FIN500: Problem Set 9

*Due: Wednesday 20th November before **11:59pm**, please submit electronic copies through the assignment function on Canvas*

Question 1 (5 points)

On October 1, 2024 you observe the following statistics about the Russell 2000 index and futures on the Russell 2000 index.

Stock index level: **2250**
Market futures price for stock index deliverable on July 1, 2025: **2216.50**
You can borrow and lend at a **0.5%** continuously compounded annual rate.
October 1 to July 1: **270 days**
One year is **360** days long (makes life easier)

a) Explain briefly why it is reasonable to model dividend payments as continuous in this example.

b) Assuming that there are no arbitrage opportunities, what is the estimate of the continuous dividend between October and July implied by the futures price?

Based on your research, **you estimate the correct dividend to be 2%.**

c) Given your estimate. Is the market futures price too high or too low?

d) Assuming that your dividend estimate is correct (i.e 2%), construct an **arbitrage strategy** using the stock index, the futures contract and risk-free borrowing and lending. Briefly explain your strategy and show your cash flows.

e) If you were to attempt to execute these arbitrage trades yourself, what would be the biggest challenge you would face? How would it likely impact your arbitrage profits?

a) It's reasonable, since the Russell 2000 index includes many stocks that pay dividends at different time.

b). $F_{0,T} = S_0 e^{(r-d)T}$ $r = 0.005$ $S_0 = 2250$ $F_{0,2025} = 2216.5$ $T = \frac{270}{360}$

$$2216.5 = 2250 e^{(0.005-d)\frac{270}{360}}$$

$$d = 0.025$$

c). $d = 2\% = 0.02$ $F_{0,T} = 2250 e^{(0.005-0.02)\frac{270}{360}}$

$F_{0,T} = S_0 e^{(r-d)T}$ $F_{0,T} = 2224.83$ $2224.83 > 2216.5$

the market futures is too low.

d). borrow 2250 at $T=0$ with $r=0.005$ $FV(D) = 2250(e^{-0.02(\frac{270}{360})} - 1)$ $= -33.498$

short futures contract $F_{0,2025} = 2216.5$

$2250 \times e^{0.005(0.75)} = 2258.453$

cash flow $= 2258.453 - 2216.5 - 33.498 = 8.455$

e). transaction costs and market frictions.

Question 3 (2 points)

It is currently September 19, 2024. Mini-S&P500 Futures contracts are available at the Chicago Mercantile Exchange. These contracts are for $50 multiplied by the S&P index. The contracts have expiry dates of December 15, 2024 and March 15, 2025.

The current spot and futures prices, as well as the 3-and 6-month "risk-free" rates (but for simplicity assume **continuous compounding** and **use actual day counts t = Actual/365**) are given below.

S&P 500:	4446.85
December Futures:	4501.50
March Futures:	4552.75
3-month "SOFR"	5.3970%
6-month "SOFR"	5.4686%

a) Estimate the continuous dividend yield (quote it in annual terms) for
   a. September – December
   b. September – March
   c. December – March (you will need to think carefully about this one)

$F_{0,T} = S_0 e^{(r-d)T}$ $11+31+30+15=87$

a. Dec.15.24 − Sep.19.24 = 87 $F_{0,Dec} = 4501.5$ $S_0 = 4446.85$

$T=87$ $r=0.05397$

$4501.5 = 4446.85 e^{(0.05397-d)\frac{87}{365}}$

$d = 0.00273$

b. Mar.15,2025 − Sep.19.24 = 177

$4552.75 = 4446.85 e^{(0.054686-d)\frac{177}{365}}$

$d = 0.00615$

$16+31+28+15$

c. Mar.15,2025 − Dec.15.24 = 90

SOFR(Dec − Mar): $e^{r_{6m}\times\frac{6}{12}} = e^{r_{3m}\times\frac{3}{12}} \cdot e^{r_{f3m}\times\frac{3}{12}}$

$r_{f3m} = 0.055402$

$4552.75 = 4501.5 e^{(0.055402-d)\frac{90}{365}}$

$d = 0.00948$

Question 2: Arbitrage in Futures Contracts

Part (a): Calendar Spread Strategy

The arbitrage profit can be calculated using the following formula:

Cash Flow Table

Date	Action	Cash Flow
Sep 19, 2024	Enter futures contracts	0 (no initial cash flow)
Mar 14, 2025	Buy stock through March futures	-337.87 (borrowed at r)
May 16, 2025	Receive dividend and invest	$0.74 \cdot e^{r(t_{\text{Sept}} - t_{\text{May}})}$
Aug 15, 2025	Receive dividend and invest	$0.74 \cdot e^{r(t_{\text{Sept}} - t_{\text{Aug}})}$
Sep 19, 2025	Deliver stock and receive September futures price	346.751

$$\text{Profit} = F_{\text{Sept}} + \left(d \cdot e^{r(t_{\text{Sept}} - t_{\text{May}})} + d \cdot e^{r(t_{\text{Sept}} - t_{\text{Aug}})}\right) - F_{\text{March}} \cdot e^{r(t_{\text{Sept}} - t_{\text{March}})}.$$

Given Data:

$$\begin{aligned}
F_{\text{March}} &= 337.87, \\
F_{\text{Sept}} &= 346.751, \\
r &= 0.053 \text{ (continuously compounded)}, \\
t_{\text{March}} &= \frac{177}{365}, \quad t_{\text{May}} = \frac{240}{365}, \quad t_{\text{Aug}} = \frac{330}{365}, \quad t_{\text{Sept}} = 1, \\
d &= 0.74 \text{ (per dividend payment)}.
\end{aligned}$$

Calculation:

$$\begin{aligned}
\text{Total Dividends} &= 0.74 \cdot e^{0.053 \cdot (35/365)} + 0.74 \cdot e^{0.053 \cdot (111/365)}, \\
&\approx 1.50. \\
\text{Cost of Stock} &= -337.87 \cdot e^{0.053 \cdot (1 - 189/365)}, \\
&\approx -347.22. \\
\text{Profit} &= 0.9759.
\end{aligned}$$

Thus, the arbitrage profit is approximately **0.9759**.

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Part (b): Finding the Dividend Size that Makes Arbitrage Profits Zero

The dividend size d that eliminates arbitrage profit is given by:

$$d = \frac{F_{\text{March}} \cdot e^{r(t_{\text{Sept}} - t_{\text{March}})} - F_{\text{Sept}}}{e^{r(t_{\text{Sept}} - t_{\text{May}})} + e^{r(t_{\text{Sept}} - t_{\text{Aug}})}}.$$

Substituting the values:

$$\begin{aligned}
F_{\text{March}} \cdot e^{r(t_{\text{Sept}} - t_{\text{March}})} - F_{\text{Sept}} &\approx 337.87 \cdot e^{0.053 \cdot (1 - 177/365)} - 346.751, \\
&\approx -1.73. \\
e^{r(t_{\text{Sept}} - t_{\text{May}})} + e^{r(t_{\text{Sept}} - t_{\text{Aug}})} &\approx e^{0.053 \cdot (1 - 240/365)} + e^{0.053 \cdot (1 - 330/365)}, \\
&\approx 7.46. \\
d &= \frac{-1.73}{7.46}, \\
&\approx 0.2572.
\end{aligned}$$

The dividend size that makes the arbitrage profit zero is approximately **0.2572**.

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Question 4: Foreign Exchange Arbitrage

Part (a): Risk-Free Trading Strategy

To construct a risk-free trading strategy, we use the following steps: 1. Borrow 1 DM at an interest rate of r_{DM}:

$$\text{Cost of borrowing DM} = 1 \cdot (1 + r_{\text{DM}}).$$

2. Convert to GBP at the maximum exchange rate (3 DM/GBP):

$$\text{DM obtained} = 1/3.$$

3. Invest the GBP at an interest rate of r_{GBP}:

$$\text{GBP after investment} = \text{DM obtained} \cdot (1 + r_{\text{GBP}}) = 0.37666.$$

4. Convert back to DM at the lower bound exchange rate (2.773 DM/GBP):

$$\text{DM received} = 0.37666 \cdot 2.773 = 1.044497.$$

5. Profit is:

$$\text{Profit} = \text{DM received} - \text{Cost of borrowing DM}.$$

Using the given values:

$$\text{Profit} = 1.044497 - 1.04 = 0.004497\text{DM}.$$

Concerns: The profit relies on the pegged exchange rate boundaries. If the British government cannot maintain this, this strategy may introduce risks.

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Part (b): No-Arbitrage Relationship

The no-arbitrage condition for the spot rate bounds and interest rates is:

$$\frac{d}{u} \leq \frac{1 + r_{GBP}}{1 + r_{DM}} \leq \frac{u}{d}$$

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Part (c): Maximum No-Arbitrage GBP Interest Rate

Using the no-arbitrage relationship:

$$r_{\text{GBP}} = \frac{(1 + r_{\text{DM}}) \cdot u}{d} - 1.$$

Substituting the given values ($u = 1.03, d = 0.07, r_{\text{DM}} = 0.04$):

$$\begin{aligned} r_{\text{GBP}} &= \frac{(1 + 0.04) \cdot 1.03}{0.07} - 1, \\ &\approx 14.3029. \end{aligned}$$

The maximum possible GBP interest rate is approximately **14.30**.