

Outcome	AEq11	Student can consistently:	Solve more complicated equations using numerical methods (e.g. iteration)
How the topic is examined	<ul style="list-style-type: none"> • Examined through test paper questions. • Questions are most likely to appear on a calculator paper due to the complexity of the calculations. • This is a new topic to GCSE and so it is difficult to predict exactly what questions they could ask. The specification requires students to be aware of iterative methods to solve equations numerically. • It is likely that students will be given an iterative equation to use with a particular starting value. Students will then have to go through several iterations to get the answer. 		
Prior knowledge	<ul style="list-style-type: none"> • Students should be confident with: <ul style="list-style-type: none"> ◦ Simplifying expressions (AEx2) ◦ Substituting into expressions (AEx5) ◦ Solving basic equations (AEq1) • In addition questions involving this topic can have links to: <ul style="list-style-type: none"> ◦ Sequences ◦ Trial and improvement 		
approaches	<ul style="list-style-type: none"> • Many equations in maths cannot be solved algebraically. This might surprise students. • Mathematicians therefore need ways of solving these type of equations. One method for doing this to use an iterative formula. • For example in order to solve the equation $x^3 + x = 19$ using an iterative method, students might use the following formula. $x_{n+1} = \sqrt{\frac{19 - x_n}{x_n}}$ • Although students don't need to know where this equation comes from, some of the higher attainers might be interested to know how this equation is formed. It is simply a rearrangement of the original formula, where you are writing x in terms of x. For a more detailed explanation and why iterative methods work have a look at http://www.cimt.org.uk/projects/mepres/alevel/pure_ch19.pdf • You will need a starting value to ensure that the formula leads to a solution, not all starting values do. The question should give the starting value. This will be called either x_0 or x_1. 		

- To work out the next values in the iteration we use the iterative formula given.
- The best way to present your results is in the form of a table.

x_n	Notes
$x_1 = 2$	This is the starting value that will be given in the question
$x_2 = \sqrt{\frac{19 - x_1}{x_1}} = \sqrt{\frac{19 - 2}{2}} = 2.915476$	We now put this starting value into our iterative formula. We end up with another value out of it.
$x_3 = \sqrt{\frac{19 - x_2}{x_2}} = \sqrt{\frac{19 - 2.915476}{2.915476}} = 2.348818$	We now put this new value into the iterative formula and get out a new value.
$x_4 = \sqrt{\frac{19 - x_3}{x_3}} = \sqrt{\frac{19 - 2.348818}{2.348818}} = 2.662550$	We now put this new value into the iterative formula and get out a new value.
$x_5 = 2.477098$	<p>We continue to do the same process over and over until the numbers starts to converge.</p> <p>We can see from this list of values that the numbers are starting to converge on 2.5.</p> <p>2.5 is a solution to the original equation to one decimal. If we wanted the solution to 2 decimal places we would have to keep going with the iteration.</p>
$x_6 = 2.582686$	
$x_7 = 2.521246$	
$x_8 = 2.556551$	
$x_9 = 2.536116$	
$x_{10} = 2.547895$	
$x_{11} = 2.541089$	
$x_{12} = 2.545016$	

- You can see from the above table that calculations can become cumbersome and time consuming. To ensure accurate values, ensure you write down at least 6 decimal places in your answers.
- Most modern calculators allow you to do this. The link here will show you this <https://www.youtube.com/watch?v=ro5M92KCmak>
- Higher attaining students might want to choose a different starting value to see what happens.
- Students should understand the word converge and diverge. Converge is when numbers get closer and closer to a particular value. Diverge is when they go away from it usually getting wider and wider apart.

Common errors and

- This question can be prone to numerical errors. This is due to students rounding prematurely. Remember to get them to

misconceptions	<p>write at least 6 decimal places (possibly more) to ensure that their subsequent answers are accurate.</p> <ul style="list-style-type: none"> • Students should organise their work so that each iteration is clear. • If they are using a calculator to do the iteration then they should ensure they show ALL the interim values and not just the final value that they get. • Some students give the answer to the wrong number of decimal places or significant figures. Encourage them to read the question carefully. • Students get the substitution wrong and either put the wrong value in or incorrectly work it out. Where x appears there should be a substituted value.
Suggested resources	<ul style="list-style-type: none"> • Questions <ul style="list-style-type: none"> ◦ https://corbettmaths.files.wordpress.com/2013/02/iteration-pdf.pdf • Past GCSE Questions <ul style="list-style-type: none"> ◦ https://www.examsolutions.net/tutorials/exam-questions-iteration/?level=International&board=Edexcel&module=C34&topic=1458 • Video tutorial <ul style="list-style-type: none"> ◦ https://www.youtube.com/watch?v=G0vyTSxNCLs ◦ https://www.examsolutions.net/tutorials/solutions-of-equations-by-iteration/?level=A-Level&board=Edexcel&module=Pure%20Maths%2012&topic=1458