Guidance for tutors

The table below outlines useful information for tutors as well as some suggested approaches and resources.

Outcome	AEx13	**Student can consistently:**	Use the factor and remainder theorem to factorise polynomials of higher order.

How the topic is examined	• This topic is not currently examined on GCSE but is on Level 2 Further Maths, AS/A2 mathematics and additional maths qualifications.
	• It is likely that students would have a calculator to solve these problems, but it is not necessary.
	• Questions may ask students to:
	  o Find the remainder when a polynomial p(x) is divided by a linear function f(x)
	  o Show that a particular linear function f(x) is a factor of p(x)
	  o Divide p(x) by f(x) in order to factorise higher order polynomials.
	• It is unlikely that students would ever have to factorise any polynomials above degree 3 (cubic)		

Prior knowledge	• Students should be confident with:
	  • Expanding brackets (AEx1)
	  • Simplifying expressions (AEx2)
	  • Substituting into formulae (AEx5)
	  • Factorising a quadratic (AEx7)		

Suggested tuition approaches

- Before teaching this topic, students should be very confident in factorising any quadratic expression.
- Introduce the remainder theorem to students, which says that “If a polynomial p(x) is divided by (x − a), the remainder is given by p(a)”
- You might want to ask students what a factor is. Explain that a factor is a number that goes into another number with no remainder. Therefore the factor theorem says that “If a polynomial p(x) is divided by (x − a), and p(a) = 0 then (x − a) is a factor of the p(x)”
- Two possible questions:

Find the remainder when $p(x) = x^3 + 2x^2 - 3x + 1$ is divided by $(x - 3)$	Using the remainder theorem we need to find $p(3)$ $p(3) = (3)^3 + 2(3)^2 - 3(3) + 1 = 37$ The remainder therefore is 37
Show that $(x + 2)$ is a factor of $p(x) = x^3 - x^2 - 24x - 36$	To show this we use the remainder theorem. First we will find $p(-2)$ $p(-2) = (-2)^3 - (-2)^2 - 24(-2) - 36$ $= -8 - 4 + 48 - 36$ $= 0$ Since the remainder is 0, this means that $(x + 2)$ is a factor.

- Students can then use the factor theorem to factorise expressions of a higher order. The steps involved are:
    - Use trial and error to find a factor if one is not already given.

      Factorise fully $p(x) = x^3 - x^2 - 24x - 36$

      In this case we already know from above that $(x + 2)$ is a factor.
    - Write this next to a general polynomial of one lower degree

$(x + 2)(Ax^2 + Bx + c)$

    - Expand this out and compare coefficients between this expansion and $p(x)$ to work out missing values.

$Ax^3 + Bx^2 + Cx + 2Ax^2 + 2Bx + 2C$

      Comparing coefficients

      Coefficient of x^3 $\quad A = 1$
      Coefficient of x^2 $\quad B + 2A = -1$ since $A = 1, B = -3$
      Coefficient of x $\quad C + 2B = -24$ since $B = -3, C = -18$

      You can check this by comparing the constants 2 x -18 = -36 as required.

      So therefore
$x^3 - x^2 - 24x - 36 = (x + 2)(x^2 - 3x - 18)$

    - If the polynomial remaining is quadratic, try to factorise. If not you could repeat the same process through again.

      Since our polynomial is quadratic we can try to factorise.

$x^3 - x^2 - 24x - 36 = (x + 2)(x - 6)(x + 3)$

	• An alternative method to the one above would be to try to find other factors by trial and error using the remainder/factor theorem. • Some students might want to go on to try to do polynomial division. This site provides a detailed worked example of polynomial division https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html
Common errors and misconceptions	• The binomial expansion formula needs to be used very carefully; there are so many areas where students can go wrong. • Encourage students to take their time over their work and list each term separately. Extra care should be taken with any negatives in the expansion. • When you have an expression like $(3m^2)^4$ students tend to multiply the 3 x 4 = 12 and write $12m^8$. They need to realise that they have to raise any coefficient to the power of the bracket. One way to explain this is to write $(3m^2)^4 = 3m^2 \times 3m^2 \times 3m^2 \times 3m^2 = 81m^8$ Using the multiplication law.
Suggested resources	• Questions o http://www.cimt.org.uk/projects/mepres/alevel/pure_ch6.pdf (pp103-105) o http://www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf o http://www.mathssite.com/resources/docs/maths/alevel/c2/c2-polynomial-factor-remainder-theorem.pdf • Past Questions o https://www.examsolutions.net/tutorials/exam-questions-remainder-theorem/ • Video tutorials o https://www.khanacademy.org/math/algebra2/polynomial_and_rational/polynomial-remainder-theorem-tutorial/v/polynomial-remainder-theorem