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HW3 Fin 500

Question 1

$U = D_1 + D_2 + D_3 + D_4$

$D_1 = \dfrac{0.25 \times (0.055 - 0.0525) \times 100}{1 + 0.25 F_0(0, 0.25)} = 0.06168$

$D_2 = \dfrac{0.25 \times (0.055 - F_0(0.25, 0.5)) \times 100}{1 + 0.5 F_0(0, 0.5)} = 0.03652$

$D_3 = \dfrac{0.25 \times (0.055 - F_0(0.5, 0.75)) \times 100}{1 + 0.75 F_0(0, 0.75)} = \dfrac{0.0125}{1.040425} = 0.01201$

$D_4 = \dfrac{0.25 \times (0.055 - F_0(0.75, 1)) \times 100}{(1+0.25\times0.0530)(1+0.25\times0.0530)(1+0.25\times0.0545)(1+0.25 F_0(0.75,1))}$

Add 1 basis point to $f_0(0.75, 1)$, equals 5.56%

$U_+ = 0.06168 + 0.03652 + 0.01201 - 0.014214 = 0.095995$ million USD

$D_4 = -0.0015 \times 100 / 1.054513 = -0.014214$

Minus 1 basis point, equals 5.54%

$D_4' = -0.0094868$

$U_- = D_1 + D_2 + D_3 + D_4$

$\dfrac{\partial U}{\partial f_0(0.75, 1)} \approx \dfrac{U_+ - U_-}{2 \times 0.0001} = \dfrac{D_4 - D_4'}{0.0002} = \dfrac{-0.014214 + 0.0094768}{0.0002} = -23.686$

Question 2:

a) Based on expectation hypothesis, $f_{0.25}(0.25, 0.5) = f_0(0.25, 0.5) = 0.0525$

$f_{0.25}(0.5, 0.75) = f_0(0.5, 0.75) = 0.05$

$f_{0.25}(0.75, 1) = f_0(0.75, 1) = 0.0475$

b) $f_{0.5}(0.5, 0.75) = f_0(0.5, 0.75) = 0.05$

$f_{0.5}(0.75, 1) = f_0(0.75, 1) = 0.0475$

$f_{0.75}(0.75, 1) = f_0(0.75, 1) = 0.0475$

Question 3 (Luenberger 4.13 or 4.15)

First calculate the PV and quasi-modified duration of the stream of cash flows, following the approach in Table 4.4 on p. 94 of Luenberger:

Year	Spot	Discount factor	Cash Flow	PV	-PV'
1	7.67%	0.92876	500	464.381908	431.301112
2	8.27%	0.85307	900	767.761333	1418.23466
3	8.81%	0.77624	600	465.741373	1284.09532
4	9.31%	0.70042	500	350.211519	1281.53515
5	9.75%	0.62803	100	62.8025612	286.116452
6	10.16%	0.55957	100	55.9572595	304.778102
7	10.52%	0.49649	100	49.6493842	314.464069
8	10.85%	0.43865	50	21.9323125	158.284619
Total:				**2,238.44**	5478.80949
Quasi-Modified Duration:					**2.4476**

The prices and durations of the 12-year and 5-year bond are provided in Table 4.4 on p. 94 of Luenberger:

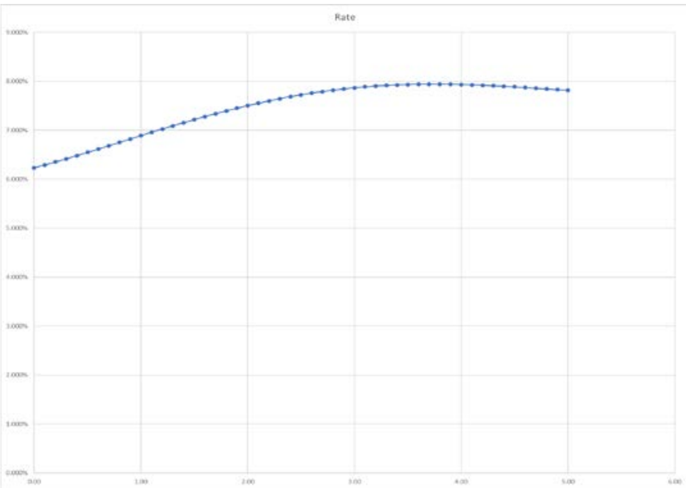
	Price	Quasi-Modified Duration
12-year bond:	65.95	7.07
5-year bond:	101.66	3.8

Solutions for x1, x2:	x1:	**-14.0374**
	x2:	**31.1254**

Check that values match:	0.00
Check that durations match:	0.0000

Question 4

Yield Curve Parameters	
a0	**0.062301798**
a1	**0.005734129**
a2	**0.001521117**
a3	**-0.00073538**
a4	**6.57E-05**
Sum of squared pricing errors	**0.0046**



Question 5 $\frac{dV}{dr} = -100(T-t_0)e^{-r(t_0,T)(T-t_0)}$

$\frac{d^2V}{dr^2} = 100(T-t_0)^2 e^{-r(t_0,T)(T-t_0)}$, $\therefore$ Convexity $= (T-t_0)^2$

Therefore, the convexity of bonds increase by the $T-t_0$, which means it's increasing with the remaining periods' decreasing.

Question 6 (a) $D=10$, $\Delta r = 2\%$, $\frac{\Delta P}{P} = -D \times \Delta r = -0.2$

(b) $\frac{\Delta P}{P} = -D \cdot P + \frac{1}{2} C \cdot (\Delta r)^2 = -10 \times 2\% + \cancel{\frac{4}{2}}\ \frac{1}{2} \times 0.02^2 \times 100$

$\cancel{C = t^2 = 0.02^2}$ $C = \frac{1}{V(\tau)} \cdot \frac{d^2V(\tau)}{d\tau^2} = \tau^2 = 100$

$\therefore \frac{\Delta P}{P} = -0.2 + 0.02 = -0.18$

Question 7							
Bonds	Maturity	Rate	Value	Duration	Convexity	Dollar Duration	Dollar Convexity
5-year:	5	0.015	927.743486	5.000	25	4638.71743	23193.5872
10-year	10	0.030	740.818221	10.000	100	7408.18221	74081.8221
20-year	20	0.035	496.585304	20.000	400	9931.70608	198634.122
solutions:	n_5 =	-1.0647					
	n_{20}=	-0.2486					
check:	duration=	0.000					
	convexity=	0.000					
	target=	0.00000					