Fin537 Homework 1 Solution

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Question1

Q1

(a) $$\text{modified duration} = -\frac{\frac{\Delta P}{\Delta Y}}{P}$$

$$= -\frac{\frac{\Delta P}{0.023}}{\$97 \text{ billion}} = 6.3$$

$\Delta P = \$-14.0553$ billion

As a result, the portfolio loss 14.0553 billion dollar

(b) $$\Delta V = -D \cdot V \cdot \Delta r + \frac{1}{2} C \cdot V \cdot (\Delta r)^2$$

$$= -6.3 \cdot 97 \cdot 0.023 + \frac{1}{2} \cdot (-50) \cdot 97 \cdot (0.023)^2$$

$$= -14.0553 - 1.282825$$

$$= \$ -15.338125 \text{ billion}$$

Quetsion2

Q2

(a) $$R = r_{t+1} + r_{t+2} + r_{t+3}$$

$$\sigma_R^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$$

(b) $$\sigma_R^2 = \sigma^2 + \sigma^2 + \sigma^2 = 3\sigma^2$$

$$\sigma_R = \sqrt{3}\,\sigma$$

Quetsion3

Quetsion4

$\sigma_3 = \sigma$, then what are the variance and standard deviation of the three-day return?

3. (1 point) The theoretical price of a stock index futures contract is given by the formula

$$F(S, t, T, r, \delta) = e^{(r-\delta)(T-t)}S,$$

where S is the value of the underlying index, t is the current date, T is the delivery date (that is, the maturity date), r is the continuously compounded interest rate, and δ is the dividend yield.

Your colleague Ben Jidan tells you that the vega of the index futures contract is negative, because the index tends to go down when volatility increases. Is your colleague correct? Please briefly explain.

3. No. According to the formula, this index future's price is irrelevant to volatility σ. Therefore, its vega should be 0

1

4. $\theta = \frac{\partial F}{\partial t} = -(r-\delta)e^{(r-\delta)(T-t)} \cdot S$, which is $-(r-\delta) \cdot F$

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4. (1/2 point) What is the theta of the stock index futures contract in Question 3? (Your answer should be a formula, not a number.)

Quetsion5

```python
from scipy.stats import norm
import numpy as np

# Given data
S = 40       # Stock price
K = 40       # Strike price
sigma = 0.3  # Volatility
r = 0.04     # Risk-free rate
delta = 0.02 # Dividend yield
T = 1/12     # Time to expiration

# Compute d1 and d2
d1 = (np.log(S/K) + (r - delta + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
d2 = d1 - sigma * np.sqrt(T)

# Compute call option price using BSM formula
call_price = S * np.exp(-delta * T) * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2

# Compute Greeks
delta_call = np.exp(-delta * T) * norm.cdf(d1)
gamma_call = np.exp(-delta * T) * norm.pdf(d1) / (S * sigma * np.sqrt(T))
theta_call = (- (S * np.exp(-delta * T) * norm.pdf(d1) * sigma) / (2 * np.sqrt(T))
              - r * K * np.exp(-r * T) * norm.cdf(d2)
              + delta * S * np.exp(-delta * T) * norm.cdf(d1))

# Output results
print("Call Price: ")
print(call_price)
print('Greeks: ')
print(delta_call, gamma_call, theta_call)
```

] ✓ 0.0s Python

```
Call Price:
1.41160092518626
Greeks:
0.5240619308725167 0.11474826576854238 -8.624660643025614
```

Quetsion6

```python
import numpy as np
import pandas as pd


# Given data
S0 = 40
S1 = 41
K = 40
sigma = 0.3
r = 0.04
delta_div = 0.02
T = 1/12
dt = 1/252

# Given Greeks (Question 5 calculations)
delta_0 = 0.52406 # Initial delta
gamma = 0.11474   # Gamma
theta = -8.62466   # Theta

# Question 6: Approximate change in call option value using Greeks
delta_change = delta_0 * (S1 - S0) + 0.5 * gamma * (S1 - S0)**2 + theta * dt

delta_change
```

```
0.5472051587301587
```

The calculated approximate change in the option price is 0.5472.

Quetsion7

```python
[6] # Question 7(a): Delta-neutral hedge at S=40
    num_options = -10 * 100   # Selling 10 contracts, each on 100 shares
    delta_position_0 = num_options * delta_0   # Total delta of option position
    shares_needed_0 = -delta_position_0   # Shares to buy for delta-neutral position

    # Question 7(b): New delta at S=41 (approximating using gamma)
    delta_1 = delta_0 + gamma * (S1 - S0)
    delta_position_1 = num_options * delta_1   # New total delta of option position
    shares_needed_1 = -delta_position_1   # Shares to buy/sell for new delta-neutral position

    shares_needed_0, shares_needed_1
```

```
(524.06, 638.8)
```

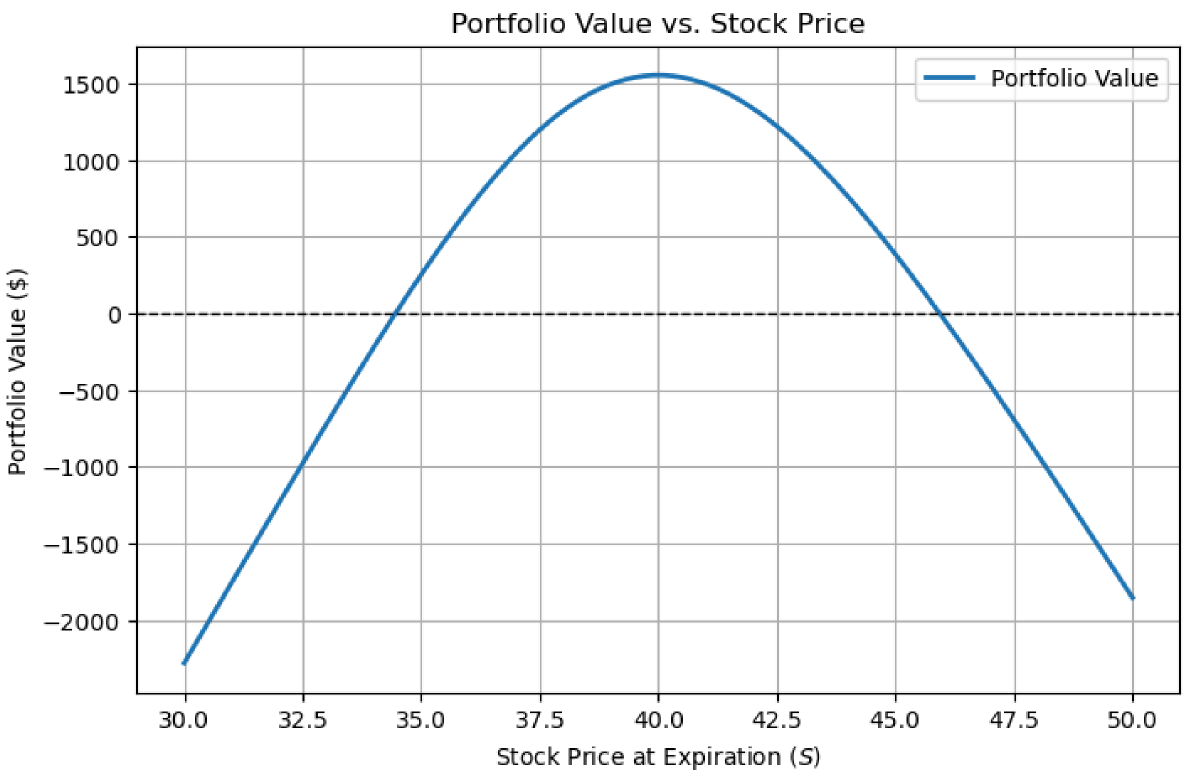
(a)To make the portfolio delta-neutral, one must **buy 524 shares**.

(b)To maintain a delta-neutral portfolio, one must adjust holdings to **638 shares**.

Quetsion8

```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
def call(S,K,T,r,d,sigma):
    d1 = (np.log(S / K) + (r - d + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    c =  S * np.exp(-d * T) * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)
    return c


K = 40
T = 1/12
sigma = 0.3
r = 0.04
d = 0.02
shares_held = 524.06
borrowed_amount = 18000
num_options_sold = 10
contract_size = 100

S_range = np.linspace(30, 50, 100)

option_value = -call(S_range,K,T,r,d,sigma)*num_options_sold*contract_size

stock_value = shares_held * S_range

borrowing_value = -borrowed_amount

portfolio_value = stock_value + option_value + borrowing_value

plt.figure(figsize=(8, 5))
plt.plot(S_range, portfolio_value, label="Portfolio Value", linewidth=2)
plt.axhline(0, color='black', linestyle='--', linewidth=1)
plt.xlabel("Stock Price at Expiration ($S$)")
plt.ylabel("Portfolio Value ($)")
plt.title("Portfolio Value vs. Stock Price")
plt.legend()
plt.grid(True)
plt.show()
```

Result:



9.

(a)

Q9

(a) $0 = -1000\Delta + n \times 1 \Rightarrow n = 500$ (shares)

Buy 500 shares

I should buy 500 shares of DGT.

(b)

(b) $\Gamma_p = -1000\Gamma_1 + 200 \times 0 = -40$

(c)

(c) $\Delta_p \times (35-40) + \frac{1}{2} \times \Gamma_p \times (35-40)^2 = 0 - 500 = -\$500$

Approximately, I will lose $500.

(d)

The theta will be greater than zero because the portfolio consists of a short position in call options and a long position in stocks. The theta of the short option position is greater than zero and the theta of stocks is zero.