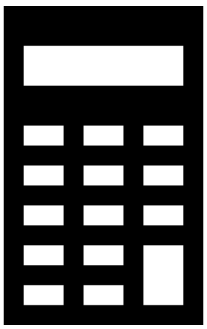
AQA, OCR, Edexcel

GCSE

GCSE Maths

Turning Points of Quadratic Graphs

Name:



Guidance

1. Read each question carefully.
2. Don't spend too long on each question.
3. Attempt every question.
4. Always show your workings.

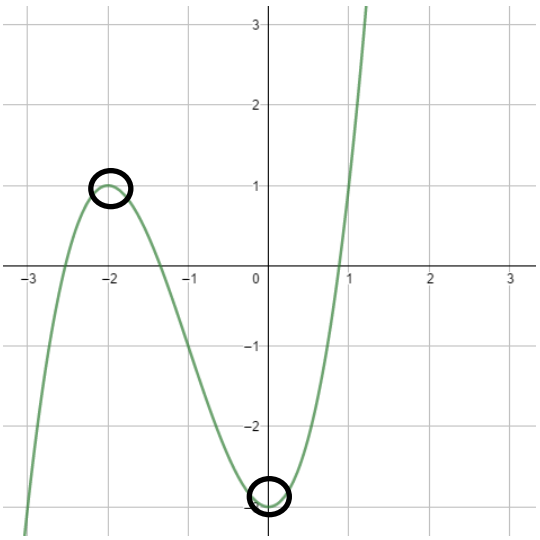
Revise GCSE Maths:
www.MathsMadeEasy.co.uk/gcse-maths-revision/

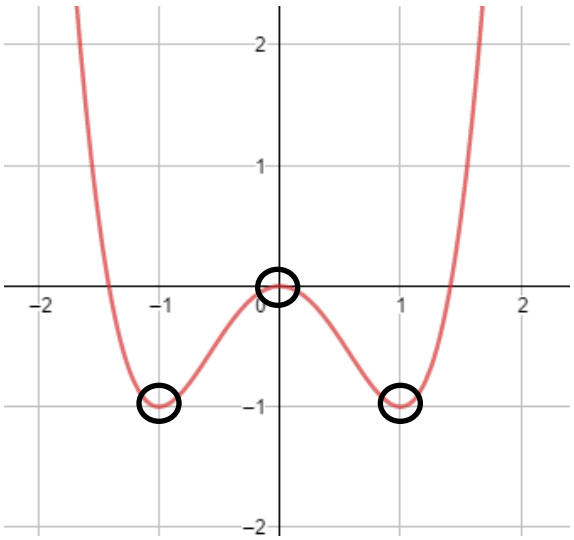
1. Define the turning point of a quadratic graph.

The turning point of a quadratic graph is where the gradient is equal to 0.

(1 mark)

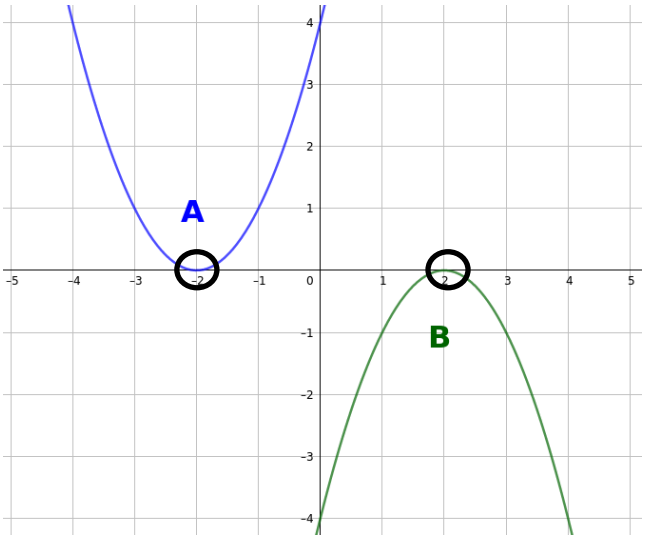
2. Circle the turning points of the two graphs below.





(2 marks)

3. Circle the turning points on the two quadratic graphs below.



Belle looks at graph A and says, “The turning point is always the minimum point of any quadratic graph”
Comment on her statement.

The minimum point of a quadratic graph is the small value it can be. Looking at graph B, we can see that the turning point is the highest point on the graph.

(2 marks)

4. Find the turning point of the following equations by completing the square.

$y = x^2 + 4x + 7$

$$y = (x + 2)^2 + 3$$

$x = \ldots\ldots -2 \ldots\ldots$, $y = \ldots\ldots 3 \ldots\ldots$

$y = 3x^2 + 36x + 99$

$$\begin{aligned} y &= 3(x^2 + 12x + 33) \\ &= 3[(x + 6)^2 - 3] \\ &= 3(x + 6)^2 - 9 \end{aligned}$$

$x = \ldots\ldots -6 \ldots\ldots$, $y = \ldots -9 \ldots\ldots$

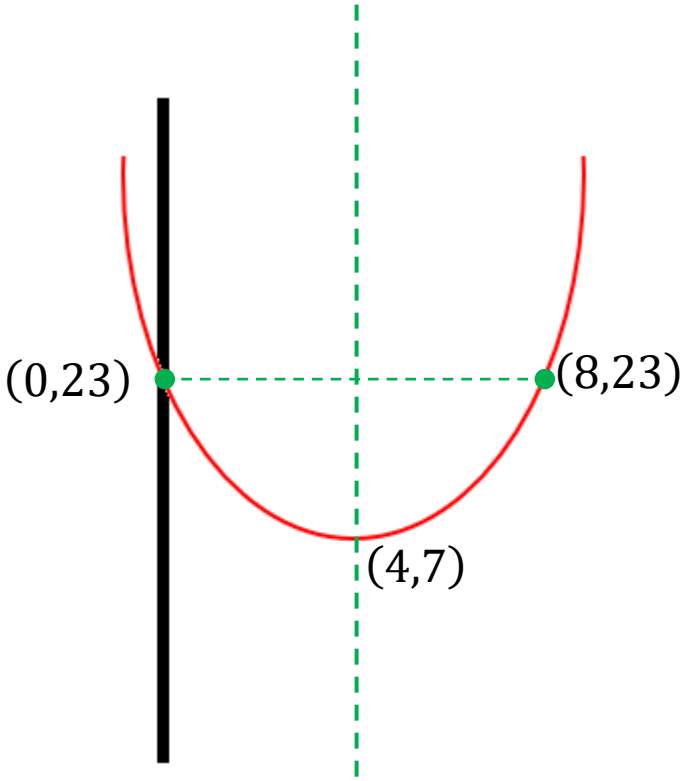
$y = 2x^2 + 7x - 10$

$$\begin{aligned} y &= 2\left(x^2 + \frac{7}{2} - 5\right) \\ &= 2\left[\left(x + \frac{7}{4}\right)^2 - \frac{129}{16}\right] \\ &= 2\left(x + \frac{7}{4}\right)^2 - \frac{129}{8} \end{aligned}$$

$x = \ldots\ldots -\frac{7}{4} \ldots\ldots$, $y = \ldots -\frac{129}{8} \ldots\ldots$

(8 marks)

5. Two points on a quadratic curve $f(x)$ are $(0,23)$ and $(8,23)$.
Given that $f(4) = 7$, what is the minimum point on the curve?
You must give your reasoning.



…… $x =$ ……4…… , $y =$ ……7……

(3 marks)

6. By writing $y = 2x^2 + 4x - 2$ as $y = 2x(x + 2) - 2$ find two symmetrical points on y. Hence find the turning point of the curve.

if $x = 0$

$$y = 2x(x + 2) - 2$$
$$y = 2 \times 0 \times (0 + 2) - 2$$
$$y = -2$$

$$(0, -2)$$

$$y = -2$$
$$-2 = 2x(x + 2) - 2$$
$$2x(x + 2) = 0$$
$$x = -2$$

Symmetrical points are:

$$(-2, -2) \; and \; (0, -2)$$

Minimum and line of symmetry:

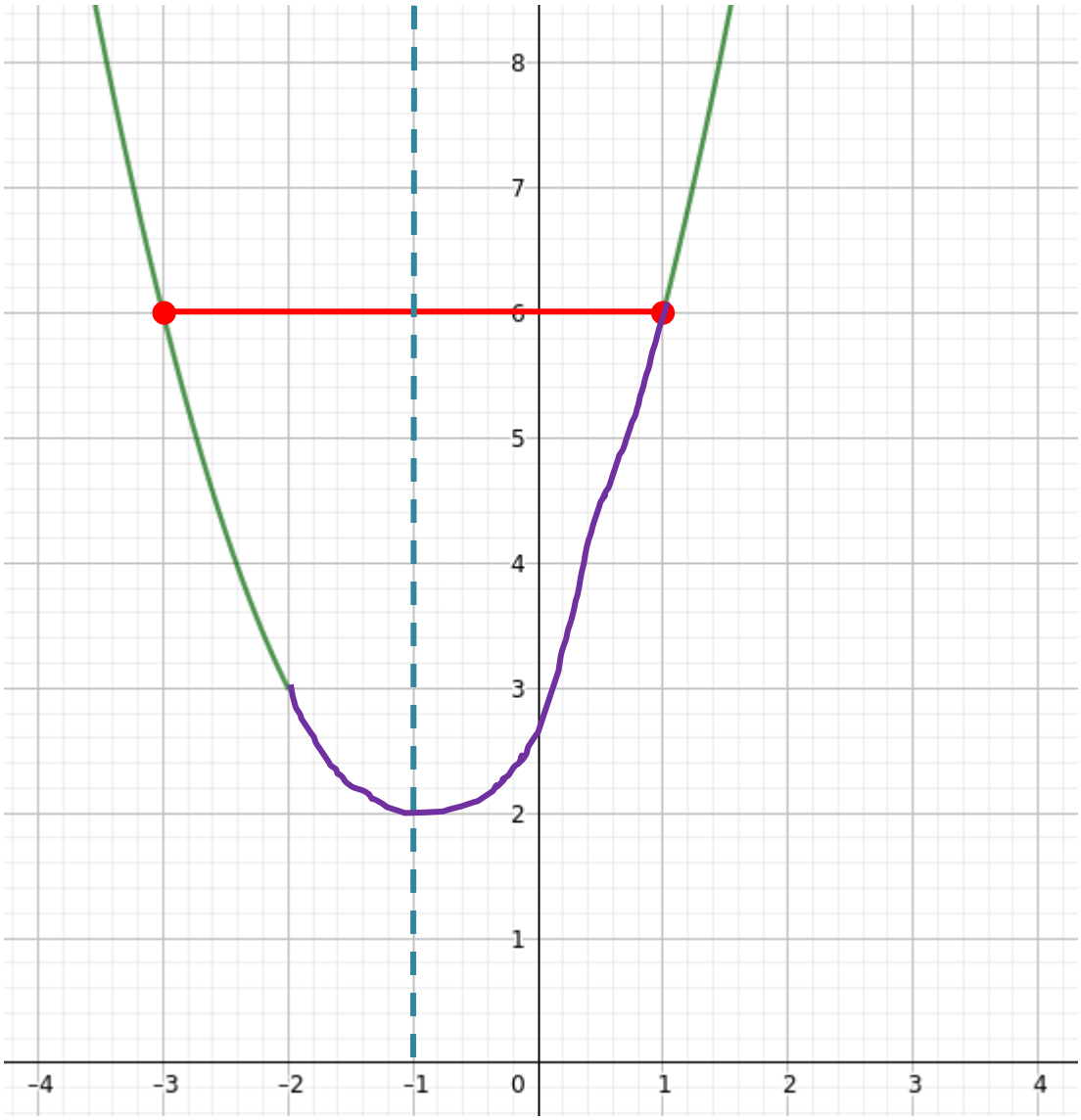
$$x = -1$$

$$y = 2 \times -1 \times (-1 + 2) - 2$$
$$y = -2 \times 1 - 2$$
$$y = -4$$

…… $x =$ ……-1…… , $y =$ …-4…

(1)

7. The graph below shows a quadratic function with the region between $-2 \leq x < 1$ missing.
   Find the line of symmetry of the quadratic and use this to plot the rest of the curve.



What are the coordinates of the turning point of the curve?

$x =$ ……-1…… , $y =$ ……2 ± 0.25……

(3 marks)

8. Given that:

$$f(x) = x - 4$$
$$g(x) = x^2$$

Find $fg(x)$ and $gf(x)$.

$$fg(x) = x^2 - 4$$

$$gf(x) = (x-4)^2$$

Find the turning point of each curve and comment on them with relation to $f(x)$.

$fg(x)$: ……

Graph of x^2 has turning point $(0,0)$

$fg(x) = x^2 - 4$ is the graph of x^2 but moved down 4, so that is how the turning point has moved too. The x stays the same, but the y goes down by 4.

$$y = 0 - 4 = -4$$

$x = \ldots\ldots 0 \ldots\ldots$, $y = \ldots -4 \ldots\ldots$

$gf(x)$: …….

Graph of x^2 has turning point $(0,0)$

$gf(x) = (x-4)^2$ is the graph of x^2 but moved right 4, so that is how the turning point has moved too. The x increased by 4, but the y stays the same.

$$x = 0 + 4 = 4$$

…… $x = \ldots\ldots 4 \ldots\ldots$, $y = \ldots\ldots 0 \ldots\ldots$

Comment:

The turning point of $fg(x)$ is where $f(x)$ intercepts the y-axis.
The turning point of $gf(x)$ is where $f(x)$ intercepts the x-axis.

(5 marks)