<table>
<tr><td>Outcome</td><td>NF8</td><td>Student can consistently:</td><td colspan="2">Solve problems involving upper and lower bounds on numbers.</td></tr>
<tr><td>How the topic is examined</td><td colspan="4">

Examined through the exam paper.
This topic is examined on both calculator and non-calculator papers.
There are two types of problems that may be asked:

Simple – students are to give an appropriate upper or lower bound/limit for a number.
Harder – they may need to work with these limits to solve a problem.

The majority of problems that require students to work with limits will be set in a real-life context. Occasionally they involve algebraic substitution.

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<tr><td>Prior knowledge</td><td colspan="4">

Students should be confident:

Rounding to decimal places and significant figures (NF3)
Substituting into expressions (AEx5)

In addition questions involving upper and lower bounds can have links to:

Continuous and discrete data.

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<tr><td>Suggested tuition approaches</td><td colspan="4">

Students need to recognise that any measurements given to the nearest whole unit (e.g. nearest 10, to 2 d.p. etc…) may be inaccurate by up to one half in either direction.
Students should be able to write down a minimum and maximum value or upper and lower limit for numbers that have been given to a particular degree of accuracy.

<table>
<tr><th>15cm has been rounded to the nearest whole number.</th><th>9.5kg has been rounded to 1 decimal place</th><th>180m has been rounded to 2 significant figures</th><th>180m has been rounded to 3 significant figures</th></tr>
<tr><td>14.5cm is the lower bound
15.5cm is the upper bound

15.4$\dot{9}$ or 15.49…. are acceptable for the upper bound.</td><td>9.45kg is the lower bound
9.55kg is the upper bound

9.54$\dot{9}$ or 9.549…. are acceptable for the upper bound.</td><td>175 is the lower bound
185 is the upper bound

184.9… or 184.$\dot{9}$ are acceptable for the upper bound.</td><td>179.5 is the lower bound
180.5 is the upper bound

180.49….. or 180.4$\dot{9}$ are acceptable for the upper bound.</td></tr>
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- For continuous data students may write the upper bounds/limits as recurring ones. See the individual examples in the table above. Some students prefer this as they struggle to justify why an upper bound/limit can be .5 etc…
- Students need to be careful when data is discrete. For example the number of people at a rugby match is 12000 to the nearest thousand. The lower bound is 11500 and the upper bound is 12499. It is not 12499.9….. as you cannot have a decimal number of people. Also 12500 would not be accepted.
- Students may need to solve problems involving upper and lower bounds.

Addition $a + b$	Upper bound	Upper bound of a + upper bound of b
	Lower bound	Lower bound of a + lower bound of b
Subtraction $a - b$	Upper bound	Upper bound of a – lower bound of b
	Lower bound	Lower bound of a – upper bound of b
Multiply $a \times b$	Upper bound	Upper bound of a x upper bound of b
	Lower bound	Lower bound of a x lower bound of b
Divide $a \div b$	Upper bound	Upper bound of a ÷ lower bound of b
	Lower bound	Lower bound of a ÷ upper bound of b

- For example 12 cm and 15 cm have been rounded to the nearest whole number. What is the upper bound of the sum of these two lengths? Upper bound on the sum is 12.5 + 15.5 = 28cm. This is the maximum that the answer could be if these two lengths were added together.
- Regardless of whether they know how to solve a question, encourage students to make a table of upper and lower bounds before they attempt any problems like this. The reason for doing so is that students are awarded marks for seeing upper and lower bounds on individual numbers.

Common errors and misconceptions

- Students put .49 at the end of the number and don't imply it should be recurring. This would be marked wrong.
- They put decimal bounds when a number has to be whole (e.g. attendance at a match, population money etc…).
- Student's do the wrong calculation on problem solving questions or don't know where to get started. Encourage students regardless to make a table of upper and lower bounds. They can then use these to try to find an appropriate answer. Even if they get it wrong, they can still be awarded marks for individual bounds.

Suggested

- Questions

resources	
	○ http://www.cimt.org.uk/projects/mepres/allgcse/pr6-es.pdf (pp 95 - 96) ○ https://www.tes.com/teaching-resource/upper-and-lower-bounds--higher-gcse--exam-style-questions-11047628 • Past GCSE Questions ○ https://keshgcsemaths.files.wordpress.com/2013/11/97_upper-and-lower-bounds.pdf • Video tutorial ○ http://corbettmaths.com/2013/10/24/rounding-smallest/ ○ https://www.youtube.com/watch?v=_UsB0vbnlMQ (basic) ○ https://www.youtube.com/watch?v=agQVpLo9Fb4 (worded problems)