

Luck: A more natural statistic

Warren D. MacEvoy

October 16, 2024

Introduction to Luck

► What is Luck?

- Luck quantifies how likely an outcome is compared to others.
- Formal definition:

$$|\Omega(x)| = \text{Prob}(\text{outcomes more likely than } x)$$

$$|\omega(x)| = \text{Prob}(\text{outcomes equally likely to } x)$$

$$L(x) = |\Omega(x)| + \frac{1}{2}|\omega(x)|$$

- **Range of Luck:** 0 (unlucky) to 1 (very lucky), average luck is 0.5.
- **Why Study Luck?**
 - Luck connects mathematical probability to real-world scenarios.
 - Examples: Password guessing, dating, randomness.

Example: Luck in Coin Tosses

- ▶ Consider 8 fair coin flips.
- ▶ What is the luck of getting exactly 4 heads?

$$p(x) = \frac{8!}{x!(8-x)!} \left(\frac{1}{2}\right)^8$$

- ▶ Luck for different outcomes is computed using the binomial distribution.
- ▶ **Table:** Luck values for outcomes

x	$p(x)$	$\Omega(x)$	$ \Omega(x) $	$\omega(x)$	$ \omega(x) $	$L(x)$
4	0.2734	$\{ \}$	0.0000	$\{4\}$	0.2734	0.1367
3 or 5	0.2188	$\{4\}$	0.2734	$\{3,5\}$	0.4375	0.4922
2 or 6	0.1094	$\{3, \dots, 5\}$	0.7109	$\{2,6\}$	0.2188	0.8203
1 or 7	0.0313	$\{2, \dots, 6\}$	0.9297	$\{1,7\}$	0.0625	0.9609
0 or 8	0.0039	$\{1, \dots, 7\}$	0.9922	$\{0,8\}$	0.0078	0.9961

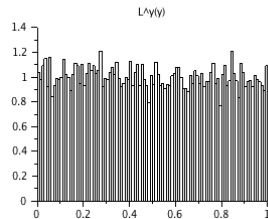
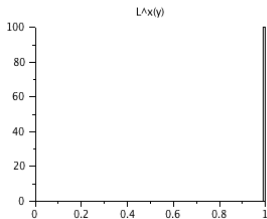
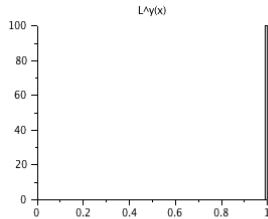
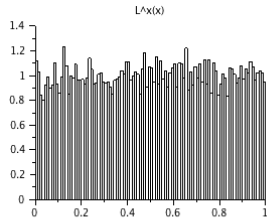
Luck in the Normal Distribution

In large dimensions, normal observations are *not* crowded near $x \approx \mu$, but almost certainly (99.99%) within ± 3 of the elliptical shell

$$|\Sigma^{-1/2}(x - \mu)| = \sqrt{d_f - 1/2}.$$

This is a pretty good statement even for d_f of 3....

$N = 1$ statistics



Z-Luck: Combining Luck from Experiments

- ▶ Z-luck allows us to combine results from multiple independent experiments.
- ▶ Formula for z-luck:

$$z_L = \sqrt{\|\Sigma^{-1}(x - \mu)\|} - \sqrt{d_f - \frac{1}{2}}$$

- ▶ Example: Combining observations from normal distributions across different dimensions.
- ▶ Application in determining whether results from independent experiments align or not.

Coin Luck: Natural Unit for Luck

$$L_C(x) = \log_2 \frac{L(x)}{1 - L(x)}$$

This is expressing luck in “coin tosses” where negative is bad luck (nearly zero) and positive is good luck (nearly one).

Conclusion

- ▶ Luck is a versatile concept connecting probability and real-world events.
- ▶ Applications range from testing randomness to real-world problem-solving.
- ▶ Key insights:
 - ▶ Uniformity of luck across distributions.
 - ▶ Elliptical shell behavior in multinomial distributions.
- ▶ Thank you! Questions?