Guidance for tutors

Outcome	AG12	**Student can consistently:**	Find and use quadratic and geometric sequences.

How the topic is examined

- Examined through test paper questions.
- Questions are most likely to appear on a calculator paper due to the complexity of the calculations.
- This is a new topic to GCSE and so it is difficult to predict exactly what questions they could ask. The specification requires students to be aware of quadratic sequences and geometric sequences.

Prior knowledge

- Students should be confident with:
  - Substituting into expressions (AEx5)
  - Sequences (AG3)
  - Simultaneous equations (AEq8)

Suggested tuition approaches

- Quadratic sequences are sequences that take the form $^2 + +$
- Students should be encouraged to explore quadratic sequences,
  - Start with the sequence of square numbers 1, 4, 9, 16, 25 …. They should understand that the terms are generated by the expression ^{2}.
  - Students find the difference between successive terms and then find the difference of the differences (called the second difference). They should see that the second difference is 2.
  - Now ask them to find the terms of the following quadratic sequences (e.g. $^2 + 5$, $2\ ^2$, $3\ ^2$, $^2 + 3 - 1$)
  - Students should notice that the second difference is twice as much as the coefficient of ^{2}
- Some quadratic sequences are easy to spot, for example 2, 5, 10, 17, 26 is one more than ^{2}. Other ones need a more formal method. To find the nth term of a quadratic sequence here are steps involved:

Step	Notes
Find the nth term of the following sequence 5, 9, 17, 29, 45, …..	Because the sequence does not go up by the same amount each time, it is not linear. We notice however that the difference increases by 4 each time. So the second difference is 4. So it is quadratic. 5 9 17 26 45 Sequence

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	4 8 12 16 — 1st difference 4 4 4 — 2nd difference
A quadratic sequence must be of the form $^2 + \ + $, the task is to find the values of a, b and c. Find the second difference and then divide by 2. This will give you the value of a.	The second difference is 4, we need to divide by 2, so 4 $=\overline{2} \ = 2$ So we currently have $2\ ^2 + \ +$
To find the values of b and c we now substitute two terms into the equation. We know the 1st term is 5, so we know that when n = 1 the value of the sequence is 5 We know the 2nd term is 9, so we know that when n = 2 the value of the sequence is 9	Substitute $= 1$ into $2\ ^2 + \ +$ and then answer should be 5. We get the following equation $2 + \ + \ = 5$ So $+ \ = 3$ Substitute $= 2$ into $2\ ^2 + \ +$ and then answer should be 9. We get the following equation. $8 + 2 \ + \ = 9$ So $2 \ + \ = 1$
You should now have two equations. Solve these equations simultaneously. Once a students has got an answer, always encourage them to check their formula by putting $= 3$ into the equation and seeing if the answer is correct.	Solving the above two equations, it is clear that $= -2$ and $= 5$ So the expression for the nth term is $2\ ^2 - 2 \ + 5$ You can check this by substituting $= 3$ into the equation and so you should get 17, which is the next term of the sequence

- Geometric sequences are ones where you multiply by the same number to move from term to term. For example.2, 6, 18, 54, 162, …. In this sequence we multiply by 3 each time.

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	• Students may need
	  ○ To find a particular term in a sequence
	  ○ Find the common ratio (i.e. that is the number you multiply by each time)
	• The highest attaining students might want to look at taking a more algebraic approach to geometric sequences. The following page proves some more detailed notes and examples http://www.mathsisfun.com/algebra/sequences-sums-geometric.html

Common errors and misconceptions	• Students try to use trial and improvement with quadratic sequences. This is an acceptable approach, although it can be time consuming for complicated sequences. They should try to take the more algebraic approach described above. • Substituting into quadratic terms can lead to a few mistakes. E.g. find 5 ^{2} when = 2. A common wrong answer is 100 as students multiply by 5 first and then square their answer. Instead it is only the that is squared, so students should square 2 and then multiply by 5. • Students make mistakes when solving the simultaneous equations. For example they add instead of subtracting (See AEq8 for full list of errors students might make with simultaneous equations)

Suggested resources	• Questions and notes
	  ○ http://www.cimt.org.uk/projects/mepres/allgcse/pr12-es.pdf
	  ○ http://www.cimt.org.uk/projects/mepres/book9/bk9i10/bk9_10i3.html
	  ○ https://www.tes.com/teaching-resource/quadratic-sequences-worksheet-6042191
	• Video and online tutorials
	  ○ https://www.youtube.com/watch?v=FfCq7bGAFoY