5 Probability

5.1 Probabilities

1. Describe the probability of the following events happening, using the terms

    Certain

    Very likely

    Possible

    Very unlikely

    Impossible

    (a)   The next Prime Minister will be Sir Cliff Richard.

    (b)   It will rain tomorrow.

    (c)   England will win the next Football European Cup.

    (d)   You will be late for school tomorrow.

    (e)   You will have a cold next winter.

    (f)   You will get maths homework tonight

    (g)   You will get full marks in your next maths test.

2. If I toss a fair coin 50 times, how many times would you expect to get heads?

3. If I throw a fair die 60 times, how many times would you expect to get

    (a)   6

    (b)   1

    (c)   an even number?

5.2 Simple Probability

1. The probability that you will be late for school is $\frac{1}{10}$.
    What is the probability of not being late?

2. With a fair die, the probability of throwing a 6 is $\frac{1}{6}$.
    What is the probability of not throwing a 6?

3. The probability of it raining tomorrow is $\frac{2}{5}$.
    (a)   What is the probability of it not raining tomorrow?
    (b)   Is it more likely to rain or not to rain?

4.	The probability of a 'white' Christmas is 0.05.

	What is the probability of it not being a 'white' Christmas?

5.	The probability of Exeter City football team coming last in Division 3 next year is estimated as 0.2.

	What is the probability of Exeter City not coming last?

6.	The probability of Newcastle United football team beating Manchester United is estimated as 0.3. The probability of Manchester United beating Newcastle United is 0.4. Why do these two probabilities *not* add up to 1?

7.	'The probability that Nottingham Forest will win the F.A. Cup is 1.2.'
	'The probability that Birmingham City will win the F.A. Cup is –0.5.'

	Explain why the value of probability in each of these statements is not possible.

	(NEAB)

8.	Imran plays a game of chess with his friend.
	A game of chess can be won or drawn or lost.

	The probability that Imran wins the game of chess is 0.3.
	The probability that Imran draws the game of chess is 0.25.

	Work out the probability that Imran loses the game of chess.

	(Edexcel)

9.	There are blue, red and yellow discs in a bag.

	When a disc is picked out at random, the probability of it being red is 0.4 and the probability of it being blue is 0.3.

	(a)	What is the probability of picking a yellow disc?

	(b)	There are 60 discs in the bag. Work out how many of them are red.

	(OCR)

10.	A bag contains only red, blue and white counters.

	The table shows the probability of taking a red or blue counter from the bag at random.

Colour	Red	Blue	White
Probability	0.25	0.3	

	Work out the probability of taking a white counter from the bag.

	(AQA)

5.3 Outcome of Two Events

1. A coin is tossed, and a die is thrown. List all the possible outcomes.

2. A die is thrown twice. Copy the diagram below which shows all the possible outcomes.
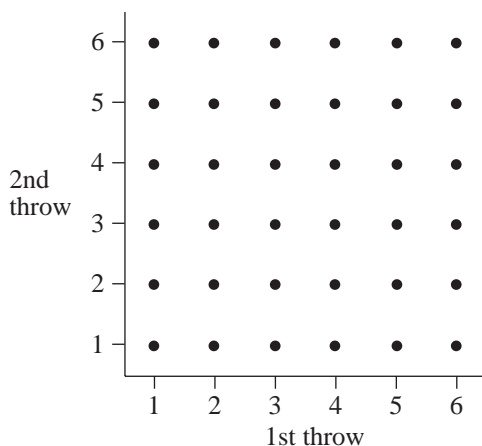
On your diagram, show outcomes which have

(a) the same number on both throws,   (b) a total score of 8.

3. When this spinner is used, the scores 1, 2, 3, 4 and 5 are equally likely.

   (a) For one spin,

       (i) what is the probability of scoring a 2,

       (ii) what is the probability of *not* scoring a 2?

   (b) When playing a game the spinner is spun twice and the scores are added to give a total.

       Write down all the different ways of getting a total of 7.
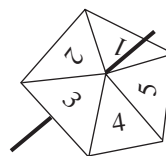
(SEG)

4. The diagram shows a spinner, labelled A.

   The result shown is Blue.

   Spinner A is a fair spinner.

   (a) What is the probability of *not* getting Green with spinner A?
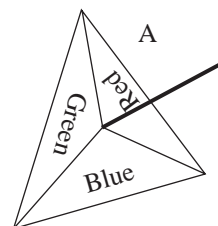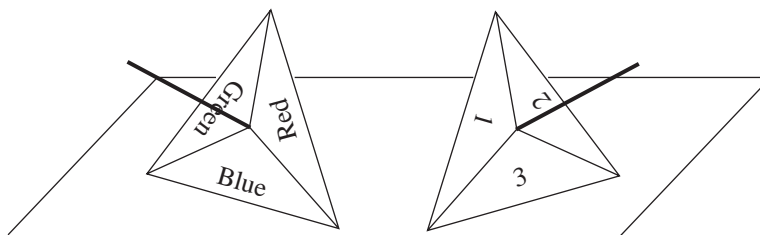

   The diagram shows another spinner, labelled B.

   The result shown is 3.

   Spinner B is weighted (biased).

   The probability of getting a 3 is 0.2 and the probability of getting a 1 is 0.1.

   (b) What is the probability of getting a 2 with spinner B?
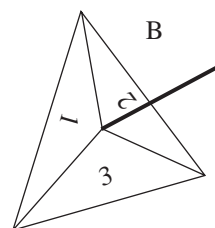
A game is played with the two spinners. They are spun at the same time. The combined result shown in the diagram is Blue 3.

(c)   Write down the total number of different possible combined results.

5.   A coin is tossed 4 times. List all the possible outcomes.

5.4 Finding Probabilities Using Relative Frequency

1.   Last year it rained on 150 days out of 365.

Estimate the probability of it raining on any one day next year.

How could your estimate be improved?

2.   Throw a die 120 times. How many times would you expect to obtain the number 6?

In an experiment, the following frequencies were obtained.

Number	Frequency
1	31
2	15
3	14
4	16
5	15
6	29

Do you think that the die is fair? If not, give an explanation why not and estimate what you think are the probabilities of obtaining each number.

3.   There are 44 students in a group. Each student plays either hockey or tennis but not both.

	Hockey	Tennis	Total
Girls	8		20
Boys	18		24
Total			44

(a)   Complete the table.

(b)   A student is chosen at random from the whole group.
        Calculate the probability that this student is a girl.

(c)   A girl is chosen at random. Calculate the probability that she plays hockey.

<div align="right">*(SEG)*</div>

4.   John recorded the results of his football team's last 24 matches.

W	W	D	L	W	L	W	D
D	L	L	W	W	W	L	L
D	W	L	W	W	L	W	L

Key:	W	Win
	D	Draw
	L	Lose

(a)   Organise and display this information in a table.

(b)   Janet told John that, since there are three possible results of any match, the
        probability that the next match would be drawn was $\frac{1}{3}$.

        (i)   Explain why Janet's argument is wrong.

        (ii)   What might John suggest for the probability of a draw, based on the
                past performance of his team?

(c)   Julia estimates that the probability that her hockey team will win their next
        match is 0.6 and that the probability they will lose is 0.3

   What is the probability that her team will draw?

<div align="right">*(MEG)*</div>

5.   The number of serious accidents on a stretch of motorway in each month of one
        year are given below.

January	16
February	12
March	9
April	10
May	6
June	5
July	7
August	8
September	6
October	10
November	9
December	12

(a)   Estimate the average number of accidents per month over the whole year.

(b)   Estimate the probability of an accident happening on any particular day.
        Would your estimate change if you know that the particular day is in
        January?

6.   Julie has a square-shaped spinner with the letters A, B, C and D on it.
      She spins the spinner and records the letter on which the spinner lands.
      She plots the relative frequence of the letter C after every 10 spins.
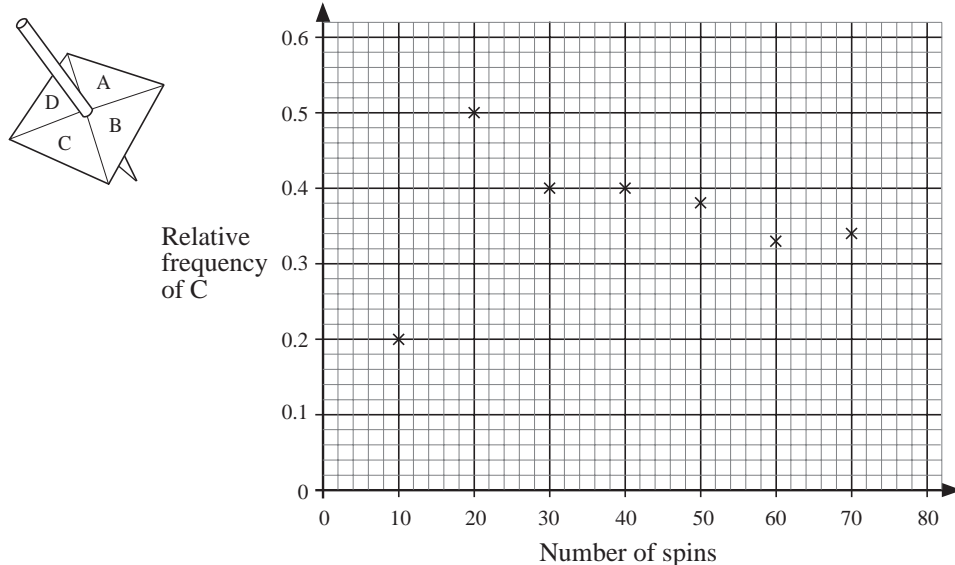
      (a)   How many times did the letter C occur in the first 40 spins?

      (b)   After 80 spins the letter C occurred 30 times.
             Plot the relative frequency for 80 spins on a copy of the diagram.

      (c)   Is the spinner biased? Give a reason for your answer.

                                                                          (AQA)

5.5 Determining Probabilities

1.   In a raffle 200 tickets are sold. Peter buys 40 tickets. What is the probability that
      he wins first prize? Give your answer as a fraction in its simplest form.

                                                                          (SEG)

2.   A box contains only blue pencils and red pencils.

      6 of the pencils are blue and 5 are red.

      A pencil is taken at random from the box.

      Write down the probability that

      (a)   a blue pencil will be taken,

      (b)   a blue pencil will *not* be taken.

                                                                          (LON)

3.   A bag contains 8 marbles of which 2 are green, 3 are red and the rest yellow.
      A marble is taken out at random.

      Find the probability that the marble is

      (a)   green   (b)   not yellow.

4. In an assortment of 36 calculators, 7 have defective switches, 12 have scratched screens and no calculator has both defects. A calculator is chosen at random for inspection.

    Find the probability that it has

    (a)   a defective switch   (b)   no defects.

5. In a raffle, a winning ticket is to be drawn from 200 tickets numbered 1 to 200. Yusof holds 1 ticket, Yanling holds 9 tickets and Sam holds 4 tickets.

    What is the probability of each of them winning the prize?

6. Each letter of the word 'MATHEMATICS' is written on a separate card. The 11 cards are placed face downwards. A card is drawn at random.

    What is the probability of picking a card with

    (a)   the letter C   (b)   the letter A

    (c)   a vowel   (d)   a consonant?

7. One hundred raffle tickets, numbered from 1 to 100 are placed in a drum.

    A ticket is taken from the drum at random.

    (a)   What is the probability that the number on the ticket is a multiple of 5?

    (b)   What is the probability that the number on the ticket is a square number?

    (SEG)

8. Zaheda conducted a probability experiment using a packet of 20 sweets.
    She counted the number of sweets of each colour.

    Her results are shown in the table.

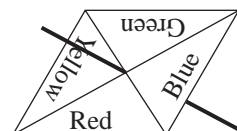 | Red | Green | Orange |
    |-----|-------|--------|
    | 12 | 3 | 5 |

    Zaheda is going to take one sweet at random from the packet.

    Write down the probability that

    (a)   Zaheda will take a green sweet from the packet

    (b)   the sweet Zaheda takes will *not* be red.

    (LON)

9. The spinner shown is biased.

    The probabilities of getting a particular colour are shown in the table below.

    (a)   Complete the table to show the probability of getting GREEN.

Colour	RED	YELLOW	BLUE	GREEN
Probability	0.4	0.1	0.3	

(b)   The spinner is spun once.
       What is the probability of getting either RED or BLUE?

(c)   The spinner is spun 50 times.
       Approximately how many times would you expect to get RED?

(NEAB)

10. A bag contains 50 discs numbered 1 to 50. A disc is selected at random.
     Find the probability that the number on the disc

     (a)   is an even number   (b)   is an odd number   (c)   has the digit 1.

11. A box contains a number of counters.
     Each counter is coloured red (R) or white (W).
     Each counter is also numbered 1 or 2.

     The table shows the probabilities of picking the
     different colours and numbers when a counter is
   picked at random from the box.

	Number	
	1	2
R	$\frac{1}{5}$	$\frac{1}{10}$
W	$\frac{1}{4}$	$\frac{9}{20}$

Colour

     (a)   Sam says that there are 50 counters in the box.
           Explain why Sam must be wrong;

     (b)   Show that the probability of picking a red counter (R) at random is $\frac{3}{10}$.

(AQA)

12. A fair spinner has 10 equal sections.
     Five sections are yellow, three are blue and two are red.

     (a)   The arrow is spun.
           (i)   What is the probability of the arrow
                  landing on blue?
           (ii)   What is the probability of the arrow
                  landing on green?
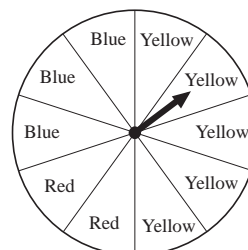
     (b)   The arrow is spun 100 times. How many times do you expect the arrow to
           land on yellow?

(AQA)

13. A spinner has coloured sections. The sections are
     different sizes.   When the spinner is spun, the
     pointer lands on a colour.

     The table shows the probability for the pointer
     landing on yellow and blue.

Diagram is not accurately drawn.

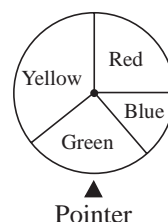| Number	Red	Yellow	Blue	Green
Probability	x	0.35	0.15	x

The probability of the
pointer landing on red is
equal to the probability
of it landing on green.

     (a)   Work out the value of *x*.

     Sarah is going to spin the wheel 400 times.

     (b)   Work out an estimate for the number of times it will land on **blue.**

(Edexcel)

5.6 Probability of Two Events

1.   A fair dice is thrown twice.

   (a)   What is the probability of obtaining *two* sixes?

   (b)   What is the probability of obtaining exactly *one* six?

2.   A coin is biased so that the probability that it lands showing heads is $\frac{2}{3}$.

   The coin is tossed three times. Find the probability that:

   (a)   no heads are obtained   (b)   more heads than tails are obtained.

3.   If a coin and a die are tossed together, calculate

   (a)   the probability of getting a tail with the coin and an even number with the die,

   (b)   the probability of a head with the coin and a number less than three on the die,

   (c)   the probability of a head with the coin and a multiple of 3 on the die.

4.   A box contains 5 red, 3 yellow and 2 blue discs. Two discs are drawn at random from the box one after another.

   (a)   What is the probability that the first disc drawn will be red?

   (b)   If the first disc drawn is blue and it is not replaced, what is the probability of drawing a yellow disc on the second draw?

5.   Consider the experiment of rolling two dice and noting the two values uppermost. The score is the sum of these two numbers.

   Complete the table of outcomes, as shown below.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4				
3						
4						
5						
6						

   From your table, deduce the probability that the score:

   (a)   equals 12,

   (b)   is less than 12,

   (c)   equals 7,

   (d)   is less than 7.

   Remember that each of the 36 entries in the table is equally likely.

6. Two bags contain 9 marbles each. In each bag, there are 4 red marbles, 3 white marbles and 2 green marbles.

    (a) One marble is drawn from the first bag. Find the probability that it is white.

    (b) One marble is drawn from the second bag. Find the probability that it is either red or green.

    These marbles are then returned to their original bags.

    (c) One marble is drawn from each bag. Calculate the probability that the two marbles are

        (i) red,

        (ii) of different colours.

7. When throwing a dice, the possible outcomes are 1, 2, 3, 4, 5 or 6.

    A particular dice is biased so that the probability of throwing a 6 is 0.25.

    (a) What is the probability of *not* throwing a 6?

    (b) The outcomes 1, 2, 3, 4 and 5 have the same probability as each other.

        What is the probability of throwing a 4?

    (c) The dice is thrown twice.

        (i) How many ways are there of reaching a total score of 10?

        (ii) What is the probability that the total score is 12?

        (MEG)

8. In the film *Shipwreck*, the captain and five passengers remain on board a sinking ship. There are three lifejackets remaining.

    The Captain knows that three of the passengers cannot swim.

    In his panic he hands out the lifejackets randomly to three of the five passengers.

    Calculate the probability that he gives the lifejackets to just two of the three non-swimmers.

    (OCR)

9. Two boxes contain coloured bricks.

    Box A contains 2 red bricks, 3 blue bricks and 1 yellow brick.

    Box B contains 3 red bricks, 2 yellow bricks and 1 green brick.

    Janet selects one brick from box A and one brick from box B.

    Calculate the probability that the two bricks will be of the same colour.

    (Edexcel)

5.7 Use of Tree Diagrams

1. A fair coin is tossed three times. By drawing a tree diagram, determine the probability of obtaining

    (a)   exactly two heads,

    (b)   at least two heads.

2. George passes three sets of traffic lights on his way to work.

    The lights work independently of each other.

    The probability that he has to stop at any set of traffic lights is 0.35.

    What is the probability that George stops at two or three sets of traffic lights?

    (SEG)

3. The faces of a die are marked with the numbers 2, 2, 4, 4, 6, 6. If the die is rolled twice what is the probability of getting

    (a)   a 4 each time,

    (b)   either a 2 or a 6 each time, or a 2 and a 6?

    If the die is rolled three times, what is the probability of getting

    (c)   a 2 each time,

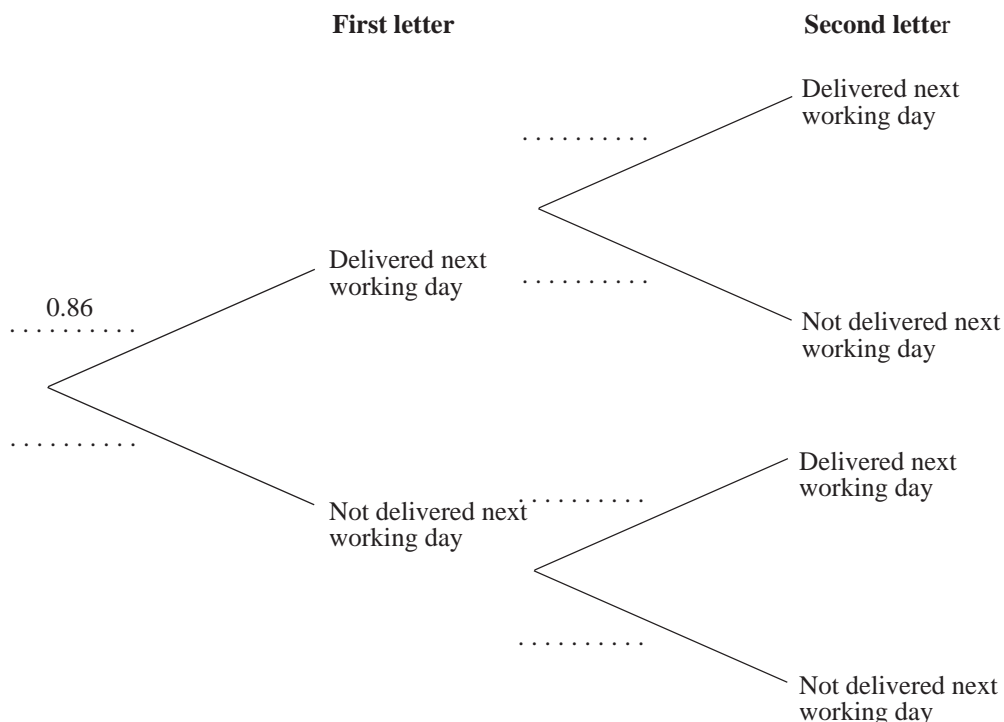   (d)   either a 4 or a 6 each time, or a combination of 4s and 6s?

4. There are two spinners, one marked into equal sections numbered 1, 2, 3, 4, 5 and the second spinner marked into equal sections A, B, C.

    Calculate the probability of getting

    (a)   a 2 and a B,

    (b)   a 5 and an A,

    (c)   an even number and an A,

    (d)   an odd number and either B or C.

5. Rob has a bag containing 3 blue balls, 4 red balls and 1 green ball.
    Sarah has a bag containing 2 blue balls and 3 red balls.
    The balls are identical except for colour.

    Rob chooses a ball at random from his bag and Sarah chooses a ball at random from her bag.

    (a)   Draw a tree diagram and write the probability of each of the events on each of the branches of the diagram.

    (b)   Calculate the probability that both Rob and Sarah will choose a blue ball.

    (c)   Calculate the probability that the ball chosen by Rob will be a different colour from the ball chosen by Sarah.

    (MEG)

6.   A letter has a first class stamp on it.

The probability that it will be delivered on the next working day is 0.86.

(a)   What is the probability that the letter will *not* be delivered on the next working day?

Sam posts 2 letters with first class stamps.

(b)   Copy and complete the tree diagram. Write all the missing probabilities on the appropriate branches.

First letter   **Second letter**

Delivered next working day

. . . .

Not delivered next working day

0.86

Delivered next working day

. . . .

. . . .

Not delivered next working day

. . . .

Delivered next working day

. . . .

Not delivered next working day

(c)   Calculate the probability that both letters will be delivered on the next working day.
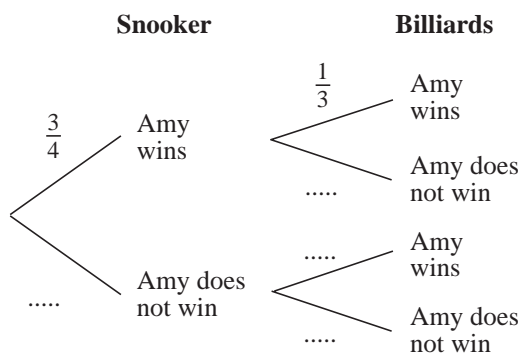
(LON)

7.   Amy is going to play one game of snooker and one game of billiards.

The probability that she will win the game of snooker is $\dfrac{3}{4}$.

The probability that she will win the game of billiards is $\dfrac{1}{3}$.

(a)   Copy and complete the probability tree diagram.

Snooker   **Billiards**

$\dfrac{1}{3}$ Amy wins

$\dfrac{3}{4}$ Amy wins

Amy does not win

.....

.....   Amy wins

.....   Amy does not win

Amy does not win

.....

5.7

(b)   Work out the probability that Amy will win **exactly** one game.

Amy played one game of snooker and one game of billiards on a number of Fridays. She won at **both** snooker and billiards on 21 Fridays.

(c)   Work out an estimate for the number of Fridays on which Amy did not win either game.

(Edexcel)

8.   Emma has a box of counters. The counters are green, red or blue.

She picks a counter at random.

The table shows the probability that she picks a green counter and the probability that she picks a red counter.

Colour	Probability
Green	0.6
Red	0.25
Blue	

(a)   What is the probability that Emma picks a blue counter?

(b)   There are 10 red counters in the box. How many green counters are in the box?

(c)   Emma picks a counter at random. She replaces it in the box and then picks another counter at random.

(i)   Copy and complete the tree diagram.

First counter   **Second counter**
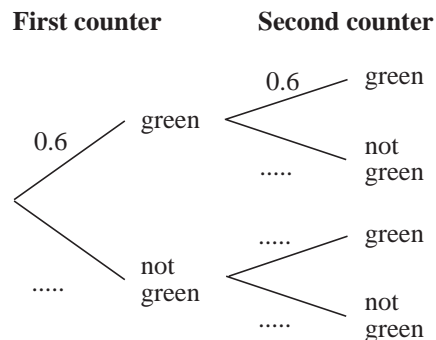
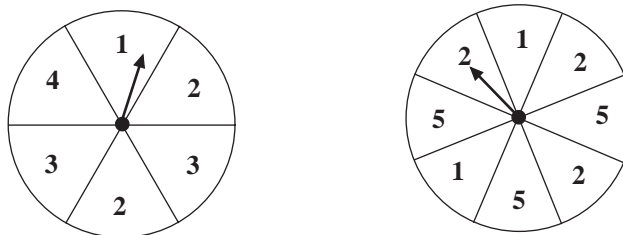(ii)   What is the probability that at least one of the counters is green?

(AQA)

5.8 Multiplication for Independent Events

1.   A die is thrown and a coin is tossed. What is the probability of obtaining an even number on the die and a Head on the coin?

2.   Three dice are thrown and their scores are added. What is the probability of scoring in total:

(a)   18   (b)   17   (c)   16?

3. A day which is fine has probability $\frac{3}{4}$ of being followed by another fine day.

    A day which is wet has a probability $\frac{2}{3}$ of being followed by another wet day.

    Given that days are classified either fine or wet, and that June 6th is fine, set out a tree diagram for June 7th, 8th and 9th.

    Calculate the probability that at least two of the three days are fine.

4. On a stretch of main road there are 4 independent sets of traffic lights, each phased for 120 seconds red, 60 seconds green.

    What is the probability that a motorist arriving at random will have to stop at least once?

5. Four balls are drawn at random, one after the other and without replacement, from a bag containing

    5 Red, 4 White , 8 Blue and 3 Purple balls.

    Find the probability that you obtain one ball of each colour.

6. A fair dice is thrown three times.

    (a) What is the probability of throwing 3 sixes?

    (b) What is the probability of throwing a six on the first throw, a six on the second throw but not a six on the third throw?

    (c) What is the probability of throwing exactly two sixes in the three throws?

    (d) What is the probability of throwing at least two sixes in the three throws?

    (SEG)

7. The diagrams show two fair spinners. Both spinners are spun and the scores are added together.

    What is the probability that the sum of the scores is at least 5?

8. Mrs Collins drives to work. On her way to work she has to cross two sets of traffic lights marked A and B in the diagram.

5.8
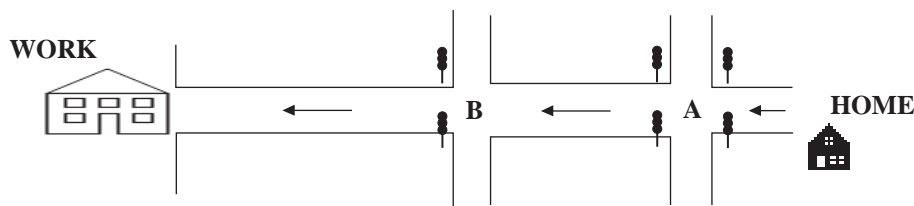
The probability of having to stop at the traffic lights is shown in the table.

Traffic	Probability of having to stop
A	0.3
B	0.6

On Monday Mrs Collins drives to work.

(a)   What is the probability that she will *not* have to stop at traffic lights A?

(b)   What is the probability that she will *not* have to stop at either set of traffic lights?

(c)   What is the probability that she will have to stop at only *one* set of traffic lights?

(SEG)

9.   A car driver has 4 keys, only one of which will open the car door. Given that the keys are otherwise indistinguishable, find the probability (before he starts trying them) that the door will open on the first, second, third and fourth attempts.

(a)   Consider two cases where

(i)   he discards each key which fails to open the door,

(ii)   he returns each key to the collection before choosing the next one at random.

(b)   Consider the cumulative probabilities with each strategy. i.e. the probability that he will have succeeded by the first, second, third and fourth attempts.

10.   A company secretary carries out a survey of incoming post to compare the delivery times of 1st and 2nd class letters. His results are shown below.

Days to deliver	1	2	3	4
1st class letter	92%	7%	1%	0%
2nd class letter	5%	55%	34%	6%

Use the information in the table to find the probability of

(a)   a 2nd class letter taking more than two days to deliver,

(b)   two 1st class letters taking two days to deliver,

(c)   a 1st and a 2nd class letter taking the same number of days to deliver.

(SEG)

11. At the village fete, Susan helps on a stall where radios can be won. She makes the following poster explaining the rules.

WIN A RADIO

Throw 3 dice

Score a total of 18 and the radio is yours!
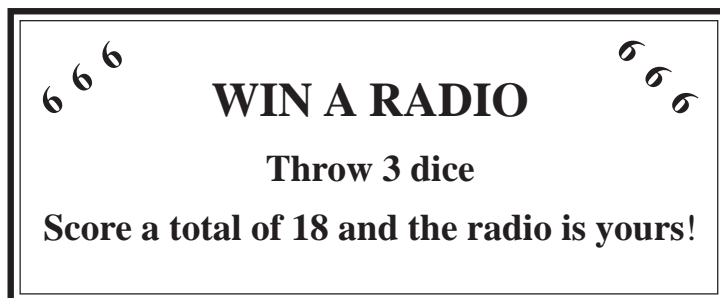
(a) The first person to try their luck was told that they must throw a six with each dice to win. Calculate the probability of this person winning the radio.

(b) During the day 648 people tried to win a radio. How many radios would you expect to be won during the day of the fete?
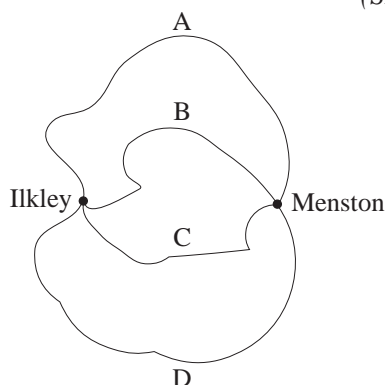
(SEG)

12. Helen lives in Ilkley.

She cycles to work in Menston.

Peter lives in Menston.

He cycles to work in Ilkley.

Ilkley and Menston are connected by four roads, A, B, C and D.

(a) Make a list of all the possible combinations of roads which they can take to go to work.

Write them in pairs with the road Helen takes written down first.

For example, A, C means that Helen goes along road A, and Peter goes along road C.

(b) Each day, Helen chooses the road she takes to go to work at random. So too does Peter. All four roads are equally likely to be chosen.

Calculate the probability that on any given day both of them will go to work on the same road.

(NEAB)

5.8

13.

START➡	1	2	3	
	8	7		5
	9		11	12
	16	15	14	13

'SWEET SIXTEEN' is a game for any number of players. To play the game, players take it in turns to throw a fair die and then move their counter the number of places shown uppermost on the die. If a player lands on one of the shaded squares the player must start again. The first player to *land on a square 16* is the winner. If a player would move past square 16 on a throw, the player is not allowed to move and misses that turn.

(a)   What is the probability that a player lands on a shaded square on the first throw?

(b)   A player moves to square 3 on the first throw. What is the probability that the player lands on a shaded square on the second throw?

(c)   (i)   A player is on square 12 after three turns. Write, in the order thrown, three scores the player could have had.

(ii)   In how many different ways could a player have reached square 12 with three throws? Show working to support your answer.

(d)   (i)   What is the minimum number of turns necessary to complete the game?

(ii)   What is the probability of this happening?

(SEG)

14.   100 tickets are sold in a raffle. There is one prize.

(a)   Dave buys one ticket. What is the probability that he wins the prize?

(b)   Joanna buys five tickets. What is the probability that she wins the prize? Give your answer as a fraction in its simplest form.

(AQA)

15.

Throw five sixes and win a holiday

(a)   What is the probability of throwing 5 sixes with one throw of the 5 ordinary dice?

(b)   The number of dice is now changed so that n dice are thrown.

You win a holiday if all n dice show sixes.

Ian throws the n dice once.

Write down an expresssion for the probability that Ian **does not** win a holiday.

Give your answer in its simplest form.

(OCR)

5.9 Mutually Exclusive Events

1. A man throws a die and a coin. Find the probability that he will get

   (a) number 3 followed by a head   (b) an even number followed by a tail.

2. In an experiment, a card is drawn from a pack of playing cards and a coin is tossed. Find the probability of obtaining

   (a) a card which is a king and a head on the coin,

   (b) the ace of diamonds and a tail on the coin.

3. In an experiment consisting of throwing a die followed by drawing a card from a pack of playing cards, find the probability of obtaining

   (a) an odd number on the die and a card which is an ace,

   (b) a six on the die and a picture card,

   (c) a six on the die and a club.

4. In a certain class, $\frac{1}{3}$ of the pupils read the local newspaper and $\frac{2}{3}$ watch the local news on television. None of these pupils read the local newspaper and also watch the local news on television. What is the probability that a pupil chosen at random reads the local newspaper or watch television?

5. In an inter-school mathematics quiz, the probability of school A winning the competition is $\frac{1}{2}$, the probability of school B winning is $\frac{1}{6}$ and the probability of school C winning is $\frac{1}{10}$.

   Find the probability that

   (a) B or C wins the competition,   (b) A, B or C wins the competition,

   (c) none of these wins the competition.

6. A box contains buttons of various colours. The probability of drawing a red button at random is $\frac{1}{5}$ and the probability of drawing a white button at random in $\frac{2}{7}$.

   What is the probability of drawing neither a red nor a white button?

7. A box contains eight marbles: 1 is red, 2 are blue and 5 are green,

   One marble is drawn at random from the box. A second marble is drawn at random from the remaining seven marbles in the box.

   (a) Find the probability that both marbles are green.

   (b) If the first marble is red, find the probability that the second marble is blue.

8. Nine slips of paper are numbered 1 to 9. A slip is drawn at random. This is replaced before a second slip is drawn. Find the probability that one is an odd number and the other is an even number.