Pythagoras and Trigonometry

Guidance for tutors

Outcome	SPT10	Student can consistently:	Solve trigonometric equations and use radian measure for angles.
How the topic is examined		- This topic is not currently examined on GCSE but certain aspects are on Level 2 Further Maths, AS/A2 mathematics and additional maths qualifications. - It is likely that students would have a calculator to solve trig equations and work with angles in radians. - Questions may ask students to: - Solve a trig equation involving sin, cos or tan and find all the solutions within a given interval. - Convert an angle between radians and degrees and vice-versa.	
Prior knowledge		- Students should be confident: - Solving basic equations (AEq1) - Rearranging simple formulae (AEx8) - Trigonometry (SLT2 and SPT3) - Graphs of trig functions (SPT8) - In addition questions on this topic can have links to: - Circles (SLAV2) - Area of sectors (SLAV6)	
Suggested tuition approaches		- The content of this outcome follows on from all the previous work with trigonometry. It is essentially that students understand how to find a side and angle in right angled triangle. - Students have already seen basic trigonometric equations and solved them. - Remind students how they found the missing angle in a triangle. They have already solved equations such as $\sin x = \frac{2}{5}$ or $\cos x = 0.75$ etc… Explain to students that they used the inverse sine, cosine and tangent functions ($sin^{-1}(x)$, $cos^{-1}(x)$ and $tan^{-1}(x)$). However we only found one solution. - Explain this is solving trig equations. Ask students are there any more in the range 0 to 360 degrees. - Show students using the graph of $\sin x$ that there are more than one solution to $\sin x = 0.4$. You could draw a line and show that there is more than one solution. - Explain to students that they can find this other solution by using the symmetry of the sine graph. Indeed the second solution for sin x = 0.4 can be found by subtracting the first solution that the calculator gives from 180°.	

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	• The steps involved in solving a basic trig equation are:
	  ○ Rearrange the equation so that $\sin x = ...$ or $\cos x = ...$ or $\tan x = ...$
	  ○ Now use $sin^{-1}(x)$, $cos^{-1}(x)$ and $tan^{-1}(x)$ to find the first (principal) solution.
	  ○ Draw the graph of the relevant function. Show this first solution on the graph.
	  ○ Identify where the other solutions lie and show them on the graph.
	  ○ Use the symmetry of the graph to calculate all the other solutions in the given range.
	• The above method is a graphical method for solving trig equations. There are others. The CAST method is a common method. Details about this are here http://mathonweb.com/help_ebook/html/cast.htm
	
	Radian measure
	• Students should understand that angles can be measured in radians as well as degrees.
	• Radians are a preferred unit for angles.
	• The key to converting between the two units is that 2π radians = 360°
	• From this you can convert any angle from degrees to radians and vice-versa.
	• Students at A-level are expected to solve equations with answers to be given in radians.

Common errors and misconceptions	• Students find the principal solution and then don't find any more. • They incorrectly sketch the correct trigonometric graph.

Suggested resources	• Questions and notes
	  ○ http://www.brentwood.k12.ny.us/HTMLpages/Curriculum/Math/AlgebraTrig/14411C13.pgs.pdf (trig equations)
	  ○ https://www.tes.com/teaching-resource/a-level-maths-c2-trig-equations-worksheets-6146809 (trig equations)
	  ○ http://www.cimt.org.uk/projects/mepres/alevel/pure_ch10.pdf (radians and trig equations)
	• Past Questions
	  ○ https://www.examsolutions.net/tutorials/exam-questions-mixed-trigonometry/?board=IB&module=Higher%20Level&topic=11043
	• Video and online tutorials
	  ○ https://www.khanacademy.org/math/trigonometry
	  ○ https://www.examsolutions.net/tutorials/trig-equations-multiple-angles/