2D SHAPES

Quadrilaterals 4 sides: Rectangle, Parallelogram, Kite, Square, Rhombus, Trapezium

Triangles 3 sides: Isosceles, Equilateral, Scalene, Right-angled

Pentagon 5 sides: Non-regular, Regular

Hexagon 6 sides: Non-regular, Regular

Octogon 8 sides: Non-regular, Regular

Decagon 10 sides: Non-regular, Regular

Symmetry

Reflection: 5 lines

Rotational: order 4

CIRCLES

Tangent, Chord, Diameter = d, Radius = r, Circumference, Segment, Sector, Arc

3D SHAPES

Plan view, Cube, Cuboid, Pyramid, Cylinder, Sphere, Cone, Triangular prism, Plans and elevations, Side view, Front view

Vertices = 6
Edges = 9
Faces = 5

ANGLE RULES

$a+b+c=180°$ — Angles at a point on a straight line

$a+b+c+d=360°$ — Angles at a point

Parallel lines: Alternate angles, Corresponding angles, Vertically opposite angles

Sum of angles in an n-sided polygon $= 180(n-2)$

Angle sum of a hexagon $= 180(6-2) = 180 \times 4 = 720°$

Sum of angles in a triangle $a+b+c=180°$

AREA AND PERIMETER

Area of a circle $= \frac{\theta}{360} \times \pi r^2$, Arc length $= \frac{\theta}{360} \times 2\pi r$

Circumference $= 2\pi r$

Area $= \frac{1}{2}bh$, Area $= bh$, Area $= \frac{1}{2}(a+b)h$

The amount of space inside a 2D shape: Area

Distance around the outside of a 2D shape: Perimeter

Combining shapes: Area

Perimeter: $12.6+3+10+16+9 = 50.6$ cm

Area: $\frac{\pi \times 4^2}{2} = 25.1\text{ cm}^2 + 9\times8 = 72\text{ cm}^2 + \frac{1}{2}\times8\times6 = 24\text{ cm}^2 = 121.1\text{ cm}^2$

VOLUME

Prisms: Volume $= \frac{1}{2}bhl$, Volume $= bhl$, Volume $= \pi r^2 l$, Volume $= Al$

Pyramids: Volume $= \frac{1}{3}abh$, Volume $= \frac{1}{3}\pi r^2 h$, Volume $= \frac{1}{3}Ah$

Sphere: Volume $= \frac{4}{3}\pi r^3$, Surface area $= 4\pi r^2$, Curved surface area $= \pi r l$

Frustum of a cone: $= \frac{1}{3}\times\pi\times6^2\times15 - \frac{1}{3}\times\pi\times2^2\times5 = 544.5\text{ cm}^3$

Combining shapes: $= \frac{1}{3}\pi\times4^2\times9 = 150.8 + \pi\times4^2\times6 = 301.6 + 5\times15\times10 = 750$, 1202 cm^3

The space inside a 3D shape

BEARINGS

The bearing of B from A

The bearing of A from B: 095°, 275°

TRANSFORMATIONS

Reflection: Line of reflection $y=6$

Translation: Right 4 up 3; As a vector $\begin{pmatrix}4\\3\end{pmatrix}$

Rotation: 90° clockwise, Centre of rotation

Enlargement: Scale factor 2, Centre of enlargement

RIGHT-ANGLED TRIANGLES

$x^2 = 13^2 + 9^2$, $x^2 = 169 + 81$, $x^2 = 250$, $x = \sqrt{250}$, $x = 15.8$ — Find the hypotenuse

$x^2 + 7^2 = 12^2$, $x^2 + 49 = 144$, $x^2 = 144 - 49$, $x^2 = 95$, $x = \sqrt{95} = 9.75$ — Find a short side

$a^2 + b^2 = h^2$ — Pythagoras — 3 sides

Question involves... 2 sides + 1 angle — Trigonometry

Hypotenuse, Opposite, Adjacent, θ

$\sin\theta = \frac{Opp.}{Hyp.}$, $\cos\theta = \frac{Adj.}{Hyp.}$, $\tan\theta = \frac{Opp.}{Adj.}$

Find a side: $\sin 27° = \frac{x}{16}$, $x = 16\sin 27°$, $x = 7.26$

Find an angle: $\tan x = \frac{11}{15}$, $x = \tan^{-1}\left(\frac{11}{15}\right)$, $x = 36.3°$

Exact values for 30°, 45°, 60°: $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$; $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \frac{\sqrt{3}}{1}$; $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = \frac{1}{1}$

VECTORS

$3\underline{a} = 3\begin{pmatrix}6\\-2\end{pmatrix} = \begin{pmatrix}18\\-6\end{pmatrix}$ — Scalar × vector

$-\underline{b} = -\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-4\\-5\end{pmatrix}$ — Negative vector

$\underline{a} = \overrightarrow{PQ} = \begin{pmatrix}6\\-2\end{pmatrix}$

$\underline{b} = \overrightarrow{ST} = \begin{pmatrix}4\\5\end{pmatrix}$

Horizontal movement, Vertical movement

Addition of vectors: $\underline{a}+\underline{b} = \begin{pmatrix}6\\-2\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}10\\3\end{pmatrix}$

Subtraction of vectors: $\underline{a}-\underline{b} = \begin{pmatrix}6\\-2\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}2\\-7\end{pmatrix}$

COMPASS AND RULER CONSTRUCTIONS

Shortest distance from a point to the line

Perpendicular to a line from a point

Bisecting an angle

Perpendicular to a line at a point

Construct an angle of 60°

Perpendicular bisector of a line segment

Locus of points which are equidistant from A and B

CONGRUENT TRIANGLES

Side Angle Side

Side Side Side

Angle Side Angle

Right Hypotenuse Side

Same shape and size

SIMILAR SHAPES

Same shape different size

$y = 10 \div 2.2 = 4.55$ cm

Scale factor $= \frac{11}{5} = 2.2$

$x = 4 \times 2.2 = 8.8$ cm

MORE TRANSFORMATIONS

Combining transformations: Reflection in line $y=x$; 90° rotation about (3,3)

Invariant points: unchanged by the transformation

Negative scale factor: Scale factor −2

SINE RULE

$\frac{\sin x}{9} = \frac{\sin 80}{10}$, $\sin x = \frac{9\sin 80}{10}$, $x = \sin^{-1}(0.8863)$, $x = 62.4°$

$\frac{\sin 50}{12} = \frac{\sin 75}{x}$, $x = \frac{12\sin 75°}{\sin 50°}$, $x = 15.1$

Find an angle: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Find a side: $\frac{a}{\sin A} = \frac{b}{\sin B}$

2 sides + 2 angles

NON-RIGHT ANGLED TRIANGLES

Area $= \frac{1}{2}ab\sin C$

$A = \frac{1}{2}\times8\times9\times\sin 70° = 33.8\text{ cm}^2$

COSINE RULE

3 sides + 1 angle

Find an angle: $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos x = \frac{10^2+7^2-8^2}{2\times10\times7}$, $x = \cos^{-1}(0.607)$, $x = 52.6°$

Find a side: $a^2 = b^2+c^2-2bc\cos A$, $x^2 = 8^2+9^2-2\times8\times9\times\cos 70°$, $x^2 = 64+81-49.25$, $x^2 = 95.75$, $x = 9.79$

VECTOR PROOF

$\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underline{a} + \underline{b}$

$\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB} = \frac{2}{5}(-\underline{a}+\underline{b}) = -\frac{2}{5}\underline{a} + \frac{2}{5}\underline{b}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \underline{a} - \frac{2}{5}\underline{a} + \frac{2}{5}\underline{b} = \frac{3}{5}\underline{a} + \frac{2}{5}\underline{b} = \frac{1}{5}(3\underline{a}+2\underline{b})$

Show that $\overrightarrow{OP}$ is parallel to $3\underline{a}+2\underline{b}$

AP : PB = 2 : 3

$\overrightarrow{OP}$ is parallel to $3\underline{a}+2\underline{b}$

3D TRIG. AND PYTHAGORAS

Find the length BH: $BH = \sqrt{89+6^2} = \sqrt{125} = 11.2$

Find the angle DBH: $\angle DBH = \tan^{-1}\left(\frac{6}{\sqrt{89}}\right) = 32.5°$

CIRCLE THEOREMS

Tangent is perpendicular to the radius at that point

Tangents from an external point are equal in length

Angle subtended at the centre is twice that at the circumference

Alternate segment theorem

Perpendicular from centre bisects the chord

Angle subtended by a semicircle is 90°

Angles in the same segment are equal

Opposite angles in a cyclic quadrilateral sum to 180°

MORE SIMILAR SHAPES

Scale factors for area and volume

Length ×3, Area $\times3^2$, Volume $\times3^3$

Volume $= 70\text{ cm}^3$, Volume $= 1890\text{ cm}^3$

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