

Outcome	AEx8	Student can consistently:	Change the subject of a formula
How the topic is examined	<ul style="list-style-type: none"> <li>• Examined through test paper questions.</li> <li>• Questions are equally likely to appear on calculator or non-calculator papers.</li> <li>• Questions will ask students to 'Make .... the subject' or 'Rearrange the formula to make ... the subject'</li> <li>• There are two levels of problems, the first is where the required subject appears a single time in the expression and the second type is where the subject appears more than once.</li> <li>• Questions that require rearranging formulae can be hidden within other problems and it may be necessary to rearrange a formula without being explicitly told.</li> </ul>		
Prior knowledge	<ul style="list-style-type: none"> <li>• Students should be confident with: <ul style="list-style-type: none"> <li>◦ Simplifying expressions (AEx2)</li> <li>◦ Factorising simple expressions (AEx4)</li> <li>◦ Solving equations.</li> </ul> </li> <li>• In addition questions involving this topic can have links to: <ul style="list-style-type: none"> <li>◦ Substituting into formulae.</li> </ul> </li> </ul>		
Suggested tuition approaches	<ul style="list-style-type: none"> <li>• Students should be made aware of what the subject of a formula is. It is the letter that the formula equals. (e.g. <math>y = mx + c</math>. In this example <math>y</math> is the subject).</li> <li>• Students should understand that when we change the subject of the formula we are simply rearranging the formula so that it starts with the required letter.</li> <li>• There are two levels of problems, the first is where the required subject appears a single time in the expression and the second type is where the subject appears more than once.</li> </ul> <p>#</p>		

**1) Required subject appears only once.**

- There are two main approaches that teachers tend to use in the classroom to explain this.

Using the inverse	Balancing
<p>Rearrange <math>y = mx + c</math> to make <math>x</math> the subject</p> <p>Think about how you build this formula up</p> <ul style="list-style-type: none"> <li>• Start with <math>x</math></li> <li>• Multiply by <math>m</math></li> <li>• Then add <math>c</math></li> <li>• and you get <math>y</math></li> </ul> <p>If we go backwards and do the inverse of this.</p> <ul style="list-style-type: none"> <li>• Start with <math>y</math></li> <li>• Subtract <math>c</math></li> <li>• Now divide by <math>m</math></li> <li>• You get <math>x</math></li> </ul> <p>As a function this is</p> $\frac{y - c}{m} = x$ <p>Sometimes this might be presented as a flow diagram.</p>	<p>Rearrange <math>y = mx + c</math> to make <math>x</math> the subject</p> <p>Subtract <math>c</math> from both sides</p> $y - c = mx$ <p>Divide both sides by <math>m</math></p> $\frac{y - c}{m} = x$

- Try and cover examples that include
  - Squares (e.g.  $y = x^2 + k$ , make  $x$  the subject)
  - Divisions (fractions) (e.g.  $m = \frac{x}{2} + 5$ , make  $x$  the subject)
  - Square roots (e.g.  $t = \sqrt{x + u}$ , – make  $x$  the subject)

## 2) Required subject appears more than once.

- These problems generally are quite easy to solve and there is a standard method for solving them. This method should help rearrange the majority of formulae.

Make  $x$  the subject of the following

$$y = \frac{5x + a}{px - 9}$$

<b><u>Step 1</u></b> Multiply and then expand – if necessary	$y(px - 9) = 5x + a$ $ypx - 9y = 5x + a$
<b><u>Step 2:</u></b> Rearrange so that all terms with required subject appear on left hand side of equation and all terms without appear on the right hand side of the equation.	$ypx - 5x = 9y + a$
<b><u>Step 3:</u></b> Factorise out the required subject from the left hand side.	$x(yp - 5) = 9y + a$
<b><u>Step 4:</u></b> Divide through by the bracket on the left hand side.	$x = \frac{9y + a}{yp - 5}$

- Sometimes the question may ask you to find the inverse function or  $f^{-1}(x)$ , this is the same as the above methods. The only difference is that the function will be called  $f(x)$  and you should rename it  $y$  before changing the subject.

## Common errors and misconceptions

- The errors with these questions tend to arrive when students are not confident with solving algebraic equations and simplifying expressions.
- Students need to be confident with these topics before tackling rearrangement questions.
- With the formulae where the subject appears once, students try to remember how to solve each different type of equation instead of thinking about following algebraic steps to change the subject. Try to avoid them remembering things by rote. They should tackle each formula separately.
- Where the  $x$  appears on both sides, students forget to factorise and they end up with the subject on both sides of the equation.
- Students should always show all steps in their working and double check their answer.

## Suggested resources

- Questions
  - <http://www.cimt.org.uk/projects/mepres/allgcse/pr2-es.pdf> (pp 14 - 17)
  - <https://corbettmaths.files.wordpress.com/2013/02/changing-the-subject-exercise-7-pdf.pdf>
  - <https://corbettmaths.files.wordpress.com/2013/02/changing-the-subject-exercise-8-pdf.pdf>
- Past GCSE Questions
  - [https://keshgcsemaths.files.wordpress.com/2013/11/100\\_more-difficult-rearranging-formulae.pdf](https://keshgcsemaths.files.wordpress.com/2013/11/100_more-difficult-rearranging-formulae.pdf)
  - [https://keshgcsemaths.files.wordpress.com/2013/11/60\\_algebra\\_changing-the-subject.pdf](https://keshgcsemaths.files.wordpress.com/2013/11/60_algebra_changing-the-subject.pdf)
- Video tutorial
  - <http://corbettmaths.com/2013/12/23/changing-the-subject-video-7/>
  - <http://corbettmaths.com/2013/12/28/changing-the-subject-advanced-video-8/>