

| Outcome | NF6 | Student can consistently: | Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM). | | | | | | |
|--|---|---|--|-------------------------------|--------------|----------------|--|---|---|
| How the topic is examined | <ul style="list-style-type: none">Examined through test paper questions.This topic is usually examined on non-calculator examination papers.The questions will ask students to find the lowest common multiple of two (or more) numbers and the highest common factor of two (or more numbers)Often questions asking students to find a LCM or HCF can be asked in the form of a word problem. These are increasingly common at GCSE.Questions on these topics are usually asked after students have been asked to write a number as a product of its prime factors. (see NF5) | | | | | | | | |
| Prior knowledge | <ul style="list-style-type: none">Students should be confident with:<ul style="list-style-type: none">Factors and multiplesWriting a number as a product of its prime factors (NF5)In addition questions involving HCF and LCM can have links to:<ul style="list-style-type: none">Venn diagrams | | | | | | | | |
| Suggested tuition approaches | <ul style="list-style-type: none">Students should be confident with what factors and multiples are.There are three approaches that students take to answer these problems:<ul style="list-style-type: none">Listing multiples and factors – only suitable if the numbers are small.Venn diagrams – easy way to see what is going on.Common factors – for students who are not sure of the Venn diagram approach. It is essentially the same idea. <table><tr><th>Listing multiples and factors</th><th>Venn diagram</th><th>Common factors</th></tr><tr><td>Find the HCF of 45 and 60 Write down the factors of each number 45 : 1, 3, 5, 9, 15, 45 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60</td><td>First write each number as a product of its prime factors 45 = 3 x 3 x 5 60 = 2 x 2 x 3 x 5 We can then put these numbers into a Venn diagram.</td><td>First write each number as a product of its prime factors 45 = 3 x 3 x 5 60 = 2 x 2 x 3 x 5</td></tr></table> | | | Listing multiples and factors | Venn diagram | Common factors | Find the HCF of 45 and 60 Write down the factors of each number 45 : 1, 3, 5, 9, 15, 45 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 | First write each number as a product of its prime factors 45 = 3 x 3 x 5 60 = 2 x 2 x 3 x 5 We can then put these numbers into a Venn diagram. | First write each number as a product of its prime factors 45 = 3 x 3 x 5 60 = 2 x 2 x 3 x 5 |
| Listing multiples and factors | Venn diagram | Common factors | | | | | | | |
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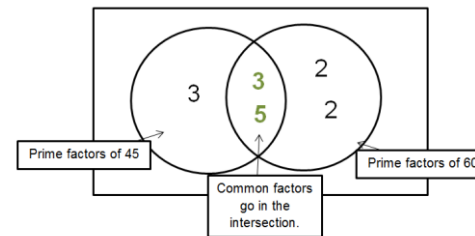
The highest factor that appears in both lists is 15. This is the HCF
Find the LCM of 45 and 60.

Simply list out the multiples of each number

45, 90, 135, 180, 235,

60, 120, 180, 240

The LCM of 45 and 60 is 180 as this is the first number that appears in both lists.



The **HCF** is the numbers in the intersection of the two circles multiplied together $3 \times 5 = 15$

The **LCM** is all the numbers across the Venn diagram multiplied together

$$\text{LCM} = 3 \times 3 \times 5 \times 2 \times 2 = 180$$

Circle the common factors

The **HCF** is one set of the circled numbers multiplied together $3 \times 5 = 15$

The **LCM** can now be calculated by multiplying all of the numbers that are not circled by the HCF.

$$\text{LCM} = 15 \times 3 \times 2 \times 2 = 180$$

- Students can check their product by multiplying the numbers together and they should get the original number.
- There are an increasing number of word problems being asked that require finding either the LCM or HCF. Students should meet some of these problems in their working.

LCM example

A bus leaves to Bradford every 12 minutes.
 A bus leaves to Leeds every 15 minutes
 A bus leaves to Manchester every 25 minutes.

At 9.00am three buses leave, one to Bradford, one to Leeds and one to Manchester. What is the next time the three buses will leave together?

HCF example

Two pieces of wood 120cm long and 270cm long are to be cut into equally sized smaller pieces.

What is the greatest length of the smaller pieces?

Common errors and misconceptions

- When using the listing factors method, they miss out factors because they don't use a systematic approach. Get them to write down factor pairs as opposed to single factors (e.g. 15×4 , 9×5 etc..)
- For larger numbers students miss out factors or multiples. Encourage students to use one of the other methods to solve problems with larger numbers.
- In the Venn diagram method students can double up the numbers in the intersection. Remember the intersection is the common numbers from the prime factor decompositions.
- When writing a number as a product of its prime factors:
 - Students put the addition symbol (+) between the numbers as opposed to the multiply sign (\times). Students need to remember it is product and product means multiply.
 - Students tend to stop too early and circle numbers that are not prime numbers. Some particular numbers that are circled are 9, 15, 25. Students often circle these instead of realising that these numbers are actually products of other numbers ($9 = 3 \times 3$, $15 = 3 \times 5$ and $25 = 5 \times 5$). This usually happens because students think it doesn't divide by 2.

Suggested resources

- Questions
 - <https://corbettmaths.files.wordpress.com/2013/02/lcm-hcf-pdf.pdf>
 - <https://corbettmaths.files.wordpress.com/2013/02/product-of-primes-pdf1.pdf>
- Past GCSE Questions
 - https://keshgcsemaths.files.wordpress.com/2013/11/52_hcf-lcm-product-of-primes.pdf
- Video tutorial
 - <http://corbettmaths.com/2012/08/11/lcm-and-common-multiples/>
 - <http://corbettmaths.com/2012/08/24/common-factors-and-hcf/>
 - <http://corbettmaths.com/2012/08/20/lcm-and-hcf-using-product-of-primes/>