Expressions

Guidance for tutors

The table below outlines useful information for tutors as well as some suggested approaches and resources.

Outcome	AEx9	Student can consistently:	Simplify algebraic fractions by cancelling common factors.
How the topic is examined		- Examined through test paper questions. - Questions are equally likely to appear on calculator or non-calculator papers. - Questions will ask students to 'Simplify' or 'Write in its simplest form' or 'Reduce'	
Prior knowledge		- Students should be confident with: - Expanding brackets (AEx1) - Simplifying expressions (AEx2) - Factorising simple expressions (AEx4) - Factorising quadratic expressions (AEx7) - Multiplying and dividing fractions - In addition questions involving this topic can have links to: - Solving algebraic fraction equations.	
Suggested tuition approaches		- Students need to understand that to simplify an algebraic fraction they need to factorise the numerator and denominator when they can and cancel common factors where necessary. - There are four common types of problems that students may meet, these are summarised in the table below.	

$$\frac{x+2}{(x+2)(x-5)} = \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-5)} = \frac{1}{(x-5)}$$

Here we have put $(x+2)$ in brackets and cancelled it with the $(x+2)$ on the denominator.

$$\frac{x^2+3x}{(x+3)(x+8)} = \frac{x\cancel{(x+3)}}{\cancel{(x+3)}(x+8)} = \frac{x}{x+8}$$

Here we factorise the numerator and cancelled the $(x+3)$ on top and bottom.

$$\frac{x^2-6x-16}{x-8} = \frac{\cancel{(x-8)}(x+2)}{\cancel{(x-8)}} = (x+2)$$

Opposite to the first one. We factorise the numerator and put a bracket around $(x-8)$ and then cancel.

$$\frac{x^2+7x+12}{x^2-9} = \frac{\cancel{(x+3)}(x+4)}{(x-3)\cancel{(x+3)}} = \frac{x+4}{x-3}$$

Factorise numerator and denominator and cancel the $(x+3)$ factors.

- A linear expression as the numerator or denominator may or may not have a bracket around it. Encourage students to put the brackets in to help them to see the common factors.
- These questions can involve any expression that can be factorised including single brackets (AEx4) and all types of quadratic expressions (AEx7)
- Sometimes it may be necessary to **multiply** or **divide** fractions before cancelling and simplifying.

1) **Multiplying fractions**
    o Multiply numerators and then multiply denominators.
    o It is not usually necessary to expand any brackets, as to simplify you will need to factorise.
    o Check then for common factors.

$$\frac{x}{(x+5)} \times \frac{3x+15}{x} = \frac{3x^2+15x}{x^2+5x} = \frac{3\cancel{x}\cancel{(x+5)}}{\cancel{x}\cancel{(x+5)}} = 3$$

    o In this example it wasn't necessary to multiply. Students could be encouraged to cancel terms straight away or write the brackets straight away (see division example)

2) **Dividing fractions**

	o To divide two fractions, remember you multiply by the inverse of the second fraction. o Then the above rules apply. e.g. $$\frac{x-7}{(x+5)} \div \frac{2x-14}{x} = \frac{(x-7)}{(x+5)} \times \frac{x}{2(x-7)} = \frac{x}{2(x+5)}$$ o You can see in this example that it is not necessary to multiply the expressions. Instead we can cancel straight away.

Common errors and misconceptions	• Students cancel x's when they see them. For example it is common for students to cancel further down by crossing out x from numerator and denominator. This is wrong.
	
	$$\frac{x+4}{x-3} = \frac{4}{-3}$$
	
	• Mistakes with expanding and simplifying happen frequently. Ask students to take care when expanding.
	• In the first example in the table, students forget that they are left with a 1 on the numerator of the fraction. Get them to think of $(x+2)$ as 1 x $(x+2)$
	• When factorising expressions:
	  o Students fail to realise that they need to use two pairs of brackets as opposed to the examples in AEx4. They need to do many different examples to help them see when to use one pair of brackets and when to use two.
	  o Students struggle with negatives when they appear in expressions.
	  o A very common mistake is that students just guess which sign goes with which number in the factorised expression. To avoid this get students to check their answer by expanding.
	  o Students can struggle to find the two numbers because they try to do it all in their head. Encourage them to write down all the factor pairs.

Suggested resources	• Questions o http://www.cimt.org.uk/projects/mepres/allgcse/pr10-es.pdf (pp 73 - 75)

- o https://corbettmaths.files.wordpress.com/2013/02/algebraic-fractions.pdf
- Past GCSE Questions
  - o https://keshgcsemaths.files.wordpress.com/2013/11/99_algebraic-fractions.pdf
- Video tutorial
  - o http://corbettmaths.com/2013/01/19/simplifying-algebraic-fractions/