Outcome	AEq5	Student can consistently:	Solve equations involving algebraic fractions.

How the topic is examined	• Examined through test paper questions. • Questions are equally likely to appear on calculator or non-calculator papers. • These questions involve finding an unknown. The questions will ask students to "Solve".

Prior knowledge	• Students should be confident with:
	    o Expanding brackets (AEx1)
	    o Simplifying expressions (AEx2)
	    o Factorising quadratic equations (AEx7)
	    o Solving basic equations (AEq1)
	    o Simplifying algebraic fractions (AEx6)
	• In addition questions involving this topic can have links to:
	    o Solving quadratic equations (AEq6) (AEq7)

Suggested tuition approaches	• There are two types of fractional equations that students are commonly asked to solve. The first ones are quite basic, where students have to simply find the missing value. The harder problems involve students having to use their knowledge of simplifying fractions first. • As with any other equation, there are different methods that students use to tackle this topic. (See AEq1) • Here are some basic equations that students might be asked to solve:

$$\frac{2x+1}{5} = 3$$
$$(\times 5)$$
$$2x+1 = 15$$
$$(-1)$$
$$2x = 14$$
$$(\div 2)$$
$$x = 7$$

$$\frac{2x}{5} + 1 = 3$$
$$(-1)$$
$$\frac{2x}{5} = 2$$
$$(\times 5)$$
$$2x = 10$$
$$(\div 2)$$
$$x = 5$$

$$\frac{3x-1}{4} = 2x+5$$
$$(\times 4)$$
$$3x-1 = 8x+20$$
$$(-3x)$$
$$-1 = 5x+20$$
$$(-20)$$
$$-21 = 5x$$
$$(\div 5)$$
$$x = -4.2$$

• In the last example above, students need to multiply through by 4 to solve. Notice that it was necessary to multiply all terms on the right hand side of the equation.

- There are three further types of equations that students might come across. Solving these problems brings together lots of areas of mathematics and students ideally need to be confident with these before solving these.

$\dfrac{2m-5}{3} = \dfrac{4-3m}{5}$	$\dfrac{3x+5}{4} + \dfrac{x-4}{6} = 2$	$\dfrac{4}{3x+5} + \dfrac{6}{x-4} = 1$
To solve this equation a common method is to 'cross multiply'. This is where students multiply the numerator of the LHS by the denominator of the RHS and multiply the numerator of the RHS by the denominator of the LHS. $$\dfrac{2m-5}{3} \diagdown\diagup \dfrac{4-3m}{5}$$ Some teachers use arrows to show what is happening. Essentially you are multiplying through by 15. You might want to explain this to students. You then get $$10m - 25 = 12 - 9m$$ We can solve this using standard methods (See AEq1)	In order to do this you need to write this first as a single fraction. (See notes AEx6) If you do this you end up with $$\dfrac{11x+7}{12} = 2$$ You can then solve this using the methods in the previous table.	In order to do this you need to write this first as a single fraction. (See notes AEx6) $$\dfrac{10x-6}{(3x+5)(x-4)} = 1$$ In order to solve this, you need to form a quadratic equation. Expand the denominator first. (AEx1) $$\dfrac{10x-6}{3x^2 - 7x - 20} = 1$$ Multiply through by $3x^2 - 7x - 20$ $$10x - 6 = 3x^2 - 7x - 20$$ Form an equation so it equals 0 $$0 = 3x^2 - 17x - 14$$ You can then try to factorise and solve (AEq6) or use the quadratic formulae (AEq7) These are some of the hardest problems at GCSE.

Common errors and misconceptions	- There are the usual errors and misconceptions of solving equations. These include - Students balancing incorrectly (e.g. they add on instead of subtracting etc…). They need to think about doing the opposite. A lot of these mistakes happen when students become complacent; they need to take their time. - They struggle with the number work without a calculator. Encourage students to leave their final answer as a fraction if it does not cancel equally. They will always be given full marks for this. - Students get the first two equations (discussed above) wrong and they always multiply first. In the second example you had to subtract first before multiplying. To help students overcome this, you could talk to them about how the equation is formed and then doing the inverse. - Errors creep in the more difficult the equation. Encourage students to take their time and go through the problem step by step. Some of the questions on this topic can have 5+ marks associated with them, so it is worth taking time to get them correct.
Suggested resources	- Questions - http://www.cimt.org.uk/projects/mepres/allgcse/pr10-es.pdf (pp 77 - 78) - https://corbettmaths.files.wordpress.com/2013/02/cross-multiplication-pdf.pdf - https://corbettmaths.files.wordpress.com/2013/02/equations-fractional-pdf.pdf - https://corbettmaths.files.wordpress.com/2013/02/equations-fractional-advanced-pdf.pdf - Past GCSE Questions - https://corbettmaths.files.wordpress.com/2013/02/cross-multiplication-pdf.pdf - https://corbettmaths.files.wordpress.com/2013/02/equations-fractional-pdf.pdf - Video tutorial - http://corbettmaths.com/2013/05/25/algebraic-equations/ - http://corbettmaths.com/2013/05/19/equations-cross-multiplication/