Guidance for tutors

Outcome	AG9	**Student can consistently:**	Estimate the gradient of a curve and area underneath a curve.

How the topic is examined	• Examined through test paper questions.
	• Questions are most likely to appear on a calculator paper due to the complexity of the calculations.
	• This is a new topic to GCSE and so it is difficult to predict exactly what questions they could ask. The specification requires students to be able to
	  ○ Estimate the gradient of a curve at a particular point
	  ○ Estimate the area under a curve between two points.
	• It is also expected that students should be able to find the gradient and area under the curve on a velocity-time graph to find the acceleration and distance travelled.

Prior knowledge	• Students should be confident with:
	  ○ Finding the gradient of a line (AG4)
	  ○ Area of basic shapes (including trapeziums)

Suggested tuition approaches	• This section covers estimating the gradient at a particular point, area under a curve and then how these rules relate specifically to velocity time graphs.
	
	1) Estimating gradient of a curve
	
	• Students should be confident in finding the gradient of a straight line. Explain to students that it is not as easy to find the gradient of a curve as it changes for every point. Therefore we can only estimate the gradient.
	• Students should know that the rate of change at a particular instance is represented by the gradient of the tangent to the curve at that point.
	• To estimate the gradient at a particular point:
	  ○ Draw a tangent to the curve at the point given. A tangent is a straight line that just touches the curve at a particular point.
	  ○ Find the gradient of tangent (See AG4 for the steps).
	  ○ The following page provides a worked example http://www.mathsrevision.net/gcse-maths-revision/algebra/gradients-and-graphs or see the video tutorial link.

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2) Estimating the area under the curve

- Students should use trapeziums, triangles and/or rectangles to estimate the area under a curve.
- Students are not expected to know or use the trapezium rule, however it is recommended as the most efficient method for calculating the area under the curve.
- To estimate the area under a curve using the trapezium rule between two points:
  - Divide the distance between the two points up into a suitable number of strips (usually 3, 4 or 5). The strips should have equal width.
  - Draw vertical lines up to curve.
  - Connect the ends of the lines to form a series of trapeziums
  - The use the trapezium rule to find the area. The rule is given by:

$$\overline{1} = {}_2 \times \quad h \times ({}_0 + + 2({}_1 + {}_2 + \ldots \ldots \ldots + {}_{-1}))$$

Where $_0$, $_1$, $_2$ etc… are heights of the vertical lines (read off the values for this)

  - An alternative to using the trapezium rule is simply to find the area of each of the trapeziums using the formula $\frac{1}{2} \times h \times (a + b)$ (See SLAV1)

- It is not necessary to use the trapezium rule. You can split up into triangles, rectangles and trapeziums. Label each shape a letter and work out each area, put your results in a table. Then add up all the areas to get an estimate of the area.

3) Velocity-time graphs

- Students need to know how these two rules specifically relate to velocity time graphs. Specifically:
  - They should understand the difference between positive and negative gradients as increasing speed and decreasing speed on a distance-time graph.
  - Students should know that the area under a velocity-time graph represents distance.
  - Students should know that if the vertical axis represents distance on a distance-time graph, then the gradient will represent velocity.

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	○ Students should know that if the vertical axis represents velocity on a velocity-time graph, then the gradient will represent acceleration.
Common errors and misconceptions	Students draw an incorrect tangent to a curve. They should use a sharp pencil and use a ruler so that the pencil line just touches the curve at the given point. A lot of tangents end up touching at more than one point and/or going through the curve. Students make errors when finding the gradient: ○ They use the number of squares instead of the actual distances for the distance. ○ They divide x by y instead of y by x ○ They work out the value of one unit incorrectly on each of the axes (this will affect their and values) For the area under the curve, students try to use the trapezium rule but don't use trapeziums of equal width. There is great room for error in the calculations for this, it is important that students always show clear working. They could label each area a particular letter and then make a table of areas. Students should then add up the areas.
Suggested resources	Questions ○ https://www.tes.com/teaching-resource/gradients-on-a-curved-graph-6421576 Video tutorial ○ https://www.youtube.com/watch?v=PadfDurqErw (estimating gradient) ○ https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/riemann-sums/v/trapezoidal-approximation-of-area-under-curve (Trapezium rule)