Outcome	AEq2	Student can consistently:	Find approximate solutions to equations using Trial and Improvement
How the topic is examined	● No longer part of the GCSE Specification ● Questions often require a calculator due to the level of calculations required. ● Students should be told the level of accuracy required. It will usually be to 1 decimal place, but can be other degrees of accuracy. ● It can be useful to provide a table for students to enter their results.		
Prior knowledge	● Students should be confident with: o Negative numbers. o Simplifying expressions (AEx2) o Substituting into expressions (AEx5) o Solving basic equations (AEq1)		
Suggested tuition approaches	● Students need to understand that trial and Improvement is a method for finding the solution to an equation that cannot be solved using standard methods. The solutions to the questions are often not exact and given as a rounded decimal. ● The method involves substituting in values for the unknown until you find one that get closest to the required answer. ● The key to success in this question is an ordered presentation of students' values that they have substituted into the equation. The best way to present this is in the form of a table. ● Here are the steps involved in solving trial and improvement problems. o Make a table. The table should have three columns (x, equation and comment are the headings) o The first two values you should substitute in are integer values. Most of the time you are told that a solution lies between two given values. You should substitute these into the equation. You should notice that one answer is too big and the other answer is too small. o Then try numbers between these two values. These numbers should be to 1 d.p. A lot of students pick half way between the two integers. You can pick closer to one number if the first step suggested it might lie closer to one than the other. o Once you have found the two numbers (to 1 d.p.) that the solution must lie between. Then you should try exactly half way in between these numbers. This step is important to ensure that you pick the closest answer. o If this answer is smaller than the required solution to the equation, then you need to choose the higher of the two numbers. The reason for this is that the solution must be closer to this than the lower number.		

- If the answer is higher than the required solution to the equation, then choose the lower of the two numbers to 1 d.p.
  (e.g. Use trial and improvement to solve the following equation $x^3 - 3 = 85$
  The solution lies between 4 and 5. Find the solution to 1 decimal place.

x	$x^3 - 3x$	Comment
4	$(4)^3 - 3(4) = 52$	Too small
5	$(5)^3 - 3(5) = 110$	Too big
4.7	$(4.7)^3 - 3(4.7) = 89.723$	Too big
4.6	$(4.6)^3 - 3(4.6) = 83.536$	Too small
4.65	$(4.65)^3 - 3(4.65) = 86.954625$	Too big

Try first the two whole number values. This should help check whether you are substituting into the equation correctly.

Here the student jumped to 4.7 instead of 4.5 as their answer was closer to 5 than 4.

Remember we are trying to make the number 85. This answer is still too big. So the x value must be smaller. We don't have to try the smaller values though as we want to answer to 1 decimal place and any x value would round to 4.6. Therefore this is the answer.

- Calculators should be used to get the result. The most efficient way to get the correct answers is to use the bracket function on their calculator – like in the examples in the middle column. All modern calculators will now do this. To get the other solutions they just have to go back on their calculator and change the x value.
- Some students may choose to use the table button on their calculator which will work out all the solutions at once.
- If the question provides no starting values for x students will have to choose a number as a starting point.

Common errors and misconceptions

- It is important for students to be systematic. A lot of students want to jump ahead really quickly. Encourage them to slow down their working out. They should find the two integer values it lies between and evaluate then the two numbers to 1 d.p. the solution lies between. Again they should evaluate the equation for each of these values.
- Students have a tendency to jump to what they think the answer is if they get very close to the number. You should tell the student to be systematic at all times.
- Students often substitute into the equation incorrectly. The most common mistake is that they forget to change one of the values in the equation and keep it constant or they don't substitute x in throughout.
- Students often round prematurely and/or think they have to round all their answers to 1 decimal place. This is not the case. In the middle column they should show all their working and write down their full answer from their calculator. Make sure the middle column in the widest.

	• Students have a tendency of going through all of the working out correctly and then not writing down the final answer. In the above example the solution is 4.6, but some students will write down 86.95… or 87.0.

Suggested resources	• Questions
	  o http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf (pp 197-201)
	  o http://justmaths.co.uk/wp-content/uploads/2013/02/Trial-and-Improvement-Exam-questions.pdf
	• Video tutorials
	  o https://www.youtube.com/watch?v=oa9Qe6OUFk8
	• Past GCSE Questions
	  o https://keshgcsemaths.files.wordpress.com/2013/11/62_trial-and-improvement.pdf