PROBABILITY OF COMBINED EVENTS

THE PROBABILITY SCALE

Probability of being born on a Tuesday

Probability of fair coin landing on heads

Probability of choosing a black ball from the bag

Probability of rolling a seven

Probability that a multiple of 10 will be an even number: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, ...

0 — Impossible; $\frac{1}{2}$ — 50/50 or even; 1 — Certain; Unlikely; Likely

PROBABILITY OF A SINGLE EVENT

Roll a fair die once

The probability of an event $= \frac{\text{Number of outcomes where the event occurs}}{\text{Total number of equally likely outcomes}}$

Probability of rolling more than 4 $= \frac{2}{6} = \frac{1}{3}$

Estimate how many times you would roll more than 4 if you roll the die 150 times: $\frac{1}{3} \times 150 = 50$

PROBABILITIES SUM TO ONE

The probability of an event not happening $= 1 -$ the probability the event happens

The probabilities of all the possible outcomes must sum to 1

Choose a ball at random from the bag

Probability of choosing a black ball $= \frac{2}{9}$

Probability of not choosing a black ball $= 1 - \frac{2}{9} = \frac{9}{9} - \frac{2}{9} = \frac{7}{9}$

Result	Win	Draw	Lose
Probability	0.6	0.15	?

Probability of losing $= 1 - 0.6 - 0.15 = 0.25$

RELATIVE FREQUENCY

Experimental probability

Estimating the probability from repeated trials

Spin to win!

Theoretical probability

Assuming all sectors have an equal chance

Probability of win $= \frac{2}{5} = 40\%$

Number of wins	5	18	42	196	403
Number of trials	10	50	100	500	1000
Relative frequency	50%	36%	42%	39.2%	40.3%

Relative frequency $= \frac{\text{Number of wins}}{\text{Number of trials}} = \frac{18}{50}$

More trials ⇒ more reliable the estimate

TREE DIAGRAMS

Spin the spinner twice

The first spin has no impact on the second spin

The spins are independent

Probabilities for the second spin stay the same

Black AND black

Multiply the probabilities

1st spin: $\frac{3}{8}$ Black, $\frac{5}{8}$ White

2nd spin: $\frac{3}{8}$ Black, $\frac{5}{8}$ White; $\frac{3}{8}$ Black, $\frac{5}{8}$ White

Probability of Black → Black $= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

Probability of Black → White $= \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

Probability of White → Black $= \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

Probability of White → White $= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

Probability of the same colour twice = Black → Black OR White → White $= \frac{9}{64} + \frac{25}{64} = \frac{34}{64} = \frac{17}{32} = 53.1\%$

Add the probabilities

Choose two balls from the bag (without replacing the first one)

The first choice impacts the second choice

The choices are dependent

Probabilities for the second choice change depending on the first choice

1st ball: $\frac{3}{8}$ Black, $\frac{5}{8}$ White

2nd ball: $\frac{2}{7}$ Black, $\frac{5}{7}$ White; $\frac{3}{7}$ Black, $\frac{4}{7}$ White

Probability of Black → Black $= \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

Probability of Black → White $= \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

Probability of White → Black $= \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Probability of White → White $= \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

Probability of the same colour twice = Black → Black OR White → White $= \frac{6}{56} + \frac{20}{56} = \frac{26}{56} = \frac{13}{28} = 46.4\%$

SPACE DIAGRAMS

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Probability of two heads and one tail in any order $= \frac{3}{8}$

8 equally likely outcomes

Throw 3 fair coins

List outcomes

Roll the die

Sum the scores

Spin the spinner

	1	2	3	4	5	6
1	1+1 = 2	1+2 = 3	1+3 = 4	1+4 = 5	1+5 = 6	1+6 = 7
2	2+1 = 3	2+2 = 4	2+3 = 5	2+4 = 6	2+5 = 7	2+6 = 8
3	3+1 = 4	3+2 = 5	3+3 = 6	3+4 = 7	3+5 = 8	3+6 = 9
4	4+1 = 5	4+2 = 6	4+3 = 7	4+4 = 8	4+5 = 9	4+6 = 10

The probability the sum is greater than 7 = probability of sum of 8, 9 or 10 $= \frac{6}{24} = \frac{1}{4} = 0.25$

VENN DIAGRAMS

Everyone is in the rectangle

A and J do both

B does neither

Music: C K A J D F B; Sport: A J E I L H I G

The probability that a random student from the class plays sport but not music $= \frac{5}{12}$

Play music ⇒ A C D F J K

Play sport ⇒ A E G H I J L

12 students in the class: A B C D E F G H I J K L

TWO-WAY TABLES

How students come to school

	Walk	Bus	Cycle	Total
Boys	42	70	29	141
Girls	59	37	13	109
Total	101	107	42	250

The probability a random student is a boy who cycles $= \frac{29}{250}$

FREQUENCY TREES

Year 12: 105 — Male 65, Female 40

Year 13: 87 — Male 38, Female 49

192

The probability a randomly chosen student is female $= \frac{40+49}{192} = \frac{89}{192}$

MEAN, MEDIAN, MODE + RANGE

Mean, median, mode — measure the centre of the data

Range measures the spread of the data

From a list of data

Test (20): 11 8 14 10 5 19 14 8 16 14

Range = highest value − lowest value $= 19 - 5 = 14$

Mode = the number which appears the most: 14 occurs 3 times

There is no mode if all the numbers are different

Mean = $\frac{\text{the total sum of the numbers}}{\text{how many numbers there are}}$ $= \frac{11+8+14+10+5+19+14+8+16+14}{10} = \frac{119}{10} = 11.9$

Median = the middle value when the list is in order: 5 8 8 10 (11 14) 14 14 16 19

If there are two numbers in the middle go halfway between them: $\frac{11+14}{2} = 12.5$

From a frequency table

The number of goals scored by a team in each match they played in the season

20th number is a 2; median = 2; 21st number is a 2

0000 111111111111 2222222222222333333334444445 5

×3 ×10 ×12 ×8 ×5 ×2

Goals scored	Frequency
0	3
1	10
2	12
3	8
4	5
5	2

Goals scored × frequency: $0 \times 3 = 0$, $1 \times 10 = 10$, $2 \times 12 = 24$, $3 \times 8 = 24$, $4 \times 5 = 20$, $5 \times 2 = 10$

Total goals scored: 88

Range $= 5 - 0 = 5$

Mode = 2 occurs 12 times

Add up the frequencies to find the total number of data items: 40

Mean $= \frac{\text{total goals scored}}{\text{number of matches played}} = \frac{88}{40} = 2.2$ goals/game

From a grouped frequency table

The height of each student in the class is recorded in intervals

Estimate using the mid-points of each interval

Height, h (cm)	Frequency	Mid-point × freq.
130 < h ≤ 140 (135)	2	135 × 2 = 270
140 < h ≤ 150 (145)	5	145 × 5 = 725
150 < h ≤ 160 (155)	11	155 × 11 = 1705
160 < h ≤ 170 (165)	9	165 × 9 = 1485
170 < h ≤ 180 (175)	3	175 × 3 = 525

Number of students: 30; Total height: 4710

Modal class — the most common interval: 150 < h ≤ 160 occurs 11 times

Estimate of mean $= \frac{4710}{30} = 157$ cm

COLLECTING DATA

Discrete data — can only take certain values, often integers: ...8 9 10 11 12 13 14... Number of sweets = 12

Continuous data — can take any value on a scale: hand span = 24.3 cm

Primary data — collected by yourself

Secondary data — originally collected by someone else

Too expensive or time consuming to ask the whole population

Quicker and cheaper to ask a sample

Population → Sample

But... sample is only a selection of opinions and can be biased

SCATTER GRAPHS

Display pairs from two sets of data

Student	A	B	C	D	E	F	G
Height (cm)	132	139	151	157	165	173	180
Handspan (cm)	17.7	19.2	20.9	19.7	21.5	17.8	21.8

Handspan (cm) vs height (cm)

Line of best fit — the straight line which best represents the data

Outlier — a point which does not fit the pattern of the rest of the data

Extrapolation: Handspan = 23.2 cm — less reliable prediction as height 190 cm is outside the range of the data set

Interpolation: Handspan = 19.6 cm — more reliable prediction as height 150 cm is inside the range of the data set

Predict handspan for height of 150 cm

Use the line of best fit

Predict handspan for height of 190 cm

CORRELATION

Causation — correlation not causation

Ice cream sales / sunglasses sales

Strong negative correlation; weak negative correlation; no correlation; weak positive correlation; strong positive correlation

The connection between the two sets of data

REPRESENTING DATA

Bar chart

Frequency; Type of pizza: MAR, PEP, BBQ, HAW, OTH

What is your favourite type of pizza?

Frequency table

Pizza	Tally	Freq.
Margherita	𝍸 𝍸 IIII	14
Pepperoni	𝍸 IIII	9
BBQ chicken	𝍸 𝍸 𝍸 III	18
Hawaiian	III	3
Other	𝍸 I	6
		50

Pizza	Angle
Margherita	100.8°
Pepperoni	64.8°
BBQ chicken	129.6°
Hawaiian	21.6°
Other	43.2°
	360°

Frequency / 50 × 360° = angle

Pictogram

Pizza	Frequency
Margherita	
Pepperoni	
BBQ chicken	
Hawaiian	
Other	

= 4 people

Pie chart

Margherita, Pepperoni, BBQ chicken, Hawaiian, Other

HISTOGRAMS

90 students are asked how long it takes them to come to school

Time (mins)	Freq.	Class width	Frequency density
0 ≤ t < 5	9	5	9 ÷ 5 = 1.8
5 ≤ t < 10	17	5	17 ÷ 5 = 3.4
10 ≤ t < 20	15	10	15 ÷ 10 = 1.5
20 ≤ t < 30	31	10	31 ÷ 10 = 3.1
30 ≤ t < 60	18	30	18 ÷ 30 = 0.6

The width of each interval

Frequency density $= \frac{\text{frequency}}{\text{class width}}$

Frequency density; time taken (mins)

The area of each bar is equal to the frequency of that class: 30 × 0.6 = 18

CUMULATIVE FREQUENCY

The heights of 120 students in a school are recorded

Height, h (cm)	Frequency
130 ≤ h < 140	5
140 ≤ h < 150	13
150 ≤ h < 160	20
160 ≤ h < 170	39
170 ≤ h < 180	28
180 ≤ h < 190	15

Cumulative frequency is the 'running total' showing how many students are less than each height

Height h (cm)	Cumulative frequency
h < 140	5
h < 150	18
h < 160	38
h < 170	77
h < 180	105
h < 190	120

To find the upper quartile: $\frac{3}{4}$ of 120

To find the median: $\frac{1}{2}$ of 120

To find the lower quartile: $\frac{1}{4}$ of 120

Plot cumulative frequency against the highest value in each class

Cumulative frequency; height (cm)

Lower quartile = 156 cm; median = 166 cm; upper quartile = 173 cm

Join points with smooth curve

Box plots

A visual representation of the median, quartiles, maximum and minimum

Lower quartile = 156 cm; median = 166 cm; upper quartile = 173 cm; minimum = 137 cm; maximum = 188 cm

Inter-quartile range = upper quartile − lower quartile = 173 − 156 = 17

A measure of spread not affected by outlying values

CONDITIONAL PROBABILITY

Probability a student takes the bus given they are a girl $= \frac{37}{109}$

109 girls; 37 of these take the bus

	Walk	Bus	Cycle	Total
Boys	42	70	29	141
Girls	59	37	13	109
Total	101	107	42	250

From a two-way table

The probability of one event given another has already happened

From a Venn diagram

Music: 4; Both: 2; Sport: 5; Neither: 1

7 play sport; 2 of these also play music

Probability a student plays music given they play sport $= \frac{2}{2+5} = \frac{2}{7}$

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