20 *CDEF* is a quadrilateral.
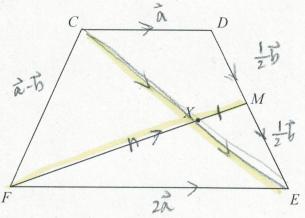

$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
   Give your answer in its simplest form.

$$2\vec{a}$$

(2)

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

First we can express $\overrightarrow{CE}$ as $-(\vec{a}-\vec{b}) + 2\vec{a} = \vec{b} + \vec{a}$ while $\overrightarrow{FM} = 2\vec{a} - \frac{1}{2}\vec{b}$

Basically there are only 4 segment we don't know their representation now =
$\overrightarrow{CX}$, $\overrightarrow{FX}$, $\overrightarrow{XE}$ and $\overrightarrow{XM}$ however we do know $\dfrac{\overrightarrow{FX}}{\overrightarrow{XM}} = \dfrac{n}{1}$

There are 2 options to solving n.

option A Constructing $\overrightarrow{FC} + \overrightarrow{CX} = \overrightarrow{FX}$

$\overrightarrow{FX} = \dfrac{n}{n+1}(2\vec{a} - \frac{1}{2}\vec{b}) = \overrightarrow{CX} + \overrightarrow{FC}$

let's pick $0 < m < 1$ such that

$\dfrac{n}{n+1}(2\vec{a} - \frac{1}{2}\vec{b}) = m\overrightarrow{CE} + \overrightarrow{FC} = m(\vec{a}+\vec{b}) + (\vec{a}-\vec{b})$

$\Longleftarrow \dfrac{2n}{n+1}\vec{a} - \dfrac{n}{2(n+1)}\vec{b} = (m+1)\vec{a} + (m-1)\vec{b}$

For the above to hold $\dfrac{2n}{n+1} = m+1$ ①

$-\dfrac{n}{2(n+1)} = m-1$ ②

① − ② gives

$\dfrac{2n}{n+1} + \dfrac{n}{2(n+1)} = 2 \Longleftrightarrow 4n+n = 4(n+1) = 4n+4$

$5n = 4n+4$

$\Longleftrightarrow \boxed{n = 4}$

option B Constructing $\overrightarrow{FX} = \overrightarrow{FE} - x\overrightarrow{E}$

$\overrightarrow{FX} = \dfrac{n}{n+1}(2\vec{a} - \frac{1}{2}\vec{b}) = \overrightarrow{FE} - x\overrightarrow{E}$

lets pick $0 < m < 1$ such that m.

$\dfrac{n}{n+1}(2\vec{a} - \frac{1}{2}\vec{b}) = \overrightarrow{FE} - m\overrightarrow{CE} = 2\vec{a} - m(b+\vec{a})$

$\dfrac{2n}{n+1}\vec{a} - \dfrac{n}{2(n+1)}\vec{b} = (2-m)\vec{a} - m\vec{b}$

For the above to hold. $\dfrac{2n}{n+1} = 2-m$ ① $\dfrac{n}{2(n+1)} = m$ ②

① + ② gives.

$\dfrac{2n}{n+1} + \dfrac{n}{2(n+1)} = 2. \Longleftrightarrow 4n+n = 4(n+1)$

$n =$

$5n = 4n+4$

(Total for Question 20 is 6 marks) $\boxed{n=4}$

TOTAL FOR PAPER IS 80 MARKS