Supplemental Material (1): Density Distribution Estimation

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Abstract

It is the supplemental material of paper “A Hough Voting based 2-Point RANSAC Solution to the Perspective-n-Point Problem”. Detailed information of the density distribution estimation is provided in this document.

Possible options to estimate joint density distribution of θ and ω_{2p}

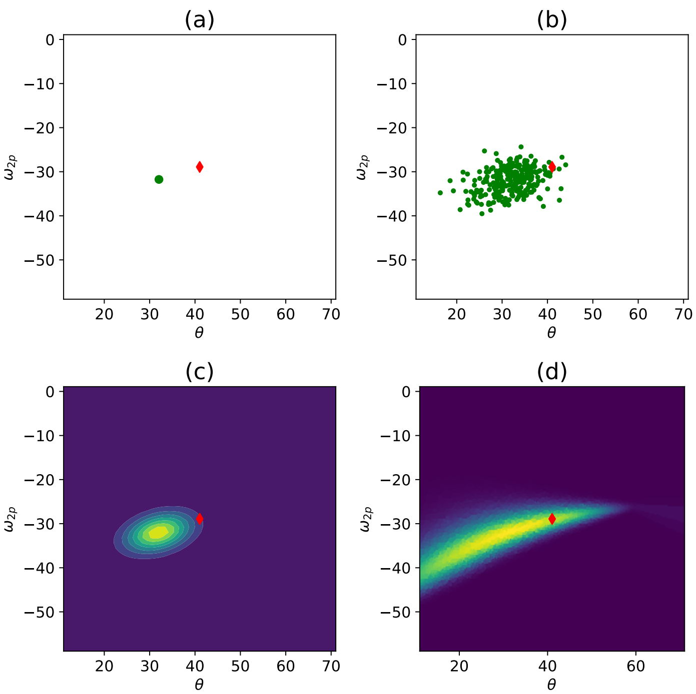

Figure 1: Options to express the joint density distributions of θ and ω_{2p}. (a) Dirac delta function, (b) multiple random sampling, (c) approximate Gaussian distribution, and (d) discrete 2D heatmap. The red denotes ground truth, and the greens denote estimation.

Let $Y^{(i)} = [\theta^{(i)}, \omega_{2p}^{(i)}]^T \in R^2$ denote the concatenation of $\theta^{(i)}$ and $\omega_{2p}^{(i)}$, and $Y^{(i)} = F(p_i)$ denote a function maps from the 2D projection p_i to $Y^{(i)}$. Many methods can be used to estimate the density distribution of $Y^{(i)}$, and we will discuss four primary options as follows (Note that, the 4th option is what we implemented in our solution).

1. it is possible to explicitly solve $Y^{(i)}$ from $PST^{(i)}$ by solving the forth order polynomial $f^{(i)}(t_1) = 0$, and then to represent the density distribution of $Y^{(i)}$ using Dirac delta function (as can be seen in Figure 1(a)). However, this option is not stable, because the observation noise of p_i may lead to big disturbance on $Y^{(i)}$.

2. it is possible to improve the robustness against noise by increasing the number of samples of p_i (as can be seen in Figure 1(b)). Assuming p_i subjects to a standard normal distribution $\mathcal{N}(\mu_{p_i}, \Sigma_{p_i})$, we repeatedly pick M samples $\{^m p_i\}_{m=1\cdots M}$ and compute the corresponding $\{^m Y^{(i)}\}$ using the PST constraint. However, this option requires huge computational cost and can hardly be applied in practice.

3. it is possible to simplify $Y^{(i)} = F(p_i)$ as a linear approximation $Y^{(i)} = F(\mu_{p_i}) + J\Delta p_i$, where $J \in R^{2\times 2}$ is the Jacobean matrix of F. As can be seen in Figure 1(c), the density distribution of $Y^{(i)}$ can be expressed as an approximate Gaussian distribution

$$p(Y^{(i)}) = \frac{1}{2\pi|\Sigma|^{-1/2}} \exp\left(-\frac{1}{2}\Delta Y^T \Sigma^{-1} \Delta Y\right)$$

   where $\Sigma = J^T J$, and $\Delta Y = Y^{(i)} - \mu_Y^{(i)}$. However, this option requires to compute the Jacobean matrix of F, which is also time consuming.

4. in order to ensure both efficiency and accuracy, in this work, we approximate the distribution of $Y^{(i)}$ using 2D discrete heatmap (as can be seen in Figure 1(d)). It is the option used in the proposed method. Details of this option have been provided in section 3.3 of the paper.

Heatmaps

We approximate the distribution of $Y^{(i)}$ to 2D discrete heatmap, and estimate the distribution using Hough voting [1]. We evenly divide $(-\pi/2, \pi/2) \times (-\pi, \pi)$ the range of $Y^{(i)}$ into $N_\theta \times N_\omega$ grid points, and each grid point corresponds to a bin. The spans of each bin are $s_\theta = \pi/N_\theta$ and $s_\omega = 2\pi/N_\omega$. Examples of 2D discrete heatmap are shown in Fig. 2. We can see that, in ordinary 3D case, normally there exists only one peak; while in near degenerate cases, there exist multiple local maxima.

References

[1] D. H. Ballard, “Generalizing the hough transform to detect arbitrary shapes,” *Pattern Recognition*, vol. 13, no. 2, pp. 111–122, 1981.

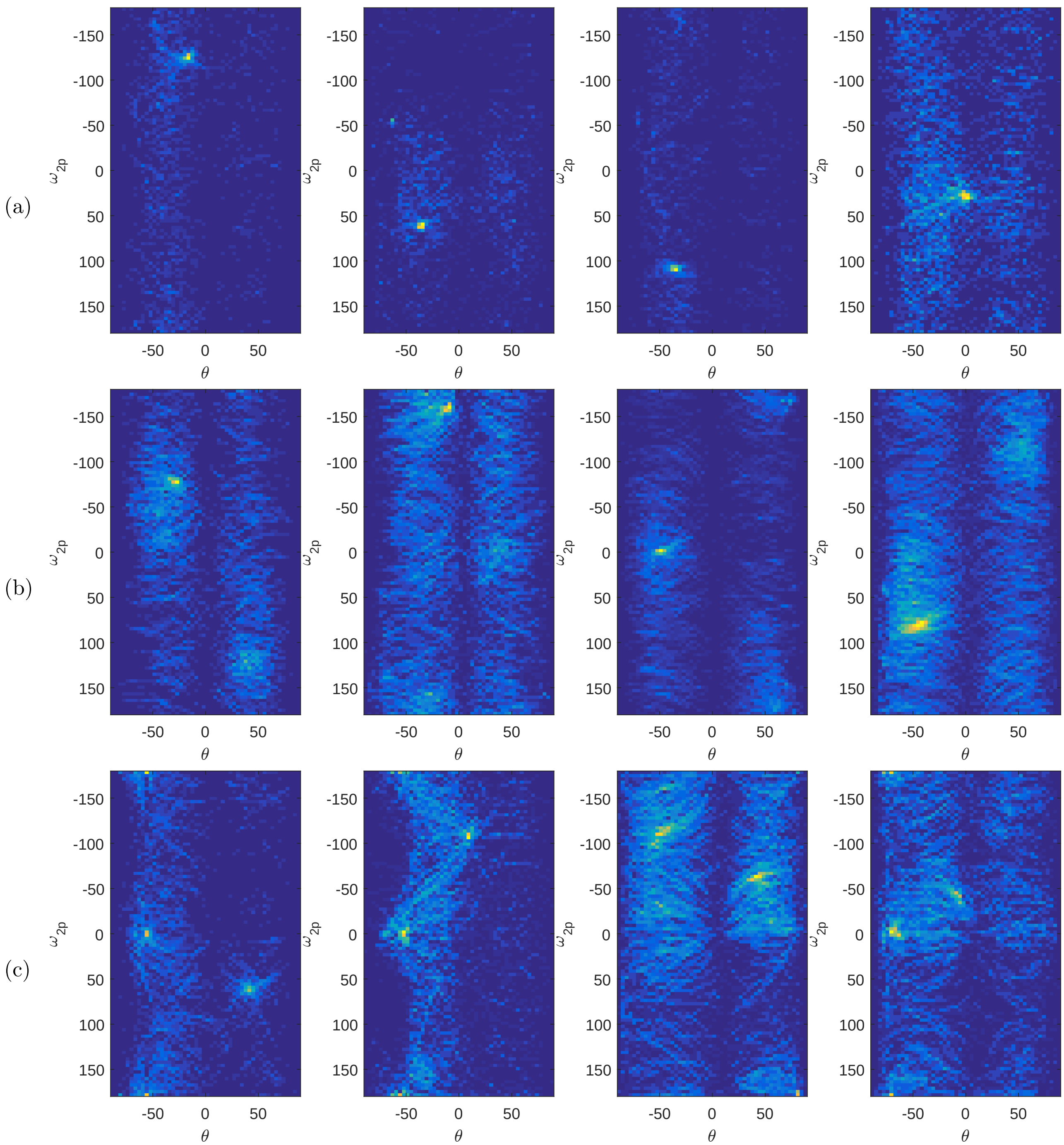

Figure 2: Examples of 2D heatmap in (a) ordinary 3D case, (b) quasi-singular case, and (c) planar case. The outlier rate is 95%.