Guidance for tutors

The table below outlines useful information for tutors as well as some suggested approaches and resources.

Outcome	AEx13	**Student can consistently:**	Use the factor and remainder theorem to factorise polynomials of higher order.

How the topic is examined	• This topic is not currently examined on GCSE but is on Level 2 Further Maths, AS/A2 mathematics and additional maths qualifications.
	• It is likely that students would have a calculator to solve these problems, but it is not necessary.
	• Questions may ask students to:
	  o Find the remainder when a polynomial $p(x)$ is divided by a linear function $f(x)$
	  o Show that a particular linear function $f(x)$ is a factor of $p(x)$
	  o Divide $p(x)$ by $f(x)$ in order to factorise higher order polynomials.
	• It is unlikely that students would ever have to factorise any polynomials above degree 3 (cubic)		

Prior knowledge	• Students should be confident with:
	  o Expanding brackets (AEx1)
	  o Simplifying expressions (AEx2)
	  o Substituting into formulae (AEx5)
	  o Factorising a quadratic (AEx7)		

Suggested tuition approaches

- Before teaching this topic, students should be very confident in factorising any quadratic expression.
- Introduce the remainder theorem to students, which says that "If a polynomial $p(x)$ is divided by $(x-a)$, the remainder is given by $p(a)$"
- You might want to ask students what a factor is. Explain that a factor is a number that goes into another number with no remainder. Therefore the factor theorem says that "If a polynomial $p(x)$ is divided by $(x-a)$, and $p(a) = 0$ then $(x-a)$ is a factor of the $p(x)$"
- Two possible questions:

Find the remainder when $p(x) = x^3 + 2x^2 - 3x + 1$ **is divided by** $(x - 3)$	Using the remainder theorem we need to find $p(3)$ $p(3) = (3)^3 + 2(3)^2 - 3(3) + 1 = 37$ The remainder therefore is 37
Show that $(x+2)$ **is a factor of** $p(x) = x^3 - x^2 - 24x - 36$	To show this we use the remainder theorem. First we will find $p(-2)$ $p(-2) = (-2)^3 - (-2)^2 - 24(-2) - 36$ $= -8 - 4 + 48 - 36$ $= 0$ Since the remainder is 0, this means that $(x+2)$ is a factor.

- Students can then use the factor theorem to factorise expressions of a higher order. The steps involved are:
  - Use trial and error to find a factor if one is not already given.

    Factorise fully $p(x) = x^3 - x^2 - 24x - 36$

    In this case we already know from above that $(x+2)$ is a factor.
  - Write this next to a general polynomial of one lower degree

$(x+2)(Ax^2 + Bx + c)$

  - Expand this out and compare coefficients between this expansion and $p(x)$ to work out missing values.

$Ax^3 + Bx^2 + Cx + 2Ax^2 + 2Bx + 2C$

Comparing coefficients

Coefficient of x^3 $A = 1$
Coefficient of x^2 $B + 2A = -1$ since $A = 1, B = -3$
Coefficient of x $C + 2B = -24$ since $B = -3, C = -18$

You can check this by comparing the constants 2 x -18 = -36 as required.

So therefore
$x^3 - x^2 - 24x - 36 = (x+2)(x^2 - 3x - 18)$

  - If the polynomial remaining is quadratic, try to factorise. If not you could repeat the same process through again.

    Since our polynomial is quadratic we can try to factorise.

$x^3 - x^2 - 24x - 36 = (x+2)(x-6)(x+3)$

	• An alternative method to the one above would be to try to find other factors by trial and error using the remainder/factor theorem. • Some students might want to go on to try to do polynomial division. This site provides a detailed worked example of polynomial division https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html
Common errors and misconceptions	• The binomial expansion formula needs to be used very carefully; there are so many areas where students can go wrong. Encourage students to take their time over their work and list each term separately. Extra care should be taken with any negatives in the expansion. • When you have an expression like $(3m^2)^4$ students tend to multiply the 3 x 4 = 12 and write $12m^8$. They need to realise that they have to raise any coefficient to the power of the bracket. One way to explain this is to write $(3m^2)^4 = 3m^2 \times 3m^2 \times 3m^2 \times 3m^2 = 81m^8$ Using the multiplication law.

Suggested resources	• Questions
	  o http://www.cimt.org.uk/projects/mepres/alevel/pure_ch6.pdf (pp103-105)
	  o http://www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf
	  o http://www.mathssite.com/resources/docs/maths/alevel/c2/c2-polynomial-factor-remainder-theorem.pdf
	• Past Questions
	  o https://www.examsolutions.net/tutorials/exam-questions-remainder-theorem/
	• Video tutorials
	  o https://www.khanacademy.org/math/algebra2/polynomial_and_rational/polynomial-remainder-theorem-tutorial/v/polynomial-remainder-theorem