3.3 Common Argument Forms

In the previous section we learned how to use truth tables to determine whether deductive arguments are valid. As arguments get longer, their truth tables would have more rows. Using truth tables to determine their validity can become quite time-consuming. For example, the truth table of the following argument has 16 rows, and can take quite a bit of time to construct.

> If interest rates are raised, the stock market will be hurt. If the stock market is hurt, the economy will slow down. But if interest rates are not raised, inflation will get worse. If inflation gets worse, the economy will slow down. So the economy will slow down. 3.3a

So using truth tables to determine validity can be tedious, and there is an incentive to find a more efficient way.

Arguments such as (3.3a) are built by combining small and basic valid argument forms. This means that if we can recognize the small forms and see how they are put together to form longer arguments, then we can determine validity without constructing truth tables.

3.3.1 Basic Valid Forms

There are six basic forms that are commonly used:

1. Disjunctive Syllogism (DS)
2. Hypothetical Syllogism (HS)
3. Modus Ponens (MP)
4. Modus Tollens (MT)
5. Constructive Dilemma (CD)
6. Destructive Dilemma (DD)

We are going to study them and learn how to recognize them.

Disjunctive Syllogism (DS)

The basic form disjunctive syllogism gets its name from the feature that one of the two premises is a disjunction. The disjunction tells us that at least one of its disjuncts must be true in order for the disjunction to be true. Now since the other premise asserts that one of the disjuncts is false (that is, its negation is true). It follows that the other disjunct must be true.

$$\begin{array}{l} p \lor q \\ \underline{\sim p \quad\quad} \\ q \end{array} \quad \text{or} \quad \begin{array}{l} p \lor q \\ \underline{\sim q \quad\quad} \\ p \end{array}$$

Notice that in Propositional Logic, the order of the premises does not matter. So the following two are treated as the same form.

$$\begin{array}{l} p \lor q \\ \underline{\sim p \quad\quad} \\ q \end{array} \quad = \quad \begin{array}{l} \sim p \\ \underline{p \lor q \quad} \\ q \end{array}$$

We can prove that the form disjunctive syllogism is valid using the truth table.

$$\begin{array}{l} p \lor q \\ \underline{\sim p \quad\quad} \\ q \end{array}$$

$p \lor q$	$\sim p$	q

After we know what the form looks like, the next step is to identify it from a written argument. Here is an example of disjunctive syllogism:

> Either interest rates go up or inflation gets worse. Since interest rates have not gone up, we can be sure that inflation is getting worse. 3.3b

After symbolizing the argument as

$$\begin{array}{l} U \lor W \\ \underline{\sim U \quad\quad} \\ W \end{array}$$

we can tell that it is an instance of disjunctive syllogism. In this way we can find out that it is valid without constructing its truth table.

Hypothetical Syllogism (HS)

A hypothetical syllogism has a distinct feature that helps us recognize it. The argument consists of three conditionals. The first conditional says that p is a sufficient condition for q. The second one says that q in turn is a sufficient condition for r. It would then follow that p is a sufficient condition for r.

$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ p \supset r \end{array}$$

The next argument

> If more prisons are built, public education will get worse due to lack of funding. If public education gets worse due to lack of funding, there will be more criminals. As a result, if more prisons are built, there will be more criminals. 3.3c

has the form

$$\begin{array}{l} B \supset W \\ \underline{W \supset C} \\ B \supset C \end{array}$$

and thus is an instance of hypothetical syllogism.

Modus Ponens (MP)

"Modus Ponens" is the Latin term for "Affirmative Mode." We can also call it "Affirming the Antecedent" because one of its premises affirms that the antecedent of the conditional is true. It is a valid form based on the concept of sufficient condition. If p is a sufficient condition of q, and p is true, then q must be true.

$$\begin{array}{l} p \supset q \\ \underline{p \qquad} \\ q \end{array}$$

Modus Ponens is one of the most commonly used valid forms. Here is an example:

> If Republicans favor free market economy, then they should oppose farm subsidies. Republicans favor free market economy. So they should oppose farm subsidies. 3.3d

The argument is symbolized as

$$\begin{array}{l} F \supset O \\ F \\ \hline O \end{array}$$

We can see that its form is Modus Ponens and thus is valid.

Modus Tollens (MT)

“Modus Tollens” means “Denying Mode” in Latin. Its English name is “Denying the Consequent” because one of its premises denies that the consequent of the conditional is true. The validity of Modus Tollens can be easily explained using the concept of necessary condition. If q is a necessary condition of p, and q is false, then p must be false.

$$\begin{array}{l} p \supset q \\ \sim q \\ \hline \sim p \end{array}$$

The next argument is an example of Modus Tollens:

> We should be against big corporations only if we are against their stock holders. We are not against the stock holders. So we should not be against big corporations. 3.3e

$$\begin{array}{l} B \supset S \\ \sim S \\ \hline \sim B \end{array}$$

Constructive Dilemma (CD)

Constructive dilemma, like Modus Ponens, is built upon the concept of sufficient condition. The two conditionals $p \supset q$ and $r \supset s$ can be joined together as a conjunction or stated separately as two premises. They assert that p is a sufficient condition for q and r is a sufficient condition for s. Consequently, if at least one of the sufficient conditions is true, then at least one of the consequents must also be true.

$$\begin{array}{l} (p \supset q) \bullet (r \supset s) \\ p \vee r \\ \hline q \vee s \end{array} \qquad \text{or} \qquad \begin{array}{l} p \supset q \\ r \supset s \\ p \vee r \\ \hline q \vee s \end{array}$$

After symbolizing the argument

> If consumers increase spending, then inflation will get worse. If consumers cut back on spending, then there will be a recession. Consumers either increase or cut back on spending. It follows that the inflation will get worse or there will be a recession. 3.3f

as

$$\begin{array}{l} (I \supset W) \bullet (C \supset R) \\ I \vee C \\ \hline W \vee R \end{array}$$

We can see that it is a constructive dilemma, and thus is valid.

A version of constructive dilemma has $p \vee {\sim}p$ as one of the premises.

$$\begin{array}{l} (p \supset q) \bullet ({\sim}p \supset s) \\ p \vee {\sim}p \\ \hline q \vee s \end{array}$$

Since $p \vee {\sim}p$ is a tautology, the argument is sound if the premise $(p \supset q) \bullet ({\sim}p \supset s)$ is true. Here is an example of this commonly seen version of constructive dilemma:

> With protectionism, prices for consumer goods would become higher. Without protectionism, jobs would be lost. Since we either adopt protectionism or reject it, prices for consumer goods would become higher or jobs would be lost. 3.3g

$$\begin{array}{l} (P \supset H) \bullet ({\sim}P \supset J) \\ P \vee {\sim}P \\ \hline H \vee J \end{array}$$

Destructive Dilemma (DD)

Destructive dilemma is another common form based on the concept of necessary condition. The two conditionals assert that q is a necessary condition for p and s is a necessary condition for r. So if q is false or s is false, then it must be the case that p is false or r is false.

$$\begin{array}{l} (p \supset q) \bullet (r \supset s) \\ {\sim}q \vee {\sim}s \\ \hline {\sim}p \vee {\sim}r \end{array}$$

or

$$\begin{array}{l} p \supset q \\ r \supset s \\ {\sim}q \vee {\sim}s \\ \hline {\sim}p \vee {\sim}r \end{array}$$

Here is an example of destructive dilemma:

> Global warming can be slowed down only if we switch to cleaner energy sources. But the current level of industrial production can be sustained only if we continue to use fossil fuels. We won't switch to cleaner energy sources or we won't continue to use fossil fuels. As a result, global warming cannot be slowed down or the current level of industrial production cannot be sustained. 3.3h

$$\begin{array}{l} (G \supset S) \bullet (P \supset F) \\ \sim S \lor \sim F \\ \hline \sim G \lor \sim P \end{array}$$

It is also common to see a destructive dilemma with a tautologous disjunction as one of its premises:

$$\begin{array}{l} (p \supset q) \bullet (r \supset \sim q) \\ q \lor \sim q \\ \hline \sim p \lor \sim r \end{array}$$

The first conditional asserts that q is a necessary condition for p. This means that if $\sim q$ is true, then $\sim p$ must be true. The second conditional says that $\sim q$ is a necessary condition for r. This means that if $\sim\sim q$ (which is logically equivalent to q) is true, then $\sim r$ must be true. Since $q \lor \sim q$ is a logical truth, $\sim p \lor \sim r$ must follow from $(p \supset q) \bullet (r \supset \sim q)$. The following argument is an instance of such a form:

> GM can be competitive only if it increases outsourcing. UAW workers can have job security only if GM does not increase outsourcing. GM either increases or does not increase outsourcing. Therefore, either GM cannot be more competitive or UAW workers cannot have job security. 3.3i

$$\begin{array}{l} (C \supset I) \bullet (J \supset \sim I) \\ I \lor \sim I \\ \hline \sim C \lor \sim J \end{array}$$

The Problem of Evil — a Destructive Dilemma

There is a well-known philosophical problem called **the problem of evil** in western philosophy and religions. It argues that God is either not all-powerful or not all-good. The argument can be constructed as a destructive dilemma:

1. If God is all-powerful, He would be able to eliminate evil.
2. If God is all-good, He would want to eliminate evil.
3. Evil exists. (This means that God is not able to eliminate evil, or God does not want to eliminate evil.)

4. Therefore, God is either not all-powerful or not all-good.

After symbolization,

$$\begin{array}{l} (P \supset A) \bullet (G \supset W) \\ \sim A \vee \sim W \\ \hline \sim P \vee \sim G \end{array}$$

we can see that the argument indeed is an instance of destructive dilemma.

3.3.2 Two Formal Fallacies

The mix-up of the sufficient condition and the necessary condition produces two common invalid forms. They are affirming the consequent and denying the antecedent. It is important not to confuse them with Modus Ponens and Modus Tollens.

Affirming the Consequent (AC)

Affirming the consequent mistakes a necessary condition for a sufficient condition. If q is a necessary condition of p, then even if q is true, p may still be false. This is why affirming the consequent is invalid.

$$\begin{array}{l} p \supset q \\ q \\ \hline p \end{array}$$

The next argument has two true premises, but its conclusion is false. It is a counterexample that illustrates the invalidity of affirming the consequent.

If Isaac Newton is a biologist, then he is a scientist. Isaac Newton is a scientist. Therefore, he is a biologist. 3.3j

$$\begin{array}{l} B \supset S \\ S \\ \hline B \end{array}$$

Denying the Antecedent (DA)

Denying the antecedent concludes that q must be false on the basis that a sufficient condition p is not true. The correct conclusion to draw from p being false should be that q can be true or false.

$$\begin{array}{l} p \supset q \\ \sim p \\ \hline \end{array}$$

$\sim q$

Here is another counterexample that shows the form is invalid.

> If Bill Gates owns Exxon Mobil, then he is a billionaire. Bill Gates does not own Exxon Mobil. Therefore, he is not a billionaire. 3.3k

$$\begin{array}{l} O \supset B \\ \underline{\sim O} \\ \sim B \end{array}$$

3.3.3 Recognizing Common Forms

In learning the basic argument forms, we use "*p*", "*q*", "*r*" and "*s*" as variables. They serve as place holders in argument forms. If we replace each variable in a basic form with a capital letter, we would of course end up with an instance of the form. But we can also replace a variable with a compound sentence. The resulting argument would also be an instance of the form. Such substitutions give us more flexibility in constructing instances of the basic forms. It also helps us identify them. Take a look at the next argument.

> The current economic growth cannot be sustained unless inflation is under control. Inflation is not under control. Therefore, the current economic growth cannot be sustained. 3.3l

The argument can be symbolized as

$$\begin{array}{l} \sim S \vee U \\ \underline{\sim U} \\ \sim S \end{array}$$

We can then see that it is an instance of disjunctive syllogism by the following substitutions:

$p = \sim S$

$q = U$

In the next example, we should recognize $A \bullet K$ as the antecedent p and $\sim D$ as the consequent q. As a result, it is a Modus Ponens.

$$
\begin{array}{l}
(A \cdot K) \supset {\sim}D \\
A \cdot K \\
\hline
{\sim}D
\end{array}
$$

To properly identify the next form as an instance of disjunctive syllogism, we need to apply the **rule of double negation**, which says that p is logically equivalent to ${\sim}{\sim}p$. Afterwards, we substitute p for $G \supset M$ and q for ${\sim}D$.

$$
\begin{array}{l}
(G \supset M) \vee {\sim}D \\
D \\
\hline
G \supset M
\end{array}
$$

3.3.4 Combining Basic Forms

We can combine basic argument forms to construct longer and more complex arguments. The argument (3.3a)

> If interest rates are raised, the stock market will be hurt. If the stock market is hurt, the economy will slow down. But if interest rates are not raised, inflation will get worse. If inflation gets worse, the economy will slow down. So the economy will slow down. 3.3a

is built by combining two hypothetical syllogisms with a constructive dilemma. It is easier to see the logical structure from the symbolization.

$$
\begin{array}{l}
R \supset S \\
S \supset E \\
{\sim}R \supset W \\
W \supset E \\
(R \vee {\sim}R) \\
\hline
E
\end{array}
$$

Notice that we include the tautology $R \vee {\sim}R$ as the last premise. The tautology goes unstated in the original argument because it is trivial that it is true. Now from the first two premises, we can derive $R \supset E$ as a conclusion based on hypothetical syllogism.

$$\begin{array}{l} R \supset S \\ S \supset E \\ \hline R \supset E \end{array}$$

Using hypothetical syllogism one more time, we can draw the conclusion $\sim R \supset E$ from the third and the fourth premises.

$$\begin{array}{l} \sim R \supset W \\ W \supset E \\ \hline \sim R \supset E \end{array}$$

Next, using the two conditionals $R \supset E$ and $\sim R \supset E$, and the tautology $R \vee \sim R$, we arrive at the conclusion E according to constructive dilemma.

$$\begin{array}{l} R \supset E \\ \sim R \supset E \\ R \vee \sim R \\ \hline E \end{array}$$

Using Basic Forms to Determine Validity

As we saw above, the argument (3.3a) is constructed by combining three basic forms. Since each of them is valid, we can determine that (3.3a) is valid without constructing a long truth table with 16 rows.

Since long arguments are often constructed out of basic forms, we can determine their validity by identifying the basic forms. If all of the basic forms are valid, then the long argument is valid. But if one of the basic forms is invalid, then the long argument is invalid. This is not a rigorous formal process like the truth table method. But it does enable us to determine validity without constructing truth tables. When we try to break up a long argument into basic forms, we need to, when possible, identify valid forms first. To decide the validity of the argument,

> A new game console is in high demand only if there are a lot of exciting games available for it. However, many game developing companies won't design new games for a new console unless it is in high demand. Since there are not a lot of exciting games available for a new console, it follows that many game developing companies won't design new games for it. 3.3m

we first symbolize it as

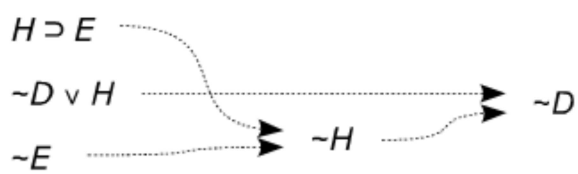
$H \supset E$

$\sim D \vee$
H
$\sim E$
―――
$\sim D$

Then we break it apart into the following two forms:

$H \supset E$ $\sim E$ ――― $\sim H$ Modus Tollens	and	$\sim D \vee H$ $\sim H$ ――― $\sim D$ Disjunctive Syllogism

Since both basic forms are valid, (3.3m) is valid. The diagram below shows how the conclusion is derived from the premises—the logical flow of the argument.



In the next example,

> If we continue the acceleration of production and consumption, we will deplete natural resources within one hundred years. If natural resources are depleted within one hundred years, life on earth will not be sustainable. We continue to accelerate production and consumption. As a result, life on earth will not be sustainable. 3.3n

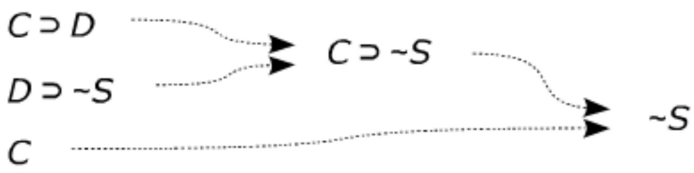
the argument is symbolized as

$C \supset D$
$D \supset \sim S$
C
―――
$\sim S$

We can see that it is constructed from the following two valid forms, and thus is valid.

$C \supset D$ $D \supset \sim S$ ――― $C \supset \sim S$ Hypothetical Syllogism	and	$C \supset \sim S$ C ――― $\sim S$ Modus Ponens

The logical flow of the argument is illustrated below.



The next argument

> If one is a fiscal conservative, then one would be against big government spending. But if one is against big government spending, then one would support budget cut on military spending. President Bush supports budget cut on military spending. Therefore, he is a fiscal conservative. 3.30

$$\begin{array}{l} F \supset A \\ A \supset S \\ S \\ \hline F \end{array}$$

is invalid because after we separate it into two forms:

$$\begin{array}{l} F \supset A \\ A \supset S \\ \hline F \supset S \end{array} \qquad \text{and} \qquad \begin{array}{l} F \supset S \\ S \\ \hline F \end{array}$$

Hypothetical Syllogism

Affirming the Consequent

we find that the second form is invalid.

Exercise 3.3

I. Decide whether each of the arguments is one of the common forms. If it is, identify the name of the form and decide its validity. If it is not a common form, label it as "No Form" and use the truth table to determine its validity.

1. $$\begin{array}{l} A \supset C \\ A \\ \hline C \end{array}$$

   Valid
   Invalid

2. $$\begin{array}{l} H \supset K \\ \sim H \\ \hline \sim K \end{array}$$

   Valid
   Invalid

3. $E \supset G$
$\sim G$
$\overline{\sim E}$

- Valid
- Invalid

4. $\sim S \supset M$
$\sim M$
$\overline{S}$

- Valid
- Invalid

5. $R \supset \sim D$
R
$\overline{\sim D}$

- Valid
- Invalid

6. $\sim F \supset \sim P$
P
$\overline{F}$

- Valid
- Invalid

7. $B \vee C$
$\sim C$
$\overline{B}$

- Valid
- Invalid

8. $A \supset \sim L$
$\sim L$
$\overline{A}$

- Valid
- Invalid

9. $G \vee \sim N$
N
$\overline{G}$

- Valid
- Invalid

10. $K \supset M$
$\sim M \supset E$
$\overline{K \supset E}$

- Valid
- Invalid

11. $\sim D \supset E$
$P \supset \sim D$
$\overline{P \supset E}$

- Valid
- Invalid

12. $(S \supset \sim V) \bullet (\sim A \supset U)$
$V \vee \sim U$
$\overline{\sim S \vee A}$

- Valid
- Invalid

13. $C \supset {\sim}F$
$\underline{{\sim}F}$
C

○ Valid
○ Invalid

14. $(R \supset L) \bullet ({\sim}R \supset E)$
$\underline{R \vee {\sim}R}$
$L \vee E$

○ Valid
○ Invalid

15. $G \vee {\sim}(H \equiv N)$
$\underline{H \equiv N}$
${\sim}G$

○ Valid
○ Invalid

16. $(A \bullet {\sim}B) \supset K$
$\underline{{\sim}(A \bullet {\sim}B)}$
${\sim}K$

○ Valid
○ Invalid

17. $(J \supset M) \bullet (P \supset {\sim}D)$
$\underline{{\sim}J \vee {\sim}P}$
${\sim}M \vee D$

○ Valid
○ Invalid

18. $F \supset ({\sim}D \vee E)$
$\underline{{\sim}({\sim}D \vee E)}$
${\sim}F$

○ Valid
○ Invalid

19. $C \vee {\sim}A$
$\underline{{\sim}A \vee N}$
$C \vee N$

○ Valid
○ Invalid

20. ${\sim}(R \bullet C) \supset {\sim}S$
$\underline{{\sim}S \supset T}$
${\sim}(R \bullet C) \supset T$

○ Valid
○ Invalid

Check Answers

II. Use the common argument forms to derive the conclusion from the premises. Give the name of the argument form used.

1. ${\sim}A \supset E$
$\underline{{\sim}E}$

2. $D \supset H$
$\underline{{\sim}D}$

~ • ∨ ⊃ ≡

~ • ∨ ⊃ ≡

3. $C \supset R$
R

~ • ∨ ⊃ ≡

4. $K \supset B$
$B \supset R$

~ • ∨ ⊃ ≡

5. $\sim G \supset \sim N$
$\sim G$

~ • ∨ ⊃ ≡

6. $(K \bullet A) \supset M$
M

~ • ∨ ⊃ ≡

7. $\sim S \vee \sim L$
L

~ • ∨ ⊃ ≡

8. $\sim B \supset R$
$D \supset \sim S$
$\sim B \vee D$

~ • ∨ ⊃ ≡

9. $C \supset \sim D$
D

~ • ∨ ⊃ ≡

10. $A \vee (C \supset U)$
$\sim(C \supset U)$

~ • ∨ ⊃ ≡

11. $\sim(P \supset \sim H) \supset N$
$P \supset \sim H$

~ • ∨ ⊃ ≡

12. $(G \supset \sim A) \bullet (B \supset \sim R)$
$A \vee R$

~ • ∨ ⊃ ≡

13.

14.

$S \supset {\sim}E$
${\sim}E \supset {\sim}P$

~ • ∨ ⊃ ≡

$(J \vee D) \supset Q$
$J \vee D$

~ • ∨ ⊃ ≡

15. ${\sim}K \supset (G \equiv {\sim}N)$
${\sim}K$

~ • ∨ ⊃ ≡

16. ${\sim}B \supset F$
$C \supset (N \vee {\sim}L)$
${\sim}F \vee {\sim}(N \vee {\sim}L)$

~ • ∨ ⊃ ≡

Check Answers

III. Decide whether each of the following arguments is valid by showing how it is constructed out of the common forms. Identify each common form with its abbreviated form name.

1. $H \supset G$
$D \vee H$
${\sim}D$

${\sim}G$

Valid
Invalid

$D \vee H$
${\sim}D$

$H \supset G$

G

~ • ∨ ⊃ ≡

2. $M \supset B$
$B \supset {\sim}C$
M

${\sim}C$

Valid
Invalid

$M \supset B$
M

$B \supset {\sim}C$

${\sim}C$

~ • ∨ ⊃ ≡

3. $A \supset E$
${\sim}A \supset {\sim}S$

$A \supset E$
${\sim}E$

${\sim}A \supset {\sim}S$

~E		~S
~S		

Valid
Invalid

~ • ∨ ⊃ ≡

4. $K \supset R$
$\sim R$
$K \supset N$
$\sim N$

$K \supset R$	$K \supset N$
$\sim R$	
	$\sim N$

Valid
Invalid

~ • ∨ ⊃ ≡

5. $D \vee C$
$\sim D \vee \sim B$
B
C

$\sim D \vee \sim B$	$D \vee C$
B	
	C

Valid
Invalid

~ • ∨ ⊃ ≡

6. $\sim H \supset (G \equiv E)$
$P \supset H$
$\sim(G \equiv E)$
P

$\sim H \supset (G \equiv E)$	$P \supset H$
$\sim(G \equiv E)$	
	P

Valid
Invalid

~ • ∨ ⊃ ≡

7. $A \supset L$
$A \vee \sim M$
$L \supset S$
$\sim S$
$\sim M$

$A \supset L$		$A \vee \sim M$
$L \supset S$	$\sim S$	
		$\sim M$

~ • ∨ ⊃ ≡

- Valid
- Invalid

8. $\sim N \supset \sim K$

$\sim N \vee R$

$R \supset \sim O$

O

$\sim K$

$R \supset \sim O$ / $\sim N \vee R$ / $\sim N \supset \sim K$ / $\sim K$

~ • ∨ ⊃ ≡

- Valid
- Invalid

9. $C \vee E$

$C \supset \sim B$

$E \supset M$

$\sim M$

$\sim B$

$C \supset \sim B$ / $\sim B$

~ • ∨ ⊃ ≡

- Valid
- Invalid

10. $W \supset (\sim G \supset H)$

W

$\sim H$

G

$W \supset (\sim G \supset H)$ / G

~ • ∨ ⊃ ≡

- Valid
- Invalid

11. $(A \supset D) \supset B$

$D \vee (B \supset H)$

$\sim H$

$\sim D$

$\sim (A \supset D)$

$D \vee (B \supset H)$ / $\sim (A \supset D)$

~ • ∨ ⊃ ≡

- Valid
- Invalid

12. $\sim(R \supset \sim L) \supset P$
    $\sim L \vee P$
    $\sim P$
    R

$\sim P$	$\sim P$	
		R

- Valid
- Invalid

~ • ∨ ⊃ ≡

13. $S \supset G$
    $N \supset D$
    $\sim G \vee \sim D$
    $(\sim S \vee \sim N) \supset K$
    K

$S \supset G$	
	K

- Valid
- Invalid

~ • ∨ ⊃ ≡

14. $(G \bullet M) \supset A$
    $\sim A \vee (\sim E \supset O)$
    $G \bullet M$
    $\sim B \supset \sim E$
    $\sim B \supset O$

		$\sim B \supset O$

~ • ∨ ⊃ ≡

- Valid
- Invalid

Check Answers

← PREVIOUS