

Guidance for tutors



Outcome	AG11	Student can consistently:	Draw a circle given its equation and find the equation of a circle.
How the topic is examined	<ul style="list-style-type: none"> <input type="checkbox"/> Examined through test paper questions. <input type="checkbox"/> Questions are equally likely to appear on calculator and non-calculator papers. <input type="checkbox"/> Although this topic was always on GCSE, it has renewed focus on the new GCSE. Because of this it is difficult to predict exactly what questions they could ask. <input type="checkbox"/> The GCSE specifications require students to recognise and draw graphs of circles with centre (0, 0). <input type="checkbox"/> Students are also expected to find the intersection of a circle and a straight line and for the highest attainers work out the equation of a tangent to a circle at a given point. <input type="checkbox"/> It is likely that students will be given an iterative equation to use with a particular starting value. Students will then have to go through several iterations to get the answer. 		
Prior knowledge	<ul style="list-style-type: none"> <input type="checkbox"/> Students should be confident with: <ul style="list-style-type: none"> ○ Simplifying expressions (AEx2) ○ Substituting into expressions (AEx5) ○ Solving basic equations (AEq1) ○ Simultaneous equations – one linear and one quadratic (AEq10) ○ Gradient of a straight line (AG4) ○ Finding the gradient of a perpendicular line (AG7) <input type="checkbox"/> In addition questions involving this topic can have links to: <ul style="list-style-type: none"> ○ Circle theorems 		
Suggested tuition approaches	<p>1) Drawing a circle and identifying the equation of a circle</p> <ul style="list-style-type: none"> <input type="checkbox"/> Students need to be aware that a circle with centre (0, 0) and radius r has an equation $x^2 + y^2 = r^2$. They should be aware that it could be written in two ways (e.g. $x^2 + y^2 = 10^2$ or $x^2 + y^2 = 100$ – both equations are for a circle with centre (0, 0) and radius 10 units. <input type="checkbox"/> Students should be able to draw these graphs using a pair of compasses. 		

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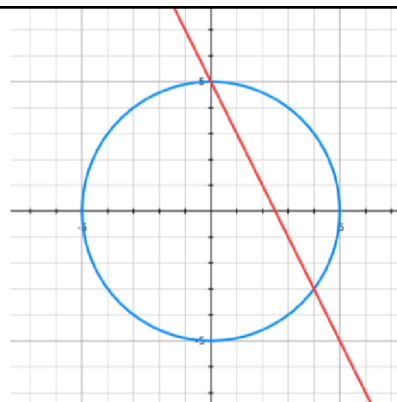
- ☐ In addition if a circle has been given, students should be able to write down the equation of the circle in either of the above forms.

2) Finding the intersection of a circle and straight line.

- ☐ Students will be given a circle and equation of a straight line. There are two methods that could be used to solve this in the exam.

Graphical method	Algebraic method
<p>This method should be used only if students are given part of the graph and told to use this method.</p> <p>Use graphs to find where the two lines below intersect</p> $y = 5 - 2x$ $x^2 + y^2 = 25$ <p>The steps involved are</p> <ul style="list-style-type: none"> - Draw the circle (if not drawn) - Draw the straight line (if not drawn) (AG1) - Find the x and y co-ordinates of the points where the lines intersect 	<p>This method is likely to be the one students should use.</p> <p>Find the points where the two lines intersect</p> $y = 5 - 2x$ $x^2 + y^2 = 25$ <p>This is the same method described in (AEq10). You will need to substitute the linear equation into the equation of the circle.</p> <ul style="list-style-type: none"> <input type="checkbox"/> Rearrange the linear equation so that either $x =$ or $y =$ (if necessary) <input type="checkbox"/> Substitute this rearranged equation into the equation with the quadratic term. <input type="checkbox"/> Expand, Simplify and rearrange to you have an equation equal to 0 <input type="checkbox"/> Solve this equation (there are usually two values to find) <input type="checkbox"/> Find the corresponding value(s) of the other unknown using the linear equation.

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You can see from the graph that the two points are (0, 5) and (4, -3)

3) Find the equation of the tangent at a particular point

- ☐ This is a topic for the highest attainers as it pulls together so many different elements of mathematics knowledge.
- ☐ Students first need to remember that the radius of a circle is perpendicular to the tangent at that point. This is one of the important circle theorems that students need to know.
(e.g. find the tangent to the circle $x^2 + y^2 = 25$ at the point (3, 4))

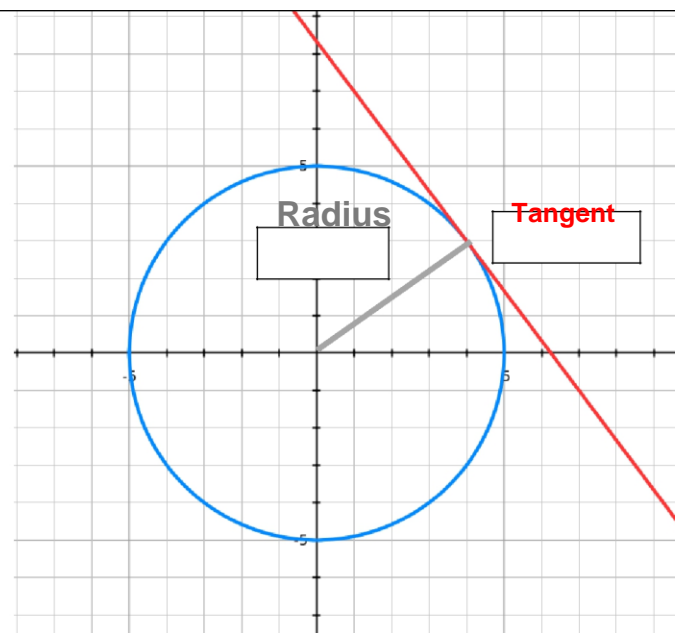
- ☐ Students should work out the gradient of the radius (see AG4)

$$\text{Gradient of radius} = \frac{3}{4}$$

- ☐ Now find the gradient of the tangent (see AG7)
Gradient of the tangent $= -\frac{4}{3}$

- ☐ Use the gradient of the tangent at the point on the circle to find the equation of the line (See last section of AG4)

$$\begin{aligned} -1 &= \left(-\frac{1}{4} \right) \\ -3 &= -4 \left(-\frac{1}{4} \right) \end{aligned}$$



Common errors and misconceptions

- ☐ Students think the radius of the circle $x^2 + y^2 = 25$ is 25 units, when it is 5. They need to remember when a circle is given in this form the right hand side is r^2 not r .
- ☐ Students make errors when solving a pair of simultaneous equations. The most common areas where students make mistakes with the algebra are:

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	<ul style="list-style-type: none"> ○ Forgetting to rearrange the equation so that it equals 0 ○ Expanding expressions like $(2 + 1)^2$, remember this needs to be thought of as $(2 + 1)(2 + 1)$ ○ Solving the quadratic equation (See notes in AEx6, 7 and 9) ○ Working with negatives. <ul style="list-style-type: none"> □ Sometimes students work out two correct values for one of the letters and then forget to work out the corresponding values for the other letter. □ When finding the gradient of the tangent they forget that $x_1 \times x_2 = -1$, instead they sometimes think it equals 1.
Suggested resources	<ul style="list-style-type: none"> □ Questions <ul style="list-style-type: none"> ○ http://www.cimt.org.uk/projects/mepres/allgcse/add_13.11c.pdf ○ http://www.cimt.org.uk/projects/mepres/allgcse/add_10.12b.pdf □ Past GCSE Questions <ul style="list-style-type: none"> ○ https://corbettmaths.files.wordpress.com/2013/02/equation-of-a-circle-pdf1.pdf □ Video tutorial <ul style="list-style-type: none"> ○ https://www.youtube.com/watch?v=HjN9TTRrQiA