Outcome	AEx12	Student can consistently:	Use the binomial theorem to expand any bracket with integer powers.
How the topic is examined	○ This topic is not currently examined on GCSE but is on Level 2 Further Maths, AS/A2 mathematics and additional maths qualifications. ○ It is likely that students would have a calculator to solve these problems, but small powers may appear on non-calculator papers. ○ Questions may ask students to: ▪ Expand and simplify ▪ Find the coefficient of a particular term ▪ Find the first 3 or 4 terms in the expansion ○ Questions will be limited to linear expressions to the power n, where n is a positive integer.		
Prior knowledge	Students should be confident with: ○ Expanding brackets (AEx1) ○ Simplifying expressions (AEx2) ○ Indices (AEx3) ○ Multiplying three or more linear terms (AEx9) ○ Permutations and combinations.		
Suggested tuition approaches	Before teaching this topic, students should have expanded three or more linear expressions by hand (See AEx11). They should see that as the number of brackets increases the time it takes to complete the expansion increases dramatically and the scope for error becomes high. This topic could be introduced through an investigation by pattern spotting. For example: ▪ Ask students to expand $(x + 1)^2$, $(x + 1)^3$ and $(x + 1)^4$. What do they notice? ▪ Can they now expand $(x + y)^2$, $(x + y)^3$ and $(x + y)^4$? How does the answer relate to the first set of expansions? ▪ Can they predict what the expansion of $(x + 1)^5$, $(x + 1)^6$ or $(x + y)^5$, $(x + y)^6$ is? ▪ Can they generalise the rule? ▪ How does this relate to combinations theory? Students should be introduced to the theory on combinations and permutations (if not already done so) and they should also explore Pascal's triangle https://en.wikipedia.org/wiki/Pascal%27s_triangle		

	• Students can then use the formula for the binomial expansion $$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n,$$ Where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ This can also be written in this way too. $$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}.$$ • Any modern calculator will be able to find the values of $\binom{n}{r}$ for any two numbers. The button is usually denoted by ^nC_r and some teachers and students may prefer to write it in this way. • The following link provides detailed worked examples of all the possible examples that students might come across. http://www.mathsisfun.com/algebra/binomial-theorem.html
Common errors and misconceptions	• The binomial expansion formula needs to be used very carefully; there are so many areas where students can go wrong. • Encourage students to take their time over their work and list each term separately. Extra care should be taken with any negatives in the expansion. • When you have an expression like $(3m^2)^4$ students tend to multiply the 3 x 4 = 12 and write $12m^8$. They need to realise that they have to raise any coefficient to the power of the bracket. One way to explain this is to write $(3m^2)^4 = 3m^2 \times 3m^2 \times 3m^2 \times 3m^2 = 81m^8$ using the multiplication law.

Suggested resources	• Questions
	  ○ http://www.cimt.org.uk/projects/mepres/alevel/pure_ch9.pdf (pp 173 onwards)
	  ○ http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/The%20Binomial%20Theorem.pdf
	• Past Questions
	  ○ http://www.examsolutions.net/maths-revision/core-maths/sequences-series/binomial/exam-questions/integer-powers/questions.php
	• Video tutorial
	  ○ https://www.khanacademy.org/math/algebra2/polynomial_and_rational/binomial_theorem/v/binomial-theorem