PES University, Bangalore

(Established Under Karnataka Act 16 of 2013)

Department of Science and Humanities

Engineering Mathematics - I (UE25MA141A)

Question Bank

Unit - 3: Partial Differential Equations

1. Eliminate the arbitrary function f from the equation:

$$z = f\left(\frac{xy}{z}\right)$$

and form the corresponding partial differential equation (PDE).

Answer: $px = qy$, where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

2. Form the partial differential equation by eliminating the arbitrary functions f and g in:

$$z = x^2 f(y) + y^2 g(x)$$

Answer: $xys + 4z = 2px + 2qy$

3. Form the partial differential equation by eliminating the arbitrary functions f and g in:

$$z = f(x^3 + 2y) + g(x^3 - 2y)$$

Answer: $9x^5 t - 4xr + 8p = 0$

4. Solve the partial differential equation:

$$\frac{y^2 z}{x}\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = y^2$$

Answer: $\Phi(x^2 - z^2, y^3 - x^3) = 0$

5. Solve the partial differential equation:

$$(x^2 + y^2 + yz)\frac{\partial z}{\partial x} + (x^2 + y^2 - zx)\frac{\partial z}{\partial y} = z(x + y)$$

Answer: $\Phi\left(2xz - \frac{z^2}{2}, x^2 + y^2 + z^2\right) = 0$

6. Solve the partial differential equation:

$$x(y^2 + z)\frac{\partial z}{\partial x} + y(x^2 + z)\frac{\partial z}{\partial y} = z(x^2 - y^2)$$

Answer: $\Phi\left(x^2 - y^2 - 2z, \frac{xz}{y}\right) = 0$

Solve the following by the method of the separation of variables (Problems 7,8, and 9)

7. $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, given $u(x,0) = e^{-x}$

   Answer: $u = e^{-\frac{1}{2}(2x-3y)}$

8. $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} - e^{-5y}$ when $x = 0$.

   Hine: Write u as the sum of two solutions. That is, $u = C_1 e^{\frac{\lambda_1}{4}x+(3-\lambda_1)y} + C_2 e^{\frac{\lambda_2}{4}x+(3-\lambda_2)y}$

   Answer: $u = 3e^{x-y} - e^{2x-3y}$

9. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$

   Answer: $u = \left(C_1 e^{(1+\sqrt{1+\lambda})x} + C_2 e^{(1-\sqrt{1-\lambda})x}\right) e^{-\lambda y}$

10. Solve $(D^3 + D^2 D' - DD'^2 - D'^3)z = 0.$

    Answer: $z = \phi_1(x+y) + x\phi_2(x-y) + \phi_3(x-y)$

11. Solve $\left[D^3 - D^2 D' - 4\,D\,(D')^2 + \left(4D'\right)^3\right] z = 0.$

    Answer: $z = \phi_1(x+y) \ + \ \phi_2(2x-y) \ + \ \phi_3(2x+y).$

12. Solve $\left[2D^2 + 5\,D\,D' + 3\,(D')^2 + D + D'\right] z = 0.$

    Answer: $z = \phi_1(x-y) \ + \ e^{-x/2}\,\phi_2(3x-2y).$

13. Solve the partial differential equation: $\left[D^2 + 3\,D\,D' + 2\,(D')^2\right] z = e^{x-y}.$

    Answer:

    $$\text{C.F.:} \quad z = \phi_1(x-y) \ + \ \phi_2(2x-y), \qquad \text{P.I.:} \quad -\,x\,e^{x-y} \quad \text{or} \quad -\,y\,e^{x-y}.$$

14. Solve the partial differential equation: $\left[3\,D^2 + 7\,D\,D' + 2\,(D')^2\right] z = 3x^2 + 2y^2.$

    Answer:

    $$z = \phi_1(2x-y) \ + \ \phi_2(x-3y) \ + \ \frac{x^2}{324}\left[113x^2 + 108y^2 - 168xy\right].$$

15. Solve the partial differential equation: $\left[4\,D^2 + 4\,D\,D' + (D')^2\right] z = 4y\,\cos(2x).$

    Answer:

    $$z = \phi_1(x-2y) \ + \ x\,\phi_2(x-2y) \ + \ \frac{1}{8}\left[\sin(2x) \ - \ 2y\cos(2x)\right].$$