Guidance for tutors

Outcome	AG3	**Student can consistently:**	Find the n^{th} term of linear sequences and related problems.

How the topic is examined

- Examined through test paper questions.
- Questions are equally likely to appear on calculator or non-calculator papers.
- There are several types of questions that could be asked involving sequences
  - Find an expression for the position to term rule of a sequence (n^{th} term)
  - Given the n^{th} term find the value of particular terms in the sequence
  - Find which term takes a particular value
  - Find terms of a sequence given a term to term rule
- Sequences may be given in the form of word, diagrammatic or real life problem, with students having to determine the sequence.

Prior knowledge

- Students should be confident with:
  - Substitution (AEx5)
  - Simplifying expressions (AEx2)
  - Solving basic equations (AEq1)
- In addition questions involving this topic can have links to:
  - Drawing linear graphs (AG1)

Suggested tuition approaches

- This section focusses on linear sequences (ones that go up or down by the same amount each time). This is sometimes referred more formally as arithmetic sequences or progressions.
- A sequence is usually described in two different forms.
  - Position to term. This is where you know a particular term number and you can find its value. It usually is given in the form $(5n + 1)$
  - Term to term. This is where you can only work out the next term if you know the one before (e.g. add 3). Sometimes these are presented using more iterative notation (e.g. $u_{n+1} = u_n - 5$). This examples says that the 'next term' is equal to the 'previous term' minus 5
- It is possible for linear sequences to move quite easily between these term to term and position to term rules.
- Students learn lots of tricks with this question. It is best though teaching this for understanding.

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Position to term sequences

Find an expression for the nth term	Find a particular term	Which term has the value ….
Find an expression for the nth term of the following sequence. 5, 9, 13, 17, 21, …. There are different methods for doing this. Here is a common approach 1. Work out what the sequence goes up in. Here it is 4 2. Now write down the four times table above the numbers in this sequence. Ask how you get from the 4 times table to the numbers in the sequence. Here you add 1 3. The 4 times table is generated by the expression 4 ; therefore this sequence is generated by the expression 4 + 1	Find the 10th term of the sequence given by the expression 5 − 4 The 10th term of the sequence can be worked out by substituting the value 10 into the expression above 10th term = 5 x 10 – 4 = 46 Another common question is asking for the first three terms of a sequence. If this is asked for, we substitute the numbers, 1, 2 and 3 in as n 1st term = 5 x 1 – 4= 1 2nd term = 5 x 2 – 4 = 6 3rd term = 5 x 3 – 4 = 11 Some students may just find the first term and then realise from the number in front of n in the expression that the sequence goes up in 5's.	Which term has the value 65 in the sequence that is given by the expression 4 + 5 Put the expression equal to 65 and solve for n. 4 + 5 = 65 4 = 60 = 15 So 65 is the 15th term. Sometimes students are asked to find if a number is in a sequence. If you do the same as above and then if the answer is a whole number then it must be in the sequence. If the answer is not whole, then it cannot be in the sequence.

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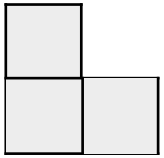
- If you have a sequence in the form of a fraction likes these $\frac{1}{2}, \frac{3}{8}, \frac{5}{14}$ … students should think of this as two separate linear sequences. The numerator sequence (1, 3, 5, …. $2n-1$) and the denominator sequence (2, 8, 14, … $6n-4$). The answer is then $\frac{2n-1}{6n-4}$

Term to term sequences

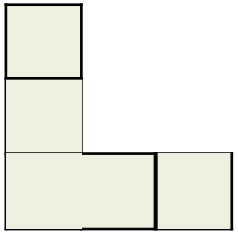
- These are usually given using iterative notation. $u_{n+1} = u_n - 5$
- The above sequence means that the next term u_{n+1} is equal to the previous term minus 5. This is the notation that will be used.
- You will need to be given a starting value. For example you might be told that the 1st term $u_1 = 7$. The next term $u_2 = 7 - 5 = 2$ etc…
- Harder questions might give students the 5th term and they have to work out earlier terms.
- Also students might be asked to find an expression for the nth term for the above sequence. If so you should encourage students to write out the terms of the sequence and then follow the method in the table above.

- Sequences may be given in the form of diagrams or word problems.

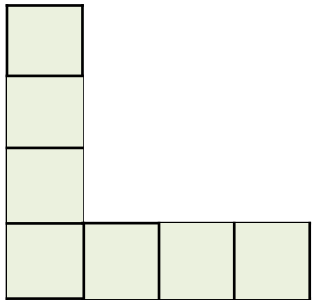
(e.g. how many squares make up pattern 40?)



Pattern 1



Pattern 2



Pattern 3

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Common errors and misconceptions	• Students learn lots of tricks with this question. It is best though teaching this for understanding. A common trick is that students work out what it goes up in and they put this in front of n and then work out the term before the 1st term. Students often get these mixed up and the wrong way around. • A common mistake is students just put n + 4 for a sequence that goes up in 4's. Again if you teach for understanding this should help overcome this. You can link a sequence that goes up in 4's to the four times table, hence why it contains 4n (or 4 x n). • Students generally struggle to remember how to work out specific terms of the sequences. Working through plenty of examples should help this.

Suggested resources	• Questions
	  ○ http://www.cimt.org.uk/projects/mepres/allgcse/bkb12.pdf (pp 273 - 281)
	  ○ https://corbettmaths.files.wordpress.com/2013/02/nth-term-exercise-288-289-pdf.pdf
	  ○ https://corbettmaths.files.wordpress.com/2013/02/nth-term-pdf.pdf
	• Video tutorials
	  ○ http://corbettmaths.com/2012/08/20/the-nth-term-for-linear-sequences/
	  ○ http://corbettmaths.com/2012/08/20/the-nth-term-for-fractional-sequences/
	• Past GCSE Questions
	  ○ https://keshgcsemaths.files.wordpress.com/2013/11/65_sequences.pdf