Pythagoras and Trigonometry

Guidance for tutors

Outcome	SPT6	Student can consistently:	Use the cosine rule to find missing sides and angles in non-right angled triangles.
How the topic is examined			• Examined through test paper questions. • It is most likely that these questions will appear on calculator papers. • Sometimes students can be asked to find a missing side or angle on a non-calculator paper. In this instance students will be given the value of the specific ratio or they could be one of the ratios that students are expected to know off by heart. • Students are likely to be provided with a diagram, but students may be expected to draw a diagram for a given question. It is important that students meet questions like this.

Prior knowledge			• Students should be confident
			    o Solving basic equations (AEq1)
			    o Rearranging simple formulae (AEx8)
			    o Trigonometry (SPT2 and SPT3)
			• In addition questions on this topic can have links to:
			    o Sine rule (SPT5)
			    o Pythagoras's Theorem (SPT1)
			    o Area and perimeter (SLAV1)

Suggested tuition approaches			• The cosine rule is used for non-right angled triangles. It is used when you cannot use the sine rule. • To be able to use the cosine rule you don't know an angle opposite a side. (e.g. you know all three sides). • Sides are labelled a, b and c and opposite angles are labelled A, B and C respectively. It is possible to use other letters. • There are two versions of cosine rule that people use:

Finding sides	Finding angles
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

This is just a rearrangement of the formula for finding sides.

- Although you can use different letters and write the formula out in many different ways students should start the cosine rule with the letter of the side they want to find.
- The steps involved in solving cosine rule problems are:
    - Draw a triangle (if necessary) and label the sides and opposite angles. Label the side you want to find as a or angle you want to find A – this means you can then use the formulae above.
    - Substitute the numbers into the cosine rule
    - Either simplify or type the calculation into your calculator and solve
    - Don't forget to square root your final answer and remember if finding an angle you will need to use cos^{-1}
- It is common for students to have to use the sine and cosine rules together in one question.
- Encourage students to show all their working and make sure they do not round answers too prematurely when working with sides and angles as this can lead to accuracy errors later on.

Common errors and misconceptions	- Questions might ask students to find particular sides or angles (e.g. side AB or angle CDE). Some students may struggle to understand the side and angle it is referring to. - The triangle's angles may already be labelled with other letters. If this is the case students can stick with these letters or rename them A, B and C. The letters are not important as long as a lowercase letter for the side is opposite an uppercase letter for the angle. - Students can round prematurely and they end up getting the wrong answer (e.g. $\cos x = \frac{7}{19}$ students then will work out $7 \div 19 = 0.4$ and then find the inverse and get 23.6, when they should have done the inverse sine of a number to at least 6 decimal places. Advise students to get the best answer by doing $x = \cos^{-1}\left(\frac{7}{19}\right)$ - Don't try to rearrange the calculation from the cosine rule as this often leads to lots of mistakes. Your calculator can deal with it as one. (e.g. $a^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \times \cos 50$ $a^2 = 100 + 225 - 300\cos 50$ $$a^2 = 325 - 300\cos 50$$

Guidance for tutors

	Some students rearrange this to $a^2 = 25 \cos 50$ which is not the case. • Ask students to double check the calculation they put into their calculator. • When they find a side students often forget to square root the final answer. • On most modern calculators it is important that students close the bracket after the fraction. • Students should check that the calculator they are using is in degrees (deg) mode.

Suggested resources	• Questions
	   o http://www.cimt.org.uk/projects/mepres/allgcse/bka4.pdf (pp 142 - 149)
	   o https://corbettmaths.files.wordpress.com/2013/02/sine-and-cosine-rule-pdf1.pdf
	• Past GCSE Questions
	   o https://keshgcsemaths.files.wordpress.com/2013/11/104_sine-and-cosine-rules-area-of-triangles-using-c2bdab-sin-c.pdf
	• Video tutorial
	   o http://corbettmaths.com/2013/04/04/cosine-rule-missing-sides/
	   o http://corbettmaths.com/2013/04/04/cosine-rule-missing-angles/