Introduction to Calculus

Definition 0.1 The line **tangent** to a curve at a point is the line that "best approximates" the curve at that point.

Definition 0.2.1 The **average velocity** of an object over a given interval of time is the change in the position divided by the change in time.

$$v_{ave} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Definition 0.2.2 The **instantaneous velocity** of an object at a given time, t = a, is the limiting value of the average velocity over the time interval from t to a, as t approaches a.

$$\lim_{t_2 \to t_1} \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Definition 1.1 The **limit** of f(x), as x approaches a, equals L. This means that as x gets close to the value of a, but never equals a, the value of f(x) gets closer to the value L.

$$\lim_{x \to \infty} f(x) = L$$

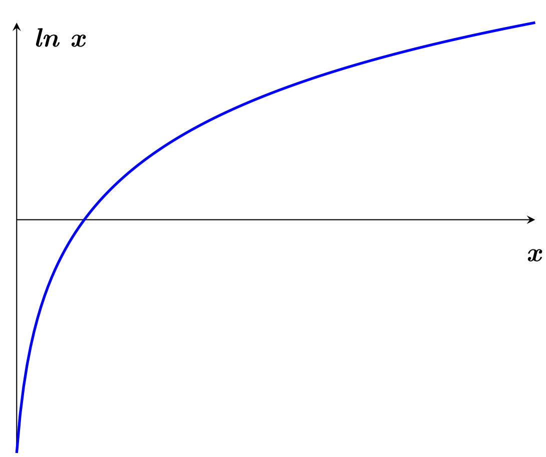
Definition 2.1 The **left limit** of f(x), as x approaches a, equals L. This means that x approaches a AND x < a

$$\lim_{x \to a^-} f(x) = L$$

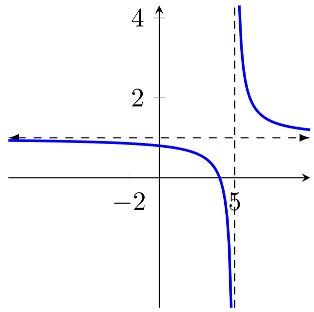
Definition 2.2 The **right limit** of f(x), as x approaches a, equals L. This means that x approaches a AND x > a

$$\lim_{x \to a^+} f(x) = L$$

Remember, when trying to see if a limit approaches infinity, try to visualize the graph.



The limit of *natural logs* will result in ∞ because the graph tends toward infinity.



Given the equation for the graph above, determine the limits as x at the given value?

$$y = \frac{x^2 - 2x - 8}{x^2 - 3x - 10}$$

$$\lim_{x \to 5^-} \frac{x^2 - 2x - 8}{x^2 - 3x - 10} = -\infty$$

$$\lim_{x \to 5^+} \frac{x^2 - 2x - 8}{x^2 - 3x - 10} = \infty$$

$$\lim_{x \to 5} \frac{x^2 - 2x - 8}{x^2 - 3x - 10} = DNE$$

$$\lim_{x \to -2^-} \frac{x^2 - 2x - 8}{x^2 - 3x - 10} = \frac{x-4}{x-5} = \frac{-2-4}{-2-5} = \frac{6}{7}$$

Theorem 1.1 The **Common Sense Limit Laws**: Given c is a constant and $\forall \lim_{x \to a} f(x) \in \mathbb{R}$

$$(1) \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$(2) \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$(3) \lim_{x \to a} cf(x) = c * \lim_{x \to a} f(x)$$

$$(4) \lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) * \lim_{x \to a} g(x)$$

$$(5) \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

$$(6) \lim_{x \to a} [f(x)^n] = [\lim_{x \to a} f(x)]^n$$

Solve the following, given that $\lim_{x \to 2} f(x) = 1$, $\lim_{x \to 2} g(x) = -5$, $\lim_{x \to 2} h(x) = 0$:

$$\lim_{x \to 2} [f(x) + 5g(x)] = \lim_{x \to 2} f(x) + 5 * \lim_{x \to 2} g(x) = 1 + 5 * (-5) = -24$$

$$\lim_{x \to 2} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)} = \sqrt{1} = 1$$

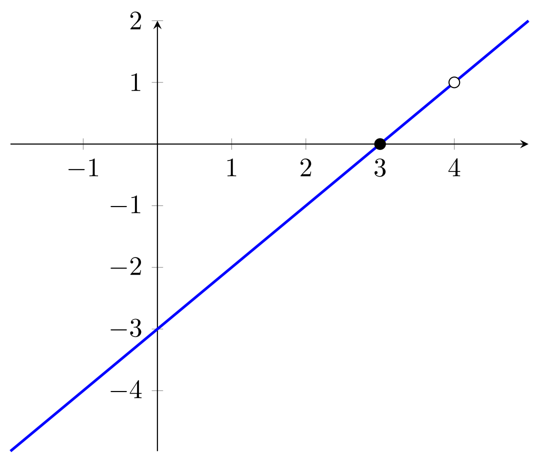
$$\lim_{x \to 2} \sqrt{\frac{3x^2 + 4}{5x - 1}} = \sqrt{\frac{\lim_{x \to 2}(3x^2 + 4)}{\lim_{x \to 2}(5x - 1)}} = \sqrt{\frac{3(2)^2 + 4}{5(2) - 1}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Theorem 2.1 if f(x) = g(x) for all x in an open interval containing a, execpt possibly at a, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$$

FACT To start a limit exercise in the form $\lim_{x \to a} f(x)$ where we have algebraic expressions for *f(x)* involving the usual operations, begin by pluggin in a to see what form you get. Depending on the result we will take different approaches.

1. $f(a)$ is a real number:
   (a) The limit of $f(a)$ is c. $\lim_{x\to a} f(a) = c$
2. $f(a)$ results in $\frac{c}{0}$ where $c \neq 0$
   (a)
$$\lim_{x\to a} f(x) \begin{cases} \infty & c > 0 \\ -\infty & c < 0 \end{cases}$$
3. $f(a)$ results in $\frac{0}{0}$, $\pm\frac{\infty}{\infty}$, or $\infty - \infty$
   (a) Perform some algrebra manipulation on original function



Theorem 5.1 **The Squeezing Theorem**: If f, g, h are functions such that: $f(x) \leq g(x) \leq h(x)$, then $\forall x$ approaching, but not equal to, a:
$$\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x) \leq \lim_{x\to a} h(x)$$
This Theorem is particularly useful when $f(x) = h(x)$

Theorem 1.1 A function f is continous on the interval (a, b) if the graph of $y = f(x)$ can be drawn over the interval (a, b) without lifting your pencil.

1. a function f is continous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$
2. a function f is continous on the interval (a, b) if f is continous at every value in (a, b)
3. a function f is left continous at x = a if $\lim_{x\to a^-} f(x) = f(a)$
4. a function f is right continous at x = a if $\lim_{x\to a^+} f(x) = f(a)$
5. a function f is continous on the interval $[a, b]$ if f is continous at every value in (a, b), f is right continous at a and left continous at b
6. a function f is *discontinous* at $x = a$ if f is not continous at $x = a$. We call a a **discontinuity of** f
7. Many discontinuities may be classified as either a **removable, jump, or infinite** discontinuity.