Fin500 HW2 Junru Wang

Question1: FRA replication

a) $F_0(0.25, 0.5) = \left(\frac{P(0.25)}{P(0.5)} - 1\right) \times \frac{1}{0.5-0.25} = \left(\frac{0.9870}{0.9742} - 1\right) \times 4 = 5.256\%$

$F_0(0.5, 0.75) = \left(\frac{P(0.5)}{P(0.75)} - 1\right) \times \frac{1}{0.75-0.5} = \left(\frac{0.9742}{0.9625} - 1\right) \times 4 = 4.862\%$

$F_0(0.75, 1) = \left(\frac{P(0.75)}{P(1)} - 1\right) \times \frac{1}{1-0.75} = \left(\frac{0.9624}{0.9524} - 1\right) \times 4 = 4.200\%$

$$F_0(0.5, 1) = \left(\frac{P(0.5)}{P(1)} - 1\right) \times \frac{1}{1-0.5} = (F_0(0.5, 0.75) \times 0.25 + 1)(F(0.75, 1) \times 0.25 + 1) - 1$$

$$= [(0.04862 \times 0.25 + 1) \times (0.04200 \times 0.25 + 1) - 1] \times 2$$

$$\cancel{= 2.667\% \times 2} = 4.582\%$$

b) We can use a combination of longing 0.25 maturity bonds and Shorting 0.5 maturity bonds to replicate the FRA. ~~(C = $1,000,000)~~

Then the cash flow: $Pay = C \times [1 + r_{0.5} \times (t_2 - t_0)] = C(1 + r_{0.25} t_1)[1 + F_0(0.25, 0.5)(t_2 - t_1)$

(at year 0.5) $Return = C \times [1 + r_{0.25} \cdot t_1] \cdot [1 + r_{0.25}(0.25, 0.5)(t_2 - t_1)]$,

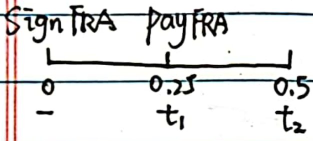
Cash Flow (CF) $= (Return - Pay) / [1 + r_{0.25}(0.25, 0.5)(t_2 - t_1)]$

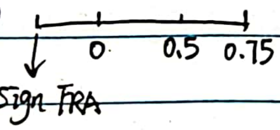
(at year 0.25)

$$= \frac{C(1 + r_{0.25} t_1)\left[r_{0.25}(0.25, 0.5) - F_0(0.25, 0.5)\right](t_2 - t_1)}{1 + r_{0.25}(0.25, -0.5) \times (t_2 - t_1)}$$

and $K = F_0(0.25, 0.5)$, $C(1 + r_{0.25} t_1) = 1,000,000 = NP$

at year 0. Value of FRA is 0.



c) 

$$F_0(0.5, 0.75) = \left(\frac{P(0.5)}{P(0.75)} - 1\right) / (0.75 - 0.5) = 4.862\%$$

$\cancel{= 0.9742}$

$\times 0.9625$

$$V_{FRA} = \frac{NP\left[F_0(0.5, 0.75) - K\right](t_2 - t_1)}{1 + \cancel{F_0(0.5, 0.75)(t_2 - t_1)}} \quad r(0, 0.75) = \frac{1,000,000 \times (4.862\% - 5\%) \times 0.25}{\cancel{1 + 4.862\% \times 0.75}}$$

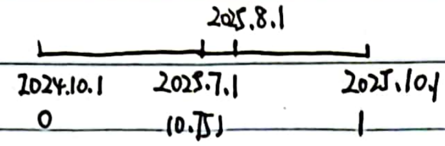
$$= NP(F_0(0.5, 0.75) - K)(t_2 - t_1) \cdot P(0.75) = -303.66$$

d) $K = F_0(0.75, 1)$, the current value of FRA is 0.

Question 2 (a) $P(0, 1) = 1/(1 + r(0, 360)) = \frac{1}{\cancel{1 + 0.0520}} = 0.95057$

(b) $F_0(0.75, 1) = \left(\frac{P(0.75)}{P(1)} - 1\right) = \frac{\left(\frac{1}{1 + r(0, 0.75)}\right)}{\left(\frac{1}{1 + r(0, 1)\cancel{\times 0.75}}\right)} - 1 = \left(\frac{\cancel{1.0520}}{\cancel{1.039375}} - 1\right) = \cancel{1.2160\%}$ 4.860%

$\frac{1}{1 - 0.75}$

c) 

If the market rate rised up to 5.75%, We pay for the ~~floating~~ fixed rate and receive the ~~floating~~ fixed rate, and FRA would pay me the hedge 5.75% − 5% = 0.75%, so you only need to pay this loan at a fixed rate 5%.

d) $P_{FRA} = \dfrac{NP \times [F_0(0.75,1) - K] \times (0.25)}{1 + r(0, \cancel{0.75}\ 1)} = NP \times [F_0(0.75,1) - K] \times 0.25 \times P(0,1)$

$= 50{,}000{,}000 \times (\overset{0.0486}{\cancel{1.2160}}\% - 5.00\%) \times 0.25 \times 0.95057$

$= \cancel{-428{,}396.1}\ -16635.5$

e) The expectation from the market is $F_0(0.75,1) = 1.2160\% > 5\%$ So most likely the FRA would urge a negative payment at time $T + \tau = 1$.

Question 3: a)

Maturity Date	T in years	Zero coupon Bond Price	Forward Rate
4/1/25	0.5	0.97442	5.25%
10/1/25	1.0	(0.94831)	5.50%
4/1/26	1.5	0.92341	5.458% (~~2.73%~~)
10/1/26	2.0	0.90045	5.10%

$P(0,1) = \dfrac{P(0,0.5)}{1 + F_0(0.5,1) \times 0.5} = \dfrac{0.97442}{1 + 0.055 \times 0.5} = 0.9483$

$F_0(1,1.5) = \left(\dfrac{P(0,1)}{P(0,1.5)} - 1\right) \times 2 = \left(\dfrac{0.94861}{0.92341} - 1\right) \times 2 = 2.729\% \times 2 = 5.458\%$

b) For the par swap, $0.5\, P(0,T_n) + K \cdot \overset{0.5}{} \sum_{i=1}^{n} P(0,T_i) = 1$

$K = \dfrac{1 - P(0,1.5)}{P(0,0.5) + P(0,1) + P(0,1.5)} = \dfrac{1 - 0.92341}{0.97442 + 0.94861 + 0.92341} = 2.69\% \times 2 = 5.38\%$

Question 4: ~~P_{swap}~~ = Note Cash Flow as CF, Fixed $CF = 0.25 \times 5.5\% \times 100 = 1.375$

$$CF_{0.25} = -0.25 \times \overset{5.25}{\cancel{5.3}}\% \times 100 + 1.375 = \cancel{0.0625}\ \cancel{0.05}\ 0.0625$$

$$CF_{0.5} = -0.25 \times 5.35\% \times 100 + 1.375 = 0.0375$$

$$CF_{0.75} = -0.25 \times 5.45\% \times 100 + 1.375 = 0.0125$$

$$CF_{1} = -0.25 \times 5.55\% \times 100 + 1.375 = -0.0125$$

Then we use price as discount factor: $p(0,T) = \frac{1}{1+F_0(0,T) \times T}$

$$P_{swap} = CF_{0.25} \times p(0,0.25) + \cdots + CF_1 \times p(0,1)$$

$$= \overset{\cancel{0.05}}{\cancel{0.0625}}\,0.0625 \times \frac{1}{1+0.053 \times 0.25} + 0.0375 \times \frac{1}{1+0.0535 \times 0.5} + 0.0125 \times \frac{1}{1+0.0545 \times 0.75}$$

$$+ (-0.0125) \times \frac{1}{1+0.055} = \cancel{0.98605}\ \cancel{0.098375}\ 0.098375 \text{ million USD}$$

Question 5:

0 — 0.5 — 1 — 1.5 — 2

Year 0

The cash flow can be decomposed into a fixed rate loan and a swap.

Time 0.5	Time 1	...	Time 2
$0.5[0.02 - r_0(0,0.5)] + 0.5 \times 0.02$	$0.5[0.02 - r_0(0,0.5)] + 0.5 \times 0.02$	...	$0.5[0.2 - r_0(0,0.5)] + 0.5 \times 0.02 + 1$

So it's a combination of a 2-year 0.02% loan and a 2% swap.

$V_{inverse\text{-}floating\text{-}note} = V_{swap} + V_{loan}$, because quoted swap rate now is 2%, which means the current value of swap is 0 ($V_{swap} = 0$). This is a par swap rate.

$V_{inverse\text{-}floating\text{-}note} = V_{loan}$ ~~$= 100 \times 0.02 \times \left(\frac{1}{1+0.02 \times 0.5} + \frac{1}{1+0.02} + \frac{1}{1+0.02 \times 1.5} + \frac{1}{1+0.02 \times 2}\right) + 100 \times \frac{1}{1+0.02 \times 2} = 103.9597$ million USD~~

= \$100 million USD