model.e = Param(initialize=0, mutable=True)
model.delta = Param(initialize=0.00001)
model.slack = Var(within=NonNegativeReals)
model.O_f1 = Objective(expr = model.f1 + model.delta * model.slack, sense=minimize)
model.C_e = Constraint(expr = model.f2 - model.slack == model.e)

n = 100
step = int((f2_max - f2_min) / n)
steps = list(range(int(f2_min),int(f2_max),step)) + [f2_max]

x1_l, x2_l, x3_l, x4_l, x5_l = [], [], [], [], []
f1_l, f2_l = [], []
for i in steps:
    model.e = i
    solver.solve(model)
    x1_l.append(value(model.X1))
    x2_l.append(value(model.X2))
    x3_l.append(value(model.X3))
    x4_l.append(value(model.X4))
    x5_l.append(value(model.X5))
    f1_l.append(value(model.f1))
    f2_l.append(value(model.f2))
    # print(i, value(model.X1), value(model.X2), value(model.f1), value(model.slack),
value(model.f2))
```

```
( X1 , X2, X3, X4, X5 ) = ( 0.27999999699979045 , 50.00000021485858 , 0.24999996237920
893 , 0.39999999572941514 , 0.8000000173812946 )
f1 = 0.49701939510293697
f2 = 10416.124260969951
( X1 , X2, X3, X4, X5 ) = ( 0.28000000999993935 , 60.000000599972694 , 0.2500000099939
384 , 0.8000000096771722 , 1.2000000119993277 )
f1 = 0.53898523398684
f2 = 8557.622612553769
```

```python

```