Outcome	NR7	Student can consistently:	Solve algebraic problems involving direct and inverse proportion, including graphical examples.

How the topic is examined	• These are common problems on GCSE exam papers and can be on either calculator or non-calculator papers. • Questions often ask students to find a relationship between two variables that are either directly or inversely proportional. • Sometimes students are given two variables that are connected and some values, other times students can be given a full table of values. • Students can be asked to draw direct proportion and inverse proportion graphs. • It may not use the word directly proportional and just say proportional. If it says proportional then assume this mean to be directly proportional.
Prior knowledge	• Students should be confident with: o Forming formulae and expressions. o Rearranging formulae and solving equations. o Substituting into a formula. • In addition questions involving proportion can have links to: o Square and square roots.
Suggested tuition approaches	• It is important for students to understand what direct and inverse proportion is: o If two variables/quantities are in direct proportion then they increase and decrease at the same rate. E.g. if you travel at 50mph, distance increases as time increases and they will increase at the same rate. o Inverse proportion is where as one quantity increases in proportion and the other decreases. • The first step with any direct or inverse proportion question is to find the relationship between the two variables given. Often this will mean setting up a formula that connects them. • Part of the formula will involve finding k, which is called the constant of proportionality. This is the multiplicative factor that is essential to work out any further values. • If two variables are (directly) proportional then we use the symbol $\propto$ (e.g $X \propto Y$) • If two variables are inversely proportional then we use the same symbol, but it is proportional to the inverse (e.g. $X \propto \frac{1}{Y}$). • The steps involved in solving problems regarding direct proportion: o Write down the connection between the variables using the $\propto$ symbol. o Introduce the constant of proportionality, k (see table below) o Substitute a pair of values that you know into this formula and then solve this equation to find k. o Substitute the value of k back into the formula. o You now have the relationship between the two quantities you have been given. To find other values you need to substitute any additional values you have been given.

- Functions are not always simple linear proportion and the most common direct proportional relations are here:

Worded example	In symbol form	The formulae
X is proportional to Y *(linear proportion)*	$X \propto Y$	$X = kY$
X is proportional to the square of Y	$X \propto Y^2$	$X = kY^2$
X is proportional to square root of Y	$X \propto \sqrt{Y}$	$X = k\sqrt{Y}$
X is proportional to Y cubed	$X \propto Y^3$	$X = kY^3$
X is proportional to the cube root of Y	$X \propto \sqrt[3]{Y}$	$X = k\sqrt[3]{Y}$

- For inverse proportional relationships you would have the same but it would say it is 'inversely proportional' and the functions would all be '1/'function''
- Students may be asked to draw the graphs of these functions. If so encourage students to substitute some values into the formula and plot the points. This should give students an idea of the shape of the graph.

Common errors and misconceptions

- The most frequent mistake that students make is they use the function $X = kY$ regardless of the question. They need to read the question carefully and see if it is any of the above variations.
- Students can struggle with rearranging the formula to find the value of k or the values of any variable.
- A common mistake is that students put k back into the number expression as opposed to the one with the letters in..

Suggested resources

- Questions
  - http://www.cimt.org.uk/projects/mepres/allgcse/pr15-es.pdf
  - http://www.cimt.org.uk/projects/mepres/allgcse/bkc15.pdf (pp 177 onwards)
- Past GCSE questions
  - https://keshgcsemaths.files.wordpress.com/2013/11/96_direct-and-inverse-proportion.pdf
- Video tutorial
  - https://www.youtube.com/watch?v=s4dV4CRgbxc