PERCENTAGES

FIND % OF A QUANTITY

FIND 42% OF £300

100% = £300 ÷100 → 1% = £3 ×42 → 42% = £126

42% = 42/100 = 0.42

OR MULTIPLY BY DECIMAL: 0.42 × £300 = £126

INCREASE OR DECREASE BY A %

THE VALUE OF A £700 WATCH INCREASES BY 15%. FIND THE NEW VALUE

100% = £700; 15% = £105; 115% = £805

QUICKER TO USE MULTIPLIER: 100% + 15% = 115% = 115/100 = 1.15

OLD VALUE £700 → ×1.15 → NEW VALUE ?

1.15 × £700 = £805

FIND THE % CHANGE

MY WAGE INCREASES FROM £500 TO £615. FIND THE % CHANGE

100% = £500; CHANGE = £615 − £500; ? % = £115; ? % = £615

% CHANGE = CHANGE / OLD VALUE × 100 = 115/500 × 100 = 23%

OR USE THE MULTIPLIER: OLD VALUE £500 → × ? → NEW VALUE £615

615/500 = 1.23 = 123% = 100% + 23% — INCREASE BY 23%

FIND THE ORIGINAL VALUE

AFTER A 20% INCREASE THE COST OF AN AIR TICKET IS NOW £360. WHAT WAS THE ORIGINAL COST BEFORE THE INCREASE?

100% = £?; 20% = £?; 120% = £360

120% = £360; ÷120; 1% = £3; ×100; 100% = £300

OR USE THE MULTIPLIER: OLD VALUE ? → × 1.20 → NEW VALUE £360; ÷1.20

100% + 20% = 120% = 120/100 = 1.20

OLD VALUE = NEW VALUE ÷ 1.20 = £360 ÷ 1.20 = £300

COMPARE QUANTITIES

75/60 = 5/4 = 1.25 = 125%

= 4/5 OF; = 0.8 OF; = 80% OF; = 5/4 OF; = 1.25 OF; = 125% OF

£60 : £75

RATIO 60 : 75, 4 : 5

60/75 = 4/5 = 0.8 = 80%

INTEREST

SIMPLE INTEREST

THE AMOUNT ADDED EACH YEAR IS THE SAME AS THE FIRST YEAR

ASH INVESTS £5000 AT AN INTEREST RATE OF 2.9% PER ANNUM FOR 4 YEARS

£5000	YEAR 0	£5000
£5145	YEAR 1	£5145
£5290	YEAR 2	£5294.21
£5435	YEAR 3	£5447.74
£5580	YEAR 4	£5605.72

2.9% OF £5000 = +£145; 2.9% OF £5145 = +£149.21; 2.9% OF £5294.21 = +£153.53; 2.9% OF £5447.74 = +£157.98

£5000 + 4 × £145

COMPOUND INTEREST

THE AMOUNT ADDED CHANGES EACH YEAR AS IT IS CALCULATED FROM THE MOST RECENT TOTAL

QUICKER TO USE MULTIPLIER: 100% + 2.9% = 102.9% = 102.9/100 = 1.029 → ×1.029

£5000 (YEAR 0) → ×1.029 → £5145 (YEAR 1) → ×1.029 → £5294.21 (YEAR 2) → ×1.029 → £5447.74 (YEAR 3) → ×1.029 → £5605.72 (YEAR 4)

£5000 (YEAR 0) → ×1.029 ×1.029 ×1.029 ×1.029 → £5605.72 (YEAR 4)

£5000 × 1.029⁴ = £5605.72

THE PRICE OF A £400 PHONE DECREASES BY 7%. FIND THE NEW PRICE

100% = £400; 7% = £28; 93% = £372

QUICKER TO USE MULTIPLIER: 100% − 7% = 93% = 93/100 = 0.93

OLD VALUE £400 → DECREASE BY 7% × 0.93 → NEW VALUE ?

0.93 × £400 = £372

THE PRICE OF A BIKE DECREASES FROM £600 TO £516. FIND THE % CHANGE

SALE! £600 £516

100% = £600; CHANGE = £600 − £516; ? % = £84; ? % = £516

% CHANGE = CHANGE / OLD VALUE × 100 = 84/600 × 100 = 14%

OR USE THE MULTIPLIER: OLD VALUE £600 → × ? → NEW VALUE £516

516/600 = 0.86 = 86% = 100% − 14% — DECREASE BY 14%

AFTER A 35% DECREASE THE PRICE OF A SOFA IS £455. WHAT WAS THE ORIGINAL PRICE OF THE SOFA?

NOW £455 — 35% OFF

100% = £?; 35% = £?; 65% = £455

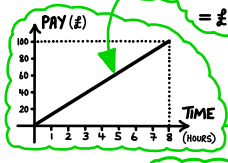
65% = £455; ÷65; 1% = £7; ×100; 100% = £700

OR USE THE MULTIPLIER: OLD VALUE ? → DECREASE BY 35% × 0.65 → NEW VALUE £455; ÷0.65

100% − 35% = 65% = 65/100 = 0.65

OLD VALUE = NEW VALUE ÷ 0.65 = £455 ÷ 0.65 = £700

GRADIENT OF THE LINE = £100 / 8 HOURS = £12.50/HOUR



£100 : 8 HOURS; £12.50 : 1 HOUR — RATE OF PAY

JUICE	APPLE	WATER	ORANGE
DENSITY	1.2 g/cm³	1 g/cm³	1.6 g/cm³

MASS OF APPLE JUICE = 1.2 × 150 = 180 g

MASS OF ORANGE JUICE = 1.6 × 250 = 400 g

MASS OF WATER = 1 × 600 = 600 g

150 ml; 600 ml; 250 ml; 1 Litre

COMBINED MASS = 180 + 400 + 600 = 1180 g

COMBINED DENSITY = 1180 g / 1000 ml = 1.18 g/ml

COMPOUND UNITS

PRESSURE = FORCE / AREA

DENSITY = MASS / VOLUME

RATIO

SHOWS THE SIZE OF DIFFERENT GROUPS RELATIVE TO EACH OTHER

2/10 = 1/5 ARE DOGS; 3/10 ARE CATS; 5/10 = 1/2 ARE RABBITS

DOGS : CATS : RABBITS = 2 : 3 : 5

SIMPLIFY RATIOS

WITH UNITS

75 cm : 2 m; 75 cm : 200 cm; 3 : 8 — DIFFERENT UNITS; CONVERT TO THE SMALLER UNIT; ÷25

WITH STANDARD FORM

1.2 × 10³ : 6 × 10⁴; 1200 : 60000; 1 : 500 — DIVIDE BY HCF OF 1200 AND 60000; ÷1200; TAKE OUT OF STANDARD FORM

WITH 2 PARTS

24 : 18; 4 : 3 — DIVIDE BOTH SIDES BY THE HCF OF 24 AND 18; ÷6

24 = 6 6 6 6 6 6 6; 18 = 6 6 6

WITH 3 PARTS

14 : 21 : 42; 2 : 3 : 6 — DIVIDE BY THE HCF OF 14, 21 AND 42; ÷7

14 = 7 7; 21 = 7 7 7; 42 = 7 7 7 7 7 7

FINDING MISSING AMOUNTS

ASH : BOB : CAT = 3 : 4 : 2

IF ASH RECEIVES £36 HOW MUCH DO BOB AND CAT RECEIVE?

3 = £36; £36 ÷ 3 = £12; 1 PART = £12

£36 = £12 £12 £12; £48 = £12 £12 £12 £12; £24 = £12 £12

BOB = 4 × £12 = £48; CAT = 2 × £12 = £24

DIVIDING INTO A RATIO

DIVIDE £80 BETWEEN ASH, BOB AND CAT

ASH : BOB : CAT = 5 : 3 : 2

DIVIDE THE MONEY BY THE TOTAL PARTS IN THE RATIO 5 + 3 + 2 = 10

£80 = £8 £8 £8 £8 £8 £8 £8 £8 £8 £8; £40; £24; £16

ASH = 5 × £8 = £40; BOB = 3 × £8 = £24; CAT = 2 × £8 = £16

£80 ÷ 10 = £8

APPLICATIONS

BEST BUY

SMALL 300 ml £2.10; MEDIUM 500 ml £3.25; LARGE 1 l £6.30

SMALL : MEDIUM : LARGE = 300 ml : 500 ml : 1000 ml = 3 : 5 : 10

SMALL = £2.10 ÷ 3 = £0.70; MEDIUM = £3.25 ÷ 5 = £0.65; LARGE = £6.30 ÷ 10 = £0.63

FIND THE LOWEST COST FOR 1 PART OF THE RATIO — LARGE IS THE BEST BUY

SCALE

LENGTHS ON THE PLAN ×150 → LENGTHS IN THE REAL BUILDING

LENGTH OF BEDROOM ON THE PLAN IS 3.7 cm

ACTUAL LENGTH OF BEDROOM = 3.7 cm × 150 = 555 cm = 5.55 m

SCALE 1 : 150

RECIPES

USE THE RATIO TO CHANGE THE AMOUNT OF EACH INGREDIENT

MAKES 18 GINGERBREAD MEN: FLOUR 250 g, BUTTER 120 g, GINGER 80 g, SUGAR 50 g

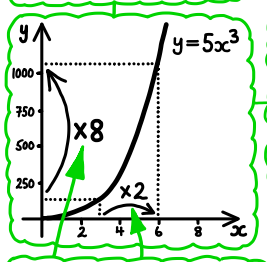
SIMPLIFY THE RATIO 18 : 45; 2 : 5; ÷2 ×5

MAKES 45 GINGERBREAD MEN: FLOUR 625 g, BUTTER 300 g, GINGER 200 g, SUGAR 125 g

PROPORTION

DIRECT PROPORTION — AS x INCREASES, y INCREASES

y IS DIRECTLY PROPORTIONAL TO x³ → y ∝ x³

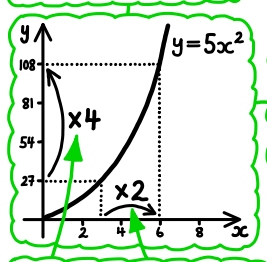


IF x = 2 THEN y = 5 × 2³ = 40; y = 5x³

IF y = 320 THEN 320 = 5x³; x = 320/5 = 64; x = ∛64 = 4

IF x INCREASES BY A FACTOR OF a THEN y INCREASES BY A FACTOR OF a³

y IS DIRECTLY PROPORTIONAL TO x² → y ∝ x²

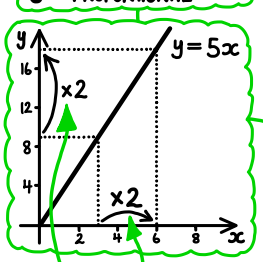


IF x = 4 THEN y = 5 × 4² = 80; y = 5x²

IF y = 245 THEN 245 = 5x²; x² = 245/5 = 49; x = √49 = 7

IF x INCREASES BY A FACTOR OF a THEN y INCREASES BY A FACTOR OF a²

y IS DIRECTLY PROPORTIONAL TO x → y ∝ x



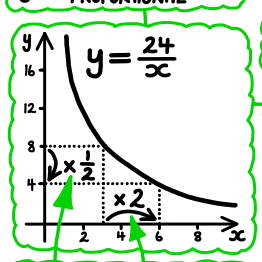
IF x = 7 THEN y = 5 × 7 = 35; y = 5x

IF y = 50 THEN x = 50/5 = 10

IF x INCREASES BY A FACTOR OF a THEN y INCREASES BY A FACTOR OF a

INVERSE PROPORTION — AS x INCREASES, y DECREASES

y IS INVERSELY PROPORTIONAL TO x → y ∝ 1/x

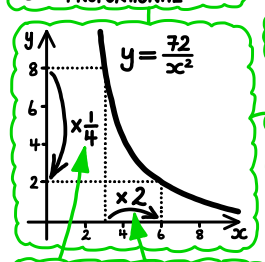


IF x = 2 THEN y = 24/2 = 12; y = 24/x

IF y = 8 THEN 8 = 24/x; x = 24/8 = 3

IF x INCREASES BY A FACTOR OF a THEN y DECREASES BY A FACTOR OF a

y IS INVERSELY PROPORTIONAL TO x² → y ∝ 1/x²



IF x = 3 THEN y = 72/3² = 8; y = 72/x²

IF y = 18 THEN 18 = 72/x²; x² = 72/18 = 4; x = √4 = 2

IF x INCREASES BY A FACTOR OF a THEN y DECREASES BY A FACTOR OF a²

MORE PROPORTION — FINDING THE EQUATION

DIRECT PROPORTION

y IS DIRECTLY PROPORTIONAL TO x² → y ∝ x² → y = kx²

WHEN x = 5, y = 100: 100 = k × 5²; 100 = 25k; k = 4

y = 4x²

INVERSE PROPORTION

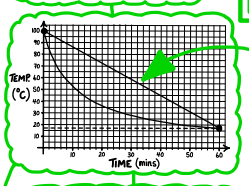
y IS INVERSELY PROPORTIONAL TO x² → y ∝ 1/x² → y = k/x²

WHEN x = 3, y = 10: 10 = k/3²; k = 10 × 3²; k = 90

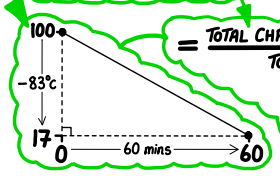
y = 90/x²

RATES OF CHANGE

AVERAGE RATE OF CHANGE OVER AN INTERVAL

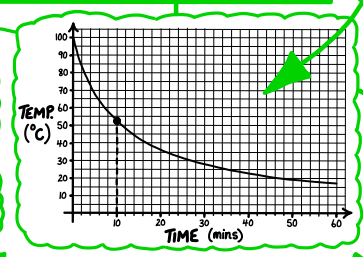


THE AVERAGE RATE OF CHANGE OF TEMPERATURE FROM 0 mins TO 60 mins

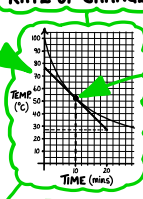


= TOTAL CHANGE IN TEMPERATURE / TOTAL TIME TAKEN = −83°C / 60 mins = −1.38 °C/min

THE TEMPERATURE OF THE WATER IN A KETTLE IS RECORDED FOR 1 HOUR AFTER IT HAS BOILED

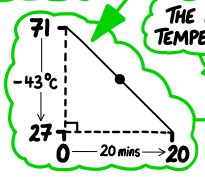


RATE OF CHANGE AT A POINT



USE THE TANGENT AT THE POINT TO ESTIMATE THE GRADIENT

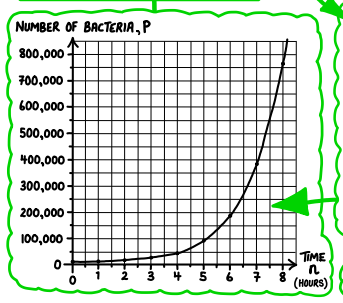
THE RATE OF CHANGE OF TEMPERATURE AT 10 mins



= −43°C / 20 mins = −2.15 °C/min

EXPONENTIAL GROWTH

INITIALLY THERE ARE 3000 BACTERIA IN THE DISH AND THE NUMBER THEN DOUBLES EVERY HOUR



n	P
0	3,000
1	6,000
2	12,000
3	24,000
4	48,000
5	96,000
6	192,000
7	384,000
8	768,000

ITERATIVE PROCESS

P_n = NUMBER OF BACTERIA AT TIME n HOURS

$P_0 = 3000$

$P_{n+1} = 2P_n$

$P_0 \xrightarrow{\times 2} P_1 \xrightarrow{\times 2} P_2 \xrightarrow{\times 2} P_3 \xrightarrow{\times 2} \dots$

3000 (HOUR 0) → ×2 → ×2 → ··· → ×2 → ×2 → ? (HOUR n); n TIMES

$P = 3000 \times 2^n$

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