Comet Tail Artifacts in Computed Tomography

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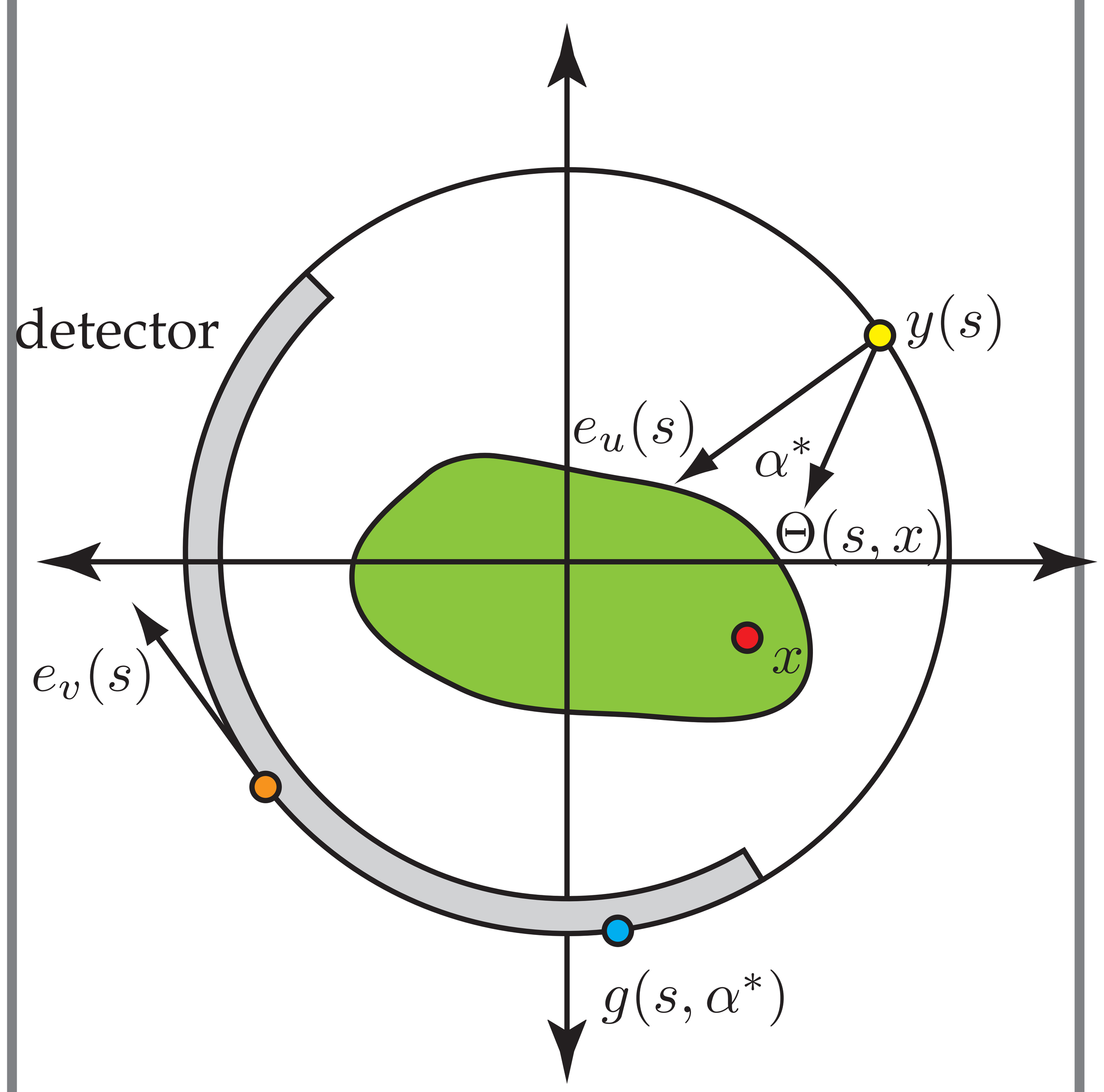
Contribution

In computed tomography (CT) we recover a function f from its x-ray data. We model our x-ray data as line integrals of f and then filter and backproject our data to recover f. We present a study of artifacts that appear in a 3D reconstruction formula for CT [2] by studying the 2D case.

Reconstruction Method

We following the work of [1]. We have a rotating detector and x-ray source traveling around an object $f(x)$. Here we assume a source curve $y(s) = (R\cos(s), R\sin(s))$ with radius R. We have detector coordinates $e_u(s)$ and $e_v(s)$. The vector $\Theta(s,x)$ points from $y(s)$ in the direction of x. The angle between $e_u(s)$ and $\Theta(s,x)$ is α^*. The point x has detector coordinate α^* and the x-ray data for x is $g(s,\alpha^*)$.



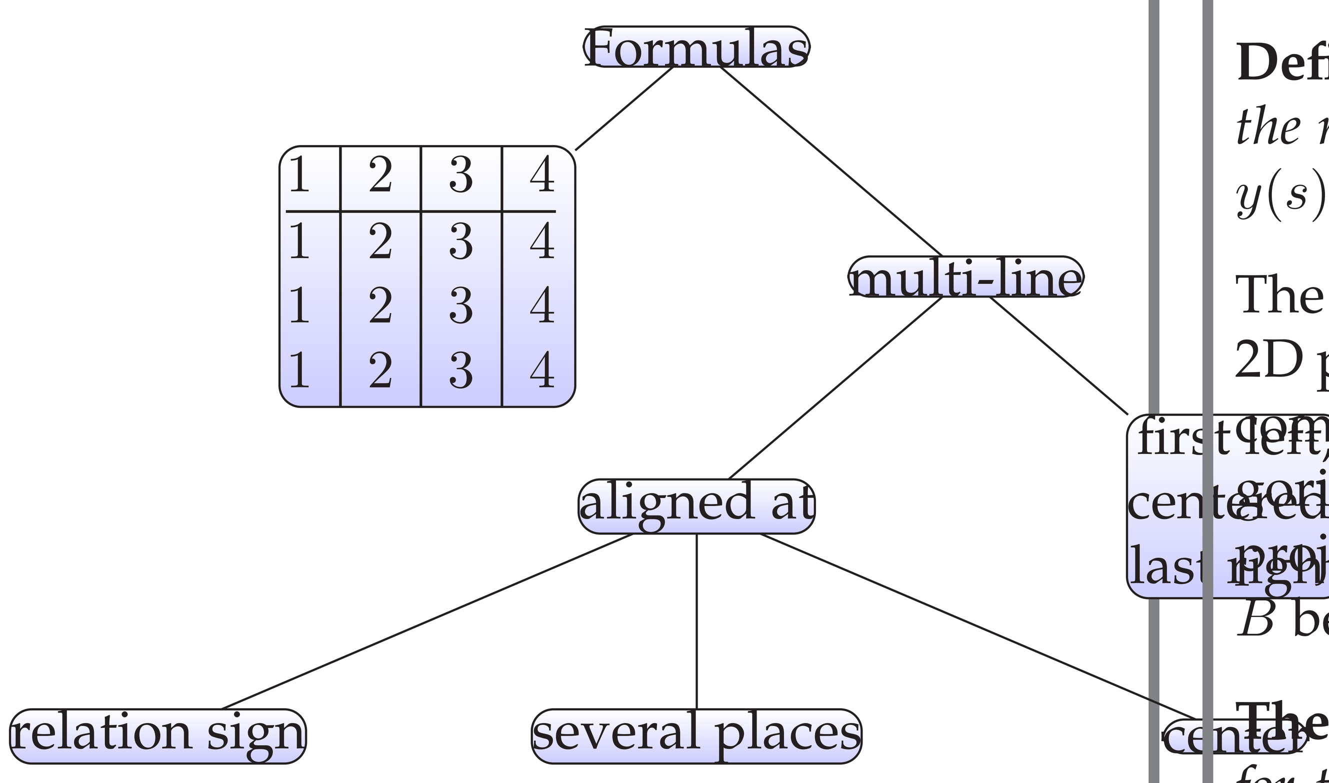
We suppose that our x-ray data is given to us by

$$g(s,\alpha^*) = \int_0^\infty f(y(s) + t\Theta(s,x))\,dt. \quad (1)$$

Our formula for f is a 2D analog of Katsevich's 3D formula,

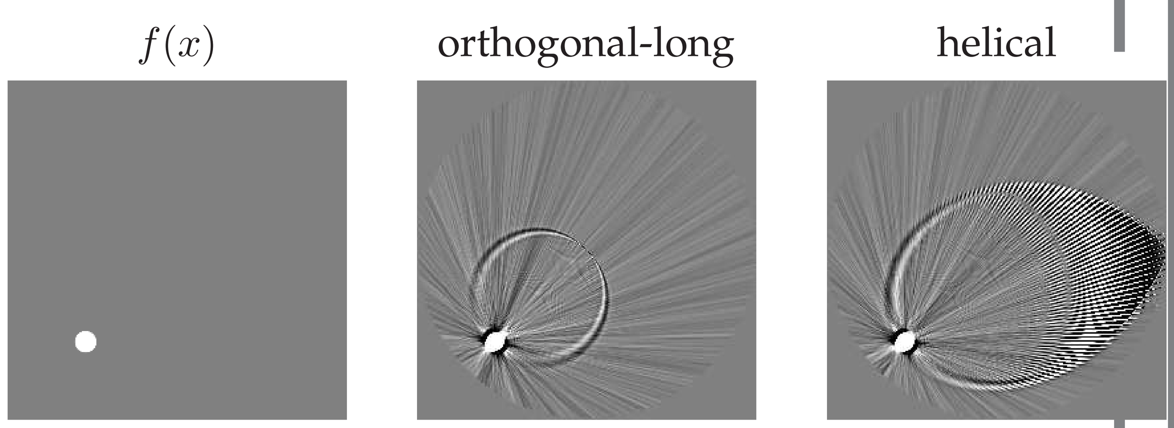
$$f(x) = \frac{1}{2\pi^2}\int_{I_{PI}(x)} \frac{1}{|x-y(s)|} \int_0^{2\pi}\left(\frac{\partial g}{\partial s}+\frac{\partial g}{\partial \alpha}\right)\frac{1}{\sin(\alpha^*-\alpha)}\,d\alpha\,ds \quad (2)$$

PI-lines



Comet Tails

Let $f(x)$ be a smooth compactly supported function. Below are $f(x)$ and reconstructions of $f(x)$ by equation 1 with orthogonal-long and helical pi-lines.



The long tail coming off of the function is called a comet tail artifact. The artifact also occurs in reconstructions from the 3D version of formula 1, however with a much stronger presence.

Results

Definition 1. *We call $RBP(s) = \{x : s \in I_{PI}(x)\}$ the region of back projection associated with position $y(s)$.*

The shape of the region of back projection in the 2D plane is directly related to the behavior of the comet tail artifact found in the 2D Katsevich algorithm. We will now describe the region of back projection for orthogonal and tilted PI-lines. Let B be the unit ball in $\mathbb{R}^2$.

Theorem 1. *Suppose $R = 1$ and let $s \in [0, 2\pi)$, then for the PI-lines of the orthogonal-long type we have $RBP(s) = B \cap D(s)$ where $D(s) = \{x : \|x - c\|_2 \geq 1/2\}$ and $c = (\cos(s)/2, \sin(s)/2)$.*

The $RBP(s)$ for the tilted-long PI-lines follows immediately.

Corollary 1. *Suppose $R = 1$ and consider B with the PI-lines of the tilted-long type. Let $s \in [0, 2\pi)$, then $s \in I_{PI}(x)$, for all $x \in B \cap D(s)$ where $D(s) = \{x : \|x - A^{-1}c\|_2 \geq \frac{1}{2\sin(\psi)}\}$ and $c = (\cos(s)/2, \sin(s)/2)$.*

Proof. (Outline) This follows from Theorem 1 and the fact that

$$\begin{aligned}
\frac{1}{4} &\leq \langle Ax - c, Ax - c\rangle \\
&= \langle Ax - AA^{-1}c, Ax - c\rangle \\
&= \langle x - A^{-1}c, A^*Ax - A^*c\rangle \\
&= \langle x - A^{-1}c, \sin^2(\psi)x - \sin^2(\psi)A^{-1}c\rangle \\
&= \sin^2(\psi)\langle x - A^{-1}c, x - A^{-1}c\rangle.
\end{aligned}$$

Hence $\|Ax - c\|_2 \geq \frac{1}{2}$ if and only if $\|x - A^{-1}c\|_2 \geq \frac{1}{2\sin(\psi)}$. □

We can now identify where along the boundary of $RBP(s)$ a point x will backproject to. This corresponds to the end points of the outer integral in equation 1.

Closing Remarks

We have shown that the selection of PI-lines will effect the shape of the RBP(s). Furthermore we have created a method to determine where along the boundary of RBP(s) x-ray data from a point source will backproject to. The curve $\gamma(s)$ in Theorem 2 corresponds to the location of the comet tail artifact associated with orthogonal-long pi-lines.