A new channel estimation method based on GPR and wavelet denosing

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Abstract—**In this paper, we propose a new method based on comb-type pilot which combines the Gaussian process regression (GPR) and the wavelet-denoising for channel estimation of orthogonal frequency-division multiplexing (OFDM) system. We use the OFDM receiver to estimation channel state information (CSI) at pilot explicitly and detect/recover the transmitted symbols using the estimated CSI. The initial channel frequency response on the pilot position was obtained by Least Square (LS) algorithm and processed by the wavelet-denoising algorithm which can reduces the influence of noise in the OFDM communication systems. Then the frequency at pilot and values of its frequency response are used as training data for the GPR algorithm to get the channel frequency response of whole carrier. The simulation results show that under the multi-path channel circumstance, the new approach significantly reduces the bit error rates (BER) compared to traditional method.**

I. Introduction

The OFDM system can effectively combat the Inter Symbol Interference (ISI) at the same time that the high-speed data business is guaranteed. It can maximize the use of frequency spectrum resources. Another advantage is that it is easy to use with other multiple addressing technologies. Due to the advantage, OFDM has been used in many communication scenarios such as Long Term Evolution (LTE), Internet of Things (IoT) [1], Internet of Vehicles (IoV) [2], underwater communications and so on. OFDM technology is not only widely used in 4G communication, but also one of the key technologies of 5G communication. Therefore, the research on OFDM system is still of practical significance

OFDM system is composed of several key technologies including synchronization [3], channel estimation, channel equalization [4], etc. Channel estimation is one of vital technologies in OFDM. Its function is to reduce the influence of multipath channel fading. The accuracy of channel estimation will seriously affect the quality of received signals. Nowadays, Temporal channel variations is still a challenge for wireless communications.

Due to the requirements of pilot symbols, there are three types of channel estimations: blind estimation [5], semi-blind estimation and pilot-aided channel estimation [6]. Generally, the pilot-aided channel estimation method is more reasonable in OFDM communication scene.

The conventional channel estimation methods for pilot symbols are LS algorithm, Linear minimum Mean Square Error (LMMSE) algorithm [7] and so on. In [8] and [9], a conventional OFDM channel estimates based on LMMSE filter is introduced with the purpose of flatting the estimation error and interpolating the channel in time and frequency domains. After obtaining the CSI on pilot symbols by estimation,interpolation is used to interpolate the channel impulse response (CIR) at the data symbols.Commonly applied interpolation methods include constant value interpolation, linear interpolation, polynomial interpolation,spline interpolation, and discrete Fourier transforming (DFT) [10] interpolation, etc.

In this paper, firstly we obtain the CSI at pilot symbols by LS algorithm, and then use a method based on wavelet analysis to de-noise the CSI. Finally, we use the GPR method [11] to predict the CIR at data symbols. As one of machine learning methods, GPR method uses a probability method to learn data under bayesian criterion. It has strong nonlinear fitting ability and small sample learning ability [12], another advantage is that it can automatically obtain the best super-parameter by an optimization process [13]. The simulation results show that the use of the GPR and wavelet analysis in communication presents obvious advantage compared to traditional approaches.

II. System Model

In this section, we present the system model, and then introduce the formula based on channel estimation. For pilot-aided channel estimation methods in OFDM systems, pilot pattern selection is the basis for our research. The current researches and applications of pilot patterns include comb-type pilot patterns, block pilot patterns and trellis pilot patterns, etc. Among them, the comb-type pilot pattern insertes the pilot symbols into part of the subcarriers. The comb-type pilot pattern is more suitable for time-varying channel [14]. So it is adopted in this paper.

We use i symbolizes the symbol time index of the OFDM system. Firstly, the sending sequence generates data symbols X^T through some processing methods such as encoding and modulation mapping. Then the serial binary sequence X^T is transformed into parallel sequences $X(i)$. Finally, pilot signals $X(k)$ are inserted into each frame of data from the frequency domain and modulated to N subcarriers by the inverse discrete fourier transform (IDFT). At this time, the OFDM signal can be expressed as:

$$x(n) = IDFT\{X(k)\} = \sum_{i=0}^{N-1} X(k)e^{j\frac{2\pi kn}{N}}, \quad n = 0, 1, ..., N-1. \tag{1}$$

A guard interval consisting of cyclic prefixes (CP) is inserted between each frame to eliminate the inter-carrier interference (ICI). Then we can get the signal $X_p(n)$. It is converted into a serial signal and sent into the channel.

The impulse response of a multipath time-varying fading channel can be expressed as:

$$h(n) = \sum_{i=0}^{q-1} h_i e^{j(\frac{2\pi}{N} f_{d_i} T_n + \theta_i)} \delta(n - \frac{\tau_i}{T}), 0 \leq n \leq N-1 \tag{2}$$

Where q represents the number of multipath, h_i represents the amplitude fading factor, f_d represents the doppler shift, τ represents the time delay, and T represents the system sampling period.

At the receiving end, the received signal is converted into a parallel signal. Received signal is shown as:

$$y_p(n) = x_p(n) \otimes h(n) + v(n) \tag{3}$$

Where $v(n)$ represents the additive white noise.

After signal synchronization, remove the CP from the signal. Then we can get the $y(n)$. After that, the discrete fourier transform (DFT) is used with the aim of obtaining the frequency-domain OFDM symbol to get the $Y(k)$:

$$Y(k) = DFT\{y(n)\} = \frac{1}{N} \sum_{n=0}^{N-1} y(n)e^{-j\frac{2\pi kn}{N}}, \quad k = 0, 1, ..., N-1 \tag{4}$$

Take the Fourier transform of both sides of formula $y_p(n)$ [15]:

$$Y(k) = X(k)H(k) + V(k), k = 0, 1, ..., N-1 \tag{5}$$

Extract the pilot signal $Y_{cp}(k)$ and take the channel estimation, then we can get the channel impulse response (CIR) $\hat{H}(k)$. So we also can get the estimation of transmit signal:

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)} \tag{6}$$

Finally, the receiving sequence can be obtained by demodulating the received signal.

III. Process of the Proposed Method

In this section, we introduce the theoretical knowledge of the new channel estimation method including the LS algorithm, wavelet denoising algorithm and the GPR algorithm. Finally, we expound the steps of the channel estimation algorithm in this paper.

A. obtain original data by LS

The expression of receiving signal can be written as:

$$Y_p(m) = X_p(m)H_p(m) + V_p(m), m = 0, 1, ..., n-1 \tag{7}$$

Where Y_p, X_p, H_p, V_p represent the received signal, transmitted signal, channel frequency domain response and the noise at the pilots.

The cost function is shown as:

$$\begin{aligned} g &= (Y_p - \hat{Y}_p)^H (Y_p - \hat{Y}_{pP}) \\ &= (Y_p - X_p\hat{H}_p)^H (Y_p - X_P\hat{H}_p) \end{aligned} \tag{8}$$

Where $\hat{H}_p$ represents the estimation of the channel frequency response at pilot.

Under the constraint of LS, the channel estimation results at pilot can be obtained:

$$\hat{H}_{LS,p} = X_p{}^{-1}Y_p \tag{9}$$

Then a new expression can be obtained from the above formulas:

$$\hat{H}_{LS,p} = X_p{}^{-1}Y_p = H_p + X_p{}^{-1}V_p \tag{10}$$

The LS algorithm is simple and only requires one division. But the $\hat{H}_{LS,p}$ is an estimation result containing noise. Since the LS algorithm does not consider the correlation between sub-channels in frequency domain and the correlation between adjacent symbols in time domain and the influence of noise, the mean square error (MSE) of LS algorithm is usually high and the BER performance is bad. If the influence of noise on estimation results can be reduced, the estimation accuracy of LS algorithm can be improved. [16] proved that the V_p follows a weighted mixture gaussian distribution. Wavelet denoising method based on threshold value can remove noise well, then we can get purer training data for GPR.

B. training data preprocessing by wavelet denosing

The wavelet denoising model can be described as:

$$X_n = f_n + e_n \tag{11}$$

Where X_n represents the signal that contains noise, f_n represents the desired signal, e_n represents the noise.

Wavelet transform (especially orthogonal wavelet transform) has strong correlation to the data. In the wavelet domain, the energy of the desired signal f_n is concentrated in some large wavelet coefficients. But the energy of the noise e_n is distributed throughout the wavelet domain. The amplitude of the wavelet coefficient of the desired signal is greater than that of the noise coefficient. We can set the wavelet domain coefficient threshold [17], a coefficient greater than the threshold is considered to be the component of the desired signal and tangentially retained. Conversely, a coefficients less than the threshold are considered to be noise components and be reset. Thus, the purpose of denoising can be achieved.

Wavelet denoising has three key steps: the selection of threshold, the selection of threshold processing mode and the selection of wavelet type.

1) The selection of threshold: There are four selection rules of threshold: Unbiased likelihood estimation, fixed threshold estimation, heuristic threshold estimation, extreme threshold estimation. Because the LS estimation results contain noise distributed over the entire frequency band, a relatively conservative unbiased likelihood estimation method is used for our threshold selection.

2) Threshold processing: There are two ways of threshold processing: soft threshold and hard threshold. Soft threshold processing can get smoother results without additional oscillation, so we adopts the soft threshold processing method in this paper.

3) The selection of wavelet type: We can get better denoising effect by using wavelet denoising which is similar to the shape of signal. Due to the frequency selectivity fading of the channel, the frequency response continuity of the whole channel is poor. To accommodate this, Haar wavelet [18] which has mutability is used in this paper.

In the case of comb-type pilot, a part of the sub-channels of OFDM system are used to send pilot sequences and LS criterion is used to estimate the frequency response $\hat{H}(m \times f+1), m=0,1,...,k-1$ at these sub-channels. Where k is the number of pilots and Δf is the pilot interval. when the $\hat{H}(m \times f+1)$ has been denoised by wavelet processing, it is renamed as $\hat{H}_w(m \times f+1)$. But when the SNR is high, the $\hat{H}_w(m \times f+1)$ loses some useful information, the accuracy is reduced instead. We can set a threshold SNR_λ [19] in practice and wavelet denoising is performed only if the system's SNR is smaller than SNR_λ.

C. channel estimation by GPR

The systems model in this paper have a nonlinear relationship between input and output, which cannot be solved by traditional linear regression methods. We choose GPR method to deal with this nonlinear relationship.

Similar to the gaussian distribution, the gaussian process can be described by the mean function $m(x)$ and the covariance function $K(x, x')$:

$$f(x) \sim GP(m(x), K(x, x')) \tag{12}$$

Where $f(x)$ represents the gaussian process.

GPR is a nonparametric model, which does not model the input data and only cares about the relationship between the input and the output. This relation can be regarded as an implicit function, and GPR gives it a priori of gaussian process. As a supervised machine learning method, GPR learns and fits this nonlinear relationship between input and output through training data. It also can generate predictive output for new input. This process is realized by probability method: GPR assigns a prior information of a gaussian process to the function to be fitted, then the posterior distribution of the function is obtained by bayesian theory and training data. At last the best estimation of the function is obtained. The adjustment of GPR model can be realized by super-parameter θ. The structure of GPR is shown in the Fig.1:

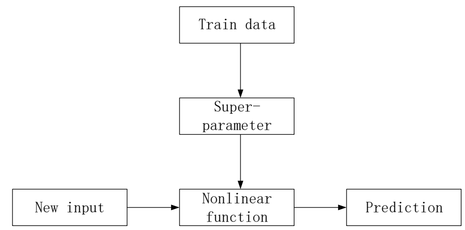


Fig. 1: Simulation results for the network.

In the training stage, GPR maps the training data to the high-dimensional feature space. And then describe the relationship between the data through different covariance functions, Gaussian priori and bayesian inference make the data distribution to be a Gaussian distribution which is easy to process. In the prediction stage, the probability distribution of the predicted output can be obtained according to the test input, which is also a Gaussian distribution. We also can get the prediction and estimation error of the test output in the prediction stage.

In order to estimate the channel frequency response of the whole channels, use the frequency at pilot and its frequency response which have been de-noised $\{m \times f+1, \hat{H}_w(m \times f+1)\}_m^{k-1}$ as the training sample of the GPR algorithm.

Our channel estimation method is summarized below:

(1). Use the LS algorithm to estimate the frequency response $\hat{H}(m \times f+1), m=0,1,...,k-1$ at the pilot.

(2). Obtain the $\hat{H}_w(m \times f+1)$ by wavelet denoising.

(3). Select the appropriate kernel function and set the initial value for the super-parameter θ.

(4). Input training sample $\{m \times f+1, \hat{H}_w(m \times f+1)\}_m^{k-1}$ and obtain the optimal super-parameter θ.

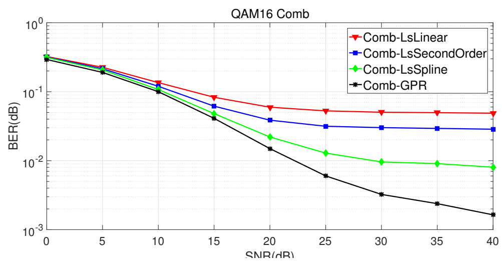
(5). Input the frequency of other sub-channels and get the whole estimated frequency response $\hat{H}(n)$.

IV. SIMULATION

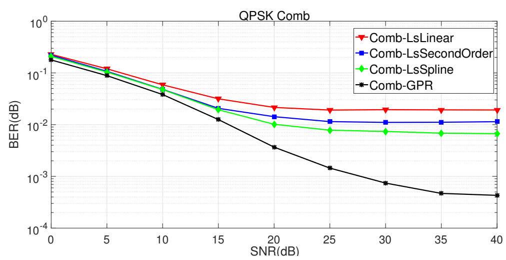
We select the Rayleigh channels with multipath fading in this paper. In our simulation, the number of subcarriers is 128, the number of multipath is 5, and the subcarrier spacing is 7.185KHz.

In this paper, the comb-type pilot is adopted and the frequency response at the pilot is obtained according to the LS method. Traditional linear interpolation method, quadratic interpolation method and cubic spline interpolation method were used for channel estimation and compared with our new method

Fig. 2 is the relationship curve between SNR and BER of each channel estimation algorithms when the modulation mode is 16QAM or QPSK and the doppler frequency shift is 80Hz. As the SNR changes from 0dB to 40dB, the BER corresponding to each algorithm gradually decreases. The performance of GPR algorithm based on machine learning algorithm is obviously better than the three traditional channel estimation algorithms.



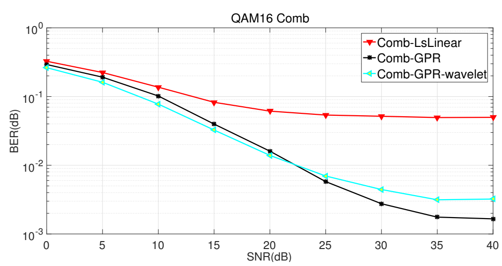
(a) based on 16QAM modulation.



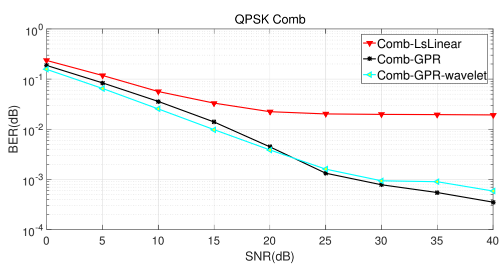
(b) based on QPSK modulation.

Fig. 2: GPR compared with traditional methods.

When the SNR is small, the LS method ignores the noise shadow, which is the main reason for the poor channel estimation effect. In order to solve this problem, we carry out wavelet denoising on the training samples of GPR algorithm. The simulation is shown in Fig. 3.



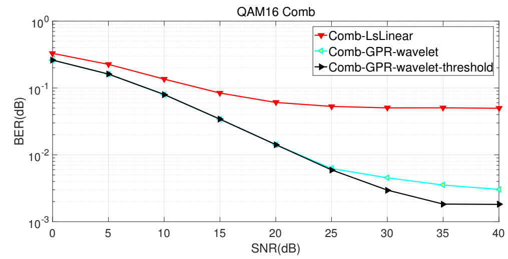
(a) based on 16QAM modulation.



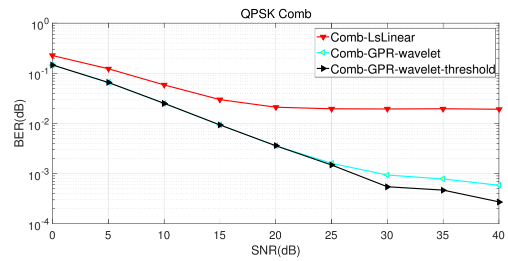
(b) based on QPSK modulation.

Fig. 3: GPR with wavelet denosing.

It can be seen that after adding the denoising algorithm, the performance of the algorithm has been improved in the case of the small SNR. But in larger SNR, the performance of BER is more poor because the wavelet denoising algorithm filters out some useful information. Therefore, we can set a threshold. When SNR is greater than this threshold, no denoising will be carried out. Then we get the simulation results as shown in Fig. 4.



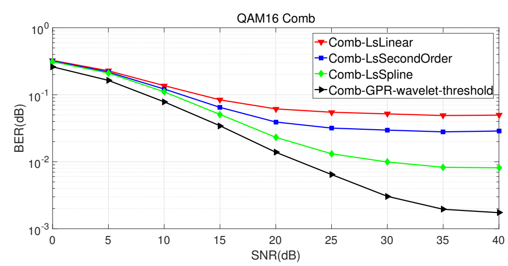
(a) based on 16QAM modulation.



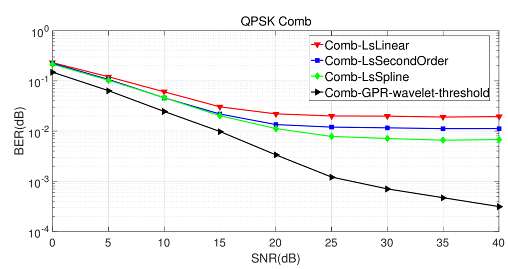
(b) based on QPSK modulation.

Fig. 4: GPR with wavelet denosing and threshold.

Finally, we compare the proposed method with three traditional channel estimation algorithms. As can be seen from figure 5, the GPR algorithm after wavelet denoising with threshold has obvious advantages.



(a) based on 16QAM modulation.



(b) based on QPSK modulation.

Fig. 5: GPR with wavelet denosing and threshold.

V. Conclusion

This paper studies the OFDM channel estimation algorithm based on pilot frequency, focusing on the application of wavelet denoising and machine learning method. For the problem of low precision of traditional interpolation algorithm, the GPR algorithm along with LS and wavelet denoising algorithm is used to obtain better system performance. This method does not need complex cross-validation process or manual selection of the best parameter values, and the best super-parameter can be obtained just by optimizing a convex function. The simulation results show that the performance of this method is improved greatly compared with the traditional estimation algorithm.

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